

**Form 4 Chapter 3**  
**Logical Reasoning**  
**Fully-Worked Solutions**

**UPSKILL 3.1**

1

<b>Quick Access</b>
The statement is a sentence that <b>can be determined whether it is true or false</b> .

- (a) Statement because we can determine that the sentence is false.
- (b) Not a statement because the sentence is a question.
- (c) Not a statement because the sentence is an instruction.
- (d) Not a statement because the sentence is an exclamation.
- (e) Statement because we can determine that the sentence is true.
- (f) Not a statement because we cannot determine whether the statement is true or false.
- (g) Statement because we can determine that the sentence is false.

2

<b>Quick Access</b>
<ul style="list-style-type: none"> <li> The quantifier '<b>all</b>' means each object or case satisfies a certain condition.</li> <li> The quantifier '<b>some</b>' means a few and not each object or case satisfies a certain condition.</li> </ul>

- (a) True
- (b) True
- (c) True
- (d) False
- (e) False
- (f) False
- (g) True
- (h) False
- (i) True
- (j) True
- (k) False
- (l) True

3

<b>Quick Access</b>
<b>Negating</b> is a process that <b>denies</b> a statement using the words ' <b>not</b> ' or ' <b>no</b> '. A true statement can be changed to a false statement and vice versa using the word ' <b>not</b> ' or ' <b>no</b> '.

- (a) False  
Sungai Rajang is not situated in the state of Sabah. [True]
- (b) True  
121 is not a perfect square. [False]

- (c) False  
The sum of all interior angles of a pentagon is not  $800^\circ$ . [True]

- (d) True  
A snake is not a reptile. [False]

- (e) False  
 $\cos 60^\circ$  is not equal to  $\frac{\sqrt{3}}{2}$ . [True]

- (f) True  
Tun Abdul Razak is not the second prime minister of Malaysia. [False]

4

<b>Quick Access</b>		
<i>p</i>	<i>q</i>	<i>p</i> and <i>q</i>
True	True	<b>True</b>
True	False	<b>False</b>
False	True	<b>False</b>
False	False	<b>False</b>

- (a) True and true is true
- (b) False and false is false
- (c) False and true is true
- (d) False and true is false
- (e) True and false is false
- (f) True and false is false

5

<b>Quick Access</b>		
<i>p</i>	<i>q</i>	<i>p</i> or <i>q</i>
True	True	<b>True</b>
True	False	<b>True</b>
False	True	<b>True</b>
False	False	<b>False</b>

- (a) True or true is true
- (b) False or true is true
- (c) True or false is true
- (d) False or false is false
- (e) True and false is true
- (f) True or false is true

6

<b>Quick Access</b>
For two statements, <i>p</i> and <i>q</i> , the statement ' <b>if <i>p</i>, then <i>q</i></b> ' is an <b>implication</b> such that <i>p</i> is the <b>antecedent</b> and <i>q</i> is the <b>consequent</b> .

- (a) Antecedent:  $a^2 + b^2 = c^2$   
Consequent: Triangle *ABC* is a right-angled triangle.
- (b) Antecedent: *ABCD* is a kite.  
Consequent: *ABCD* has one axis of symmetry.
- (c) Antecedent: *h* is an odd number.  
Consequent:  $h + 1$  is an even number.

- (d) Antecedent:  $x^2 - 81 = 0$   
Consequent:  $x = \pm 9$
- (e) Antecedent: Set  $A$  does not have any element.  
Consequent: Set  $A$  is  $\phi$ .
- (f) Antecedent:  $y > 3$   
Consequent:  $-y < -3$

- 7 (a) If  $x = 64$ , Then  $\sqrt[3]{x} = 4$ .  
(b) If  $PQRST$  is a regular pentagon, then  $PQRST$  has five axes of symmetry.  
(c) If  $-2x > 12$ , then  $x < -6$ .  
(d) If  $x$  is a prime number, then  $x$  can be divided by itself and by 1.

8

Quick Access
For two statements $p$ and $q$ , the statement ' <b><math>p</math> if and only if <math>q</math></b> ' is the abbreviation of 'if $p$ , then $q$ ' and 'if $q$ , then $p$ '.

- (a) Implication 1: If  $x > y$ , then  $\frac{1}{5}x > \frac{1}{5}y$ .  
Implication 2: If  $\frac{1}{5}x > \frac{1}{5}y$ , then  $x > y$ .
- (b) Implication 1: If  $x^2 = 36$ , then  $x = \pm 6$ .  
Implication 2: If  $x = \pm 6$ , then  $x^2 = 36$ .
- (c) Implication 1: If  $5(x+3) = 25$ , then  $5x+15 = 25$ .  
Implication 2: If  $5x+15 = 25$ , then  $5(x+3) = 25$ .
- (d) Implication 1: If  $ABC$  is an isosceles triangle, then it has two equal sides.  
Implication 2: If triangle  $ABC$  has two equal sides, then the triangle is an isosceles triangle.
- (e) Implication 1: If  $\theta$  and  $\alpha$  are alternate angles, then  $\theta = \alpha$ .  
Implication 2: If  $\theta = \alpha$ , then  $\theta$  and  $\alpha$  are alternate angles.
- (f) Implication 1: If  $x$  is the conjugate angle of  $y$ , then  $x + y = 360^\circ$ .  
Implication 2: If  $x + y = 360^\circ$ , then  $x$  is the conjugate angle of  $y$ .

- 9 (a)  $\frac{a}{b}$  is a proper fraction if and only if  $a < b$ .  
(b) The equation of a straight line is  $\frac{x}{4} + \frac{y}{2} = 1$  if and only if the  $x$ -intercept and  $y$ -intercept of the straight line are 4 and 2 respectively.  
(c)  $(x+5)^2 = 0$  if and only if  $x = -5$ .  
(d)  $\tan x = \sqrt{3}$  if and only if  $x = 60^\circ$ .

10

Quick Access
For the statement ' $p$ if and only if $q$ ', <b>two implications</b> can be written as follows: Implication 1: If $p$ , then $q$ . Implication 2: If $q$ , then $p$ .

- (a) If  $x > -5$ , then  $x > -10$ . [False]  
(b)  $y = 3x + 1$ , then  $x = \frac{y-1}{3}$ . [True]  
(c) If each interior angle of  $RSTUVW$  is  $120^\circ$ , then  $RSTUVW$  is a regular hexagon. [True]  
(d) If a number is divisible by 5, then the last digit of a number is 0 or 5. [True]  
(e) If a polygon has two pairs of opposite parallel sides, then the polygon is a trapezium. [True]  
(f) If  $p$  is a multiple of 2, the number  $p$  is a multiple of 4. [True]

11

Quick Access
The <b>inverse</b> of an implication ' $p$ , then $q$ ' is ' $\text{if not } p, \text{ then not } q$ '.

- (a) If  $h \leq \frac{9}{3}$ , then  $h \leq \frac{4}{3}$ . [False]  
(b) If  $k \geq -15$ , then  $k \geq -10$ . [False]  
(c) If  $p \geq -\frac{8}{9}$ , then  $p \geq -\frac{4}{9}$ . [False]  
(d) If  $g(x)$  is not a quadratic function, then its graph does not have the shape of a parabola. [True]  
(e) If set  $A \neq \phi$ , then  $n(A) \neq 0$ . [True]  
(f) If  $k^5$  is not negative, then  $k$  is not negative. [True]

12

**Quick Access**

The **contrapositive** of an implication 'if  $p$ , then  $q$ ' is 'if not  $q$ , then not  $p$ '.

- (a) If a polygon is not a regular pentagon, then it does not have seven equal angles. [True]
- (b) If  $\cos \theta \neq \frac{1}{\sqrt{2}}$ , then  $\theta \neq 45^\circ$ . [True]
- (c) If  $n$  is not a multiple of 3, then it is not a multiple of 12. [True]
- (d) If  $x \geq -\frac{5}{9}$ , then  $x \geq -\frac{8}{9}$ . [True]
- (e) If a number is not divisible by 2, then the number is not a multiple of 4. [True]
- (f) If  $k$  is not a factor of 48, then  $k$  is not a factor of 12. [True]

13

**Quick Access**

**Counter-example to negate** the truth value of a statement involving

- (a) quantifiers,  
 (b) compound statements,  
 (c) negations and  
 (d) implications  
 which are appropriate.

- (a) True
- (b) False. In Peninsular Malaysia, the longest river is the Pahang River and the highest mountain is Mount Tahan.
- (c) True
- (d) True
- (e) False. If  $x = 30^\circ$ , then  $\tan 30^\circ = \frac{1}{\sqrt{3}}$ .

**UPSKILL 3.2**

1

**Quick Access**

- **Deductive** argument is an argument such that its **premise** is **definitely true** to guarantee that the **conclusion** is **true**.
- In an **inductive** argument, the premises are made such that they are as **convincing** as possible in order that if they are **cogent** enough, then its conclusion is **strong**

- (a) Inductive reasoning  
 (b) Deductive argument  
 (c) Inductive reasoning  
 (d) Deductive argument

- 2 (a) Reasonable because the deductive argument is valid and both the premises and conclusion are true.
- (b) Not reasonable because the first premise is not true. The word 'complementary' should be 'supplementary'.
- (c) Not reasonable because the second premise is not true. The maximum value of  $\cos y$  is 1.
- (d) Not reasonable because the first premise is not true. A vertical straight line is undefined.

3 (a) Deductive argument form I:

$RSTU$  has four right angles.

(b) Deductive argument form II:

$$x < -\frac{9}{5}$$

(c) Deductive argument form I:

$EFGH$  has four axes of symmetry.

(d) Deductive argument form II:

$$v = 30^\circ \text{ and } w = 60^\circ$$

(e) Deductive argument form III:

Set  $Q \neq \{x : x \text{ is a two digit integer}\}$

4 (a) Deductive argument form I:

Premise 1: All kites are quadrilaterals.

(b) Deductive argument form I:

Premise II:  $PQRST$  is a regular pentagon.

(c) Deductive argument form II:

$$\text{Premise 2: } a \frac{-m}{n} = b$$

(d) Deductive argument form III:

Premise 2:  $x \neq 30^\circ$

(e) Deductive argument form II:

Premise I: If recurring decimal 0.363636... can be expressed as a fraction, then the recurring decimal 0.363636... is a rational number.

(f) Deductive argument form I:

Premise I: All reptiles are cold-blooded.

(g) Deductive argument form II:

Premise 1: If  $h$  is a complementary angle of  $k$ , then  $h + k = 90^\circ$ .

(h) Deductive argument form III:

If  $p + q = 180^\circ$ , then  $q$  is the complementary angle of  $p$ .

(i) Deductive argument form III:

Premise 2:  $y$  is not the conjugate angle of  $x$ .

5 (a) Strong because its premises and

conclusion are logical and convincing.

(b) Weak because the inductive reasoning does not state the colour of the plastic chairs in the rooms and thus its conclusion is not convincing.

- 6 (a) Many Sek Wooi's Facebook friends emphasise the value of friendship.  
 (b) All vehicles in Malaysia are driven on the left side of the road.

7 Volume of cylinder

$$= \pi r^2 h = \left(\frac{22}{7}\right)(7)^2(14) = 2\,156 \text{ cm}^3$$

8 Volume of a square-base pyramid

$$= \frac{1}{3}x^2h = \frac{1}{3} \times 7^2 \times 14 = 228\frac{2}{3} \text{ cm}^3$$

9  $5 = 5(2^{1-1})$

$$10 = 5 \times 2^{2-1}$$

$$20 = 5 \times 2^{3-1}$$

$$40 = 5 \times 2^{4-1}$$

The  $n$ th term =  $5(2^{n-1})$ ,  $n = 1, 2, 3, 4, \dots$

10  $1 = \frac{1}{2}(1)(2)$

$$3 = \frac{1}{2}(2)(2+1)$$

$$6 = \frac{1}{2}(3)(3+1)$$

$$10 = \frac{1}{2}(4)(4+1)$$

The  $n$ th term =  $\frac{1}{2}n(n+1)$ ,  $n = 1, 2, 3,$

4, ...

### Summative Practice 3

#### Multiple-Choice Questions

1 'List the factors of 12' is an instruction.

Hence, it is not a statement.

Answer: C

2 A triangle has 2 sides or 3 vertices.

False or true is true.

Answer: A

3 The contrapositive is 'If  $k$  is not a multiple of 3, then  $k$  is not a multiple of 18'.

Answer: C

4 Deductive argument form I:

Premise 1: All regular heptagons have 7 equal interior angles.

Answer: C

5  $2 = \frac{1}{2}[4+3(1-1)]$

$$7 = \frac{2}{2}[4+3(2-1)]$$

$$15 = \frac{3}{2}[4+3(3-1)]$$

The  $n$ th term =  $\frac{n}{2}[4+3(n-1)]$

Answer: A

#### Structured Questions

1 (a) Statement because we can determine that it is false.

(b) Not a statement because we cannot determine whether it is true or false.

2 (a) True (b) False

3 (a) True (b) False

4 (a) True and false is false.

(b) False atau true is true.

5 (a) If  $p > -\frac{7}{3}$ , then  $p > -\frac{2}{3}$ .

(b) If  $p \leq -\frac{2}{3}$ , then  $p \leq -\frac{7}{3}$ .

(c) If  $p \leq -\frac{7}{3}$ , then  $p \leq -\frac{2}{3}$ .

6 Implication 1: If the gradient of a straight

line is  $-\frac{b}{a}$ , then the  $x$ -

intercept and the  $y$ -intercept of the straight line are  $a$  and  $b$  respectively.

Implication 2: If the  $x$ -intercept and the  $y$ -

intercept of a straight line are  $a$  and  $b$  respectively, then the gradient of the straight line is

$$-\frac{b}{a}$$

- 7** Deductive argument form 1:  
Premise I: All negative numbers are less than 0.
- 8** Deductive argument form III:  
Premise II: Triangle  $P$  and triangle  $Q$  do not have the same shape and size.
- 9** Encik Sarip and Encik Dinna can have high quality of life in terms of their health because they emphasis of diet and exercise.
- 10** (a) Reasonable because the deductive argument is valid and both the premises and conclusion are true.  
(b) Not reasonable because the first premise is not true. 2 is an even prime number.
- 11** Strong because both premises and conclusion are cogent and logical.

- 12** (a)  $T_n = 2^{n-1}$ ,  $n = 1, 2, 3, 4, \dots$ ,  
 $S_n = 2^n - 1$ ,  $n = 1, 2, 3, 4, \dots$
- (b)  $T_8 = 2^{8-1} = 128$  buttons
- (c)  $2^n - 1 = 255$   
 $2^n = 256$   
 $2^n = 2^8$   
 $n = 8$   
All the 8 sectors can be filled because  $S_8 = 256$ .

- 13** (a) Let the area of the regions which are not shaded for the  $n$ th pattern =  $L_n$   
 $L_1 = (10 \times 1)^2 - 25\pi(1)^2$   
 $L_2 = (10 \times 2)^2 - 25\pi(2)^2$   
 $L_3 = (10 \times 3)^2 - 25\pi(3)^2$   
 $\vdots$   
 $L_n = (10n)^2 - 25n^2\pi$ ,  $n = 1, 2, 3, \dots$

(b)  $L_5 = (10 \times 5)^2 - 25(5)^2 \left( \frac{22}{7} \right)$   
 $= 535 \frac{5}{7} \text{ cm}^2$