Form 5 Chapter 1
Variation
Fully-Worked Solutions

UPSKILL 1.1

1

x	9	27	45	63
y	8	24	40	56
$\frac{y}{x}$	$\frac{8}{9}$	$\frac{8}{9}$	$\frac{8}{9}$	$\frac{8}{9}$

Since the value of $\frac{y}{x} = \frac{8}{9}$ (a constant), thus y varies directly as x.

2

x	12	20	50
y	10	64	400
$\frac{y}{x}$	$\frac{5}{6}$	$\frac{16}{5}$	2

Since $\frac{y}{x}$ is not a constant, thus y does not vary directly as x.

3 (a) $m \propto \sqrt{n}$

$m = k\sqrt{n}$ [k is a constant.]

$m = 1$ when $n = 16$

$1 = k\sqrt{16}$

$k = \frac{1}{4}$

$m = \frac{1}{4}\sqrt{n}$

(b) When $n = 36$,

$m = \frac{1}{4}\sqrt{36} = \frac{3}{2}$

4 $y \propto x^{\frac{1}{3}}$

$y = kx^{\frac{1}{3}}$

Given that $x = 8$ and $y = 10$,

$10 = k(8)^{\frac{1}{3}}$ [k is a constant.]

$10 = 2k$

$k = 5$

$y = 5x^{\frac{1}{3}}$

Given that $x = p$ and $y = 15$,

$15 = 5p^{\frac{1}{3}}$

$p^{\frac{1}{3}} = 3$

$p = 3^3 = \mathbf{27}$

5 (a) $p \propto x^3 y^{\frac{1}{2}} z^{\frac{1}{3}}$

$p = kx^3 y^{\frac{1}{2}} z^{\frac{1}{3}}$ [k is a constant.]

When $x = 3$, $y = \frac{1}{9}$ and $z = 64$, thus $p = 72$.

$72 = k(3)^3\left(\frac{1}{9}\right)^{\frac{1}{2}}(64)^{\frac{1}{3}}$

$72 = k(27)\left(\frac{1}{3}\right)(4)$

$k = \frac{72(3)}{27(4)} = 2$

$\therefore\ p = 2x^3 y^{\frac{1}{2}} z^{\frac{1}{3}}$

(b) $p = 2x^3 y^{\frac{1}{2}} z^{\frac{1}{3}}$

When $x = 2$, $y = \frac{1}{81}$, $z = 125$,

$p = 2(2)^3\left(\frac{1}{81}\right)^{\frac{1}{2}}(125)^{\frac{1}{3}}$

$p = 8\frac{8}{9}$

6 (a) $M \propto vn$

$M = kvn$ [k is a constant.]

$M = 60$ when $v = 1000$ and $n = 3$.

$60 = k(1000)(3)$

$k = \frac{60}{3000}$

$k = \frac{1}{50}$

$M = \frac{1}{50}vn$

(b) When $v = 1500$ and $n = 4$,

$M = \frac{1}{50}(1500)(4) = 120$

7 (a) $V \propto j^3$

$V = kj^3$ [k is a constant.]

When $j = 7$, $V = \frac{2156}{3}$

$\frac{2156}{3} = k(7)^3$

$k = \frac{2156}{3} \times \frac{1}{343}$

$k = \frac{44}{21}$

$V = \frac{44}{21} j^3$

(b) When $j = 3$,

$V = \frac{44}{21}(3)^3$

$V = 56\frac{4}{7}$ cm^3

8 $V \propto T$

$V = kT$ [k is a constant.]

$V = 13$ when $T = 650$

$13 = k(650)$

$k = \frac{13}{650}$

$V = \frac{13}{650} T$

When $T = 400$,

$V = \frac{13}{650}(400) = 8$

UPSKILL 1.2

1

x	2	4	5	8
y	10	5	4	2.5
xy	20	20	20	20

Since the value of $xy = 20$ (a constant), thus y varies inversely as x.

2 $m \propto \frac{1}{y^{\frac{1}{3}}}$

$m = k\left(\frac{1}{y^{\frac{1}{3}}}\right)$ [k is a constant.]

$m = 1$ when $y = 8$

$1 = k\left(\frac{1}{8^{\frac{1}{3}}}\right)$

$1 = k\left(\frac{1}{2}\right)$

$k = 2$

$m = 2\left(\frac{1}{y^{\frac{1}{3}}}\right)$

When $m = 10$,

$10 = 2\left(\frac{1}{y^{\frac{1}{3}}}\right)$

$y^{\frac{1}{3}} = \frac{1}{5}$

$y = \frac{1}{125}$

3 $y \propto \frac{1}{x^3}$

$y = \frac{k}{x^3}$ [k is a constant.]

$y = 5$ when $x = 2$

$5 = \frac{k}{2^3}$

$k = 40$

$y = \frac{40}{x^3}$

When $y = \frac{5}{8}$,

$\frac{5}{8} = \frac{40}{x^3}$

$x^3 = 40 \times \frac{8}{5}$

$x^3 = 64$

$x = 4$

4 $h \propto \frac{1}{r^2}$

$h = k\frac{1}{r^2}$ [k is a constant.]

$h = 7$ when $r = 3$

$7 = k\left(\frac{1}{3^2}\right)$

$k = 63$

$h = \frac{63}{r^2}$

When $r = 5$,

$h = \frac{63}{5^2} = 2.52$ cm

5 $I \propto \frac{1}{R}$

$I = \frac{k}{R}$ [k is a constant.]

Given that $I = 0.4$ and $R = 30$,

$0.4 = \frac{k}{30}$

$k = 12$

$I = \frac{12}{R}$

When $R = 48$,

$I = \frac{12}{48} = 0.25$

UPSKILL 1.3

1 $w \propto \frac{x^2}{\sqrt{y}}$

$w = \frac{kx^2}{\sqrt{y}}$ [k is a constant.]

Given that $w = 4$, $x = 2$ and $y = 9$,

$4 = \frac{k(2)^2}{\sqrt{9}}$

$k = 3$

$w = \frac{3x^2}{\sqrt{y}}$

When $w = 8$, $x = p$ and $y = 36$,

$8 = \frac{3p^2}{\sqrt{36}}$

$8 = \frac{3p^2}{6}$

$p^2 = 16$

$p = \pm 4$

2 $p \propto \frac{q^2}{r}$

$p = \frac{kq^2}{r}$ [k is a constant.]

Given that $p = 25$, $q = 5$, $r = 3$,

$25 = \frac{k(5)^2}{3}$

$k = 3$

$p = \frac{3q^2}{r}$

When $p = 12$, $q = m$ and $r = 9$,

$12 = \frac{3m^2}{9}$

$m^2 = 36$

$m = \pm 6$

3 (a) $K \propto mv^2$

$K = kmv^2$ [k is a constant.]

If $m = 3$ and $v = 12$, thus $K = 216$.

$216 = k(3)(12)^2$

$k = \frac{1}{2}$

$K = \frac{1}{2}mv^2$

When $K = 250$ and $v = 10$,

$250 = \frac{1}{2}m(10)^2$

$250 = 50m$

$m = 5$ kg

(b) When $m = 6$ and $K = 300$,

$300 = \frac{1}{2}(6)v^2$

$v^2 = 100$

$v = 10$ m s^{-1}

4 $F \propto \frac{q_1 q_2}{r^2}$

$F = \frac{kq_1 q_2}{r^2}$ [k is a constant.]

$125 = \frac{k\left(1.0 \times 10^{-5}\right)\left(2.0 \times 10^{-5}\right)}{0.12^2}$

$125 = \frac{k\left(1.0 \times 10^{-5}\right)\left(2.0 \times 10^{-5}\right)}{0.12^2}$

$1.8 = k\left(2.0 \times 10^{-10}\right)$

$k = \frac{1.8}{\left(2.0 \times 10^{-10}\right)}$

$k = 9 \times 10^9$

Summative Practice 1

Multiple-Choice Questions

1

x	3	5	7
y	12	20	60
$\frac{y}{x}$	4	4	$8\frac{4}{7}$

Since $\frac{y}{x}$ is not a constant, thus y does not vary directly as x.
Answer: B

2 $m \propto n$
$m = kn$ [k is a constant.]
When $n = 3$, $m = 9$
$9 = k(3)$
$k = 3$
$m = 3n$
Answer: A

3

x	2	6	8
y	24	8	6
xy	48	48	48

Since the value of xy is a constant, thus y varies inversely as x.
Answer: C

4 $h \propto \frac{1}{t^3}$
$h = \frac{k}{t^3}$ [k is a constant.]
When $t = 2$, $h = 4$
$4 = \frac{k}{2^3}$
$k = 32$
$h = \frac{32}{t^3}$
Answer: A

5 $y \propto \frac{x^2}{p}$
$y = \frac{kx^2}{p}$ [k is a constant.]
When $x = 2$, $y = 4$ and $p = 3$,
$4 = \frac{k(2)^2}{3}$
$k = 3$
$y = \frac{3x^2}{p}$
When $y = 25$ and $p = 3$,
$25 = \frac{3x^2}{3}$
$x^2 = 25$
$x = \pm 5$
Answer: B

6 $m \propto \frac{n}{\sqrt{h}}$
$m = \frac{kn}{\sqrt{h}}$ [k is a constant.]
When $n = 8$, $h = 16$ and $m = 2$,
$2 = \frac{k(8)}{\sqrt{16}}$
$2 = \frac{k(8)}{4}$
$k = 1$
$m = \frac{n}{\sqrt{h}}$
Answer: A

Structured Questions

1 $p \propto q$

$p = kq$ [k is a constant.]

$q = 5$ when $p = 20$

$20 = k(5)$

$k = 4$

$p = 4q$

If $q = 8$,

$p = 4(8) = 32$

2 $y \propto x^2$

$y = kx^2$ [k is a constant.]

When $x = 4$, $y = 48$,

$48 = k(4)^2$

$k = 3$

$y = 3x^2$

When $x = 6$ and $y = p$,

$p = 3(6)^2 = 108$

3 $p \propto \sqrt{q}$

$p = k\sqrt{q}$ [k is a constant.]

$p = 10$ when $q = 25$

$10 = k\sqrt{25}$

$k = 2$

$p = 2\sqrt{q}$

If $q = 100$,

$p = 2\sqrt{100} = 20$

4 $y \propto \dfrac{x^{\frac{1}{2}}p^{\frac{1}{3}}}{q^3}$

$y = k\dfrac{x^{\frac{1}{2}}p^{\frac{1}{3}}}{q^3}$ [k is a constant.]

When $x = 16$, $p = 125$, $q = 2$, thus $y = 5$,

$5 = k\left[\dfrac{16^{\frac{1}{2}}125^{\frac{1}{3}}}{2^3}\right]$

$5 = k\left[\dfrac{4(5)}{8}\right]$

$k = 2$

$y = \dfrac{2x^{\frac{1}{2}}p^{\frac{1}{3}}}{q^3}$

When $x = 4$, $p = 8$ and $q = 4$,

$y = \dfrac{2(4)^{\frac{1}{2}}(8)^{\frac{1}{3}}}{4^3} = \dfrac{1}{8}$

5 (a) $T \propto j$

$T = kj$ [k is a constant.]

$T = 10$ when $j = 5$.

$10 = 5k$

$k = 2$

$T = 2j$

(b) When $j = 3.5$,

$T = 2(3.5) = 7$ g cm^{-3}

6 (a) $v \propto \sqrt{T}$

$v = k\sqrt{T}$ [k is a constant.]

$T = 100$ when $v = 5$

$5 = k\sqrt{100}$

$5 = k(10)$

$k = 0.5$

$v = 0.5\sqrt{T}$

(b) (i) When $T = 64$,

$v = 0.5\sqrt{64} = 4$ m s^{-1}

(ii) When $v = 8$,

$8 = 0.5\sqrt{T}$

$\sqrt{T} = 16$

$T = 256$ J

7 $N \propto dv^2$

$N = kdv^2$ [k is a constant.]

$N = 7\dfrac{1}{2}$ when $d = 30$ and $v = 18$

$\dfrac{15}{2} = k(30)(18)^2$

$k = \dfrac{1}{1296}$

$N = \dfrac{1}{1296}dv^2$

$N = \dfrac{1}{1296}(200)(36)^2$

$N = 200$ barrels

8 $R \propto Av$

$R = kAv$ [k is a constant.]

When $A = 25$ and $v = 40$, thus $R = 225$.

$225 = k(25)(40)$

$k = \frac{9}{40}$

$R = \frac{9}{40}Av$

When $v = 350$ and $R = 236.25$,

$236.25 = \frac{9}{40}A(350)$

$A = \frac{236.25}{350} \times \frac{40}{9}$

$A = 3 \text{ m}^2$

9 $q \propto \frac{1}{p^2}$

$q = \frac{k}{p^2}$ [k is a constant.]

Given that $p = 1$ and $q = 12$,

$12 = \frac{k}{1^2}$

$k = 12$

$q = \frac{12}{p^2}$

When $p = 2$ and $q = x$,

$x = \frac{12}{2^2} = \frac{12}{4} = 3$

10 $y \propto \frac{1}{x^3}$

$y = \frac{k}{x^3}$ [k is a constant.]

$y = 5$ when $x = 2$

$5 = \frac{k}{2^3}$

$k = 40$

$y = \frac{40}{x^3}$

When $y = \frac{5}{8}$,

$\frac{5}{8} = \frac{40}{x^3}$

$x^3 = 40 \times \frac{8}{5}$

$x^3 = 64$

$x = 4$

11 (a) $b \propto \frac{1}{j^3}$

$b = \frac{k}{j^3}$ [k is a constant.]

When $j = 2$, $b = 500$,

$500 = \frac{k}{2^3}$

$k = 4\,000$

$b = \frac{4\,000}{j^3}$

(b) (i) When $j = 5$,

$b = \frac{4\,000}{5^3}$

$b = 32$

(ii) When $b = 4\,000$

$4\,000 = \frac{4\,000}{j^3}$

$j^3 = 1$

$j = 1 \text{ cm}$

12 (a) $T \propto \frac{1}{d^2}$

$T = \frac{k}{d^2}$ [k is a constant.]

$T = 3.84$, when $d = 2$

$3.84 = \frac{k}{2^2}$

$k = 15.36$

$T = \frac{15.36}{d^2}$

(b) When $T = 6$,

$6 = \frac{15.36}{d^2}$

$d^2 = \frac{15.36}{6}$

$d^2 = 2.56$

$d = 1.6 \text{ cm}$

13 $w \propto \dfrac{x^2}{\sqrt{y}}$

$w = \dfrac{kx^2}{\sqrt{y}}$ [k is a constant.]

Given that $w = 4$, $x = 2$ and $y = 9$,

$4 = \dfrac{k\left(2^2\right)}{\sqrt{9}}$

$k = 3$

$w = \dfrac{3x^2}{\sqrt{y}}$

When $w = 8$, $x = 4$ and $y = p$,

$8 = \dfrac{3(4)^2}{\sqrt{p}}$

$\sqrt{p} = 6$

$p = 36$

14 $p \propto \dfrac{1}{q\sqrt{r}}$

$p = \dfrac{k}{q\sqrt{r}}$

When $p = \dfrac{2}{3}$, $q = 2$ and $r = 9$,

$\dfrac{2}{3} = \dfrac{k}{2\sqrt{9}}$ [k is a constant.]

$\dfrac{2}{3} = \dfrac{k}{6}$

$k = 4$

$p = \dfrac{4}{q\sqrt{r}}$

When $p = \dfrac{1}{9}$, $q = 4$ and $r = n$,

$\dfrac{1}{9} = \dfrac{4}{4\sqrt{n}}$

$\sqrt{n} = 9$

$n = 81$

15 $p \propto \dfrac{\sqrt{r}}{q^2}$

$p = \dfrac{k\sqrt{r}}{q^2}$ [k is a constant.]

When $p = 3$, $q = 2$ and $r = 9$,

$3 = \dfrac{k\sqrt{9}}{3^2}$

$3 = \dfrac{k(3)}{3^2}$

$k = 9$

$p = \dfrac{9\sqrt{r}}{q^2}$

When $p = \dfrac{4}{9}$, $q = k$ and $r = 16$,

$\dfrac{4}{9} = \dfrac{9\sqrt{16}}{k^2}$

$k^2 = 36 \times \dfrac{9}{4}$

$k^2 = 81$

$k = \pm 9$

16 $V \propto AR$

$V = kAR$

When $V = 8$, $A = 4$ and $R = 2$,

$8 = k(4)(2)$

$k = 1$

$V = AR$

(a) When $A = 5$ and $R = 3$,

$V = 5(3) = 15$ volt

(b) When $V = 16$ and $R = 4$,

$16 = A(4)$

$A = 4$ ampere

(c) When $A = 6$ and $V = 3$,

$3 = 6R$

$R = 0.5$ ohm

17 (a) $F \propto \dfrac{Mm}{r^2}$

$F = G\dfrac{Mm}{r^2}$

(b) $F = \left(6.670 \times 10^{-11}\right)\left[\dfrac{5.972 \times 10^{24} \times 7.348 \times 10^{22}}{\left(3.844 \times 10^{8}\right)^2}\right]$

$F = 1.981 \times 10^{20}$ N