SPM Model Paper

Paper 1

1 0.4685 = 0.47 (correct to 2 significant figures)
Answer: **C**

2 $0.0000307 = 3.07 \times 10^{-5}$
Answer: **B**

3 $4.2 \times 10^{15} + 5.3 \times 10^{14}$

$= 4.2 \times 10^{14} \times 10^{1} + 5.3 \times 10^{14}$

$= 42 \times 10^{14} + 5.3 \times 10^{14}$

$= (42 + 5.3) \times 10^{14}$

$= 47.3 \times 10^{14}$

$= 4.73 \times 10 \times 10^{14}$

$= 4.73 \times 10^{15}$

Answer: **C**

4 Volume of water

$= \frac{60}{100}(900 \times 600 \times 300)$

$= 9.72 \times 10^{7}$

Answer: **C**

5

2^4	2^3	2^2	2^1	2^0
1	1	0	0	1

2^1	2^0
1	1

$11001_2 = 2^4 + 2^3 + 1 = 25$
$11_2 = 2 + 1 = 3$

$11001_2 - 11_2 = 25 - 3 = 22$

2	22	Remainder
2	11	– 0
2	5	– 1
2	2	– 1
2	1	– 0
	0	– 1

$\therefore 11001_2 - 11_2 = 10110_2$
Answer: **A**

6

5^2	5^1	5^0
1	2	3

$123_5 = 25 + 10 + 3 = 38$

8	38	Remainder
8	4	– 6
	0	– 4

$123_5 = 46_8$
Answer: **B**

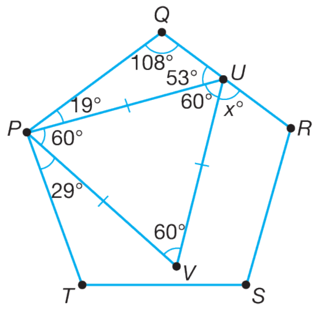
7 The sum of interior angles of a regular pentagon $= (5-2) \times 180^\circ = 540^\circ$

Each interior angle $= \frac{540^\circ}{5} = 108^\circ$

$\angle QPU = 108^\circ - 60^\circ - 29^\circ = 19^\circ$

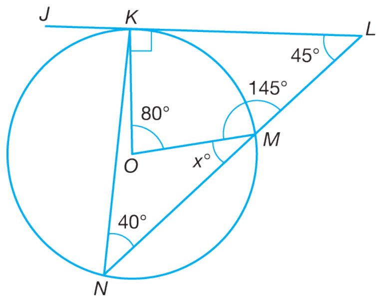
In ΔPUQ,

$\angle QUP = 180^\circ - 19^\circ - 108^\circ = 53^\circ$



QUR is a straight line.
Hence, $x = 180 - 60 - 53 = 67$
Answer: **C**

8



$\angle KOM = 2 \times \angle KNM = 2 \times 40^\circ = 80^\circ$

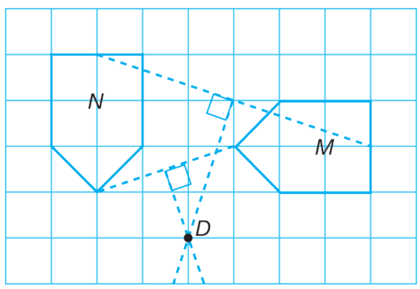
$\angle OKL = 90^\circ$

Sum of interior angles of a quadrilateral $= 360^\circ$

$\angle OML = 360^\circ - 90^\circ - 80^\circ - 45^\circ = 145^\circ$

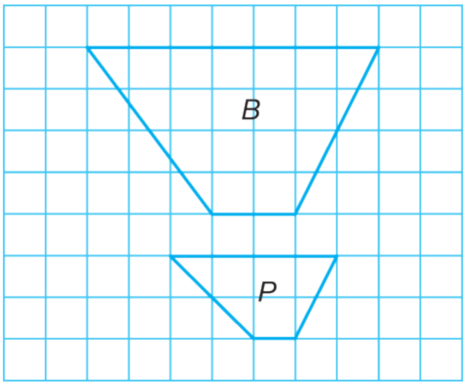
Hence, $x = 180^\circ - 145^\circ = 35^\circ$
Answer: **A**

9



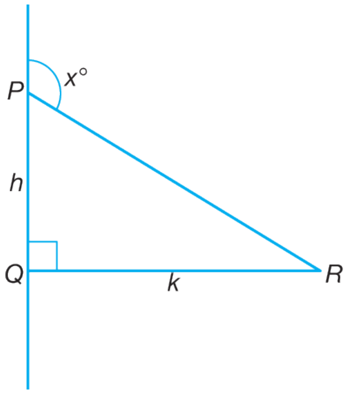
Answer: **D**

10



Answer: **B**

11



$$\cos x^{o} = -\frac{1}{2}$$

$$\text{Basic } \angle = 60^{o}$$

$$\angle QPR = 60^{o}$$

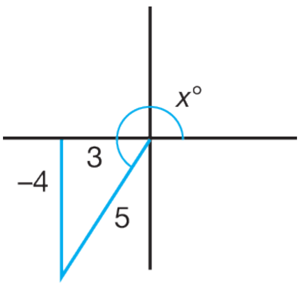
$$\tan \angle QPR = \frac{k}{h}$$

$$\tan 60^{o} = \frac{k}{h}$$

$$k = h \tan 60^{o}$$

Answer: **B**

12



$$\sin x^{o} = \frac{-4}{5}$$

Answer: **A**

13 The trigonometric function of the given graph is $y = \sin 2x + 1$

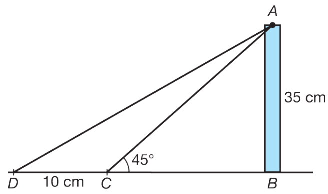
Answer: **C**

14 The taxes imposed by the State Government are property assessment tax and quit rent

Answer: **A**

15 Insurance company manage to pay high compensations although the premiums collected from policyholders are low. The principle used is the risk accumulation principle.

16



$$\tan 45^{o} = \frac{35}{BC}$$

$$BC = \frac{35}{\tan 45^{o}}$$

$$BC = \frac{35}{1}$$

$$BC = 35 \text{ m}$$

$$\tan \angle ADB = \frac{35}{35+10}$$

$$\tan \angle ADB = \frac{35}{45}$$

$$\angle ADB = 37^{o}52'$$

Answer: **A**

Vertex	*Degree*
P	10
Q	3
R	5
S	2
Sum	20

Answer: **D**

18 The contrapositive for the implication "If $y > \frac{7}{5}$, then $y > \frac{3}{5}$' is 'If $y \leq \frac{3}{5}$, then $y \leq \frac{7}{5}$.

Answer: D

19 Variance

$= \frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2$

$= \frac{1\,544}{10} - \left(\frac{110}{10}\right)^2$

$= 33.4$

Answer: **D**

20 $\frac{d-2}{d} - \frac{2(d-3)}{d^2}$

$= \frac{d(d-2)-2d+6}{d^2}$

$= \frac{d^2-2d-2d+6}{d^2}$

$= \frac{d^2-4d+6}{d^2}$

Answer: C

21 $P = 5\sqrt{\frac{1}{R+Q}}$

$P^2 = \frac{25}{R+Q}$

$RP^2 + QP^2 = 25$

$RP^2 = 25 - QP^2$

$R = \frac{25-QP^2}{P^2}$

$R = \frac{25}{P^2} - Q$

Answer: **B**

22 $2(3-k) = \frac{8k-5}{2}$

$4(3-k) = 8k-5$

$12-4k = 8k-5$

$8k+4k = 12+5$

$12k = 17$

$k = \frac{17}{12}$

Answer: A

23 $\frac{1}{a^b} = 5^{-3}$

$\frac{1}{a^b} = \frac{1}{5^3}$

$a = 5$ and $b = 3$

Answer: **D**

24 $\frac{(n^4h^8)^{-1}}{n^{-6}h^7}$

$= \frac{n^{-4}h^{-8}}{n^{-6}h^7}$

$= n^{-4+6}h^{-8-7}$

$= n^2h^{-15}$

Answer: B

25 $3x - 5 \geq 7$

$3x \geq 7+5$

$3x \geq 12$

$x \geq 4$

$6 - x > -4$

$-x > -4-6$

$x < 4+6$

$x < 10$

Answer: **A**

26 The difference between the number of gold medals and silver medals = 2

Total medals = 6 + 8 + 4 = 18

Angle of the sector = $\frac{2}{18} \times 360^\circ = 40^\circ$

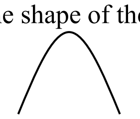
Answer: C

27 The maximum value of p is 9.

Answer: B

28 $y = -x^2 + 2x + 3$

The shape of the curve is



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The y-intercept is 3.

$y = 0$

$-x^2 + 2x + 3 = 0$

$x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$

$x - -3$ or $x = 1$

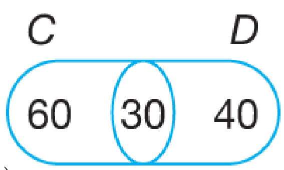
The curve passes through the points (–3, 0) and (1, 0).

Answer: B

29 $H' = \{3, 4, 7\}$

Answer: C

30



$n(\xi) = 60 + 30 + 40 = 130$

Answer: **B**

31 n(English and Mathematics societies)

$3 + 3x = 9$
$3x = 6$
$x = 2$

n(two societies only)
$= 2x + 6 + 3x$
$= 2(2) + 6 + 3(2)$
$= 16$
Answer: **C**

32 $y = mx + c$

$n = \frac{6-4}{5-(-1)} = \frac{1}{3}$

$y = \frac{1}{3}x + c$

For the point $W(5, 6)$,

$6 = \frac{1}{3}(5) + c$

$c = \frac{13}{3}$

$y = \frac{1}{3}x + \frac{13}{3}$

$3y = x + 13$

$3y - x = 13$

Answer: **A**

33 Interquartile range
$= 64 - 52$
$= 12$
Answer: **C**

34 Number of yellow cards
$= \frac{1}{6} \times 48 = 8$ pieces

Let the number of yellow cards that need to be added $= x$

P(yellow) $= \frac{4}{9}$

$\frac{8+x}{48+x} = \frac{4}{9}$

$72 + 9x = 192 + 4x$

$5x = 120$

$x = 24$

Answer: **C**

35 P(not red)
$= \frac{9}{16}$
Answer: D

36 $R \propto S$

$R = kS$ [k is a constant.]

Given $R = 18$ and $S = 12$,

$18 = k(12)$

$k = \frac{18}{12} = \frac{3}{2}$

$R = \frac{3}{2}S$

Given $R = 54$ and $S = x$,

$54 = \frac{3}{2}x$

$x = \frac{2}{3}(54)$

$x = 36$

Answer: **A**

37 $y \propto \frac{1}{x^3}$

$y = \frac{k}{x^3}$ (k is a constant.)

Given $y = \frac{1}{128}$ when $x = 4$,

$\frac{1}{128} = \frac{k}{4^3}$

$k = \frac{64}{128}$

$k = \frac{1}{2}$

Thus, $y = \frac{1}{2x^3}$

When $y = \frac{1}{54}$,

$\frac{1}{54} = \frac{1}{2x^3}$

$2x^3 = 54$

$x^3 = 27$

$x = 3$

Answer: C

38 $P \propto \frac{H^2}{\sqrt{N}}$

$P = \frac{kH^2}{\sqrt{N}}$ (k is a constant.)

Given $P = 24$ when $N = 36$ and $H = 4$,

$24 = \frac{(4^2)k}{\sqrt{36}}$

$24 = \frac{16k}{6}$

$k = \frac{24 \times 6}{16}$

$k = 9$

Answer: A

39 $\begin{pmatrix} 4 & 1 \\ 2 & 3 \\ -3 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ -4 \end{pmatrix}$

$= \begin{pmatrix} 4\times 1+(-4) \\ 2+3(-4) \\ -3+0 \end{pmatrix}$

$= \begin{pmatrix} 0 \\ -10 \\ -3 \end{pmatrix}$

Answer: **B**

40 $MV = P\left(1+\frac{r}{n}\right)^{nt} = 1\ 077\ 484$

$800\ 000\left(1+\frac{r}{4}\right)^{4(5)} = 1\ 077\ 484$

$\left(1+\frac{r}{4}\right)^{20} = \frac{1\ 077\ 484}{800\ 000}$

$\left(1+\frac{r}{4}\right) = \left(\frac{1\ 077\ 484}{800\ 000}\right)^{\frac{1}{20}}$

$1+\frac{r}{4} = 1.015$

$\frac{r}{4} = 0.015$

$r = 0.06\ \ (6\ \%)$

Answer: **A**

Paper 2

1 (a) $c \propto np$

$c = knp$ (k is a constant.)

Given n = 10 000, p = 240 and c = 50 000,

$50\ 000 = k(10\ 000)(240)$

$k = \dfrac{50\ 000}{(10\ 000)(240)} = \dfrac{1}{48}$

$c = \dfrac{1}{48}np$

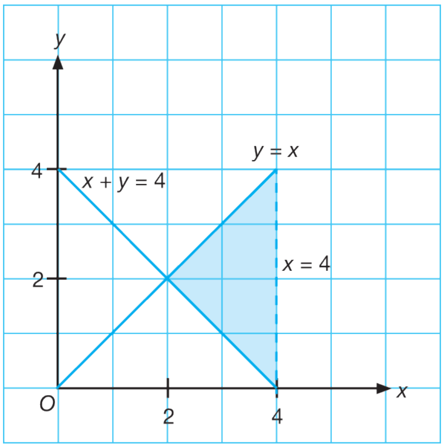
(b) Given c = 900 000 and p = 480,

$900\ 000 = \dfrac{1}{48}(480)n$

$n = \dfrac{900\ 000 \times 48}{480}$

$n = 90\ 000$ books

2



3 $\dfrac{x-1}{6} - \dfrac{2x-1}{5x} = 0$

$\dfrac{x-1}{6} = \dfrac{2x-1}{5x}$

$5x^2 - 5x = 12x - 6$

$5x^2 - 17x + 6 = 0$

$(5x-2)(x-3) = 0$

$x = \dfrac{2}{5}$ or 3

4 $h + 3k = 15$ … (1)

$\dfrac{2}{3}h - k = -8$ … (2)

(2) × 3: $2h - 3k = -24$ … (3)

$h + 3k = 15$ … (1)

(+) $2h - 3k = -24$ … (3)

$3h = -9$

$h = -3$

Substitute $h = -3$ into (1):

$-3 + 3k = 15$

$3k = 18$

$k = 6$

5 (a) P(moving the chip one box to the right)

$= \dfrac{2}{6} = \dfrac{1}{3}$

P(Moving the chip one box to the left)

$= \dfrac{2}{3}$

P(moving the chip three boxes to the right)

$= \dfrac{1}{3} \times \dfrac{1}{3} \times \dfrac{1}{3}$

$= \dfrac{1}{27}$

(b) P(moving the chip one box to the right and two boxes to the left) + P(moving the chip two boxes to the left and one box to the right)

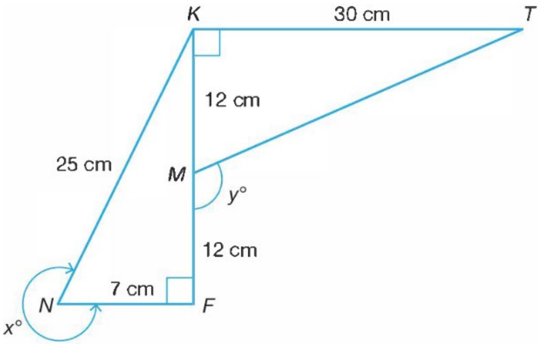
$= \dfrac{1}{3} \times \dfrac{2}{3} \times \dfrac{2}{3} + \dfrac{2}{3} \times \dfrac{2}{3} \times \dfrac{1}{3}$

$= \dfrac{8}{27}$

6 (a) $(3, -1) \xrightarrow{\mathbf{T}} (5, -6) \xrightarrow{\mathbf{P}} (-5, -6)$

(b) $(1, 2) \xrightarrow{\mathbf{E}} (3, 6) \xrightarrow{\mathbf{R}} (6, -3)$

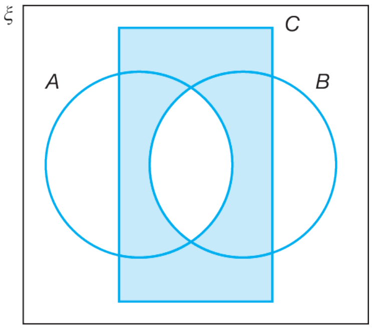
7



(a) $\sin x^\circ = -\dfrac{24}{25}$

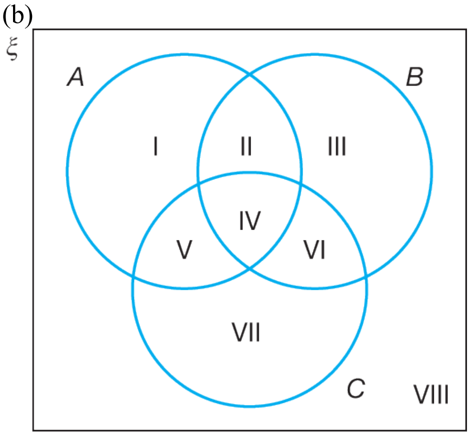
(b) $\tan y^\circ = -\dfrac{30}{12} = -\dfrac{5}{2}$

8 (a)

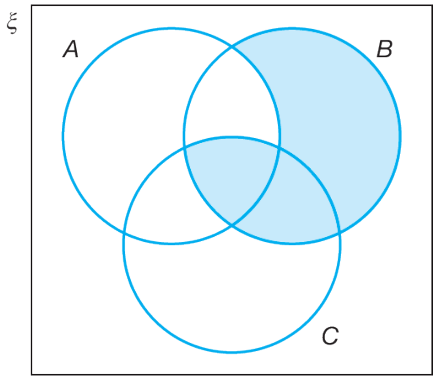


$(A \cap B)' \cap C$ lies outside of $A \cap B$ but inside the set C.

(b)



A' = {III, VI, VII, VIII}
C = {IV, V, V1, VII}
$A' \cup C$ = {III, IV, V, VI, VII, VIII}
B = {II, III, IV, VI}
$(A' \cup C) \cap B$ = {III, IV, VI}



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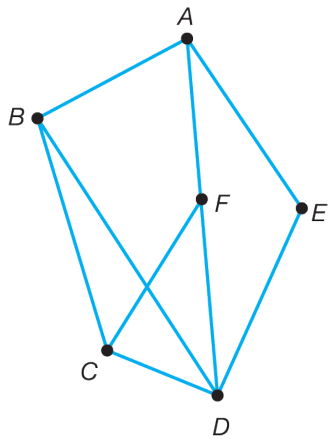
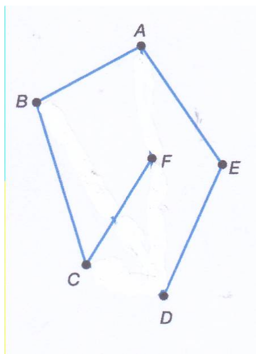


(a) (i) Total number of vertices = 6
(ii) Total number of edges = 9
(b) Number of edges extra by 4.

The sides *BD*, *CD*, *AF* and *FD* have to be removed.
The required tree is as follows:



10 (a) The insurance company will only pay compensation for the repair expenses of the car Henry hits.

(b) Sum of tax relief = Sum of incomes – Taxable income
Rebate = Total income tax – Sum of tax payable

11 (a) Number of rabbits initially

(b) Number of rabbits after 1 year

(c) $P(t) = ab^t$

$P(0) = ab^0$

$200 = a(1)$

$a = 200$

The number of rabbits increases by 20% a year

Number of rabbits after 1 year

$= \frac{120}{100} \times 200$

$= 240$

$P(1) = 200b^1$

$240 = 200b$

$b = 1.2$

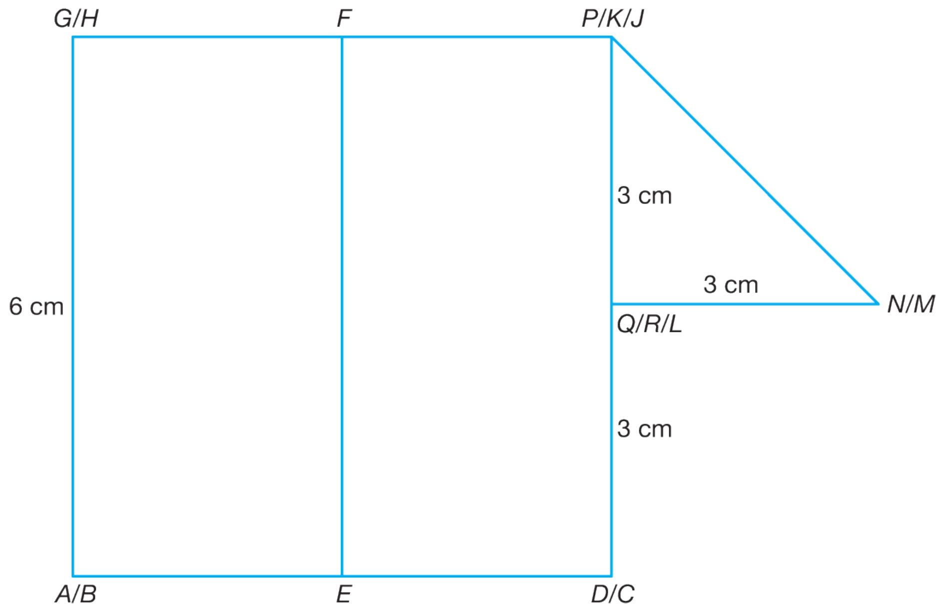
(d) The rate of increase of rabbits yearly

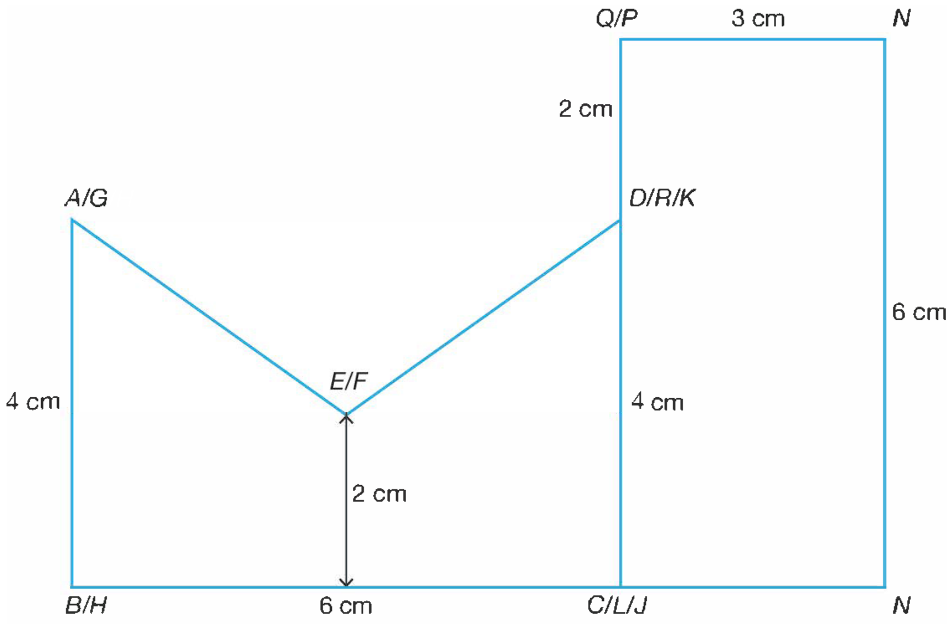
(e) $P(t) = 200(1.2)^t$

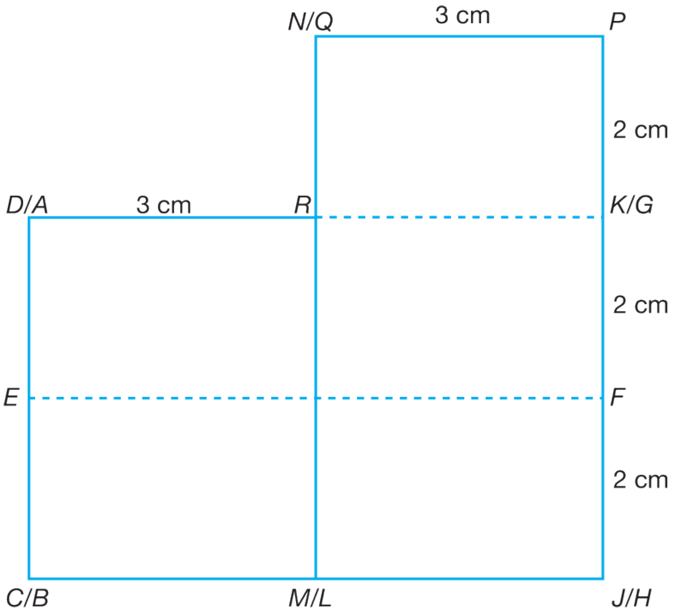
$P(4) = 200(1.2)^4$

$= 415$ rabbits

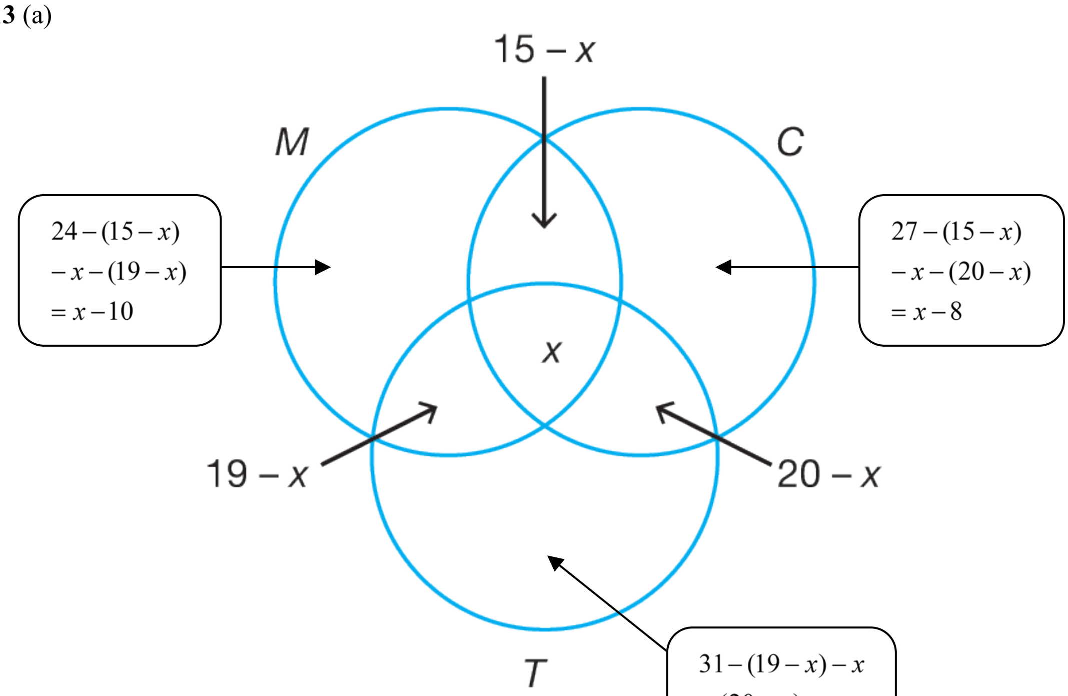
12







13 (a)

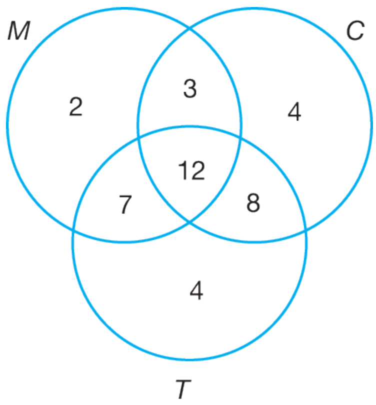


$$n(\xi) = 40$$

$$[x - 10 + x - 8 + x - 8 + 15 - x + 20 - x + 19 - x + x] = 40$$

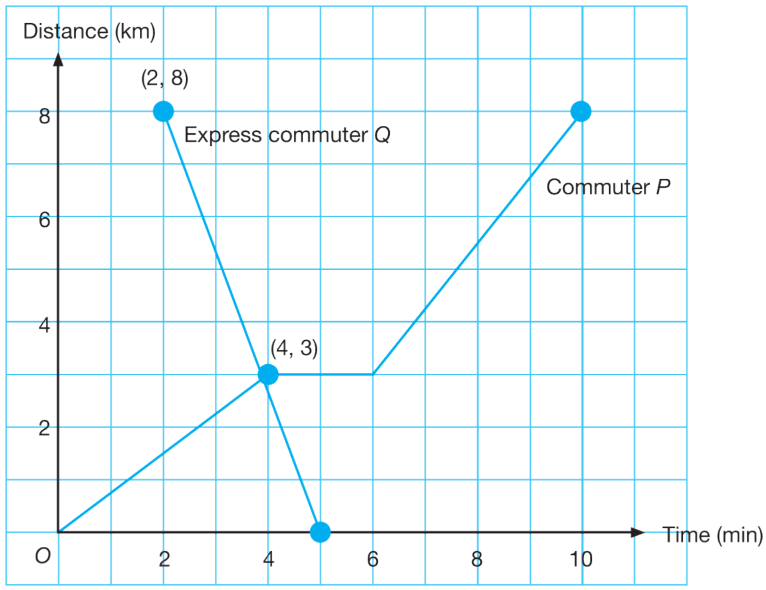
$$x + 28 = 40$$

$$x = 12$$



(b) The number of workers who like to drink all three types of drinks is 12.

14 (a) Average speed of commuter $P = \dfrac{8}{\left(\dfrac{10}{60}\right)} = 48 \text{ km h}^{-1}$



v = Maximum speed of the express commuter Q

$v = \dfrac{8-3}{\left(\dfrac{2-4}{60}\right)} = -150 \text{ km h}^{-1}$, i.e. 150 km h^{-1} from town C to town A.

(b) (i) Distance in the first h seconds = 120 m

$\frac{1}{2}(25+15)(h) = 120$

$20h = 120$

$h = 6$

(ii) Acceleration

= Gradient

$3 = \dfrac{k-15}{10-6}$

$12 = k - 15$

$k = 27$

(iii) Total distance $= 120 + \frac{1}{2}(15 + 27)(4)$

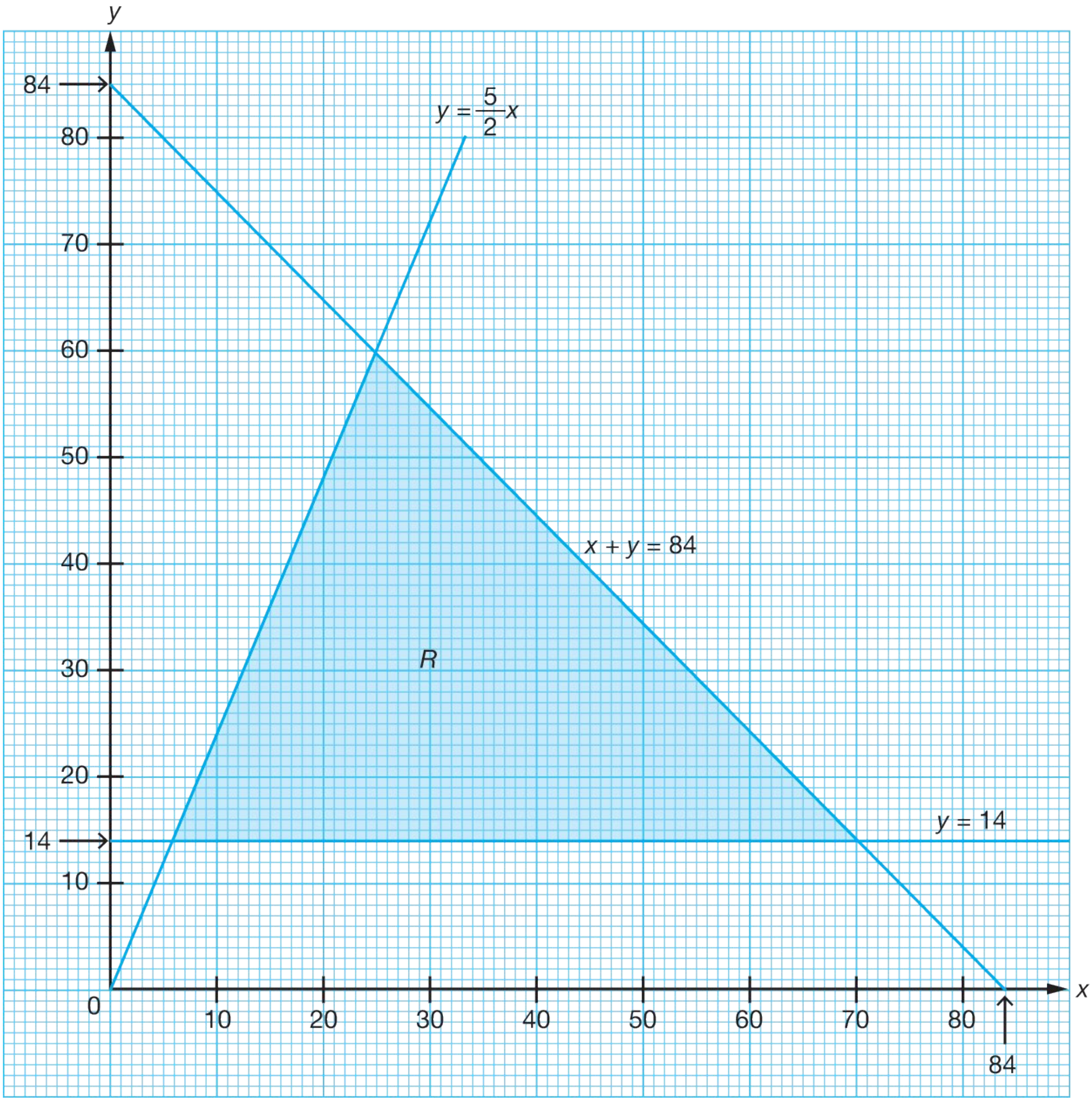
$= 120 + 84$

$= 204$ km

Average speed $= \frac{204}{10} = 20.4$ m s^{-1}

15 (a) $x + y \le 84$, $y \ge 14$, $y \le \frac{5}{2}x$

(b)

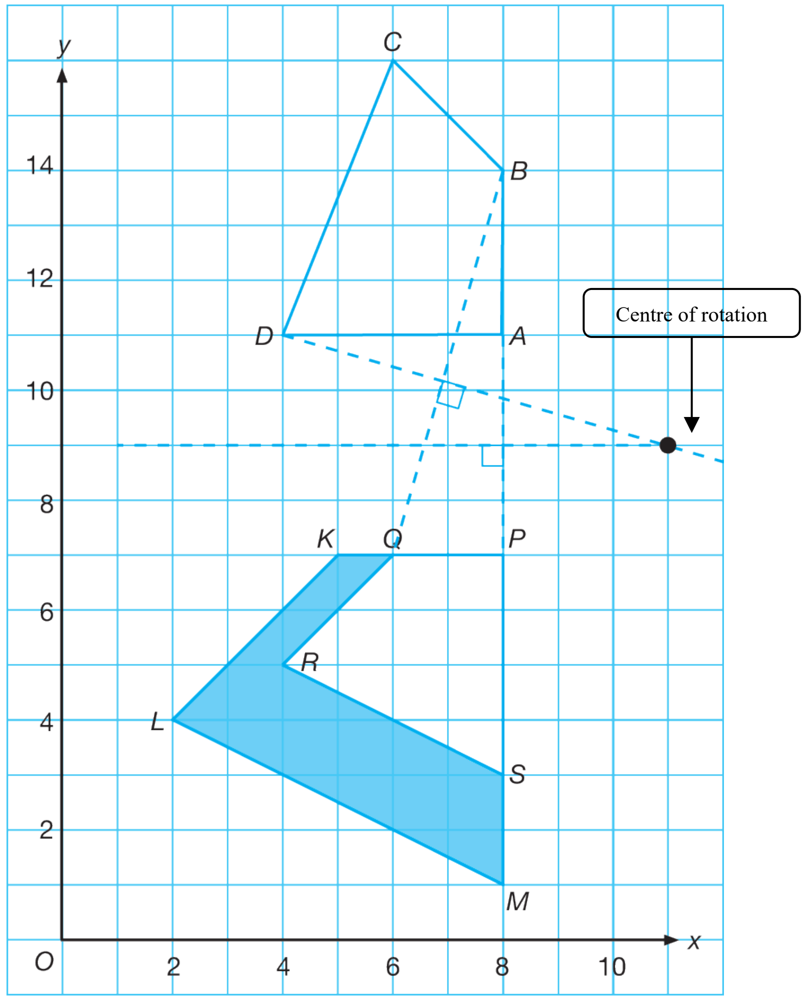


(c) 70 trainees

16 (a) (i) $D(4, 11) \xrightarrow{\mathbf{T}} D'(6, 7) \xrightarrow{\mathbf{P}} D''(0, 7)$

(ii) $D(4, 11) \xrightarrow{\mathbf{P}} D'(2, 11) \xrightarrow{\mathbf{T}} D''(4, 7)$

(b)



(i) (a) Anticlockwise rotation of 90° about the centre (10, 9)

(b) Enlargement at the centre $P(8, 7)$ with a scale factor of $\frac{3}{2}$.

$\frac{PK}{PA} = \frac{3}{2}$

(ii) Area of $PKLM = \left(\frac{3}{2}\right)^2 \times 160 = 360 \text{ cm}^2$

Area of the shaded region
$= 360 - 160$
$= 200 \text{ cm}^2$

(c) Rotation of 180° about the centre (4, 6)

17

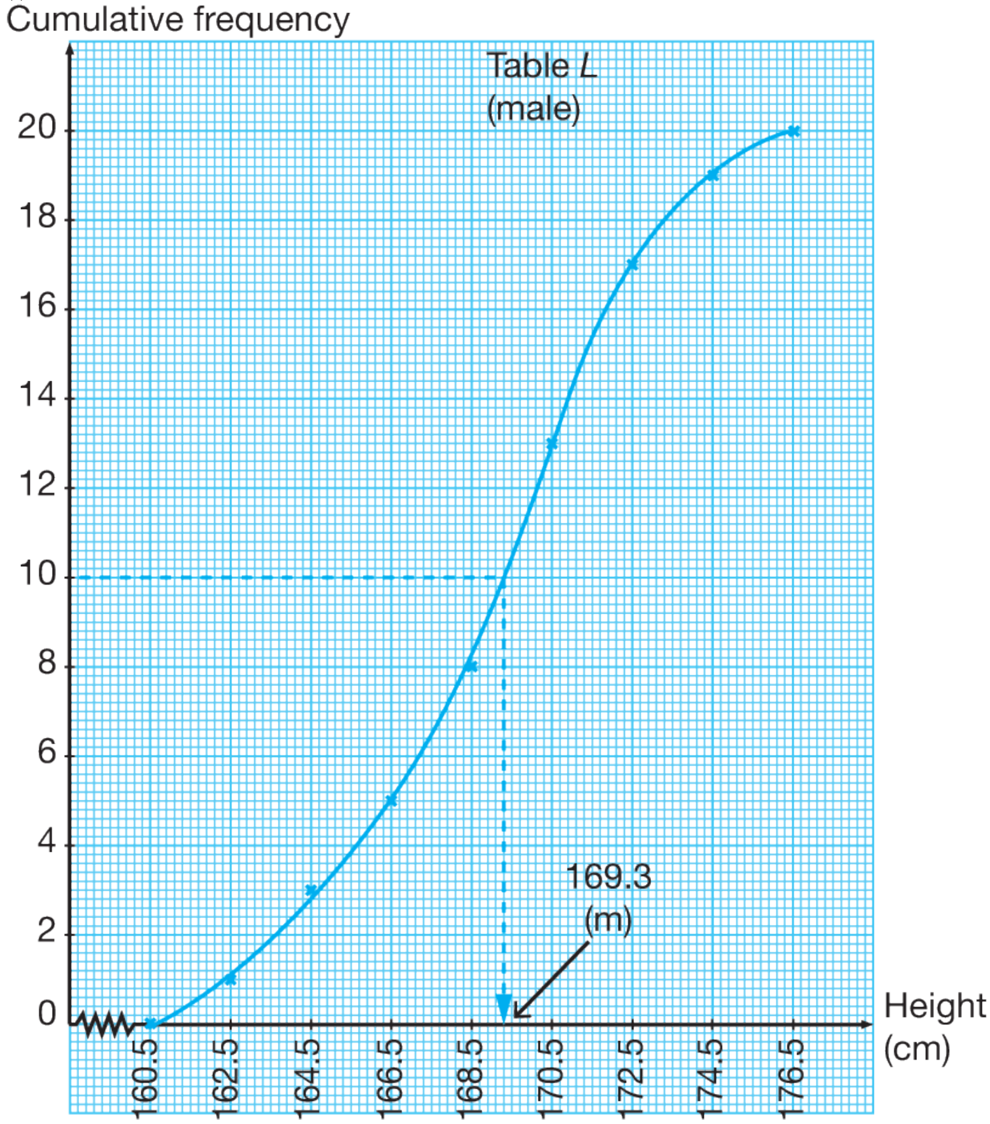
Height (cm)	*Number of males* (f)	*Upper boundary*	*Cumulative frequency*	*Midpoint* (x)	fx	fx^2
159 – 160	0	160.5	0			
161 – 162	1	162.5	1	161.5	161.5	26 082.25
163 – 164	2	164.5	3	163.5	327.0	53 464.5
165 – 166	2	166.5	5	165.5	331.0	54 780.5
167 – 168	3	168.5	8	167.5	502.5	84 168.75
169 – 170	5	170.5	13	169.5	847.5	143 651.25
171 – 172	4	172.5	17	171.5	686.0	117 649
173 – 174	2	174.5	19	173.5	347.0	60 204.5
175 – 176	1	176.5	20	175.5	175.5	30 800.25
Sum	20				3378.0	570 801

Table *L*

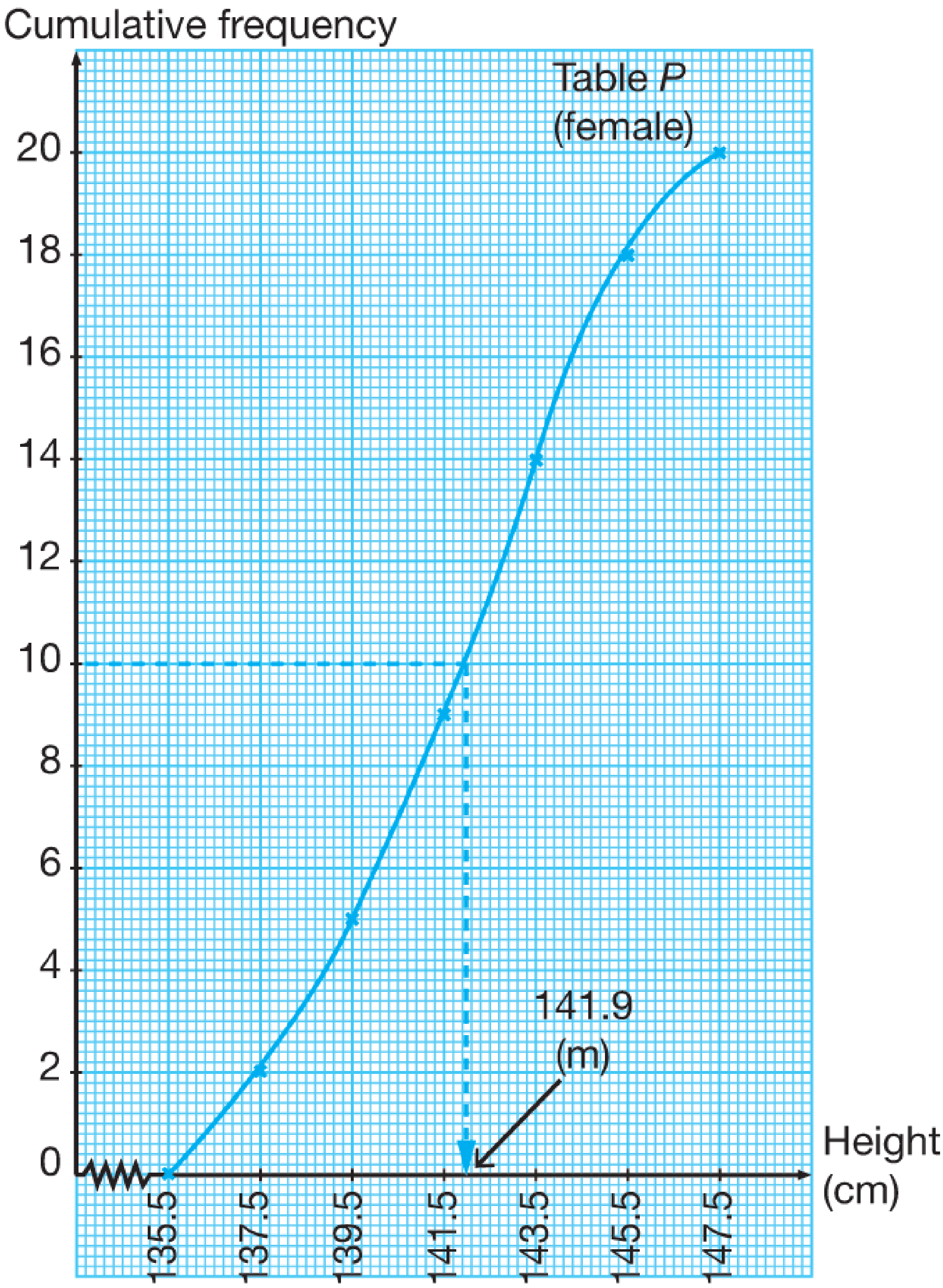
Height (cm)	*Number of females* (f)	*Upper boundary*	*Cumulative frequency*	*Midpoint* (x)	fx	fx^2
134 – 135	0	135.5	0			
136 – 137	2	137.5	2	136.5	273.0	37 264.5
138 – 139	3	139.5	5	138.5	415.5	57 546.75
140 – 141	4	141.5	9	140.5	562.0	78 961
142 – 143	5	143.5	14	142.5	712.5	101 531.25
144 – 145	4	145.5	18	144.5	578.0	83 521
146 – 147	2	147.5	20	146.5	293.0	42 924.5
Sum	20				2 834	401 749

Table *P*

(a) (i)


Cumulative frequency
Table L
(male)
20
18
16
14
12
10
8
6
4
2
0
169.3
(m)
Height
(cm)
160.5
162.5
164.5
166.5
168.5
170.5
172.5
174.5
176.5

(ii)



(b) (i) Median = 169.3 cm (ii) Median = 141.9 cm

(c) (i) Standard deviation, $\sigma = \sqrt{\dfrac{\sum fx^2}{\sum f} - \left(\dfrac{\sum fx}{\sum f}\right)^2} = \sqrt{\dfrac{570\ 801}{20} - \left(\dfrac{3\ 378}{20}\right)^2} = \sqrt{12.84} = 3.583$ cm

(ii) Standard deviation = $\sigma = \sqrt{\dfrac{401\ 749}{20} - \left(\dfrac{2\ 834}{20}\right)^2} = \sqrt{8.56} = 2.926$ cm

(d) *Interpretation*:
Both median and standard deviation of the of the height distribution of the male participants are greater than the median and standard deviation of the height distribution of the female participants
Conclusion:
The average height achieve by the male participants is higher than that of the female participants. The height distribution achieved by the male participants is more widely dispersed compared to that of the female participants.