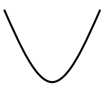
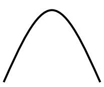
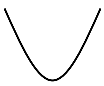
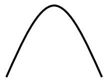
Form 4 Chapter 1
Quadratic Functions and Equations in One Variable
Fully-Worked Solutions

UPSKILL 1.1

1 (a) No because the highest power of $p + 8$ is 1 and not 2.
(b) Yes because the highest power of the variable q is 2
(c) Yes because the highest power of the variable r is 2
(d) Yes because the highest power of the variable s is 2
(e) No because the highest power of $4t^3 + 3t^2 - 6t + 8$ is 3 and not 2.
(f) No because the highest power of $0u^2 + 18u + 3 = 18u + 3$ is 1 and not 2.
(g) No because $12 - 5vw + 2v^2$ has two variables and not one variable
(h) No because $z^2 + 4z - \frac{5}{z}$ is not in the form $az^2 + bz + c$.

2 (a) Yes because the highest power of $f(d) = (3-d)(2+5d) = 6 + 13d - 5d^2$ is 2.
(b) No because the highest power of $g(e) = \frac{4}{e^2} - 5 = 4e^{-2} - 5$ is –2.
(c) Yes because the highest power of $h(j) = (2-3j)^2 = 4 - 12j + 9j^2$ is 2.
(d) No because the highest power of $m(k) = k - \frac{3}{k}$ is 1.
(e) No because the highest power of $n(p) = p(3p+1)^2 = 3p^3 + 6p^2 + p$ is 3.
(f) No because the highest power of $p(u) = \frac{u^2+5}{u^2} = 1 + 5u^{-2}$ is –2.

3 (a) Since the coefficient of x^2 is positive, then the shape of the graph is .
(b) Since the coefficient of x^2 is negative, then the shape of the graph is .
(c) Since the coefficient of x^2 is positive, then the shape of the graph is .

(b) Since the coefficient of x^2 is negative, then the shape of the graph is .

4 (a) The equation of the axis of symmetry is
$x = \frac{-2+8}{2} = 3$
(b) When $x = 2$,
$y = -(3)^2 + 6(3) + 16 = 25.$
The coordinates of the maximum point are (3, 25).
(c) When the curve is reflected in the x-axis, its function is $y = x^2 - 6x - 16$.
(d) When the curve is reflected in the y-axis, its function is $y = -x^2 - 6x + 16$.

5 (a) The equation of the axis of symmetry is
$x = \frac{-6+(-2)}{2} = -4$
(b) When $x = -4$,
$y = (-4)^2 + 8(-4) + 12 = -4$.
The coordinates of the maximum point are $(-4, -4)$.
(c) When the curve is reflected in the x-axis, its function is $y = -x^2 - 8x - 12$.
(d) When the curve is reflected in the y-axis, its function is $y = x^2 - 8x + 12$.

6
$$B(x) = (x+2)(3x+6) = 3x^2 + 12x + 12$$

$$B(x) = 300$$
$$3x^2 + 12x + 12 = 300$$
$$x^2 + 4x + 4 = 100$$
$$x^2 + 4x - 96 = 0$$

7
$$L(x) = \frac{1}{2}(4x+8)(2x+6) = \frac{1}{2}\left(8x^2 + 40x + 48\right) = 4x^2 + 20x + 24$$

$$L(x) = 80$$
$$4x^2 + 20x + 24 = 80$$
$$4x^2 + 20x - 56 = 0$$

$x^2 + 5x - 14 = 0$

8 $L(x) = \frac{1}{2}(5x + 2 + 3x)(4x)$

$= \frac{1}{2}(8x + 2)(4x)$

$= 2x(8x + 2)$

$= 16x^2 + 4x$

$L(x) = 80$

$16x^2 + 4x - 80 = 0$

$4x^2 + x - 20 = 0$

9 $V(x) = (x + 4)(5)(2x)$

$= 10x(x + 4)$

$= 10x^2 + 40x$

$V(x) = 600$

$10x^2 + 40x = 600$

$x^2 + 4x - 60 = 0$

10 Swee Ling's age 5 years ago = $x - 5$.
If Swee Ling's age 5 years ago is half of her mother's age, then her mother's age is $2(x - 5)$.

$h(x) = 2(x - 5)(x - 5)$

$= 2(x^2 - 10x + 25)$

$= 2x^2 - 20x + 50$

$h(x) = 1\ 250$

$2x^2 - 20x + 50 = 1\ 250$

$2x^2 - 20x - 1\ 200 = 0$

$x^2 - 10x - 600 = 0$

11 $3x^2 - 5x - 2 = 0$

(a) LHS

$= 3(2)^2 - 5(2) - 2$

= RHS

Thus, $x = 2$ is not a root for $3x^2 - 5x - 2 = 0$.

(b) LHS

$= 3(1)^2 - 5(1) - 2$

$= -4$

$\neq$ RHS

Thus, $x = 1$ is not a root of $3x^2 - 5x - 2 = 0$.

(c) LHS

$= 3\left(-\frac{1}{3}\right)^2 - 5\left(-\frac{1}{3}\right) - 2$

$= 0$

= RHS

Thus, $x = -\frac{1}{3}$ is a root of $3x^2 - 5x - 2 = 0$.

12 $-2x^2 + 3x - 1 = 0$

(a) LHS

$= -2(3)^2 + 3(3) - 1$

$= -10$

$\neq$ RHS

Thus, $x = 3$ is not a root of $-2x^2 + 3x - 1 = 0$.

(b) LHS

$= -2(1)^2 + 3(1) - 1$

$= 0$

= RHS

Thus, $x = 1$ is a root of $-2x^2 + 3x - 1 = 0$.

(c) LHS

$= -2\left(\frac{1}{2}\right)^2 + 3\left(\frac{1}{2}\right) - 1$

$= 0$

= RHS

Thus, $x = \frac{1}{2}$ is a root of $-2x^2 + 3x - 1 = 0$.

13 (a) $3p^2 - 5p = 0$

$p(3p - 5) = 0$

$p = 0$ or $p = \frac{5}{3}$

(b) $5q^2 + 45q = 0$

$5q(q + 9) = 0$

$q = 0$ or -9

(c) $16r^2 - 25 = 0$

$(4r + 5)(4r - 5) = 0$

$r = -\frac{5}{4}$ or $r = \frac{5}{4}$

(d) $36s^2 - 16 = 0$

$4(9s^2 - 4) = 0$

$9s^2 - 4 = 0$

$(3s + 2)(3s - 2) = 0$

$s = -\frac{2}{3}$ or $\frac{2}{3}$

(e) $12t^2 - 28t + 15 = 0$

$(2t-3)(6t-5) = 0$

$t = \frac{3}{2}$ or $t = \frac{5}{6}$

(f) $8m^2 - 51m + 18 = 0$

$(m-6)(8m-3) = 0$

$m = 6$ or $m = \frac{3}{8}$

(g) $6u^2 + 5u - 6 = 0$

$(3u-2)(2u+3) = 0$

$u = \frac{2}{3}$ or $u = -\frac{3}{2}$

(h) $10v^2 - 7v - 12 = 0$

$(2v-3)(5v+4) = 0$

$v = \frac{3}{2}$ or $v = -\frac{4}{5}$

(i) $-12w^2 - 11w + 36 = 0$

$12w^2 + 11w - 36 = 0$

$(3w-4)(4w+9) = 0$

$w = \frac{4}{3}$ or $w = -\frac{9}{4}$

14 (a) $-3z^2 = 4 - 13z$

$3z^2 - 13z + 4 = 0$

$(z-4)(3z-1) = 0$

$z = 4$ or $z = \frac{1}{3}$

(b) $(2z+1)^2 = 16$

$4z^2 + 4z + 1 = 16$

$4z^2 + 4z - 15 = 0$

$(2z-3)(2z+5) = 0$

$z = \frac{3}{2}$ or $z = -\frac{5}{2}$

(c) $3f + 1 = \frac{7}{f-1}$

$(3f+1)(f-1) = 7$

$3f^2 - 2f - 1 - 7 = 0$

$3f^2 - 2f - 8 = 0$

$(f-2)(3f+4) = 0$

$f = 2$ or $f = -\frac{4}{3}$

(d) $g - 1 = \frac{g+20}{6g}$

$6g^2 - 6g = g + 20$

$6g^2 - 7g - 20 = 0$

$(2g-5)(3g+4) = 0$

$g = \frac{5}{2}$ or $-\frac{4}{3}$

(e) $(h-3)(h+2) = \frac{1}{2}h(h-3)$

$2(h^2 - h - 6) = h^2 - 3h$

$2h^2 - 2h - 12 = h^2 - 3h$

$h^2 + h - 12 = 0$

$(h-3)(h+4) = 0$

$h = 3$ or $h = -4$

(f) $\frac{j-1}{6} - \frac{2j-1}{5j} = 0$

$\frac{5j(j-1) - 6(2j-1)}{30j} = 0$

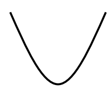
$\frac{5j^2 - 5j - 12j + 6}{60j} = 0$

$5j^2 - 17j + 6 = 0$

$(j-3)(5j-2) = 0$

$j = 3$ or $j = \frac{2}{5}$

15 (a) $y = f(x) = 2x^2 + 2$

Since the coefficient of x^2 is positive, the shape of its curve is .

At the y-axis, $x = 0$.

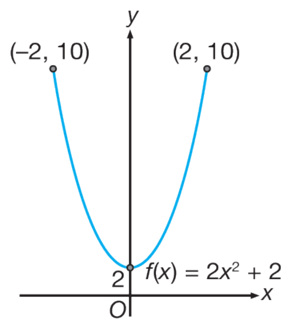
$y = 2(0)^2 + 2 = 2$

Thus, the curve will intersect the x-axis at the point (0, 2).

When $x = -2$, $y = 2(-2)^2 + 2 = 10$

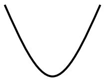
When $x = 2$, $y = 2(2)^2 + 2 = 10$

Thus, the curve will intersect the y-axis at the point $(-2, 10)$ and (2, 10).



(b) $y = g(x) = x^2 + 4x + 3$

Since the coefficient of x^2 is positive,

the shape of its curve is  .

At the y-axis, $x = 0$.

$y = 0^2 + 4(0) + 3 = 3$

Thus, the curve will intersect the y-axis at the point (0, 3).

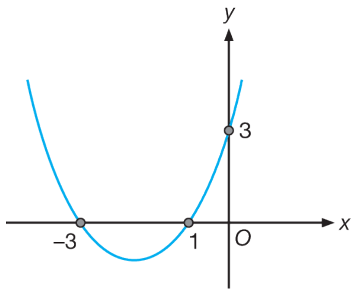
At the x-axis, $y = 0$.

$x^2 + 4x + 3 = 0$

$(x+1)(x+3) = 0$

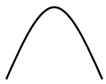
$x = -1$ or $x = -3$

Thus, the curve will intersect the x-axis at the points $(-1, 0)$ and $(-3, 0)$.



(c) $y = h(x) = -x^2 - 2x + 15$

Since the coefficient of x^2 is negative,

the shape of the curve is  .

At the y-axis, $x = 0$.

Thus, the curve will intersect the y-axis at the point (0, 3).

At the x-axis, $y = 0$.

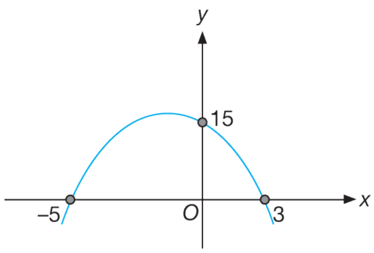
$-x^2 - 2x + 15 = 0$

$x^2 + 2x - 15 = 0$

$(x-3)(x+5) = 0$

$x = 3$ or $x = -5$

Thus, the curve will intersect the x-axis at the points $(-5, 0)$ and $(3, 0)$.



16 (a) Area of the shaded region = 28 cm^2

$8(x+2) - \frac{1}{2}x(x+2) = 28$

$8x + 16 - \frac{1}{2}x^2 - x = 28$

$16x + 32 - x^2 - 2x = 56$

$x^2 - 14x + 24 = 0$

(b) $x^2 - 14x + 24 = 0$

$(x-12)(x-2) = 0$

$x = 12$ or $x = 2$

$x = 12$ is not accepted because x has to be less than 8.

$\therefore x = 2$

(c) Area of $BAR = \frac{1}{2}x(x+2)$

$= \frac{1}{2}(2)(2+2)$

$= 4$ cm^2

17 (a) Area of the triangle ABC = 70 cm^2

$\frac{1}{2}(4x)(12-x) = 70$

$24x - 2x^2 - 70 = 0$

$2x^2 - 24x + 70 = 0$

$x^2 - 12x + 35 = 0$

(b) $x^2 - 12x + 35 = 0$

$(x-5)(x-7) = 0$

$x = 5$ or $x = 7$

(c) The smaller value of x is 5.

$AB = 12 - x = 12 - 5 = 7$ cm

$BC = 4x = 4(5) = 20$ cm

Using the Pythagoras' Theorem,

$AC = \sqrt{7^2 + 20^2} = \sqrt{449} = 21.19$ cm

18 (a) Area of the trapezium = 99 cm^2

$\frac{1}{2}(2y+y+7)(2y-1)=99$

$(3y+7)(2y-1)=198$

$6y^2-3y+14y-7=198$

$6y^2+11y-205=0$

(b) $6y^2+11y-205=0$

$(y-5)(6y+41)=0$

$y=5$ or $y=-\frac{41}{6}$

$y=-\frac{41}{6}$ is not accepted.

$\therefore y=5$

(c) $UV=2y=2(5)=10$ cm

$XW=y+7=5+7=12$ cm

$VW=2y-1=2(5)-1=9$ cm

19 (a) Area of the shaded region = 120 cm^2

$(t+8)(t-6)=120$

$t^2+2t-48-120=0$

$t^2+2t-168=0$

(b) $t^2+2t-168=0$

$(t-12)(t+14)=0$

$t=12$ or $t=-14$

$t=-14$ is not accepted.

$\therefore t=12$

(c) $AC=t+8=12+8=20$ cm

20 (a) Area of the L shape = 30 cm^2

Area of the first rectangle + Area of the second triangle = 30

$6x+x(7-x)=30$

$6x+7x-x^2=30$

$13x-x^2-30=0$

$x^2-13x+30=0$

(b) $x^2-13x+30=0$

$(x-10)(x-3)=0$

$x=10$ or $x=3$

$x=10$ is not accepted because x cannot be greater than 6.

$\therefore x=3$

21 Distance = Speed × Time

$J(x)=3x(x-8)+(3x+5)(x-9)$

$J(x)=3x^2-24x+3x^2-22x-45$

$J(x)=6x^2-46x-45$

$J(x)=95$

$6x^2-46x-45=95$

$6x^2-46x-140=0$

$3x^2-23x-70=0$

$(x-10)(3x+7)=0$

$x=10$ or $x=-\frac{7}{3}$

$x=-\frac{7}{3}$ is not accepted because the question states that x has to be positive.

$\therefore x=10$

22 $W(x)=(x+5)(x-4)+(x+10)(x-5)$
$+(x-4)(x-6)$

$=x^2+x-20+x^2+5x-50+$
$x^2-10x+24$

$=3x^2-4x-46$

$W(x)=214$

$3x^2-4x-46=214$

$3x^2-4x-260=0$

$(x-10)(3x+26)=0$

$x=10$ or $x=-\frac{26}{3}$

$x=-\frac{26}{3}$ is not accepted because the question states that x has to be positive.

$\therefore x=10$

Summative Practice 1

Multiple-Choice Questions

1 $2y^2 + ky - 12 = 0$

It is given that –4 is one of the root.

$$2(-4)^2 + k(-4) - 12 = 0$$
$$-4k + 20 = 0$$
$$-4k = -20$$
$$k = 5$$

Answer: D

2 Let the age of Sazali be x.

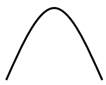
Sazali's sister (i.e. Tina) = $x + 3$

$$x(x+3) = 70$$
$$x^2 + 3x - 70 = 0$$
$$(x-7)(x+10) = 0$$
$$x = 7 \text{ or } x = -10$$
$x = -10$ is not accepted.

$\therefore x = 7$

Answer: A

3 $y = f(x) = -x^2 + 4$

Since the coefficient of x^2 is negative, the shape of the graph is  .

At the y-axis, $x = 0$.

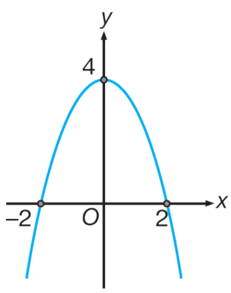
$$y = -0^2 + 4 = 4$$

Thus the curve will intersect the y-axis at the point (0, 4).

At the x-axis, $y = 0$.

$$-x^2 + 4 = 0$$
$$x^2 = 4$$
$$x = \pm 2$$

Thus, the curve will intersect the x-axis at the points (–2, 0) and (2, 0).



Answer: A

4 Surface area of the sphere = 616 cm^2

$$4\pi j^2 = 616$$
$$4 \times \frac{22}{7} \times (x+3)^2 = 616$$
$$(x+3)^2 = \frac{616 \times 7}{4 \times 22}$$
$$x^2 + 6x + 9 = 49$$
$$x^2 + 6x - 40 = 0$$
$$(x-4)(x+10) = 0$$
$$x = 4 \text{ or } x = -10$$
$x = -10$ is not accepted.

$\therefore x = 4$

Answer: A

5 Let $AB = x$ cm

Height of the triangle = $x + 5$

Area of the triangle = 75 cm^2

$$\frac{1}{2}(x)(x+5) = 75$$
$$x(x+5) = 150$$
$$x^2 + 5x - 150 = 0$$
$$(x-10)(x+15) = 0$$
$$x = 10 \text{ or } x = -15$$
$x = -15$ is not accepted.

$\therefore x = 10$

Answer: D

Structured Questions

1 (a)
$$m - 1 = \frac{6-m}{2m}$$
$$2m(m-1) = 6 - m$$
$$2m^2 - 2m = 6 - m$$
$$2m^2 - m - 6 = 0$$
$$(m-2)(2m+3) = 0$$
$$m = 2 \text{ or } m = -\frac{3}{2}$$

(b)
$$\frac{4}{16c+9} = \frac{1}{c(c+4)}$$
$$4c(c+4) = 16c + 9$$
$$4c^2 + 16c - 16c - 9 = 0$$
$$4c^2 - 9 = 0$$
$$(2c+3)(2c-3) = 0$$
$$c = -\frac{3}{2} \text{ or } \frac{3}{2}$$

(c) $\dfrac{p(5p+4)}{3} = 2-p$

$5p^2 + 4p = 6 - 3p$

$5p^2 + 7p - 6 = 0$

$(5p-3)(p+2) = 0$

$p = \dfrac{3}{5}$ or $p = -2$

(d) $\dfrac{3f-5}{2} = -\dfrac{3f-1}{f}$

$f(3f-5) = -2(3f-1)$

$3f^2 - 5f = -6f + 2$

$3f^2 + f - 2 = 0$

$(3f-2)(f+1) = 0$

$f = \dfrac{2}{3}$ or $f = -1$

(e) $\dfrac{3w(w+1)}{2} = 6-w$

$3w(w+1) = 2(6-w)$

$3w^2 + 3w = 12 - 2w$

$3w^2 + 5w - 12 = 0$

$(3w-4)(w+3) = 0$

$w = \dfrac{4}{3}$ or $w = -3$

(f) $\dfrac{z(z+4)-9}{z-3} = 2$

$z^2 + 4z - 9 = 2(z-3)$

$z^2 + 4z - 9 = 2z - 6$

$z^2 + 4z - 2z - 9 + 6 = 0$

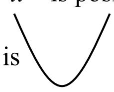
$z^2 + 2z - 3 = 0$

$(z-1)(z+3) = 0$

$z = 1$ or $z = -3$

2 (a) $y = x^2 + 6x + 8$

Since the coefficient of x^2 is positive,

The shape of the graph is  .

At the y-axis, $x = 0$.

$y = 0^2 + 6(0) + 8$

$y = 8$

Thus, the curve will intersect the y-axis at the point (0, 8).

At the x-axis, $y = 0$.

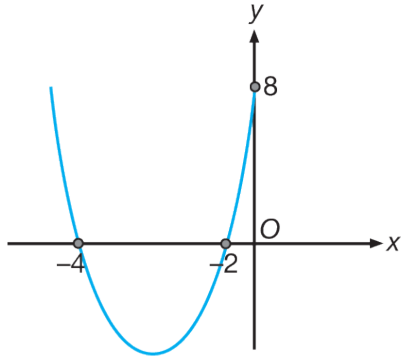
$y = 0$

$x^2 + 6x + 8 = 0$

$(x+4)(x+2) = 0$

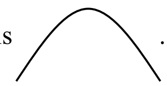
$x = -4$ or $x = -2$

Thus, the curve will intersect the x-axis at the points (–4, 0) and (–2, 0).



(b) $y = -x^2 + 2x + 3$

Since the coefficient of x^2 is negative,

the shape of the graph is  .

At the y-axis, $x = 0$.

$y = -0^2 + 2(0) + 3$

$y = 3$

Thus, the curve will intersect the y-axis at the point (0, 3).

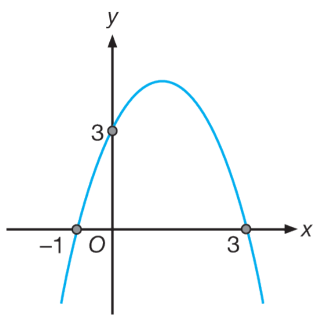
$-x^2 + 2x + 3 = 0$

$x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$

$x = 3$ or $x = -1$

Thus, the curve will intersect the y-axis at the points $(-1, 0)$ and (3, 0).



3 (a) Volume of tank = $4\frac{1}{2}$ m^3

$$(x)(2)(x)=\frac{9}{2}$$
$$2x^2=\frac{9}{2}$$
$$x^2=\frac{9}{4}$$
$$x=\frac{3}{2}$$

Hence, the width of the tank is 1.5 m.

(b) Volume of water

$= 2x\left(x-\frac{6}{5}\right)$

$= 2(1.5)(1.5-1.2)$

$= 0.9$ m^3

4 (a) $(3x+10)\left(\frac{1}{5}x+\frac{7}{4}\right)=150$

$$(3x+10)\left(\frac{4x+35}{20}\right)=150$$
$$(3x+10)(4x+35)=150(20)$$
$$12x^2+105x+40x+350=3\ 000$$
$$12x^2+145x-2\ 650=0$$
$$(x-10)(12x+265)=0$$

$x=10$ or $x=-\frac{265}{12}$

$x=-\frac{265}{12}$ is not accepted.

$\therefore x=10$

(b) Average speed = $3(10)+10=40$ km h^{-1}

5 (a) $(x+4)^2=x(8x+2)$

$$x^2+8x+16=8x^2+2x$$
$$7x^2-6x-16=0$$

(b) $7x^2-6x-16=0$

$(x-2)(7x+8)=0$

$x=2$ or $x=-\frac{8}{7}$

$x=-\frac{8}{7}$ is not accepted.

$\therefore x=2$

(c) (i) Length of the side of the square

= 2 + 4

= 6 cm

(ii) For the rectangle,

length = 8(2) + 2 = 18 cm,

width = 2 cm

6 (a) Area of the shaded region

= Area of the rectangle $PQRS$ – Area of ΔFSM – Area of ΔQPF

$= 20(2)(x+6)-\frac{1}{2}x(x+6)$

$\quad -\frac{1}{2}\times 2(x+6)(20-x)$

$= 40x+240-\frac{1}{2}x^2-3x-(14x-x^2+120)$

$= 40x+240-\frac{1}{2}x^2-3x-14x+x^2-120$

$=\frac{1}{2}x^2+23x+120$

Area of the shaded region = 168 cm^2

$$\frac{1}{2}x^2+23x+120=168$$
$$\frac{1}{2}x^2+23x-48=0$$
$$x^2+46x-96=0$$

(b) $x^2+46x-96=0$

$(x-2)(x+48)=0$

$x=2$ or $x=-48$

$x=-48$ is not accepted.

$\therefore x=2$

7 (a) Using the Pythagoras' Theorem,

$$(2x-2)^2+(2x)^2=(2x+2)^2$$
$$4x^2-8x+4+4x^2=4x^2+8x+4$$
$$4x^2-16x=0$$
$$4x(x-4)=0$$

$x=0$ or $x=4$

$x=0$ is not accepted.

$\therefore x=4$

(b) $AB=2(4)-2=6$ cm

$BC=2(4)=8$ cm

$AC=2(4)+2=10$ cm

Perimeter of $\Delta ABC=6+8+10=24$ cm

Area of $\Delta ABC=\frac{1}{2}\times 6\times 8=24$ cm^2

8 It is given that Latifah's age is x years old.

$h(x)=x+(x-2)+x^2$

$h(x)=x^2+2x-2$

$h(x)=33$

$x^2+2x-2=33$

$x^2+2x-35=0$

$(x-5)(x+7)=0$

$x=5$ or $x=-7$

$x=-7$ is not accepted.

Hence, Latifah's age is 5 years old.