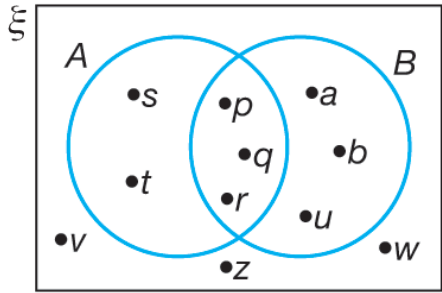


**Form 4 Chapter 4
Operations on Sets
Fully-Worked Solutions**

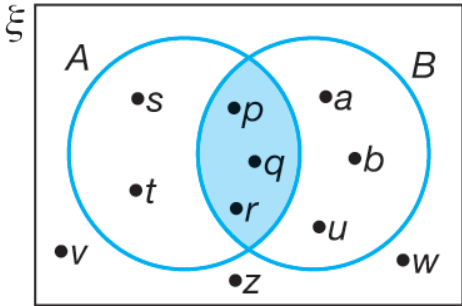
UPSKILL 4.1

- 1 (a) $\{b, h, k\}$
 (b) $\{5, 6\}$
 (c) $P = \{2, 3, 5, 7, \dots\}$
 $Q = \{1, 2, 3, 4, 5, 6\}$
 $P \cap Q = \{2, 3, 5\}$
 (d) $\{ \}$

- 2 (a) $\{p, q, r\}$
 (b) (i)



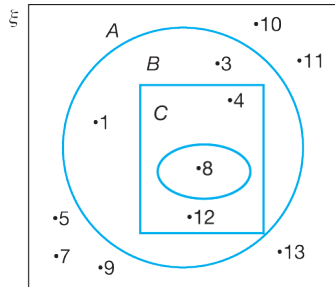
(ii)



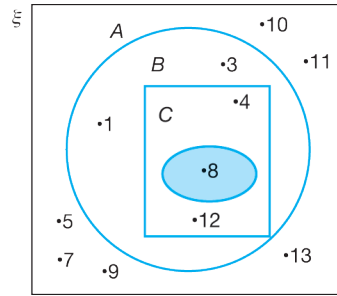
- 3 (a) $\{2, 4, 5\}$

- 4 $A = \{1, 2, 3, 4, 6, 8, 12\}$
 $B = \{4, 8, 12\}$
 $C = \{8\}$

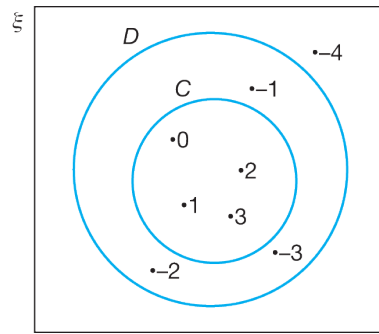
(a)



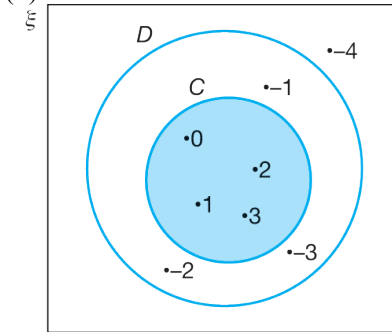
(b)



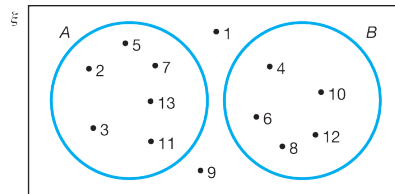
- 5 (a) $\{0, 1, 2, 3\}$



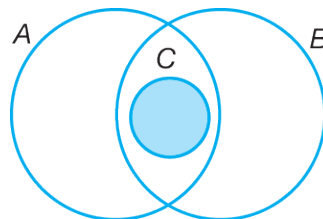
(ii)

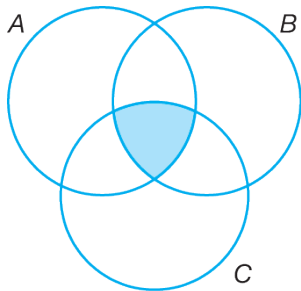
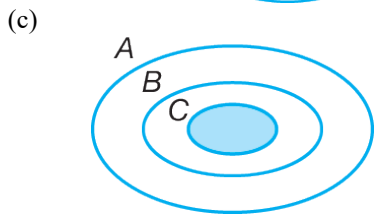
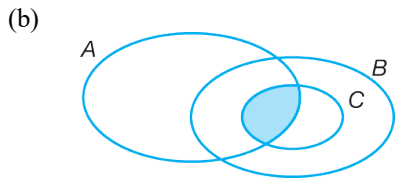


- 6 (a) $A = \{2, 3, 5, 7, 11, 13\}$

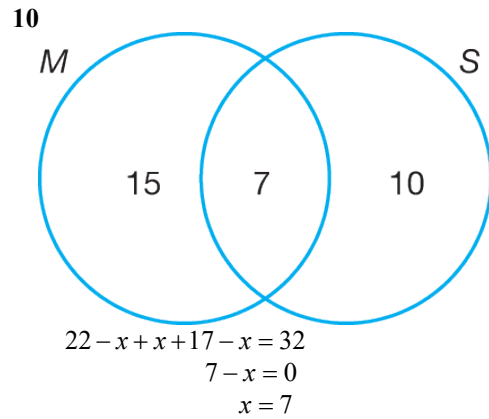
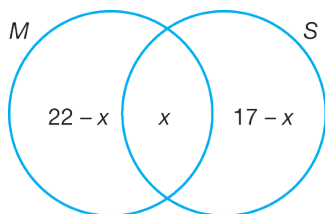
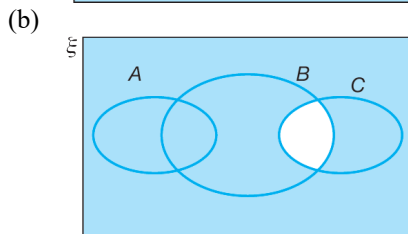
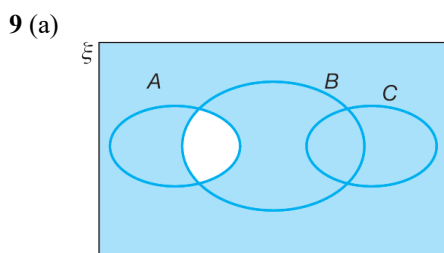
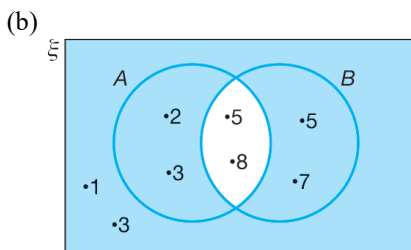


(b) $A \cap B = \emptyset$

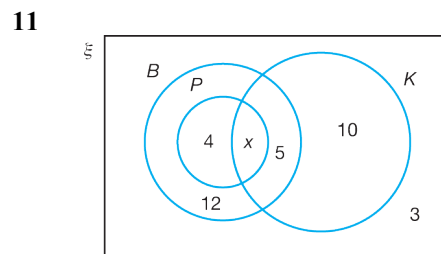




8 (a) (i) $A \cap B = \{6, 8\}$
 $(A \cap B)' = \{1, 2, 3, 4, 5, 7\}$



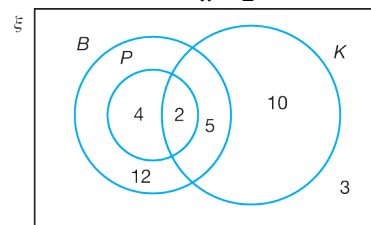
- (a) The number of students who are interested in both Mathematics and Science is 7.
 (b) The number of students who are interested in Science but not Mathematics is 10.



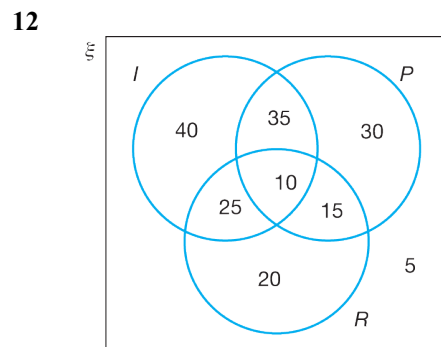
$$n(\xi) = 36$$

$$12 + 4 + x + 5 + 10 + 3 = 36$$

$$x = 2$$

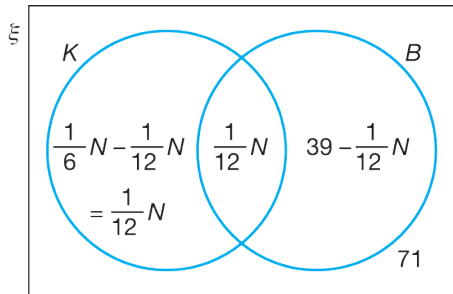


- (a) $n(B \cap P \cap K) = 2$ students
 (b) $n(P' \cap K) = 5 + 10 = 15$ students
 $n(B \cap P') = 5 + 12 = 17$ students
 (d) The number of students who like to play one type of game only = $12 + 10 = 22$



- (a) $n(\text{internet and smart phone but not reading}) = 35$ students
- (b) $n(\text{two activities only}) = 35 + 25 + 15 = 75$ students
- (c) $n(\text{one activity only}) = 40 + 30 + 20 = 90$ students
- (d) $n(\text{not interested in any of the activities}) = 5$ students

13



$$(a) \frac{1}{12}N + \frac{1}{12}N + \left(39 - \frac{1}{12}N + 71\right) = N$$

$$\frac{11}{12}N = 110$$

$$N = 120$$

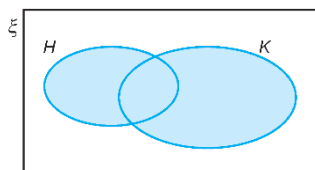
Number of undergraduates = 120

- (b) $\frac{1}{12}N = \frac{1}{12} \times 120 = 10$ undergraduates
- (c) $120 - 10 = 110$ undergraduates

UPSKILL 4.2

- 1 (a) {5, 7, 15, 20}
- (b) {a, b, c, d, x, y, z}
- (c) {p, q, r, s}
- 2 $P = \{3, 6, 9, 12\}$
- $Q = \{1, 2, 3, 4, 6, 12\}$
- $P \cup Q = \{1, 2, 3, 4, 6, 9, 12\}$

3 (a)

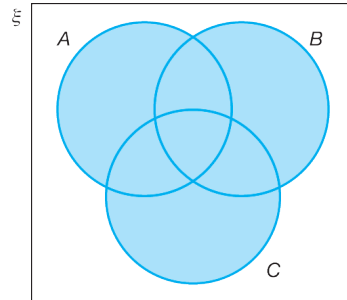


(b)

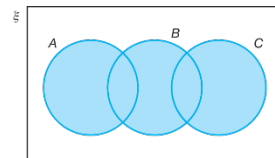


4 (a) {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12}

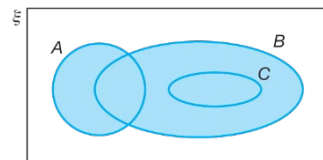
5 (a)



(b)

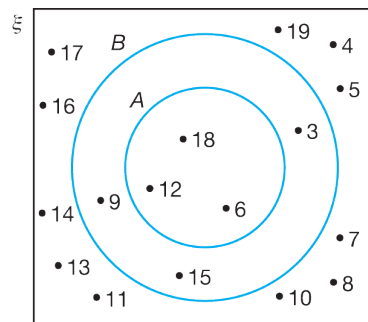


(c)

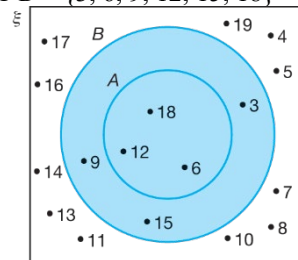


6 $A = \{6, 12, 18\}$

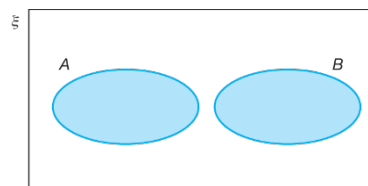
(a)

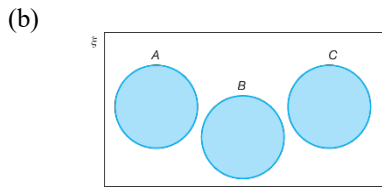


(b) $A \cup B = \{3, 6, 9, 12, 15, 18\}$

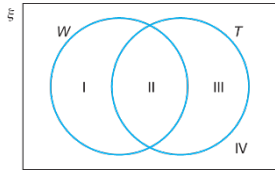


7 (a)



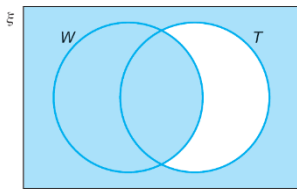


8



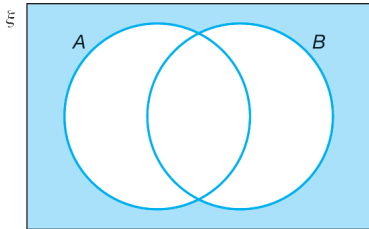
$$W = \{I, II\}$$

$$W \cup T' = \{I, II, IV\}$$

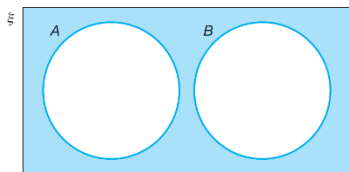


- 9 (a) $A \cup B = \{1, 3, 5, 6, 7, 9\}$
 $(A \cup B)' = \{2, 4, 8, 10\}$
 (b) $P \cup Q = \{a, b, c, d, e, f\}$
 $(P \cup Q)' =$

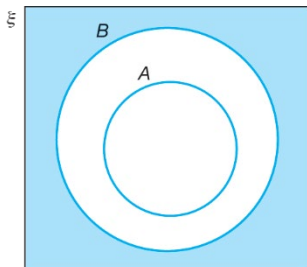
10 (a)



(b)

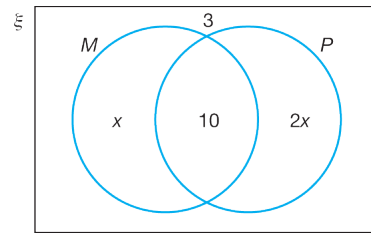


(c)



11 $n(K \cup M)' = n(L \cup M)'$
 $4 + 5 = x + 5$
 $x = 4$

12



$$x + 10 + 2x + 3 = 40$$

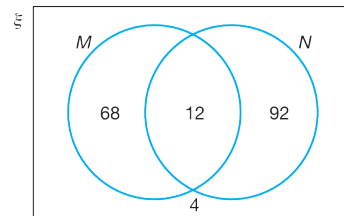
$$3x = 27$$

$$x = 9$$

$$n(\text{girl guide}) = 10 + 2x$$

$$= 28$$

13



$$n(\xi) = 68 + 12 + 92 + 4 = 176 \text{ students}$$

14 (a) $2k + k + 3 = 18$

$$3k = 15$$

$$k = 5$$

(b) $n(\xi) = 4k + 3 = 4(5) + 3 = 23$
 participants

15 (a) $n(E \cap K) = 8$

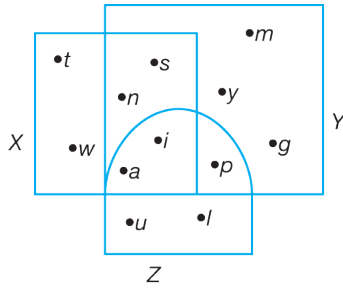
$$2 + 2h = 8$$

$$h = 3$$

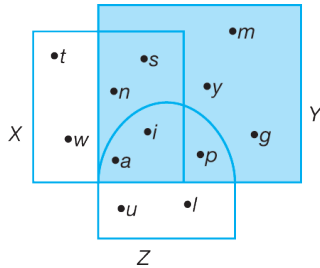
(b) Number of students who like to drink
 two types of drinks only
 $= 5 + 1 + 2(3)$
 $= 12$

UPSKILL 4.3

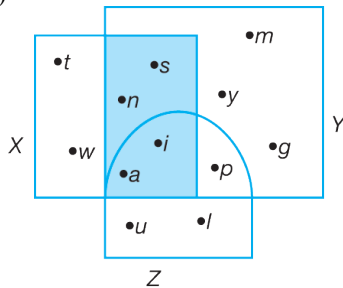
1 (a)



(b) (i)



(ii)



2 (a) (i) $A = \{1, 2, 4, 5, 10\}$

(iii) $C = \{2, 4, 6, 8, 10\}$

$$B \cap C = \{4, 8\}$$

$$A \cup (B \cap C) = \{1, 2, 4, 5, 8, 10\}$$

$$A \cup B = \{1, 2, 4, 5, 8, 10\}$$

$$C \cap (A \cup B) = \{2, 4, 8, 10\}$$

(iii) $A \cup C = \{1, 2, 4, 5, 6, 8, 10\}$

$$B \cap (A \cup C) = \{4, 8\}$$

(c) (i) $n[A \cup (B \cap C)] = 6$

(ii) $n[C \cap (A \cup B)] = 4$

(iii) $n[B \cap (A \cup C)] = 2$

3 (a) $(P \cap Q) \cup R$

(b) $(P \cap R) \cup (Q \cap R)$ or $(P \cup Q) \cap R$

4 $A = \{1, 3, 5, 7, 9\}$

$B = \{2, 4, 6, 8, 10\}$

$C = \{2, 3, 4, 5, 6\}$

(a) $A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$$A \cup B \cap C = \{2, 3, 4, 5, 6\}$$

$$(A \cup B \cap C)' = \{1, 7, 8, 9, 10\}$$

(b) $B' = \{1, 3, 5, 7, 9\}$

$$B' \cap C = \{3, 5\}$$

$$A \cup (B' \cap C) = \{1, 3, 5, 7, 9\}$$

(c) $A' = \{2, 4, 6, 8, 10\}$

$$B \cup C = \{2, 3, 4, 5, 6, 8, 10\}$$

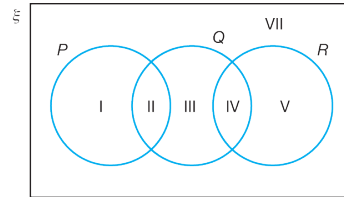
$$A' \cap (B \cup C) = \{2, 4, 6, 8, 10\}$$

(d) $A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$$C' = \{1, 7, 8, 9, 10\}$$

$$A \cup B \cap C' = \{1, 7, 8, 9, 10\}$$

5 (a)

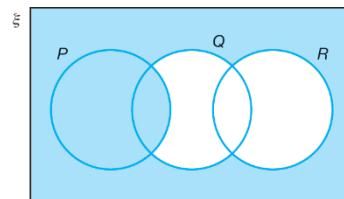


$$P = \{I, II\}$$

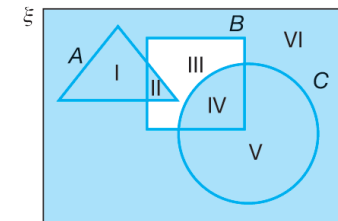
$$Q \cup R = \{II, III, IV, V\}$$

$$(Q \cup R)' = \{I, VII\}$$

$$P \cup (Q \cup R)' = \{I, II, VII\}$$



(b)

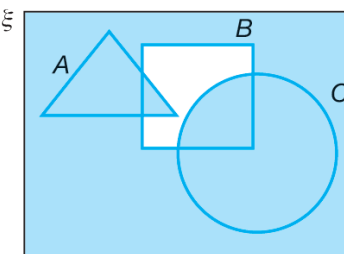


$$A = \{I, II\}$$

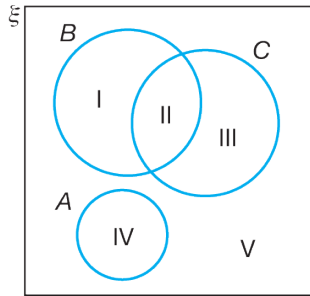
$$B' = \{I, V, VI\}$$

$$C = \{IV, V\}$$

$$A \cup B' \cup C = \{I, II, IV, V, VI\}$$



6 (a)



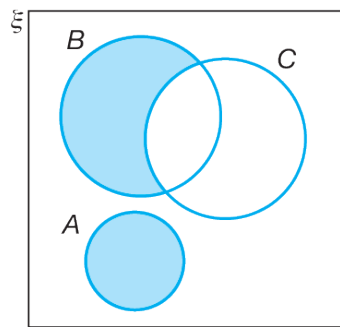
$$C' = \{I, IV, V\}$$

$$B = \{I, II\}$$

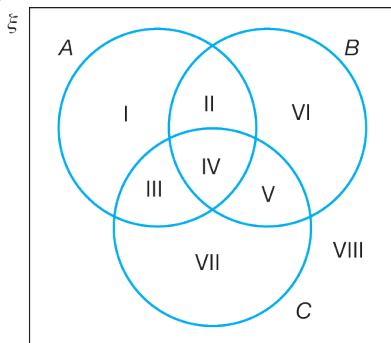
$$C' \cap B = \{I\}$$

$$A = \{IV\}$$

$$(C' \cap B) \cup A = \{I, IV\}$$



(b)



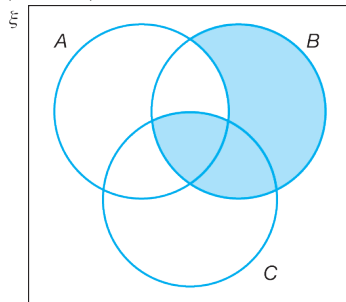
$$A' = \{V, VI, VII, VIII\}$$

$$C = \{III, IV, V, VII\}$$

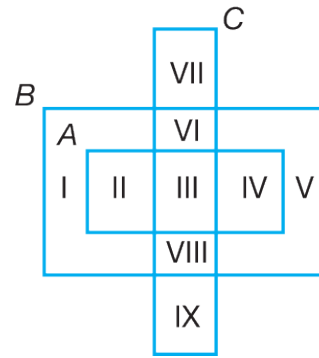
$$A' \cup C = \{III, IV, V, VI, VII, VIII\}$$

$$B = \{II, IV, V, VI\}$$

$$(A' \cup C) \cap B =$$



7

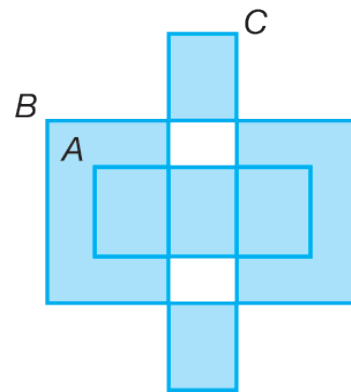


$$B \cap C = \{III, VI, VIII\}$$

$$(B \cap C)' = \{I, II, IV, V, VII, IX\}$$

$$A =$$

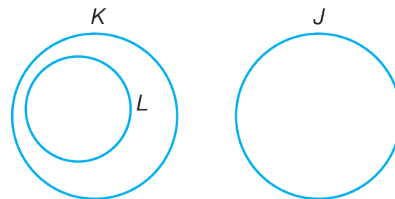
$$(B \cap C)' \cup A = \{I, II, III, IV, V, VII, IX\}$$



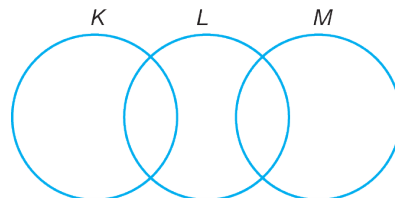
$$(Q' \cap P) \cap R$$

9 $(A \cap B) \cup (B' \cap C)$

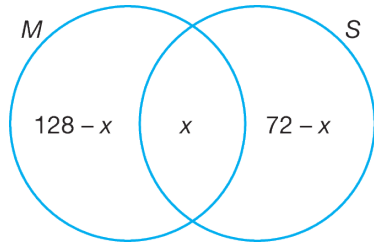
10



11



12



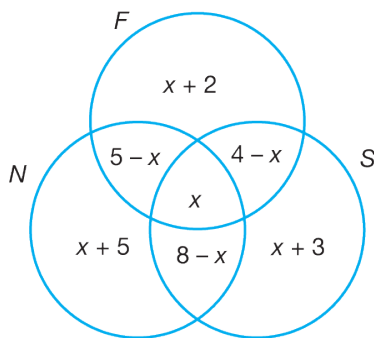
$$n(\xi) = 160$$

$$128 + 72 - x = 160$$

$$x = 40$$

Hence, the number of students who achieve grade A in both Mathematics and Science is 40.

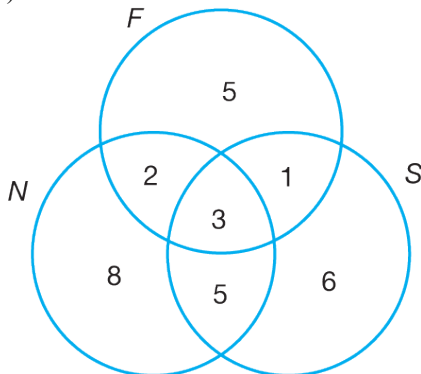
13



$$n(\xi) = x + 27 = 30$$

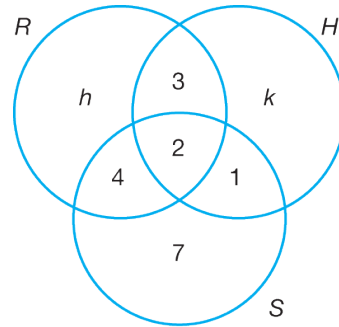
$$x = 3$$

(a)



- (b) (i) The number of students who like to read all three types of books is 3.
 (ii) The number of students who like to read non-fiction or sports but not fiction story books is $8 + 5 + 6 = 19$.

14 (a)



$$n(\xi) = 40$$

$$h + k + 17 = 40$$

$$h + k = 23 \dots (1)$$

$$n(R) = n(H)$$

$$h + 9 = k + 6$$

$$h - k = -3 \dots (2)$$

$$2k = 26$$

$$k = 13$$

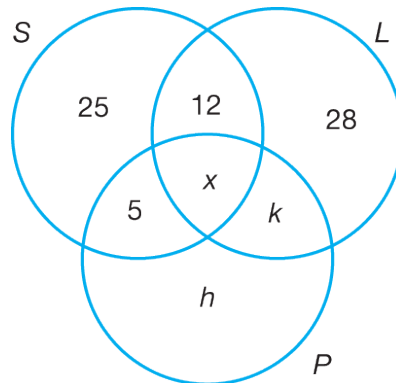
From (1) :

$$h + 13 = 23$$

$$h = 10$$

- (c) The number of students who have two characteristics only = $3 + 4 + 1 = 8$

15



(a) $n(S) = 55$

$$x + 42 = 55$$

$$x = 13$$

$$n(L) = 61$$

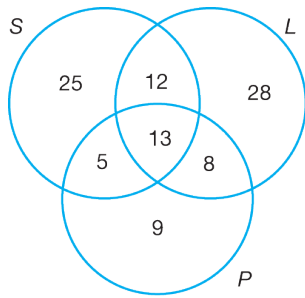
$$40 + 13 + k = 91$$

$$k = 8$$

$$n(\xi) = 100$$

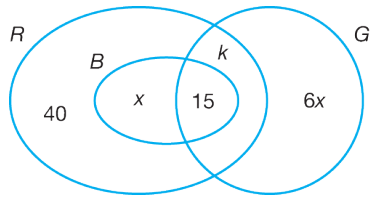
$$h + 91 = 100$$

$$h = 9$$



Number of movie viewers who like to watch scaring or comedy and investigation genres = $5 + 13 + 8 = 26$

16



(a) $n(B) = \frac{1}{6} \times n(R)$

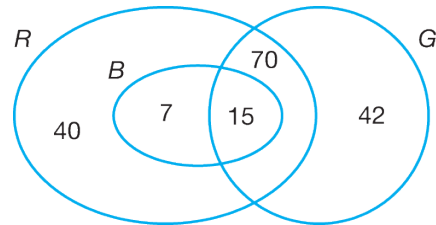
$$x + 15 = \frac{1}{6}(x + k + 55)$$

$$x + 15 = \frac{1}{6}(x + 70 + 55)$$

$$6x + 90 = x + 125$$

$$5x = 35$$

$$x = 7$$



Number of members who join not more than two clubs
 $= 40 + 7 + 70 + 42$
 $= 159$

Summative Practice 4

Multiple-Choice Questions

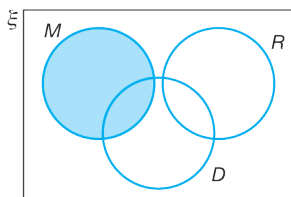
1 The set which represents the shaded region is $P' \cup Q$.

Answer: B

2 The set which represents the shaded region is $(P \cup R)' \cap Q$.

Answer: C

3 The Venn diagram which represents the students who like pineapples but not star fruits is



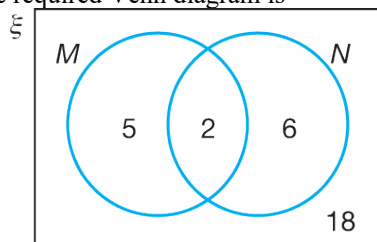
Answer: C

4 $M = \{x : x \text{ is a multiple of } 5\}$
 $M = \{10, 15, 20, 25, 30, 35, 40\}$

$N = \{x : x \text{ is a multiple of } 4\}$
 $N = \{12, 16, 20, 24, 28, 32, 36, 40\}$

$M \cap N = \{20, 40\}$
 $(M \cup N)' = \{11, 13, 14, 17, 18, 19, 21, 22, 23, 26, 27, 29, 31, 33, 34, 37, 38, 39\}$

The required Venn diagram is



5 $P = \{2, 4, 6, 8, 10\}$
 $Q = \{2, 3, 5, 7\}$
 $R = \{1, 2, 3, 4, 5\}$
 $P' = \{1, 3, 5, 7, 9\}$
 $P' \cup R = \{1, 2, 3, 4, 5, 7, 9\}$
 $Q' = \{1, 4, 6, 8, 9, 10\}$
 $Q' \cap (P' \cup R) = \{1, 4, 9\}$

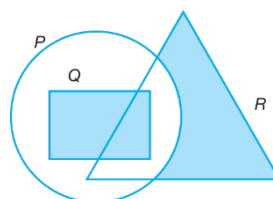
Answer: A

6 The Venn diagram which represents the shaded region is $(P \cup R)'$

Answer: C

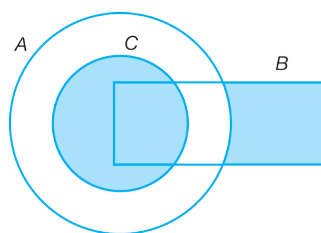
Structured Questions

1



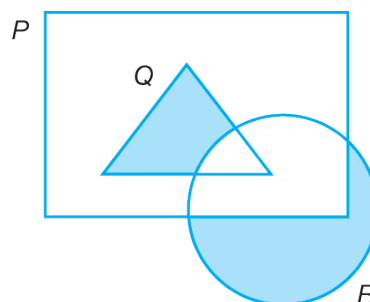
$(P' \cap R) \cup Q$

2



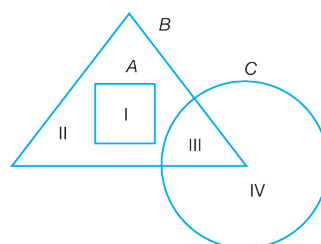
$(A' \cap B) \cup C$

3



$(P' \cap R) \cup Q$

4

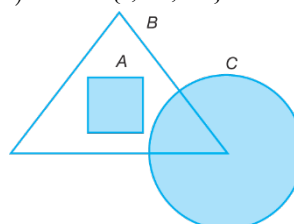


$A = \{I\}$
 $B' =$

$A \cup B' = \{1, IV\}$

$C = \{III, IV\}$

$(A \cup B)' \cup C = \{I, III, IV\}$



5 (a) $A \cup (B \cap C)$

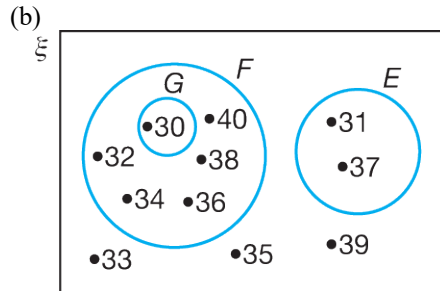
(b) $(A \cup B) \cap C'$

6 (a) $\xi = \{30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40\}$

$E = \{31, 37\}$

$F = \{30, 32, 34, 36, 38, 40\}$

$G = \{30\}$



(c) $F \cap G = \{30\}$

$E \cup (F \cap G) = \{30, 31, 37\}$

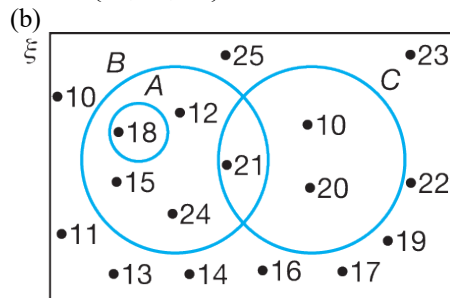
(d) $n[E \cup (F \cap G)] = 3$

7 (a) $\xi = \{10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25\}$

$A = \{18\}$

$B = \{12, 15, 18, 21, 24\}$

$C = \{10, 20, 21\}$



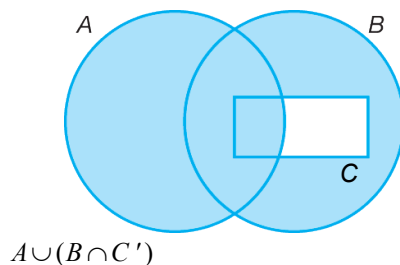
(c) $(A \cap B) \cup (B \cap C)$

$= \{18\} \cup \{21\}$

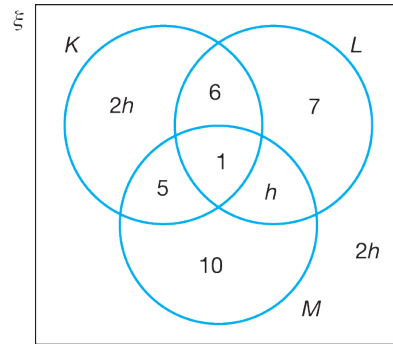
$= \{18, 21\}$

(d) $n(B \cap C)' = 15 + 1 - 1 = 15$

8



9



(a) $n(K) = n(L \cup M)'$

$2h + 12 = 4h$

$2h = 12$

$h = 6$

$n[(K \cap L) \cup (L \cap M) \cup (K \cap M)]'$

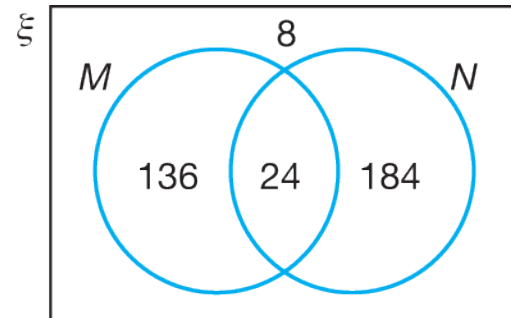
$= 2h + 7 + 10 + 2h$

$= 4h + 17$

$= 4(6) + 17$

$= 41$

10



(a) Number of students

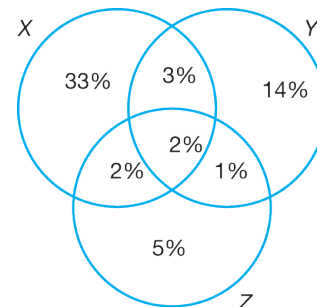
$= 136 + 24 + 184 + 8$

$= 352$

$= 136 + 184$

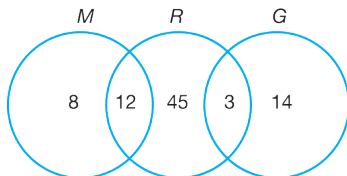
$= 320$

11 (a)



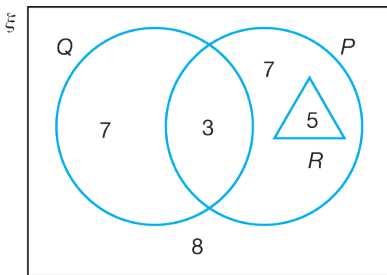
- (b) The percentage of customers who buy one brand of facial cleanser only
 $= 33\% + 14\% + 5\%$
 $= 52\%$
 The number of customers who buy one brand of facial cleanser only
 $= \frac{52}{100} \times 10\,000$
 $= 5\,200$

12 (a)



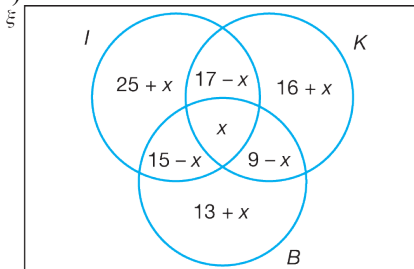
- (b) The number of students who like two types of hobbies only
 $= 12 + 3$
 $= 15$

13



- (a) $h = 7, k = 5, m = 8$
 (b) (i) The number of customers who do not buy brand R soaps but buy brand P soaps
 $= 7 + 3$
 $= 10$
 (ii) The number of customers who do not buy brand Q soaps but buy brand P soaps
 $= 7 + 5$
 $= 12$
 (c) The number of customers buy one brand of soaps only
 $= 14$
 (d) $P' \cap R = \phi$
 $\therefore n(P' \cap R) = 0$

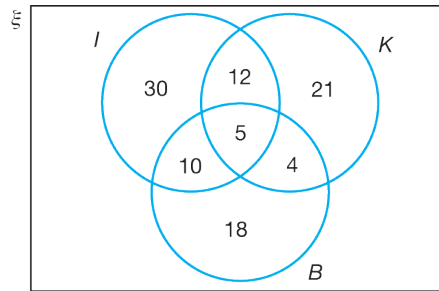
14 (a)



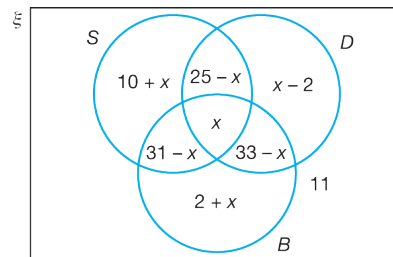
$$25 + x + 15 - x + x + 17 - x + 16 + x + 9 - x + 13 + x = 100$$

$$95 + x = 100$$

$$x = 5$$



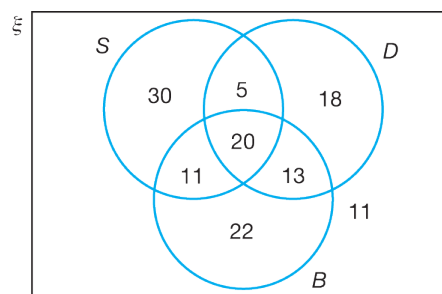
- (b) The number of housewives who like to cook
 (i) all three types of dishes = 5
 (ii) one type of dish only
 $= 30 + 21 + 18$
 $= 69$
 (iii) fried chicken or chicken curry but not broccoli
 $= 30 + 12 + 21$
 $= 63$



$$10 + x + 25 - x + x + 31 - x + x - 2 + 33 - x + 2 + x + 11 = 130$$

$$x + 110 = 130$$

$$x = 20$$



- (b) The number of customers who like
 (i) all the three types of programmes
 $= 20$
 (ii) only one type of programme
 $= 30 + 18 + 22$
 $= 70$
 (iii) at least two types of programmes
 $= 5 + 11 + 13 + 20$