

**Form 4 Chapter 9**  
**Probability of Combined Events**  
**Fully-Worked Solutions**

**UPSKILL 9.1**

- 1 (a) (i) The event of obtaining a multiple of 3 or a factor of 12, i.e.  $A \cup B$   
 (ii) The event of obtaining a multiple of 3 and a factor of 12, i.e.  $A \cap B$   
 (b) (i)  $A \cup B = \{1, 2, 3, 4, 6, 9, 12\}$   
 (ii)  $A \cap B = \{3, 6, 12\}$
- 2 (a) (i) The event such that the first digit is an even number or the last digit is an odd number, i.e.  $P \cup Q$   
 (ii) The event such that the first digit is an even number and the last digit is an odd number, i.e.  $P \cap Q$   
 (b) (i)  $P \cup Q = \{618, 681, 816, 861\}$   
 (ii)  $P \cap Q = \{681, 861\}$

**UPSKILL 9.2**

- 1 (a) Dependent  
 (b) Dependent  
 (c) Independent

2

Box A \ Box B	1	2	3
1	(1, 1)	(1, 2)	(1, 3)
2	(2, 1)	(2, 2)	(2, 3)
3	(3, 1)	(3, 2)	(3, 3)
4	(4, 1)	(4, 2)	(4, 3)

(a)  $\frac{7}{12}$

(b)  $\frac{1}{6}$

3  $S = \{(R, 2), (R, 3), (R, 7), (R, 8), (U, 2),$

$(U, 3), (U, 7), (U, 8)\}$

(a)  $\{(R, 3), (R, 7), \{(U, 2),$

$(U, 3), (U, 7), (U, 8)\}$

$\frac{3}{4}$

(b)  $\{(R, 2), (R, 3), (R, 7)\}$

$\frac{3}{8}$

- 4 (a) P(all three of them strike the target)

$$= \frac{2}{5} \times \frac{3}{4} \times \frac{2}{3} = \frac{1}{5}$$

- (b) P(only one of them strikes the target)

$$= \left(\frac{2}{5} \times \frac{1}{4} \times \frac{1}{3}\right) + \left(\frac{3}{5} \times \frac{3}{4} \times \frac{1}{3}\right) + \left(\frac{3}{5} \times \frac{1}{4} \times \frac{2}{3}\right)$$

$$= \frac{1}{30} + \frac{3}{20} + \frac{1}{10}$$

$$= \frac{17}{60}$$

- (c) P(at least one of them strike the target)

$$= 1 - \frac{3}{5} \times \frac{1}{4} \times \frac{1}{3}$$

$$= \frac{19}{20}$$

- 5 (a) P(win all the games)

$$= \frac{3}{5} \times \frac{1}{4} \times \frac{5}{6} = \frac{1}{8}$$

- (b) P(win at least two of the games)

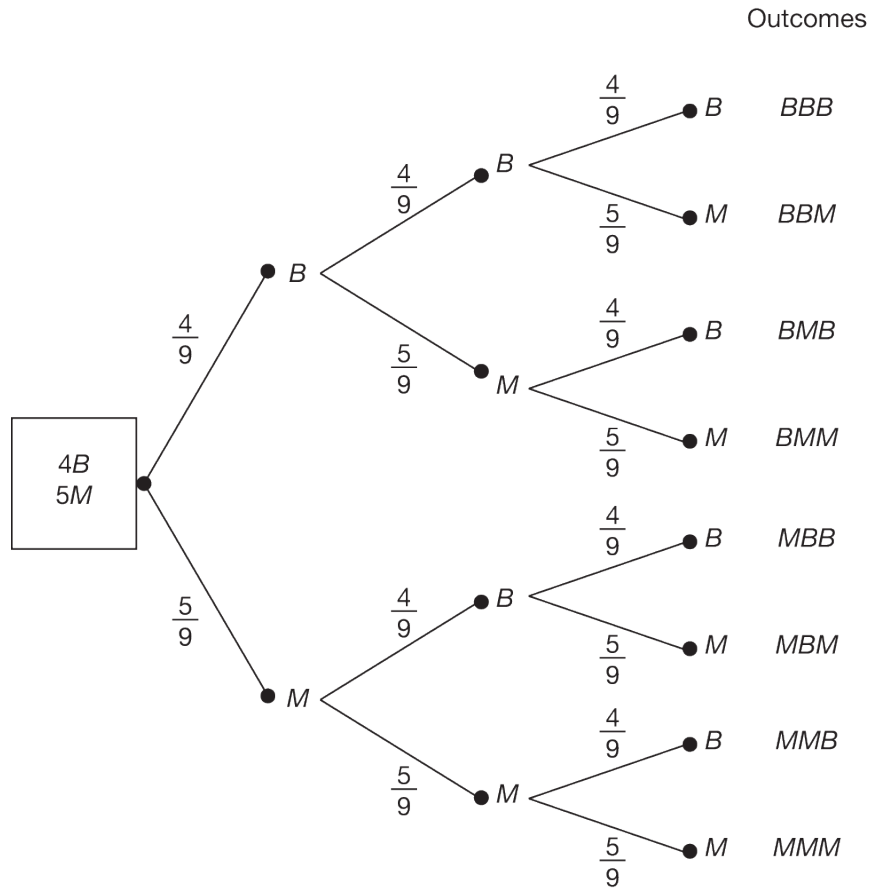
$$= \left(\frac{3}{5} \times \frac{1}{4} \times \frac{5}{6}\right) + \left(\frac{3}{5} \times \frac{1}{4} \times \frac{1}{6}\right)$$

$$+ \left(\frac{2}{5} \times \frac{1}{4} \times \frac{5}{6}\right) + \left(\frac{3}{5} \times \frac{3}{4} \times \frac{5}{6}\right)$$

$$= \frac{1}{8} + \frac{1}{40} + \frac{1}{12} + \frac{3}{8}$$

$$= \frac{73}{120}$$

6 B – Blue M – Red



(a)  $P(MM) = \frac{5}{9} \times \frac{5}{9} = \frac{25}{81}$

(b)  $P(MMM) = \frac{5}{9} \times \frac{5}{9} \times \frac{5}{9} = \frac{125}{729}$

(c)  $P(BBB) + P(BBM) + P(BMB) + P(MBB)$

$$= \left( \frac{4}{9} \times \frac{4}{9} \times \frac{4}{9} \right) + \left( \frac{4}{9} \times \frac{4}{9} \times \frac{5}{9} \right) \times 3$$

$$= \frac{304}{729}$$

(d)  $P(BBM) + P(BMB) + P(MBB)$

$$= \left( \frac{5}{9} \times \frac{4}{9} \times \frac{4}{9} \right) \times 3 = \frac{80}{243}$$

7 (a) P(both cards have the same colour)

$$= P(BB) + P(MM) + P(HH)$$

$$= \left( \frac{4}{14} \times \frac{3}{13} \right) + \left( \frac{3}{14} \times \frac{2}{13} \right) + \left( \frac{7}{14} \times \frac{6}{13} \right)$$

$$= \frac{30}{91}$$

(b) P(the cards are of different colours)

$$= 1 - \frac{30}{91} = \frac{61}{91}$$

$$8 \text{ (a) } P(BH) + P(HB) = \left(\frac{4}{15} \times \frac{5}{14}\right) \times 2 = \frac{4}{21}$$

$$(b) P(HH) + P(BB) = \left(\frac{5}{15} \times \frac{4}{14}\right) + \left(\frac{6}{15} \times \frac{5}{14}\right) = \frac{5}{21}$$

9 P(3 of them obtain the pens with the same colour)

$$\begin{aligned} &= P(BBB) + P(MMM) \\ &= \left(\frac{7}{10} \times \frac{6}{9} \times \frac{5}{8}\right) + \left(\frac{3}{10} \times \frac{2}{9} \times \frac{1}{8}\right) \\ &= \frac{7}{24} + \frac{1}{120} \\ &= \frac{3}{10} \end{aligned}$$

$$10 \text{ (a) } P(MM) = \frac{1}{6} \times \frac{4}{6} = \frac{1}{9}$$

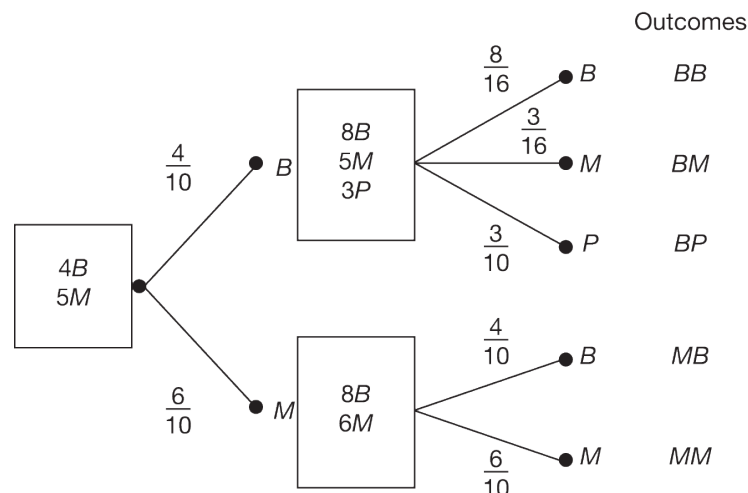
$$(b) P(MK) + P(KM) = \left(\frac{1}{6} \times \frac{1}{6}\right) + \left(\frac{3}{6} \times \frac{4}{6}\right) = \frac{13}{36}$$

$$\begin{aligned} (c) P(\text{different colours}) &= 1 - P(\text{same colour}) \\ &= 1 - \left(\frac{1}{6} \times \frac{4}{6}\right) - \left(\frac{2}{6} \times \frac{1}{6}\right) - \left(\frac{3}{6} \times \frac{1}{6}\right) \\ &= \frac{3}{4} \end{aligned}$$

$$11 \text{ (a) } P(HH) = \frac{2}{9} \times \frac{8}{14} = \frac{8}{63}$$

$$(b) P(KB) + P(BK) = \frac{3}{9} \times \frac{4}{14} + \frac{4}{9} \times \frac{2}{14} = \frac{10}{63}$$

12 *B* – Blue  
*M* – Red  
*P* – White



$$(a) P(MB) + P(BM) = \frac{6}{10} \times \frac{4}{10} + \frac{4}{10} \times \frac{5}{16} = \frac{73}{200}$$

$$(b) P(BP) = \frac{4}{10} \times \frac{3}{16} = \frac{3}{40}$$

**UPSKILL 9.3**

- 1 (a) Mutually exclusive events  
 (b) Not mutually exclusive events

2 (a) (i)  $P(A) = \frac{4}{10} = \frac{2}{5}$

(ii)  $P(B) = \frac{3}{10}$

(b) Yes.  $A$  and  $B$  are mutually exclusive events.

(c)  $P(A \cup B) = \frac{2}{5} + \frac{3}{10} = \frac{7}{10}$

- 3 (a) Yes,  $A$  and  $B$  are mutually exclusive events.

(b)  $P(A \cup B) = \frac{4}{6} + \frac{2}{6} = 1$

4

$A \backslash B$	1	2	3	4	5
1	√			×	
2		√	×		
3		×	√		
4	×			√	
5					√

Mark √ for the event both cards have the same number.

Mark × for the event the sum of the numbers on the two cards is 5.

(a)  $P(\text{both cards have the same number}) = \frac{5}{25} = \frac{1}{5}$

(b)  $P(\text{the sum of the numbers on the two cards is 5}) = \frac{4}{25}$

(c)  $P(A \cup B) = \frac{1}{5} + \frac{4}{25} = \frac{9}{25}$

5  $P(X \cup Y) = P(X) + P(Y) - P(X \cap Y)$

$$= \frac{3}{10} + \frac{2}{5} - \left( \frac{3}{10} \times \frac{2}{5} \right)$$

$$= \frac{29}{50}$$

- 6 (a) Yes, both events are mutually exclusive.

(b) (i)  $P(T \cup B) = \frac{2}{9} + \frac{4}{9} = \frac{2}{3}$

(ii)  $P(T' \cap B') = P(T \cup B)' = 1 - \frac{2}{3} = \frac{1}{3}$

**UPSKILL 9.4**

1

$B \backslash A$	1	2	3	4
1	√		√	
2		√		√×
3	√		√ ×	×
4		√×	×	√×

Mark √ for the event that the sum of the numbers is even.

Mark × for the event the sum of the numbers is greater than 5.

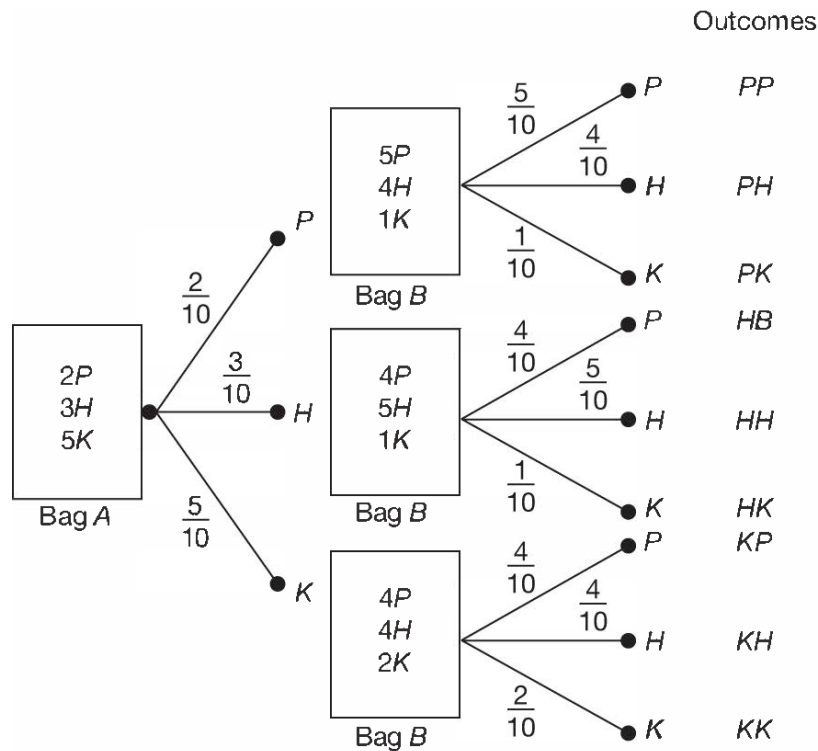
(a) P(the sum of the two numbers is even and greater than 5)

$$= \frac{4}{16} = \frac{1}{4}$$

(b) P(the sum of the two numbers is even or greater than 5)

$$= \frac{10}{16} = \frac{5}{8}$$

2 P – White  
B – Black  
K – Grey



$$\begin{aligned}
 & P(PP) + P(HH) + P(KK) \\
 &= \left(\frac{2}{10} \times \frac{5}{10}\right) + \left(\frac{3}{10} \times \frac{5}{10}\right) + \left(\frac{5}{10} \times \frac{2}{10}\right) \\
 &= \frac{7}{20}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{3 (a)} \quad & P(\overline{A} \overline{R} \overline{Z}) + P(\overline{A} \overline{R} Z) + P(\overline{A} R \overline{Z}) \\
 &= \left(\frac{9}{10} \times \frac{1}{5} \times \frac{3}{10}\right) + \left(\frac{1}{10} \times \frac{4}{5} \times \frac{3}{10}\right) \\
 &\quad + \left(\frac{1}{10} \times \frac{1}{5} \times \frac{7}{10}\right) \\
 &= \frac{23}{150}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{(b)} \quad & P(A \overline{R} \overline{Z}) + P(A \overline{R} Z) + P(\overline{A} R Z) \\
 &= \left(\frac{9}{10} \times \frac{4}{5} \times \frac{3}{10}\right) + \left(\frac{9}{10} \times \frac{1}{5} \times \frac{7}{10}\right) \\
 &\quad + \frac{1}{10} \times \frac{4}{5} \times \frac{7}{10} \\
 &= \frac{199}{500}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{(c)} \quad & P(\text{at least one of them strike the target}) \\
 &= 1 - \frac{1}{10} \times \frac{1}{5} \times \frac{3}{10} \\
 &= \frac{497}{500}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{4} \quad & P(MH) + P(HM) \\
 &= \left(\frac{6}{10} \times \frac{4}{10}\right) + \frac{4}{10} \times \frac{6}{9} \\
 &= \frac{38}{75}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{5 (a)} \quad & P(\text{both are SPM books}) \\
 &= \frac{5}{8} \times \frac{4}{7} \\
 &= \frac{5}{14}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{(b)} \quad & P(\text{both are of the same subject}) \\
 &= \left(\frac{6}{18} \times \frac{5}{17}\right) + \left(\frac{7}{18} \times \frac{6}{17}\right) + \left(\frac{5}{18} \times \frac{4}{17}\right) \\
 &= \frac{46}{153}
 \end{aligned}$$

## Summative Practice 9

### Multiple-Choice Questions

1

$A \backslash B$	1	3	5
1			
2	×	×	√×
3			
4	×	√×	×
5			

Mark √ for the event that the sum of the numbers is 7.

Mark × for the event that the product of the numbers is greater than 5.

P(the sum of the numbers is 7 or the product of the numbers is greater than 5)

$$= \frac{6}{15} = \frac{2}{5}$$

Answer: C

2 P(multiple of 3 or perfect square)

$$= \frac{3}{9} + \frac{2}{9} - \frac{1}{9} = \frac{4}{9}$$

Answer: B

3 P(two pass)

$$= \left(\frac{1}{3} \times \frac{2}{5} \times \frac{1}{4}\right) + \left(\frac{2}{3} \times \frac{2}{5} \times \frac{3}{4}\right) + \left(\frac{1}{3} \times \frac{3}{4} \times \frac{3}{5}\right)$$

$$= \frac{1}{30} + \frac{1}{5} + \frac{3}{20}$$

$$= \frac{23}{60}$$

Answer: D

4 P(same colour)

$$= \left(\frac{5}{11} \times \frac{6}{10}\right) + \left(\frac{6}{10} \times \frac{5}{11}\right)$$

$$= \frac{6}{11}$$

Answer: C

5 P(RG) + P(GR)

$$= \left(\frac{4}{8} \times \frac{4}{8}\right) + \left(\frac{4}{8} \times \frac{4}{7}\right)$$

$$= \frac{15}{18}$$

Answer: B

### Structured Questions

1 (a) P(FFF)

$$= \frac{8}{9} \times \frac{23}{25} \times \frac{19}{20}$$

$$= \frac{874}{1125}$$

(b) P(only one of them pass)

$$= \left(\frac{1}{9} \times \frac{23}{25} \times \frac{19}{20}\right) + \left(\frac{8}{9} \times \frac{2}{25} \times \frac{19}{20}\right) +$$

$$\left(\frac{1}{20} \times \frac{8}{9} \times \frac{23}{25}\right)$$

$$= \frac{437}{4500} + \frac{76}{1125} + \frac{46}{1125}$$

$$= \frac{37}{180}$$

(c) P(at least one of them pass)

$$= 1 - \text{P(all fail)}$$

$$= 1 - \frac{8}{9} \times \frac{23}{25} \times \frac{19}{20}$$

$$= 1 - \frac{874}{1125}$$

$$= \frac{251}{1125}$$

2 (a) P(Physics, Chemistry)

$$= \frac{3}{14} \times \frac{5}{13}$$

$$= \frac{15}{182}$$

(b) P(the books are of the same category)

$$= \left(\frac{3}{14} \times \frac{2}{13}\right) + \left(\frac{5}{14} \times \frac{4}{13}\right) + \left(\frac{6}{14} \times \frac{5}{13}\right)$$

$$= \frac{4}{13}$$

3 (a) Yes, T and S are mutually exclusive events.

(b) (i) P(play tennis or squash)

$$= \frac{4}{7} + \frac{2}{7}$$

$$= \frac{6}{7}$$

(ii) P(did not play tennis nor squash)

$$= 1 - \frac{6}{7}$$

$$= \frac{1}{7}$$

4

$A \backslash B$	1	2	3
1	√		×
2		√×	×
3	×		√

Mark √ for the event the numbers of the cards are the same.

Mark × for the event the sum of the numbers on the cards is 4.

(a) P(the numbers of the cards are the same)

$$= \frac{3}{9} = \frac{1}{3}$$

(b) P(the sum of the numbers on the cards is 4)

$$= \frac{3}{9} = \frac{1}{3}$$

(c) P(the numbers of the cards are the same or the sum of the numbers on the cards is 4)

$$= \frac{3}{9} + \frac{3}{9} - \frac{1}{9} = \frac{5}{9}$$

5 (a) P(both are STPM books)

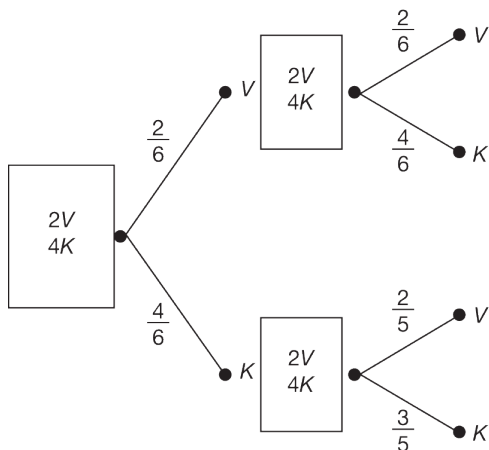
$$= \frac{7}{13} \times \frac{6}{12} = \frac{7}{26}$$

(b) P(both books are of the same subject)

$$= \left( \frac{6}{18} \times \frac{5}{17} \right) + \left( \frac{7}{18} \times \frac{6}{17} \right) + \left( \frac{5}{18} \times \frac{4}{17} \right)$$

$$= \frac{46}{153}$$

6



(a)  $P(KK) = \frac{4}{6} \times \frac{3}{5} = \frac{2}{5}$

(b)  $P(VK) + P(KV) = \frac{2}{6} \times \frac{4}{6} + \frac{4}{6} \times \frac{2}{5} = \frac{22}{45}$

7 (a)  $P(AA) = \frac{3}{8} \times \frac{2}{9} = \frac{1}{12}$

(b)  $P(A\bar{A}) + P(\bar{A}A) = \left( \frac{3}{8} \times \frac{7}{9} \right) + \left( \frac{5}{8} \times \frac{1}{9} \right) = \frac{13}{36}$

8 (a)  $P(20, 20) + P(20, 20) = \frac{2}{10} \times \frac{3}{12} = \frac{1}{20}$

(b)  $P(20, 50) + P(50, 20) = \frac{2}{10} \times \frac{4}{12} + \frac{5}{10} \times \frac{3}{12} = \frac{23}{120}$

9 (a)  $P(PP) = \frac{6}{11} \times \frac{5}{10} = \frac{3}{11}$

(b)  $P(LL) + P(PP) = \frac{5}{9} \times \frac{4}{8} + \frac{4}{9} \times \frac{3}{8} = \frac{4}{9}$

10  $S = \{(2, A), (2, B), (4, A), (4, B), (7, A), (7, B), (9, A), (9, B)\}$

(a)  $P\{(2, A), P(4, A)\}$ ,

$$P(\text{even number and letter } A) = \frac{2}{8} = \frac{1}{4}$$

(b)  $P\{(2, A), (2, B), (7, A), (7, B), (4, B), (9, B)\}$

$$P(\text{prime number or letter } B) = \frac{6}{8} = \frac{3}{4}$$

11 (a)  $P(\text{white, blue ball}) = \frac{120}{360} \times \frac{2}{6} = \frac{1}{9}$

(b)  $P(\text{purple, red card}) + P(\text{white, red ball})$

$$= \frac{240}{360} \times \frac{6}{14} + \frac{120}{360} \times \frac{4}{6}$$

$$= \frac{32}{63}$$

Or

12  $P(\text{same colour}) =$

$$\left( \frac{4}{15} \times \frac{3}{14} \times \frac{2}{13} \right) + \left( \frac{5}{15} \times \frac{4}{14} \times \frac{3}{13} \right) + \left( \frac{6}{15} \times \frac{5}{14} \times \frac{4}{13} \right)$$

$$= \frac{4}{455} + \frac{2}{91} + \frac{4}{91}$$

$$= \frac{34}{455}$$



**13 (a)** P(strike the target 1 time)

$$\begin{aligned} &= \left(\frac{4}{5} \times \frac{1}{5} \times \frac{1}{5}\right) \times 3 \\ &= \frac{12}{125} \end{aligned}$$

**(b)** P(strike the target 2 times)

$$\begin{aligned} &= \left(\frac{4}{5} \times \frac{4}{5} \times \frac{1}{5}\right) \times 3 \\ &= \frac{48}{125} \end{aligned}$$

**(c)** P(straight the target at least 1 time)

$$= 1 - \text{P}(\text{did not strike the target})$$

$$\begin{aligned} &= 1 - \left(\frac{1}{5} \times \frac{1}{5} \times \frac{1}{5}\right) \\ &= \frac{124}{125} \end{aligned}$$

**(d)** P(strike the target at least 2 times)

$$\begin{aligned} &= \frac{48}{125} + \left(\frac{4}{5} \times \frac{4}{5} \times \frac{4}{5}\right) \\ &= \frac{48}{125} + \frac{64}{125} \\ &= \frac{112}{125} \end{aligned}$$

**14** P(2 same letters)

$$\begin{aligned} &= \left(\frac{2}{9} \times \frac{1}{8}\right) + \left(\frac{2}{9} \times \frac{1}{8}\right) + \left(\frac{2}{9} \times \frac{1}{8}\right) \\ &= \frac{1}{12} \end{aligned}$$