Form 5 Chapter 4
Permutation and Combination
Fully-Worked Solutions

UPSKILL 4.1a

1 The number of ways a person can travel from town *A* to town *C* via town *B*

$= 2\times 5 = 10$

2 The number of ways a person can travel from Butterworth to Kuala Lumpur via Ipoh by taking a bus

$= 4\times 5 = 20$

3 The number of different ways Puan Tan can match her blouse, gowns and shoes

$= 5\times 4\times 2$
$= 40$

UPSKILL 4.1b

1 (a) $3! = 3\times 2\times 1 = 6$

(b) $5!\ 3!$

$= \left(5\times 4\times 3\times 2\times 1\right) \times (3\times 2\times 1)$
$= 120 \times 6$
$= 720$

(c) $0!\ 4!$

$= 1\times\left(4\times 3\times 2\times 1\right)$
$= 24$

(d) $\frac{7!}{4!} = 7\times 6\times 5 = 210$

(e) $\frac{9!}{5!} = 9\times 8\times 7\times 6 = 3\ 024$

(f) $\frac{n!}{(n-2)!}$

$= \frac{n(n-1)(n-2)\ldots}{(n-2)\ldots}$

$= n(n-1)$

(g) $\frac{n!\,0!}{(n-1)!}$

$= \frac{n(n-1)\ldots}{(n-1)\ldots}$

$= n$

(h) $\frac{3!\,(n+1)!}{2!\,n!}$

$= \frac{3\times 2\times 1\times (n+1)(n)\ldots}{2\times 1\times (n\ldots}$

$= 3(n+1)$

2 The number of ways to arrange 6 books

$= 6!$
$= 720$

3 The number of ways 7 different presents can be given to 7 children

$= 7!$
$= 5\ 040$

4 The number of ways 4 girls and 2 boys can be seated in a row that consists of 6 chairs

$= 6!$
$= 720$

5 The number of 5-digit numbers that can be formed

$= 5!$
$= 120$

6 The possible number of pairs

$= 5\times 4$
$= 20$

7 (a) (i) The number of different arrangements

$= 5!$
$= 120$

(ii) The number of different arrangements

$= 6!$
$= 720$

(iii) The number of different arrangements

$= 8!$
$= 40\ 320$

(b) (i) $\underline{O}$ _ _ _ _ … 4!

$\underline{E}$ _ _ _ _ … 4!

The number of different arrangements

$= 4!\times 2$
$= 48$

(ii) $\underline{O}$ _ _ _ _ _ … 5!

$\underline{E}$ _ _ _ _ _ … 5!

The number of different arrangements

$= 5!\times 2$
$= 240$

(iii) $\underline{I}$ _ _ _ _ _ _ _… 7!

$\underline{U}$ _ _ _ _ _ _ _… 7!

$\underline{A}$ _ _ _ _ _ _ _ … 7!

$\underline{E}$ _ _ _ _ _ _ _… 7!

The number of different arrangements
$= 7! \times 4$
= 20 160

8 _ _ _ $\underline{4}$ … 3!

_ _ _ $\underline{6}$ … 3!

The number of 4-digit even numbers that can be formed
$= 3! \times 2$
= 6

9 _ _ _ $\underline{1}$ … 3!

_ _ _ $\underline{7}$ … 3!

The number of 4-digit odd numbers that can be formed
$= 3! \times 2$
= 12

10 $\underline{3}$ _ _ _ … 3!

$\underline{4}$ _ _ _ … 3!

The number of 4-digit numbers that exceed 3 000 that can be formed
$= 3! \times 2$
= 12

11 (a) The number of arrangements

$$= \frac{6!}{2!} = 360$$

(b) The number of arrangements

$$= \frac{6!}{2!\ 2!} = 180$$

12 The number of arrangements

$$\frac{11!}{2!\,2!\,2!} = 4\ 989\ 600$$

UPSKILL 4.1c

1 The number of ways five people can sit around a circular table
= (5 – 1)!
= 24

2 _ _ _ $\underline{1}$

_ _ _ $\underline{3}$

_ _ _ $\underline{5}$

_ _ _ $\underline{7}$

The number of four-digit odd numbers
$= {}^6P_3 \times 4$
= 480

3 The number of different numbers that can be formed
$= {}^6P_1 + {}^6P_2 + {}^6P_3 + {}^6P_4 + {}^6P_5 + {}^6P_6$
= 1 956

4 The number of three-digit numbers that that can be formed
$= {}^5P_3$
= 60

5 _ _ _ _ $\underline{2}$ … 5!

_ _ _ _ $\underline{4}$ … 5!

_ _ _ _ $\underline{6}$ … 5!

_ _ _ _ $\underline{8}$ … 5!

The number of five -digit even numbers that can be formed
$= {}^8P_4 \times 4 = 6\ 720$

6 The number of different number that that can be formed
$= {}^5P_1 + {}^5P_2 + {}^5P_3 + {}^5P_4 + 5P_5$
= 325

7 (a) (i) The number of arrangements
$= {}^5P_3$
= 60

(ii) The number of arrangements
$= {}^8P_3$
= 336

(b) (i) $\underline{O}$ _ _

$\underline{E}$ _ _

The number of arrangements
$= {}^4P_2 \times 2$
= 24

(ii) $\underline{U}$ _ _ … ${}^7P_2$

$\underline{A}$ _ _ … ${}^7P_2$

The number of arrangements
$= {}^7P_2 \times 2$
= 84

Two children at the left of the table.

8 ${}^4P_2 \times 6! + {}^4P_2 \times 6!$

Two children at the right of the table.

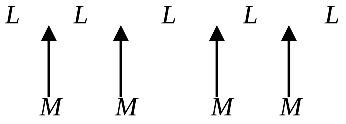
= 17 280

Three children at the left of the table.

9 ${}^5P_3 \times 7! + {}^5P_3 \times 7!$
= 604 800

Three children at the right of the table.

10 (a)



The number of arrangements
$= 5! \times 4!$
= 2 880

(b) If the four men want to be seated together, they will be counted as 1 object. Together with 5 ladies, there are 6 objects.

$\boxed{MMMM}$	L	L	L	L	L
√	√	√	√	√	√

This gives 6!.

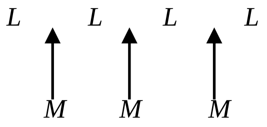
But the 4 men can also be arranged among themselves in their group.

L	L	L	L
√	√	√	√

This gives 4!.

Using the multiplication rule, the number of arrangements
$= 6! \times 4!$
= 17 280

11



If the ladies and men want to be seated alternately, the possible number of arrangements
$= 4! \times 3!$
= 144

12 If 3 girls want to be seated together, they are counted as 1 object. Together with four boys 4, there are 5 objects.

$\boxed{GGG}$	B	B	B	B
√	√	√	√	√

This gives 5!.

But the 3 girls can also be arranged among themselves in their group.

G	G	G
√	√	√

This gives 3!.

Using the multiplication rule, the number of arrangements
$= 5! \times 3!$
= 720

13 The possible number of arrangements
$= 3! \times 5! \times 4! \times 3!$
= 103 680

14 The possible number of arrangements
$= 3! \times 6! \times 4! \times 2!$
= 207 360

15 If the 4 vowels are to be together, they will be counted as 1 object. Together with the 4 consonants, there are 5 objects.

$\boxed{IUAI}$	S	M	L	S
√	√	√	√	√

This gives $\frac{5!}{2!} = 60$

But the 4 vowels can also be arranged among themselves in their group.

I U A I
√ √ √ √

This gives $\frac{4!}{2!} = 12$.

Using the multiplication rule, the number of arrangements
$= 60 \times 12$
$= 720$

16 The number of arrangements if no restriction is imposed
$= \frac{7!}{2!2!} = 1\ 260$

If the letters *O* and *E* have to be adjacent to each other, they will be counted as 1 object. Together with the other 5 letters, there are 6 object.

OE *S* *U* *P* *P* *S*
√ √ √ √ √ √

This gives $\frac{6!}{2!2!} = 180$

But *O* and *E* can also be arranged among themselves.

O E
√ √

This gives 2! = 2.

Using the multiplication rule, the number of arrangements
$= 180 \times 2$
$= 360$

17 The number of arrangements if no restriction is imposed
$= \frac{13!}{2!\ 3!\ 2!} = 259\ 459\ 200$

If 5 vowels have to be together, they will be counted as 1 object. Together with the 8 consonants, there are 9 objects.

EEAIO *T* *S* *S* *S* *L* *L* *T* *N*
√ √ √ √ √ √ √ √ √

This gives $\frac{9!}{3!2!2!} = 15\ 120$

But the 5 vowels can also be arranged among themselves in their group.

E E A I O
√ √ √ √ √

This gives $\frac{5!}{2!} = 60$.

Using the multiplication rule, the number of arrangements
$= 15\ 120 \times 60$
$= 907\ 200$

UPSKILL 4.2

1 (a) ${}^9C_5 = \dfrac{9!}{5! \times (9-5)!} = 126$

(b) ${}^8C_3 = \dfrac{8!}{3! \times (8-3)!} = 56$

(c) ${}^6C_4 = \dfrac{6!}{4! \times (6-4)!} = 15$

(d) ${}^{10}C_6 = \dfrac{10!}{6! \times (10-6)!} = 210$

3 The number of pairs of double table tennis players that can be formed $= {}^8C_2 = 28$

4 (a) The number of combinations to answer 6 questions = ${}^9C_6 = 84$

(b) The number of combinations to answer 6 questions $= {}^5C_4 \times {}^4C_2 = 30$

5 The number of different committees that can be formed

$= {}^9C_5 \times {}^7C_4$

= 4 410

6

HOT TIPS

Three points on a straight line cannot form a triangle.

The number of triangles that can be formed

$= {}^{10}C_3 - {}^4C_3 - {}^6C_3 = 120 - 4 - 20 = 96$

7 (a) The number of different committees that can be formed

$= {}^1C_1 \times {}^{12}C_6$

= 924

(b) The number of different committees that can be formed

$= {}^8C_5 \times {}^5C_2$

= 560

(c)

	Male	*Female*	*Number of ways*
Available	8	5	
Required	4	3	${}^8C_4 \times {}^5C_3$
	3	4	${}^8C_3 \times {}^5C_4$
	2	5	${}^8C_2 \times {}^5C_5$
	1	6	Impossible
	0	7	Impossible

The number of different committees that can be formed

$= {}^8C_4 \times {}^5C_3 + {}^8C_3 \times {}^5C_4 + {}^8C_2 \times {}^5C_5$

= 700 + 280 + 28

= 1 008

8 (a) The number of different ways to form the group

$= {}^{12}C_5 \times {}^{10}C_7$

= 95 040

(b) The number of different ways to form the group

$= {}^9C_4 \times {}^{13}C_8$

= 162 162

(c) The number of different ways to form the group

$= {}^4C_3 \times {}^5C_3 \times {}^6C_3 \times {}^7C_3$

= 28 000

9 (a) The number of different committees that can be formed

$= {}^{10}C_6$

= 210

(b)

	Teacher	*Student*	*Number of ways*
Available	4	6	
Required	2	4	${}^4C_2 \times {}^6C_4$
	1	5	${}^4C_1 \times {}^6C_5$
	0	6	${}^4C_0 \times {}^6C_6$

The number of different committees that can be formed

$= {}^4C_2 \times {}^6C_4 + {}^4C_1 \times {}^6C_5 + {}^4C_0 \times {}^6C_6$
$= 90 + 24 + 1$
$= 115$

10 The number of different choices
$= {}^6C_1 + {}^6C_2 + {}^6C_3 + {}^6C_4 + {}^6C_5 + {}^6C_6$
$= 63$

Summative Practice 4

1 The number of ways to choose the team
$= 5 \times 6$
$= 30$

2 The number of permutations
$= 6!$
$= 720$

3 3 _ _ 1 … 2!

4 _ _ 1 … 2!

4 _ _ 3 … 2!

The number of four-digit odd numbers that are greater than 3 000 that can be formed
$= 2! \times 3$
$= 6$

4 (a) The number of six--digit numbers that can be formed = 6! = 720

(b) (i) 5 _ _ _ _ _ … 5!

6 _ _ _ _ _ … 5!

The number of numbers that are greater than 500 000
$= 5! \times 2$
$= 240$

(ii) 1 _ _ _ _ 1 … 4!

1 _ _ _ _ 3 … 4!

1 _ _ _ _ 5 … 4!

The number of even numbers that are less than 200 000
$= 4! \times 3$
$= 72$

5 5 _ _ _ … ${}^4P_3$

6 _ _ _ … ${}^4P_3$

7 _ _ _ … ${}^4P_3$

The number of four-digit numbers that are smaller than 8 000
$= {}^4P_3 \times 3$
$= 72$

6 E _ _ _ … ${}^5P_3$

I _ _ _ … ${}^5P_3$

U _ _ _ … ${}^5P_3$

The number of arrangements
$= {}^5P_3 \times 3$
$= 180$

7 The prime numbers are 2, 3, 5 and 7.

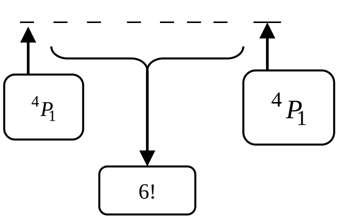
2 _ _ _ … 3!

3 _ _ _ … 3!

5 _ _ _ … 3!

The number of four-digit numbers that are less than 6 000
$= 3! \times 3$
$= 18$

8



The number of different arrangements
$= {}^4P_1 \times {}^4P_1 \times 6!$
$= 11\ 520$

9 (a) The number of arrangements if no restriction is imposed
$= \dfrac{10!}{5!\ 2!} = 15\ 120$

(b) If 5 vowels have to be together, they are counted as 1 object. Together with the 7 consonants, there are 8 objects.

$\boxed{EAE}$ S S S S S L F

√ √ √ √ √ √ √ √

This gives $\frac{8!}{5!} = 336$

But the 5 vowels can also be arranged among themselves in their group.

$\boxed{\begin{matrix} E & A & E \\ \surd & \surd & \surd \end{matrix}}$

This gives $\frac{3!}{2!} = 3$.

Using the multiplication rule, the number of arrangements
= 336 × 3
= 1 008

10 The number of ways 3 books can be chosen from 8 books = ${}^8C_3 = 56$

11 The number of teams that can be formed
$= {}^8C_5 \times {}^6C_4$
= 840

12 (a) The number of triangles that can be formed
$= {}^8C_3 - {}^6C_3$
= 36

(b) The number of triangles (with point B only)
$= {}^6C_2 = 15$
The number of triangles (with points A and B)
$= {}^6C_1 = 6$
The total number of triangles
= 15 + 6
= 21

13 The number of ways the team can be formed
$= {}^8C_4 \times {}^4C_3 \times {}^{10}C_4$
= 58 800

14 Number of combinations to invite colleagues
$= {}^2C_2 \times {}^{10}C_6$
= 210

15 Number of combinations for the travelling plan
$= {}^1C_1 \times {}^{10}C_4 \times {}^6C_6$
= 210

16 The total number of ways of selections
$= {}^{10}C_3 \times {}^7C_5 \times {}^2C_2$
= 2 520

17 (a) The number of ways the committees can be formed
$= {}^{13}C_6$
= 1 716

(b) The number of ways the committees can be formed
$= {}^7C_3 \times {}^6C_3 + {}^7C_4 \times {}^6C_2$
= 1 225

18 (a) The number of ways the presents can be wrapped
$= {}^5C_2 \times {}^7C_4$
= 350

(b)

	Stationery	*Story books*	*Number of ways*
Available	5	7	
Required	3	3	${}^5C_3 \times {}^7C_3$
	4	2	${}^5C_4 \times {}^7C_2$
	5	1	${}^5C_5 \times {}^7C_1$
	6	0	Impossible

The number of ways
$= {}^5C_3 \times {}^7C_3 + {}^5C_4 \times {}^7C_2 + {}^5C_5 \times {}^7C_1$
= 350 + 105 + 7
= 462

19 (a) The number of arrangements
$= {}^5P_3$
= 60

(b) The number of combinations
$= {}^5C_2$
= 10

20 (a) The number of teams that can be formed

$= {}^{10}C_3$

$= 120$

(b)

	Teacher	*Student*	*Number of ways*
Available	4	6	
Required	2	1	${}^4C_2 \times {}^6C_1$
	1	2	${}^4C_1 \times {}^6C_2$
	0	3	${}^4C_0 \times {}^6C_3$

The number of teams

$= {}^4C_2 \times {}^6C_1 + {}^4C_1 \times {}^6C_2 +$

${}^4C_0 \times {}^6C_3$

$= 36 + 60 + 20$

$= 116$