Form 5 Chapter 2 Differentiation Fully-Worked Solutions

UPSKILL 2.1a

1 (a) $\lim_{n\to 0}\left(\frac{n^2+n}{n}\right)$

$= \lim_{n\to 0}\left(\frac{n(n+1)}{n}\right)$

$= \lim_{n\to 0}(n+1)$

$= 0+1$

$= 1$

(b) $\lim_{n\to 7}\left(\frac{n^2-49}{n-7}\right)$

$= \lim_{n\to 7}\left(\frac{(n+7)(n-7)}{n-7}\right)$

$= \lim_{n\to 7}(n+7)$

$= 7+7$

$= 14$

(c) $\lim_{n\to 1}\left(\frac{n^2+n-2}{n-1}\right)$

$= \lim_{n\to 1}\left(\frac{(n+2)(n-1)}{n-1}\right)$

$= \lim_{n\to 1}(n+2)$

$= 1+2$

$= 3$

(d) $\lim_{n\to 2}\left(\frac{n^2-n-2}{n^2-5n+6}\right)$

$= \lim_{n\to 2}\left(\frac{(n-2)(n+1)}{(n-3)(n-2)}\right)$

$= \lim_{n\to 2}\left(\frac{n+1}{n-3}\right)$

$= \frac{2+1}{2-3}$

$= -3$

UPSKILL 2.1b

1 (a) $y = x^2 - 2 \ \ldots (1)$

$y+\delta y = (x+\delta x)^2 - 2$

$y+\delta y = x^2 + 2x\delta x + (\delta x)^2 - 2 \ \ldots (2)$

$(2)-(1):$

$\delta y = 2x\delta x + (\delta x)^2$

$\frac{\delta y}{\delta x} = 2x + \delta x$

$\frac{dy}{dx} = \lim_{\delta x\to 0}\left(\frac{\delta y}{\delta x}\right)$

$\frac{dy}{dx} = \lim_{\delta x\to 0}(2x+\delta x)$

$\frac{dy}{dx} = 2x+0$

$\frac{dy}{dx} = 2x$

(b) $y = x^2 + x - 6 \ \ldots (1)$

$y+\delta y = (x+\delta x)^2 + x + \delta x - 6$

$y+\delta y = x^2 + 2x\delta x + (\delta x)^2 + x + \delta x - 6 \ \ldots (2)$

$(2)-(1): \ \delta y = 2x\delta x + (\delta x)^2 + \delta x$

$\frac{\delta y}{\delta x} = 2x + \delta x + 1$

$\frac{dy}{dx} = \lim_{\delta x\to 0}\left(\frac{\delta y}{\delta x}\right)$

$\frac{dy}{dx} = \lim_{\delta x\to 0}(2x+\delta x+1)$

$\frac{dy}{dx} = 2x+0+1$

$\frac{dy}{dx} = 2x+1$

2 (a) $y = \frac{3}{x} \ \ldots (1)$

$y+\delta y = \frac{3}{x+\delta x} \ \ldots (2)$

$(2)-(1):$

$\delta y = \frac{3}{x+\delta x} - \frac{3}{x}$

$\delta y = \frac{3x - 3(x+\delta x)}{(x+\delta x)^2}$

$\delta y = \frac{-3\delta x}{(x+\delta x)^2}$

$\frac{\delta y}{\delta x} = \frac{-3}{(x+\delta x)^2}$

$$\frac{dy}{dx}=\lim_{\delta x\to 0}\left(\frac{\delta y}{\delta x}\right)$$
$$\frac{dy}{dx}=\lim_{\delta x\to 0}\frac{-3}{(x+\delta x)^2}$$
$$\frac{dy}{dx}=\frac{-3}{(x+0)^2}$$
$$\frac{dy}{dx}=\frac{-3}{x^2}$$

(b) $y=\frac{5}{x^2}$ … (1)

$$y+\delta y=\frac{5}{(x+\delta x)^2} \quad \ldots (2)$$

(2) – (1) :
$$\delta y=\frac{5}{(x+\delta x)^2}-\frac{5}{x^2}$$
$$\delta y=\frac{5x^2-5\left[(x+\delta x)^2\right]}{x^2(x+\delta x)^2}$$
$$\delta y=\frac{5x^2-5\left[x^2+2x\delta x+(\delta x)^2\right]}{x^2(x+\delta x)^2}$$
$$\delta y=\frac{5x^2-5x^2-10x\delta x-5(\delta x)^2}{x^2(x+\delta x)^2}$$
$$\delta y=\frac{-10x\delta x-5(\delta x)^2}{x^2(x+\delta x)^2}$$
$$\frac{\delta y}{\delta x}=\frac{-10x-5\delta x}{x^2(x+\delta x)^2}$$
$$\frac{dy}{dx}=\lim_{\delta x\to 0}\left(\frac{\delta y}{\delta x}\right)$$
$$\frac{dy}{dx}=\lim_{\delta x\to 0}\left(\frac{-10x-5\delta x}{x^2(x+\delta x)^2}\right)$$
$$\frac{dy}{dx}=\frac{-10x-5(0)}{x^2(x+0)^2}$$
$$\frac{dy}{dx}=\frac{-10x}{x^2(x)^2}$$
$$\frac{dy}{dx}=\frac{-10}{x^3}$$

UPSKILL 2.2a

1 (a) $y=24$
$$\frac{dy}{dx}=0$$

(b) $y=6x$
$$\frac{dy}{dx}=6$$

(c) $y=x^8$
$$\frac{dy}{dx}=8x^7$$

(d) $y=5x^4$
$$\frac{dy}{dx}=20x^3$$

(e) $y=\frac{4}{x}$
$$\frac{dy}{dx}=-\frac{4}{x^2}$$

(f) $y=-\frac{4}{x^3}$
$$\frac{dy}{dx}=\frac{12}{x^4}$$

(g) $y=\frac{x^5}{15}$
$$\frac{dy}{dx}=\frac{1}{15}\left(5x^4\right)=\frac{x^4}{3}$$

(h) $y=-\frac{3}{4x^8}=-\frac{3}{4}\left(x^{-8}\right)$
$$\frac{dy}{dx}=-\frac{3}{4}\left(-8x^{-9}\right)=6x^{-9}=\frac{6}{x^9}$$

2 (a) $\frac{d}{dx}\left(x^6-3x^3+5\right)=6x^5-9x^2$

(b) $\frac{d}{dx}\left(\frac{1}{2}t^4+3t^2+6\right)=2t^3+6t$

(c) $\frac{d}{dj}\left(5j^3-4j^2-3j\right)=15j^2-8j-3$

3 (a) $f(x)=3x^2+\frac{3}{x}+\frac{3}{x^2}$
$$f\,'(x)=6x-\frac{3}{x^2}-\frac{6}{x^3}$$

(b) $f(x)=2x+\frac{2}{x}-\frac{2}{x^3}$

$f'(x)=2-\frac{2}{x^2}+\frac{6}{x^4}$

(c) $f(x)=\frac{1}{x}+\frac{1}{x^2}+\frac{1}{x^3}$

$f'(x)=-\frac{1}{x^2}-\frac{2}{x^3}-\frac{3}{x^4}$

4 (a) $s=(t-1)(2t+5)$

$s=2t^2+3t-5$

$\frac{ds}{dt}=4t+3$

(b) $s=(2t-5)^2=4t^2-20t+25$

$\frac{ds}{dt}=8t-20$

(c) $s=3t^2(t-2)^2$

$s=3t^2\left(t^2-4t+4\right)$

$s=3t^4-12t^3+12t^2$

$\frac{ds}{dt}=12t^3-36t^2+24t$

(d) $s=\left(t-\frac{2}{t}\right)^2=t^2-4+\frac{4}{t^2}$

$\frac{ds}{dt}=2t-\frac{8}{t^3}$

5 (a) $v=\frac{2t^2-9}{t}=2t-\frac{9}{t}$

$\frac{dv}{dt}=2+\frac{9}{t^2}$

(b) $v=\frac{t^2+6t-12}{t^2}=1+\frac{6}{t}-\frac{12}{t^2}$

$\frac{dv}{dt}=-\frac{6}{t^2}+\frac{24}{t^3}$

(c) $v=\frac{(t+1)(2-3t)}{t^2}=\frac{2-t-3t^2}{t^2}$

$v=\frac{2}{t^2}-\frac{1}{t}-3$

$\frac{dv}{dt}=-\frac{4}{t^3}+\frac{1}{t^2}$

UPSKILL 2.2b

1 (a) $y=2x^3\left(2-x^5\right)=4x^3-2x^8$

$\frac{dy}{dx}=12x^2-16x^7$

(b) $y=\left(6+x^2\right)(5-3x)$

$=30-18x+5x^2-3x^3$

$\frac{dy}{dx}=-9x^2+10x-18$

(c) $y=\left(5x^2-4\right)(3-x)$

$=15x^2-5x^3-12+4x$

$\frac{dy}{dx}=-15x^2+30x+4$

(d) $y=\left(x^4+1\right)\left(x^3-2x\right)$

$=x^7-2x^5+x^3-2x$

$\frac{dy}{dx}=7x^6-10x^4+3x^2-2$

(e) $y=(2x-1)\left(x^2+3x-2\right)$

$\frac{dy}{dx}=(2x-1)(2x+3)+\left(x^2+3x-2\right)(2)$

$=4x^2+4x-3+2x^2+6x-4$

$=6x^2+10x-7$

(f) $y=\left(2x^2+1\right)\left(3x^2+x-4\right)$

$\frac{dy}{dx}=(2x^2+1)(6x+1)+(3x^2+x-4)(4x)$

$=12x^3+2x^2+6x+1+$

$12x^3+4x^2-16x$

$=24x^3+6x^2-10x+1$

(g) $y=\left(\frac{1}{x}+1\right)\left(\frac{2}{x^2}-3\right)$

$y=\frac{2}{x^3}-\frac{3}{x}+\frac{2}{x^2}-3$

$\frac{dy}{dx}=-\frac{6}{x^4}+\frac{3}{x^2}-\frac{4}{x^3}$

(h) $y=\left(2x^2+x^3\right)\left(x-\frac{2}{x}\right)$

$=2x^3-4x+x^4-2x^2$

$\frac{dy}{dx}=6x^2-4+4x^3-4x$

2 (a) $f(x) = x(x^2 + 4x - 3)$

$= x^3 + 4x^2 - 3x$

$f'(x) = 3x^2 + 8x - 3$

(b) $f'(x) = 0$

$3x^2 + 8x - 3 = 0$

$x = \frac{1}{3}$ or -3

3 (a) $y = \frac{5x}{x+3}$

$\frac{dy}{dx} = \frac{5(x+3) - 5x}{(x+3)^2}$

$= \frac{5x + 15 - 5x}{(x+3)^2}$

$= \frac{15}{(x+3)^2}$

(b) $y = \frac{6x^2}{x-5}$

$\frac{dy}{dx} = \frac{(x-5)(12x) - 6x^2}{(x-5)^2}$

$= \frac{12x^2 - 60x - 6x^2}{(x-5)^2}$

$= \frac{6x^2 - 60x}{(x-5)^2}$

(c) $y = \frac{2x-5}{2x+3}$

$\frac{dy}{dx} = \frac{(2x+3)(2) - (2x-5)(2)}{(2x+3)^2}$

$= \frac{4x + 6 - 4x + 10}{(2x+3)^2}$

$= \frac{16}{(2x+3)^2}$

(d) $y = \frac{x^2 - 5}{x+3}$

$\frac{dy}{dx} = \frac{(x+3)(2x) - (x^2 - 5)}{(x+3)^2}$

$= \frac{2x^2 + 6x - x^2 + 5}{(x+3)^2}$

$= \frac{x^2 + 6x + 5}{(x+3)^2}$

(e) $y = \frac{4x^2}{7 - x^2}$

$\frac{dy}{dx} = \frac{(7 - x^2)(8x) - 4x^2(-2x)}{(7 - x^2)^2}$

$= \frac{56x - 8x^3 + 8x^3}{(7 - x^2)^2}$

$= \frac{56x}{(7 - x^2)^2}$

(f) $y = \frac{x^2}{x^2 - 5}$

$\frac{dy}{dx} = \frac{(x^2 - 5)(2x) - (x^2)(2x)}{(x^2 - 5)^2}$

$= \frac{2x^3 - 10x - 2x^3}{(x^2 - 5)^2}$

$= \frac{-10x}{(x^2 - 5)^2}$

(g) $y = \frac{3x^4}{9 - x^2}$

$\frac{dy}{dx} = \frac{(9 - x^2)(12x^3) - 3x^4(-2x)}{(9 - x^2)^2}$

$= \frac{108x^3 - 12x^5 + 6x^5}{(9 - x^2)^2}$

$= \frac{108x^3 - 6x^5}{(9 - x^2)^2}$

(h) $y = \frac{x^2 - 2x}{x^2 + 3x}$

$\frac{dy}{dx} = \frac{(x^2 + 3x)(2x - 2) - (x^2 - 2x)(2x + 3)}{(x^2 + 3x)^2}$

$= \frac{2x^3 - 2x^2 + 6x^2 - 6x - [2x^3 + 3x^2 - 4x^2 - 6x]}{(x^2 + 3x)^2}$

$= \frac{2x^3 + 4x^2 - 6x - [2x^3 - x^2 - 6x]}{(x^2 + 3x)^2}$

$= \frac{5x^2}{(x^2 + 3x)^2}$

4 (a) $f(x)=\frac{2-x}{x^2+5}$

$f'(x)=\frac{(x^2+5)(-1)-(2-x)(2x)}{(x^2+5)^2}$

$=\frac{-x^2-5-4x+2x^2}{(x^2+5)^2}$

$=\frac{x^2-4x-5}{(x^2+5)^2}$

(b) $f'(x)=0$

$x^2-4x-5=0$

$x=5$ or -1

5 (a) $y=(4x+7)^5$

$\frac{dy}{dx}=5(4x+7)^4(4)=20(4x+7)^4$

(b) $y=(6-x^2)^4$

$\frac{dy}{dx}=4(6-x^2)^3(-2x)=-8x(6-x^2)^3$

(c) $y=\left(\frac{1}{3}x^3-9\right)^6$

$\frac{dy}{dx}=6\left(\frac{x^3}{3}-9\right)^5(x^2)=6x^2\left(\frac{x^3}{3}-9\right)^5$

(d) $y=(x^2+5x-3)^3$

$\frac{dy}{dx}=3(x^2+5x-3)^2(2x+5)$

6 (a) $s=\frac{2}{5t+1}=2(5t+1)^{-1}$

$\frac{ds}{dt}=-2(5t+1)^{-2}(5)$

$=-\frac{10}{(5t+1)^2}$

(b) $s=\frac{3}{t^2+4}=3(t^2+4)^{-1}$

$\frac{ds}{dt}=-3(t^2+4)^{-2}(2t)$

$=\frac{-6t}{(t^2+4)^2}$

(c) $s=\frac{5}{(t^2-2)^3}=5(t^2-2)^{-3}$

$\frac{ds}{dt}=-15(t^2-2)^{-4}(2t)=\frac{-30t}{(t^2-2)^4}$

(d) $s=\frac{1}{(2t^2-t+2)^3}=(2t^2-t+2)^{-3}$

$\frac{ds}{dt}=-3(2t^2-t+2)^{-4}(4t-1)$

$\frac{ds}{dt}=\frac{-12t+3}{(2t^2-t+2)^4}$

7 (a) $y=x^2(x+1)^4$

$\frac{dy}{dt}=x^2(4)(x+1)^3(1)+(x+1)^4(2x)$

$=2x(x+1)^3[2x+(x+1)]$

$=2x(x+1)^3(3x+1)$

(b) $y=(2x+1)(x+3)^3$

$\frac{dy}{dx}=(2x+1)(3)(x+3)^2+(x+3)^3(2)$

$=(x+3)^2[6x+3+2(x+3)]$

$=(x+3)^2(8x+9)$

(c) $y=(x+2)^4(2x-3)^3$

$\frac{dy}{dx}=(x+2)^4(3)(2x-3)^2(2)+$

$(2x-3)^3(4)(x+2)^3$

$\frac{dy}{dx}=6(x+2)^4(2x-3)^2+4(2x-3)^3(x+2)^3$

$\frac{dy}{dx}=2(x+2)^3(2x-3)^2[3(x+2)+2(2x-3)]$

$\frac{dy}{dx}=2(x+2)^3(2x-3)^2[7x]$

$\frac{dy}{dx}=14x(x+2)^3(2x-3)^2$

8 (a) $f(x)=x(3-x)^4$

$f'(x)=x(4)(3-x)^3(-1)+(3-x)^4$

$f'(x)=-4x(3-x)^3+(3-x)^4$

$f'(x)=(3-x)^3[-4x+3-x]$

$f'(x)=(3-x)^3(3-5x)$

(b) $f'(x)=0$

$(3-x)^3[3-5x]=0$

$x=3$ or $\frac{3}{5}$

9 (a) $y = \dfrac{3-x}{x^2+1}$

$\dfrac{dy}{dx} = \dfrac{(x^2+1)(-1)-(3-x)(2x)}{(x^2+1)^2}$

$\dfrac{dy}{dx} = \dfrac{(-x^2-1)-(6x+2x^2)}{(x^2+1)^2}$

$\dfrac{dy}{dx} = \dfrac{(-x^2-1)-6x-2x^2}{(x^2+1)^2}$

$\dfrac{dy}{dx} = \dfrac{-3x^2-6x-1}{(x^2+1)^2}$

(b) $y = \dfrac{(3x+1)^3}{2x+1}$

$\dfrac{dy}{dx} = \dfrac{(2x+1)(3)(3x+1)^2(3)-(3x+1)^3(2)}{(2x+1)^2}$

$\dfrac{dy}{dx} = \dfrac{9(2x+1)(3x+1)^2-2(3x+1)^3}{(2x+1)^2}$

$\dfrac{dy}{dx} = \dfrac{(3x+1)^2[18x+9-2(3x+1)]}{(2x+1)^2}$

$\dfrac{dy}{dx} = \dfrac{(3x+1)^2(12x+7)}{(2x+1)^2}$

(c) $y = \dfrac{2x-1}{(x^2+3)^2}$

$\dfrac{dy}{dx} = \dfrac{(x^2+3)^2(2)-(2x-1)(2)(x^2+3)(2x)}{(x^2+3)^4}$

$\dfrac{dy}{dx} = \dfrac{2(x^2+3)[x^2+3-2x(2x-1)]}{(x^2+3)^4}$

$\dfrac{dy}{dx} = \dfrac{2(-3x^2+2x+3)}{(x^2+3)^3}$

10 (a) $y = t^3-2t$

$\dfrac{dy}{dx} = 3t^2-2$

$x = t^2+3t$

$\dfrac{dx}{dt} = 2t+3$

$\dfrac{dy}{dx} = \dfrac{\frac{dy}{dt}}{\frac{dx}{dt}} = \dfrac{3t^2-2}{2t+3}$

(b) $y = (t-1)^2 = t^2-2t+1$

$\dfrac{dy}{dt} = 2t-2$

$x = t^2-1$

$\dfrac{dx}{dt} = 2t$

$\dfrac{dy}{dx} = \dfrac{\frac{dy}{dt}}{\frac{dx}{dt}} = \dfrac{2t-2}{2t} = \dfrac{t-1}{t}$

11 (a) $y = 3t^2$

$\dfrac{dy}{dt} = 6t$

$x = 2t+1$

$\dfrac{dx}{dt} = 2$

$x = 2t+1$

$t = \dfrac{x-1}{2}$

$\dfrac{dy}{dt} = \dfrac{\frac{dy}{dt}}{\frac{dx}{dt}} = \dfrac{6t}{2} = 3t = 3\left(\dfrac{x-1}{2}\right)$

(b) $y = \dfrac{1}{4}u^8 = \dfrac{1}{4}(5x-2)^8 = 2(5x-2)^7(5)$

$= 10(5x-2)^7$

UPSKILL 2.3

1 (a) $y = 2x^3+4x^2-6x-3$

$\dfrac{dy}{dx} = 6x^2+8x-6$

$\dfrac{d^2y}{dx^2} = 12x+8$

(b) $y = \dfrac{2x+5}{x} = 2+\dfrac{5}{x}$

$\dfrac{dy}{dx} = -\dfrac{5}{x^2}$

$\dfrac{d^2y}{dx^2} = \dfrac{10}{x^3}$

(c) $y=\frac{3x}{x+3}$

$\frac{dy}{dx}=\frac{(x+3)(3)-3x(1)}{(x+3)^2}$

$\frac{dy}{dx}=\frac{9}{(x+3)^2}=9(x+3)^{-2}$

$\frac{d^2y}{dx^2}=-18(x+3)^{-3}=-\frac{18}{(x+3)^3}$

(d) $y=(2x+1)\left(3x^2-2\right)=6x^3-4x+3x^2-2$

$\frac{dy}{dx}=18x^2-4+6x$

$\frac{d^2y}{dx^2}=36x+6$

(e) $y=(2x+1)^5$

$\frac{dy}{dx}=5(2x+1)^4(2)=10(2x+1)^4$

$\frac{d^2y}{dx^2}=40(2x+1)^3(2)=80(2x+1)^3$

2 (a) $f(x)=x^3-6x^2+9x+2$

$f'(x)=3x^2-12x+9$

$f''(x)=6x-12$

(b) $f(x)=x^2-\frac{2}{x}$

$f'(x)=2x+\frac{2}{x^2}$

$f''(x)=2-\frac{4}{x^3}$

(c) $f(x)=\frac{1}{3x+4}=(3x+4)^{-1}$

$f'(x)=-(3x+4)^{-2}(3)$

$f''(x)=6(3x+4)^{-3}(3)=\frac{18}{(3x+4)^3}$

(d) $f(x)=\frac{4x}{3x+1}$

$f'(x)=\frac{(3x+1)(4)-4x(3)}{(3x+1)^2}$

$=\frac{4}{(3x+1)^2}=4(3x+1)^{-2}$

$f''(x)=-8(3x+1)^{-3}(3)=-\frac{24}{(3x+1)^3}$

(e) $f(x)=\left(2x+\frac{1}{x}\right)^2=4x^2+4+x^{-2}$

$f'(x)=8x-2x^{-3}$

$f''(x)=8+6x^{-4}=8+\frac{6}{x^4}$

UPSKILL 2.4a

1 (a) $y=\left(x^2+3\right)^3$

$\frac{dy}{dx}=3(x^2+3)^2(2x)$

$\frac{dy}{dx}=6x\left(x^2+3\right)^2$

$m=6(-1)\left[(-1)^2+3\right]^2=-96$

(b) $y=\frac{x^2+3}{4x+1}$

$\frac{dy}{dx}=\frac{(4x+1)(2x)-(x^2+3)(4)}{(4x+1)^2}$

$\frac{dy}{dx}=\frac{8x^2+2x-4x^2-12}{(4x+1)^2}$

$\frac{dy}{dx}=\frac{4x^2+2x-12}{(4x+1)^2}$

$m=\frac{-12}{1^2}=-12$

(c) $y=\left(9-x^3\right)(1+x^2)=9+9x^2-x^3-x^5$

$\frac{dy}{dx}=18x-3x^2-5x^4$

$m=18(2)-3(2)^2-5(2)^4=-56$

(d) $y=\frac{x+1}{x^2}=x^{-1}+x^{-2}$

$\frac{dy}{dx}=-x^{-2}-2x^{-3}$

$\frac{dy}{dx}=-\frac{1}{x^2}-\frac{2}{x^3}$

$m=-\frac{1}{2^2}-\frac{2}{2^3}=-\frac{1}{4}-\frac{1}{4}=-\frac{1}{2}$

(e) $y = \frac{x-2}{x+1}$

$\frac{dy}{dx} = \frac{(x+1)-(x-2)}{(x+1)^2} = \frac{3}{(x+1)^2}$

$m = \frac{3}{(2+1)^2} = \frac{3}{9} = \frac{1}{3}$

2 $y = hx^2 + kx$

The curve passes through the point $\left(\frac{1}{4}, \frac{7}{8}\right)$.

$\frac{7}{8} = h\left(\frac{1}{4}\right)^2 + k\left(\frac{1}{4}\right)$

$\frac{7}{8} = h\left(\frac{1}{16}\right) + k\left(\frac{1}{4}\right)$

$14 = h + 4k \ \dots (1)$

$\frac{dy}{dx} = 2hx + k$

$4 = 2h\left(\frac{1}{4}\right) + k$

$8 = h + 2k \ \dots (2)$

(1) – (2) : $2k = 6$

$k = 3$

Substitute $k = 3$ into (2) :

$8 = h + 2(3)$

$h = 2$

3 $y = \frac{a}{x} + bx$

The curve passes through the point (2, 7).

$7 = \frac{a}{2} + 2b$

$14 = a + 4b \ \dots (1)$

$\frac{dy}{dx} = \frac{-a}{x^2} + b$

$\frac{5}{2} = \frac{-a}{2^2} + b$

$10 = -a + 4b \ \dots (2)$

(1) + (2) : $24 = 8b$

$b = 3$

Substitute $b = 3$ into (1) :

$14 = a + 4(3)$

$a = 2$

4 $y = fx + \frac{g}{x^2}$

The curve passes through the point (2, 5).

$5 = 2f + \frac{g}{2^2}$

$20 = 8f + g \ \dots (1)$

$\frac{dy}{dx} = f - \frac{2g}{x^3}$

$1 = f - \frac{2g}{2^3}$

$1 = f - \frac{g}{4}$

$4 = 4f - g \ \dots (2)$

(1) + (2) : $24 = 12f$

$f = 2$

Substitute $f = 2$ into (2) :

$4 = 4(2) - g$

$g = 4$

UPSKILL 2.4b

1 (a) $y = 3x^2 - 2x - 5$

$m = \frac{dy}{dx} = 6x - 2$

$m = 6(2) - 2 = 10$

The equation of the tangent is

$y - 3 = 10(x - 2)$

$y = 10x - 20 + 3$

$y = 10x - 17$

(b) $y = x + \frac{3}{x^2}$

$m = \frac{dy}{dx} = 1 - \frac{6}{x^3}$

$m = 1 - \frac{6}{1^3} = -5$

The equation of the tangent is

$y - 4 = -5(x - 1)$

$y = -5x + 5 + 4$

$y = -5x + 9$

2 When $x = 4$, $y = x^3 - 8x^2 + 14x$

$y = 4^3 - 8(4)^2 + 14(4) = -8$

$y = x^3 - 8x^2 + 14x$

$m = \frac{dy}{dx} = 3x^2 - 16x + 14$

$m = 3(4)^2 - 16(4) + 14 = -2$

The equation of the tangent is

$y - (-8) = -2(x - 4)$

$y + 8 = -2x + 8$

$y = -2x$

3 $y = \dfrac{3 + x}{3 - 2x}$

$4 = \dfrac{3 + x}{3 - 2x}$

$12 - 8x = 3 + x$

$9 = 9x$

$x = 1$

$m = \dfrac{dy}{dx} = \dfrac{(3 - 2x)(1) - (3 + x)(-2)}{(3 - 2x)^2}$

$m = \dfrac{3 - 2x + 2(3 + x)}{(3 - 2x)^2}$

$m = \dfrac{3 - 2x + 6 + 2x}{(3 - 2x)^2}$

$m = \dfrac{9}{(3 - 2x)^2}$

When $x = 1$, $m = \dfrac{9}{(3 - 2)^1} = 9$

(1, 4), $m = 9$

The equation of the tangent is

$y - 4 = 9(x - 1)$

$y - 4 = 9x - 9$

$y = 9x - 5$

4 (a) $y = \dfrac{1}{3}x^3 - \dfrac{1}{2}x^2 - 3x - 9$

$m = \dfrac{dy}{dx} = x^2 - x - 3$

When $x = 6$, $m = 6^2 - 6 - 3 = 27$

Gradient of normal = $-\dfrac{1}{27}$

The equation of the normal is

$y - 27 = -\dfrac{1}{27}(x - 6)$

$27y - 729 = -x + 6$

$27y = -x + 735$

(b) $y = 2x + \dfrac{8}{x}$

$m = \dfrac{dy}{dx} = 2 - \dfrac{8}{x^2}$

$m = 2 - \dfrac{8}{(-1)^2} = -6$

Gradient of normal = $\dfrac{1}{6}$

The equation of the normal is

$y - (-10) = \dfrac{1}{6}(x - (-1))$

$6y + 60 = x + 1$

$6y = x - 59$

5 $y = \dfrac{x^2 + 2}{x} = x + \dfrac{2}{x}$

$\dfrac{dy}{dx} = 1 - \dfrac{2}{x^2} = \dfrac{1}{2}$

$\dfrac{2}{x^2} = \dfrac{1}{2}$

$x^2 = 4$

$x = \pm 2$

$y = \dfrac{(\pm 2)^2 + 2}{\pm 2}$

$y = \dfrac{4 + 2}{2}$ or $y = \dfrac{4 + 2}{-2}$

$y = 3$ or -3

The required points are (2, 3) or (–2, –3).

6 $y = 2x^2 - x + 5$

$m = \dfrac{dy}{dx} = 4x - 1$

$m = 4(1) - 1 = 3$

The equation of the tangent is

$y - 6 = 3(x - 1)$

$y = 3x + 3$

Gradient of perpendicular line = $-\dfrac{1}{3}$.

$4x - 1 = -\dfrac{1}{3}$

$4x = \dfrac{2}{3}$

$x = \dfrac{1}{6}$

7 The gradient of the line $10y = x + 5$ is $\frac{1}{10}$.

Thus, the gradient of the tangent is -10.

$y = 3x^2 + hx + k$

$\frac{dy}{dx} = 6x + h$

$-10 = 6(-2) + h$

$h = 2$

The curve passes through the point $(-2, 9)$.

$y = 3x^2 + 2x + k$

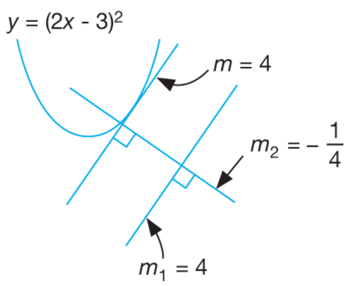
$9 = 3(-2)^2 + 2(-2) + k$

$9 = 12 - 4 + k$

$k = 1$

8 The gradient of the line $y = 4x - 2$ is 4.

Gradient of normal is $-\frac{1}{4}$.



Thus, the gradient of the tangent is 4.

$y = (2x - 3)^2$

$\frac{dy}{dx} = 2(2x - 3)(2) = 4$

$8x - 12 = 4$

$x = 2$

When $x = 2$, $y = (2 \times 2 - 3)^2 = 1$

The required point is (1, 2).

The equation of the normal is

$y - 1 = -\frac{1}{4}(x - 2)$

$4y - 4 = -x + 2$

$4y = -x + 6$

9 (a) Gradient of tangent $= \frac{18 - 2}{7 + 1} = 2$

$m = \frac{dy}{dx} = 2$

$2x - 4 = 2$

$x = 3$

When $x = 3$, $y = 3^2 - 4(3) - 3 = -6$

The coordinates of point R are (3, –6).

(b) Gradient of normal $= -\frac{1}{2}$

The equation of the normal is

$y - (-6) = -\frac{1}{2}(x - 3)$

$2y + 12 = -x + 3$

$2y = -x - 9$

UPSKILL 2.4c

1 (a) $y = 3x^2 - x^3$

$\frac{dy}{dx} = 6x - 3x^2$

$\frac{d^2y}{dx^2} = 6 - 6x$

At the turning points,

$\frac{dy}{dx} = 0$

$6x - 3x^2 = 0$

$3x(2 - x) = 0$

$x = 0$ or $x = 2$

When $x = 0$,

$y = 3(0)^2 - (0)^3 = 0$

Thus, (0, 0) is a turning point.

$\frac{d^2y}{dx^2} = 6 - 6(0) = 6$ (positive)

Hence, the minimum point is (0, 0).

When $x = 2$,

$y = 3(2)^2 - (2)^3 = 4$

Thus, (2, 4) is a turning point.

$\frac{d^2y}{dx^2} = 6 - 6(2) = -6$ (negative)

Hence, (2, 4) is a maximum point.

(b) $y = 4x + \frac{9}{x}$

$\frac{dy}{dx} = 4 - \frac{9}{x^2}$

$\frac{d^2y}{dx^2} = \frac{18}{x^3}$

At the turning points,

$\frac{dy}{dx} = 0$

$4 - \frac{9}{x^2} = 0$

$$\frac{9}{x^2} = 4$$
$$x^2 = \frac{9}{4}$$
$$x = \pm\frac{3}{2}$$
$$\frac{d^2y}{dx^2} = \frac{18}{x^3}$$
When $x = \frac{3}{2}$,
$$y = 4\left(\frac{3}{2}\right) + \frac{9}{\left(\frac{3}{2}\right)} = 12$$
$\left(\frac{3}{2}, 12\right)$ is a turning point.
$$\frac{d^2y}{dx^2} = \frac{18}{\left(\frac{3}{2}\right)^3} = \frac{16}{3} \text{ (positive)}$$
Hence, $\left(\frac{3}{2}, 12\right)$ is a minimum point.

When $x = -\frac{3}{2}$,
$$y = 4\left(-\frac{3}{2}\right) + \frac{9}{\left(-\frac{3}{2}\right)} = -12$$
$$\frac{d^2y}{dx^2} = \frac{18}{\left(-\frac{3}{2}\right)^3} = -\frac{16}{3} \text{ (negative)}$$
$\left(-\frac{3}{2}, -12\right)$ is a maximum point.

(c) $y = 4 - x^2 - \frac{16}{x^2}$
$$\frac{dy}{dx} = -2x + \frac{32}{x^3}$$
$$\frac{d^2y}{dx^2} = -2 - \frac{96}{x^4}$$

At the turning points,
$$\frac{dy}{dx} = 0$$
$$-2x + \frac{32}{x^3} = 0$$
$$\frac{32}{x^3} = 2x$$
$$x^4 = 16$$
$$x = \pm 2$$

When $x = 2$,
$$y = 4 - 2^2 - \frac{16}{2^2}$$
$$y = -4$$
Hence, (2, –4) is a turning point.

At the turning points,
$$\frac{dy}{dx} = 0$$
$$-2x + \frac{32}{x^3} = 0$$
$$\frac{32}{x^3} = 2x$$
$$x^4 = 16$$
$$x = \pm 2$$

When $x = 2$,
$$y = 4 - 2^2 - \frac{16}{2^2} = -4$$
(2, –4) is a turning point.
$$\frac{d^2y}{dx^2} = -2 - \frac{96}{2^4} = -8 \text{ (negative)}$$
Hence, (2, –4) is a maximum point.

When $x = -2$,
$$y = 4 - (-2)^2 - \frac{16}{(-2)^2} = -4$$

(–2, –4) is a turning point.
$$\frac{d^2y}{dx^2} = -2 - \frac{96}{(-2)^4} = -8 \text{ (negative)}$$
Hence, (–2, –4) is a maximum point.

(d) $y = x(x-3)^2$
$$y = x(x^2 - 6x + 9)$$
$$y = x^3 - 6x^2 + 9x$$
$$\frac{dy}{dx} = 3x^2 - 12x + 9$$
$$\frac{d^2y}{dx^2} = 6x - 12$$

At the turning points,
$$\frac{dy}{dx} = 0$$
$$3x^2 - 12x + 9 = 0$$
$$x^2 - 4x + 3 = 0$$

$(x-3)(x-1)=0$
$x=3$ or $x=1$

When $x = 3$,
$y=3(3-3)^2=0$
Thus, (3, 0) is a turning point.
$\frac{d^2y}{dx^2}=6(3)-12=6$ (positive)
Hence, (3, 0) is a minimum point.

When $x=1$,
$y=(1)(1-3)^2=4$
Hence, (1, 4) is a turning point.
$\frac{d^2y}{dx^2}=6(1)-12=-6$ (negative)
Hence, (1, 4) is a maximum point.

(e) $y=x^3+3x^2-9x-1$
$\frac{dy}{dx}=3x^2+6x-9$
$\frac{d^2y}{dx^2}=6x+6$

At the turning points,
$\frac{dy}{dx}=0$
$3x^2+6x-9=0$
$x^2+2x-3=0$
$(x+3)(x-1)=0$
$x=-3$ or $x=1$

When $x=-3$,
$y=(-3)^3+3(-3)^2-9(-3)-1=26$
Thus, (–3, 26) is a turning point.
$\frac{d^2y}{dx^2}=6(-3)+6=-12$ (negative)
Hence, $(-3, 26)$ is a maximum point.

When $x=1$, $y=1+3-9-1=-6$
Thus, (1, –6) is a turning point.
$\frac{d^2y}{dx^2}=6+6=12$ (positive)
Hence, (1, −6) is a minimum point.

2 (a) $y=hx^2+\frac{k}{x^2}$
The curve passes through the point (1, 17).
$17=h+k$ … (1)

$\frac{dy}{dx}=2hx-\frac{2k}{x^3}$
$-30=2h-2k$
$-15=h-k$ … (2)

(1) + (2) : $2h = 2$
$h = 1$

From (1) :
$17=1+k$
$k=16$

(b) $y=x^2+\frac{16}{x^2}$
$\frac{dy}{dx}=2x-\frac{32}{x^3}$
$\frac{d^2y}{dx^2}=2+\frac{96}{x^4}$

At the turning points,
$\frac{dy}{dx}=0$
$2x-\frac{32}{x^3}=0$
$2x=\frac{32}{x^3}$
$x^4=16$
$x=\pm 2$

When $x = 2$,
$y=2^2+\frac{16}{2^2}=8$
Thus, (2, 8) is a turning point.
$\frac{d^2y}{dx^2}=2+\frac{96}{2^4}=8$ (positive)
Hence, (2, 8) is a minimum point.

When $x = -2$,
$y=(-2)^2+\frac{16}{(-2)^2}=8$
Thus, (–2, 8) is a turning point.
$\frac{d^2y}{dx^2}=2+\frac{96}{(-2)^4}=8$ (positive)
Hence, (–2, 8) is a minimum point.

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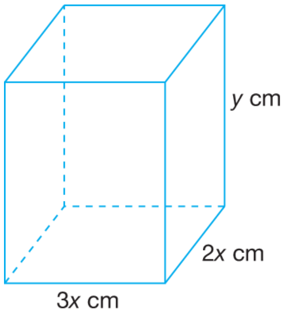
1



Let the height of the cuboid = y cm

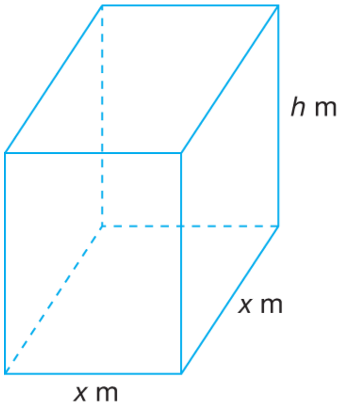
$$4(3x)+4(2x)+4y=200$$
$$3x+2x+y=50$$
$$y=50-5x$$

$$V=(3x)(2x)(y)$$
$$V=6x^2y$$
$$V=6x^2(50-5x)$$
$$V=30x^2(10-x)\ \text{[Shown]}$$
$$V=300x^2-30x^3$$

$$\frac{dV}{dx}=600x-90x^2=0$$
$$30x(20-3x)=0$$
$$x=\frac{20}{3}$$

$$V_{\text{maximum}}=300\left(\frac{20}{3}\right)^2-30\left(\frac{20}{3}\right)^3$$
$$=\frac{40\ 000}{9}\text{ cm}^3$$

2

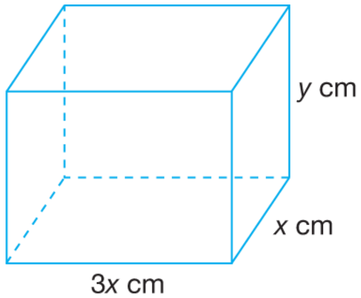


$$V=x^2h=8$$
$$h=\frac{8}{x^2}$$
$$A=2x^2+4hx$$
$$A=2x^2+4x\left(\frac{8}{x^2}\right)$$
$$A=2x^2+\frac{32}{x}$$
$$\frac{dA}{dx}=4x-\frac{32}{x^2}$$

When $\frac{dA}{dx}=4x-\frac{32}{x^2}=0$,
$$\frac{32}{x^2}=4x$$
$$x^3=8$$
$$x=2$$

$$h=\frac{8}{x^2}=\frac{8}{2^2}=2$$

3



$$A=2(3x^2)+2xy+2(3xy)=1274\text{ cm}^2$$
$$6x^2+8xy=1274$$
$$y=\frac{1274-6x^2}{8x}$$

$$P=4(3x)+4x+4y$$
$$P=16x+4y$$
$$P=16x+4\left(\frac{1274-6x^2}{8x}\right)$$
$$P=16x+\left(\frac{1274-6x^2}{2x}\right)$$
$$P=16x+\left(\frac{637-3x^2}{x}\right)$$
$$P=16x+\left(\frac{637}{x}-3x\right)$$

$$P = 13x + \frac{637}{x}$$

$$\frac{dP}{dx} = 13 - \frac{637}{x^2} = 0$$

$$\frac{637}{x^2} = 13$$

$$x^2 = 49$$

$$x = 7$$

$$P_{\text{minimum}} = 13(7) + \frac{637}{7} = 182 \text{ cm}$$

4 (a) Let $AB = DC = y$ cm

$$P = 2y + 2\pi r = 120$$

$$y + \pi r = 60$$

$$y = 60 - \pi r$$

$$L = 2ry - \pi r^2$$

$$L = 2r(60 - \pi r) - \pi r^2$$

$$L = 120r - 3\pi r^2 \text{ [Shown]}$$

(b) $\frac{dL}{dr} = 120 - 6\pi r = 0$

$$6\pi r = 120$$

$$r = \frac{120}{6\pi}$$

$$r = \frac{20}{\pi}$$

$$\frac{d^2L}{dr^2} = -6\pi \ [< 0]$$

Thus, the value of L is a maximum.

5 (a) $V = 96 \text{ cm}^3$

$$\frac{1}{2}(4x)(3x)(y) = 96$$

$$6x^2y = 96$$

$$y = \frac{96}{6x^2}$$

$$y = \frac{16}{x^2}$$

$$A = (4x)(3x) + 3xy + 4xy + 5xy$$

$$A = 12x^2 + 12xy$$

$$A = 12x^2 + 12x\left(\frac{16}{x^2}\right)$$

$$A = 12x^2 + \frac{192}{x} \text{ [Shown]}$$

(b) $\frac{dA}{dx} = 24x - \frac{192}{x^2} = 0$

$$\frac{192}{x^2} = 24x$$

$$x^3 = 8$$

$$x = 2$$

$$h = 3x = 3(2) = 6 \text{ cm}$$

6 (a) $V = \pi r^2 h = 192\pi$

$$h = \frac{192}{r^2}$$

$$A = \pi r^2 + 2\pi rh + 2\pi r^2$$

$$A = 3\pi r^2 + 2\pi r\left(\frac{192}{r^2}\right)$$

$$A = 3\pi r^2 + 2\pi\left(\frac{192}{r}\right)$$

$$A = 3\pi r^2 + \pi\left(\frac{384}{r}\right) \text{ [Shown]}$$

(b) $\frac{dA}{dr} = 6\pi r - \frac{384\pi}{r^2} = 0$

$$6\pi r = \frac{384\pi}{r^2}$$

$$r^3 = 64$$

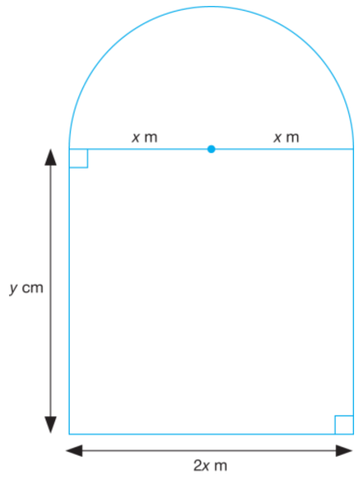
$$r = 4$$

$$A_{\text{maximum}} = 3\pi(4)^2 + \pi\left(\frac{384}{4}\right)$$

$$= 48\pi + 96\pi$$

$$= 144\pi \text{ cm}^2$$

7



(a) Perimeter = 6 m

$2y + 2x + \pi x = 6$

$2y = 6 - 2x - \pi x$

$y = \dfrac{6 - 2x - \pi x}{2}$

A = Area of rectangle + Area of semicircle

$= 2xy + \dfrac{1}{2}\pi x^2$

$= 2x\left(\dfrac{6 - 2x - \pi x}{2}\right) + \dfrac{1}{2}\pi x^2$

$= 6x - 2x^2 - \pi x^2 + \dfrac{1}{2}\pi x^2$

$= 6x - 2x^2 - \dfrac{1}{2}\pi x^2$

(b) $\dfrac{dA}{dx} = 6 - 4x - \pi x$

When A has a stationary value,

$\dfrac{dA}{dx} = 0$

$6 - 4x - \pi x = 0$

$4x + \pi x = 6$

$(4 + \pi)x = 6$

$x = \dfrac{6}{4 + \pi}$

$\dfrac{d^2A}{dx^2} = -4 - \pi$ (negative)

Hence, A is a maximum.

Hence, when the surface area of the window is a maximum, the width of the window

$= 2x$

$= 2\left(\dfrac{6}{4 + 3.142}\right)$

$= 1.680$ m

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1 $A = \pi r^2$

$\dfrac{dA}{dr} = 2\pi r$

$\dfrac{dA}{dt} = \dfrac{dA}{dr} \times \dfrac{dr}{dt} = 2\pi(4) \times 0.5 = 4\pi \text{ cm}^2\text{s}^{-1}$

2 (a) $V = \dfrac{4}{3}\pi r^3$

$\dfrac{dV}{dr} = 4\pi r^2$

$\dfrac{dV}{dt} = \dfrac{dV}{dr} \times \dfrac{dr}{dt} = 4\pi r^2 \times 0.5$

$= 4\pi(4)^2 \times 0.5$

$= 32\pi \text{ cm}^3\text{s}^{-1}$

(b) $A = 4\pi r^2$

$\dfrac{dA}{dr} = 8\pi r$

$\dfrac{dA}{dt} = \dfrac{dA}{dr} \times \dfrac{dr}{dt} = 8\pi(4) \times 0.5 = 16\pi \text{ cm}^2\text{s}^{-1}$

3 $V = \pi r^2 h = \pi(30)^2 h = 900\pi h$

$\dfrac{dV}{dh} = 900\pi$

$\dfrac{dh}{dt} = \dfrac{dh}{dV} \times \dfrac{dV}{dt}$

$= \dfrac{1}{900\pi} \times 90$

$= \dfrac{1}{10\pi} \text{ cms}^{-1}$

4 $V = 6\pi h^2 + \dfrac{1}{3}\pi h^3$

$\dfrac{dV}{dh} = 12\pi h + \pi h^2$

$\dfrac{dh}{dt} = \dfrac{dh}{dV} \times \dfrac{dV}{dt}$

$= \dfrac{1}{12\pi h - \pi h^2} \times 3$

$= \dfrac{1}{12\pi(2) - \pi(2)^2} \times 3$

$= \dfrac{1}{20\pi} \times 3$

$= \frac{3}{20\pi}$ cms^{-1}

5 Let the area of $\Delta ABC = L$ cm^2

$L = \frac{1}{2}(x)(3x)\sin 150^o$

$L = \frac{3}{2}x^2\left(\frac{1}{2}\right)$

$L = \frac{3}{4}x^2$

$\frac{dL}{dx} = \frac{3}{2}x$

$\frac{dL}{dt} = \frac{dL}{dx} \times \frac{dx}{dt}$

$= \frac{3}{2}x \times (0.5)$

$= \frac{3}{4}x$

$= \frac{3}{4} \times 4$

$= 3$ cm^{2}s^{-1}

6 $V = \frac{4}{3}\pi r^3$

$\frac{dV}{dr} = 4\pi r^2$

$\frac{dr}{dt} = \frac{dr}{dV} \times \frac{dV}{dt}$

$= \frac{1}{4\pi r^2} \times 10\pi$

$= \frac{1}{4\pi(4)^2} \times 10\pi$

$= \frac{5}{32}$ cms^{-1}

7 (a) $V = x^3$

$\frac{dV}{dx} = 3x^2$

$\frac{dV}{dt} = \frac{dV}{dx} \times \frac{dx}{dt} = 3(5)^2 \times (-2) = -150$ cm^{3}s^{-1}

(b) $A = 6x^2$

$\frac{dA}{dx} = 12x$

$\frac{dA}{dt} = \frac{dA}{dx} \times \frac{dx}{dt} = 12(5) \times (-2) = -120$ cm^{2}s^{-1}

8 $\frac{h}{16} = \frac{r}{4}$

$r = \frac{4}{16}h = \frac{h}{4}$

$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\left(\frac{h}{4}\right)^2 h = \frac{1}{48}\pi h^3$

$\frac{dV}{dh} = \frac{1}{16}\pi h^2$

$\frac{dh}{dt} = \frac{dh}{dV} \times \frac{dV}{dt}$

$= \frac{16}{\pi(8)^2} \times 2$

$= \frac{1}{2\pi}$ cms^{-1}

UPSKILL 2.4e

1 $y = 2x^3 - 8x^2 + 11$

$\frac{dy}{dx} = 6x^2 - 16x$

$\delta y = \frac{dy}{dx} \times \delta x$

$= (6x^2 - 16x) \times 0.02$

$= (6(2)^2 - 16(2)) \times 0.02$

$= -0.16$

The value of y decreases.

2 $y = 4x - \frac{6}{x}$

$\frac{dy}{dx} = 4 + \frac{6}{x^2}$

$\delta y = \frac{dy}{dx} \times \delta x = \left(4 + \frac{6}{x^2}\right) \times (2 + k - 2)$

$\delta y = \left(4 + \frac{6}{2^2}\right) \times k = \frac{11}{2}k$

3 $A = \pi r^2$

$\frac{dA}{dr} = 2\pi r$

$\delta A = \frac{dA}{dr} \times \delta r = 2\pi(5) \times (0.03) = 0.3\,\pi$ cm^2

4 $V=\frac{1}{3}\pi r^2 h$

$V=\frac{1}{3}\pi r^2(5)$

$V=\frac{5}{3}\pi r^2$

$\frac{dV}{dr}=\frac{10}{3}\pi r$

$\delta V=\frac{dV}{dr}\times\delta r$

$=\frac{10}{3}\pi r\times(3.15-3)$

$=\frac{10}{3}\pi(3)\times(3.15-3)$

$=1.5\pi\ \text{cm}^3$

5 $V=\pi r^2 h$

$V=\pi(4)^2 h$

$V=16\pi h$

$\delta V=\frac{dV}{dh}\times\delta h$

$=16\pi\times0.25$

$=4\pi\ \text{cm}^3$

6 $V=\frac{4}{3}\pi r^3$

$\frac{dV}{dr}=4\pi r^2$

$\delta V=\frac{dV}{dr}\times\delta r$

$=4\pi r^2\times(-0.2)$

$=4\pi(10)^2\times(-0.2)$

$=80\pi\ \text{cm}^3$

7 $A=6x^2$

$\frac{dA}{dx}=12x$

$\delta A=\frac{dA}{dx}\times\delta x=12x(-0.1)=-1.2(6)$

$=-7.2\ \text{cm}^2$

8 $y=\frac{5}{x^3}$

$\frac{dy}{dx}=-\frac{15}{x^4}$

When $x=4$, $\frac{dy}{dx}=-\frac{15}{4^4}=-\frac{15}{256}$

(a) $y_{\text{new}}=y_{\text{original}}+\frac{dy}{dx}\times\delta x$

$\frac{5}{4.02^3}=\frac{5}{4^3}+\left(-\frac{15}{x^4}\right)(0.02)$

$=\frac{5}{64}+\left(-\frac{15}{4^4}\right)(0.02)$

$=0.0770$

(b) $y_{\text{new}}=y_{\text{original}}+\frac{dy}{dx}\times\delta x$

$\frac{5}{3.99^3}=\frac{5}{4^3}+\left(-\frac{15}{x^4}\right)(-0.01)$

$=\frac{5}{64}+\left(-\frac{15}{4^4}\right)(-0.01)$

$=0.0787$

Summative Practice 2

1 $\lim_{x\to-4}\left(\frac{x^2-16}{x^4+4x}\right)$

$=\lim_{x\to-4}\left[\frac{(x+4)(x-4)}{x(x+4)}\right]$

$=\lim_{x\to-4}\left[\frac{(x-4)}{x}\right]$

$=\frac{-4-4}{-4}$

$=2$

2 $y=x+\frac{1}{x}\ \ldots(1)$

$y+\delta y=x+\delta x+\frac{1}{x+\delta x}\ \ldots(2)$

$(2)-(1):$

$\delta y=\delta x+\frac{1}{x+\delta x}-\frac{1}{x}$

$\delta y=\delta x+\frac{x-(x+\delta x)}{x(x+\delta x)}$

$\delta y=\delta x+\frac{-\delta x}{x(x+\delta x)}$

$\frac{\delta y}{\delta x}=1+\frac{-1}{x(x+\delta x)}$

$\frac{dy}{dx}=\lim_{\delta x\to0}\frac{\delta y}{\delta x}$

$\frac{dy}{dx}=\lim_{\delta x\to0}\left[1+\frac{-1}{x(x+\delta x)}\right]$

$\frac{dy}{dx}=1+\frac{-1}{x(x+0)}$

$\frac{dy}{dx} = 1 - \frac{1}{x^2}$

3 $y = x^2(1-3x)^3$

$\frac{dy}{dx} = x^2(3)(1-3x)^2(-3) + (1-3x)^3(2x)$

$= x(1-3x)^2[-9x + 2(1-3x)]$

$= x(1-3x)^2[-9x + 2 - 6x]$

$= x(1-3x)^2(2-15x)$

4 $y = \frac{2}{t}$ | $x = 2t - 1$

$\frac{dy}{dt} = -\frac{2}{t^2}$ | $\frac{dx}{dt} = 2$

$t = \frac{x+1}{2}$

$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{-\frac{2}{t^2}}{2} = -\frac{1}{t^2}$

$\frac{dy}{dx} = -\frac{1}{\left(\frac{x+1}{2}\right)^2} = -\frac{4}{(x+1)^2}$

5 $y = (3-x)^2(2x-3)^4$

$\frac{dy}{dx} = (3-x)^2(4)(2x-3)^3(2) + (2x-3)^4(2)(3-x)(-1)$

$\frac{dy}{dx} = 2(3-x)(2x-3)^3[4(3-x) - (2x-3)]$

$\frac{dy}{dx} = 2(3-x)(2x-3)^3(15-6x)$

$\frac{dy}{dx} = 2(3-x)(2x-3)^3(3)(5-2x)$

$\frac{dy}{dx} = 6(3-x)(2x-3)^3(5-2x)$

6 $y = \frac{1}{3}u^6$

$y = \frac{1}{3}(3x-6)^6$

$\frac{dy}{dx} = \frac{6}{3}(3x-6)^5(3)$

$= 6(3x-6)^5$

7 $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{14t^3}{2} = 7t^3$

From $x = 2t + 3$, $t = \frac{x-3}{2}$

Hence, $\frac{dy}{dx} = 7\left(\frac{x-3}{2}\right)^3$

$= \frac{7}{8}(x-3)^3$

8 $y = \frac{3x}{2x+5}$

$\frac{dy}{dx} = \frac{(2x+5)(3) - (3x)(2)}{(2x+5)^2}$

$= \frac{15}{(2x+5)^2}$

$\frac{dy}{dx} = 15(2x+5)^{-2}$

$\frac{d^2y}{dx^2} = -30(2x+5)^{-3}(2)$

$= \frac{-60}{(2x+5)^3}$

9 $y = px^2 + \frac{q}{x}$

The curve passes through the point (2, 3).

$3 = p(2)^2 + \frac{q}{2}$

$6 = 8p + q$ … (1)

$\frac{dy}{dx} = 2px - \frac{q}{x^2}$

$\frac{3}{2} = 2p(2) - \frac{q}{2^2}$

$6 = 16p - q$ … (2)

(1) + (2) :

$24p = 12$

$p = \frac{1}{2}$

From (1) :

$6 = 8\left(\frac{1}{2}\right) + q$

$q = 2$

10 If the gradient of normal is $-\frac{1}{9}$, hence the gradient of the tangent is 9.

$y = (3x-2)^3$

$\frac{dy}{dx} = 3(3x-2)^2(3)$

$\frac{dy}{dx} = 9(3x-2)^2$

$9 = 9(3-2x)^2$

$(3x-2)^2 = 1$

$9x^2 - 12x + 4 = 1$

$9x^2 - 12x + 3 = 0$

$3x^2 - 4x + 1 = 0$

$(x-1)(3x-1) = 0$

$x = 1$ or $x = \frac{1}{3}$ $y = (3x-2)^3$

When $x = 1$, $y = (3-2)^3 = 1$

When $x = \frac{1}{3}$, $y = \left(3 \times \frac{1}{3} - 2\right)^3 = -1$

Hence, the required points are (1, 1) and $\left(\frac{1}{3}, -1\right)$.

11 (a) The equation of the normal is

$4y + x = p$

$4y = -x + p$

$y = -\frac{1}{4}x + \frac{p}{4}$

Gradient of normal = $-\frac{1}{4}$

Gradient of tangent = 4

$y = (2x-3)^2 - 4$

$\frac{dy}{dx} = 2(2x-3)(2)$

$\frac{dy}{dx} = 8x - 12$

$4 = 8x - 12$

$8x = 16$

$x = 2$

When $x = 2$,

$y = (2 \times 2 - 3)^2 - 4 = -3$

Hence, the coordinates of point Q are $(2, -3)$.

$4y + x = p$

At $(2, -3)$,

$4(-3) + 2 = p$

$p = -10$

(b) The equation of the tangent is

$y - (-3) = 4(x-2)$

$y + 3 = 4x - 8$

$y = 4x - 11$

12 $y = \frac{2x-6}{x+3}$

$\frac{dy}{dx} = \frac{(x+3)(2) - (2x-6)(1)}{(x+3)^2}$

$\frac{dy}{dx} = \frac{2x+6-2x+6}{(x+3)^2}$

$\frac{dy}{dx} = \frac{12}{(x+3)^2}$

At point P (x-axis), $y = 0$

$y = \frac{2x-6}{x+3}$

$0 = \frac{2x-6}{x+3}$

$2x - 6 = 0$

$x = 3$

$\frac{dy}{dx} = \frac{12}{(3+3)^2}$

$m = \frac{12}{36} = \frac{1}{3}$

13 $y = \frac{1}{x^2} - \frac{1}{x^3}$

$\frac{dy}{dx} = -\frac{2}{x^3} + \frac{3}{x^4}$

$\frac{d^2y}{dx^2} = \frac{6}{x^4} - \frac{12}{x^5}$

$x^4\left(\frac{dy}{dx} + \frac{d^2y}{dx^2}\right) + x^2y + 5 = 0$

$x^4\left(-\frac{2}{x^3} + \frac{3}{x^4} + \frac{6}{x^4} - \frac{12}{x^5}\right) + x^2\left(\frac{1}{x^2} - \frac{1}{x^3}\right) + 5 = 0$

$-2x + 9 - \frac{12}{x} + 1 - \frac{1}{x} + 5 = 0$

$-2x + 15 - \frac{13}{x} = 0$

$-2x^2 + 15x - 13 = 0$

$2x^2 - 15x + 13 = 0$

$(2x-13)(x-1) = 0$

$x = \frac{13}{2}$ or $x = 1$

14 $y=(x+1)^2(x-2)$

$y=\left(x^2+2x+1\right)(x-2)$

$y=x^3+2x^2+x-2x^2-4x-2$

$y=x^3-3x-2$

$\frac{dy}{dx}=3x^2-3$

$\frac{d^2y}{dx^2}=6x$

At the turning points,

$\frac{dy}{dx}=0$

$3x^2-3=0$

$x^2-1=0$

$x^2=1$

$x=\pm1$

When $x=1$, $y=1^3-3(1)-2=-4$

Thus, $(1,-4)$ is a turning point.

$\frac{d^2y}{dx^2}=6(1)=6$ (positive)

Hence, $(1,-4)$ is a minimum point.

When $x=-1$, $y=(-1)^3-3(-1)-2=0$

$\frac{d^2y}{dx^2}=6(-1)=-6$ (negative)

Hence, $(-1,0)$ is a maximum point.

15 (a) $y=ax^3+bx+c$

$\frac{dy}{dx}=3ax^2+b$

At the turning points,

$\frac{dy}{dx}=0$

$3ax^2+b=0$

$3a(1)^2+b=0$

$3a+b=0$ … (1)

The curve passes through the point (1, 1).

$1=a(1)^3+b(1)+c$

$a+b+c=1$ … (2)

The curve passes through the point (–1, 5).

$5=a(-1)^3+b(-1)+c$

$-a-b+c=5$ … (3)

(2) – (3) : $2a+2b=-4$

$a+b=-2$ … (4)

$3a+b=0$ … (1)

(–) $a+b=-2$ … (4)

$2a=2$

$a=1$

From (1) :

$3(1)+b=0$

$b=-3$

From (2) :

$1-3+c=1$

$c=3$

(b) $y=x^3-3x+3$

$\frac{dy}{dx}=3x^2-3$

$\frac{d^2y}{dx^2}=6x$

For (1, 1):

$\frac{d^2y}{dx^2}=6(1)=6$ (positive)

Hence, (1, 1) is a minimum point.

For (–1, 5):

$\frac{d^2y}{dx^2}=6(-1)=-6$ (negative)

Hence, (–1, 5) is a maximum point.

16 (a) $y=\frac{x^3}{3}-\frac{x^2}{2}-2x+\frac{1}{6}$

$\frac{dy}{dx}=\frac{3x^2}{3}-\frac{2x}{2}-2$

$=x^2-x-2$

At the point $A(1, -2)$,

$m=\frac{dy}{dx}=1^2-1-2=-2$

(b) m (tangent) = –2

m (normal) = $\frac{1}{2}$

Equation of normal at the point $A(1,-2)$ is

$y-(-2)=\frac{1}{2}(x-1)$

$2y+4=x-1$

$2y=x-5$

(c) At the turning points,

$$\frac{dy}{dx}=0$$
$$x^2-x-2=0$$
$$(x-2)(x+1)=0$$
$$x=2 \text{ or } -1$$

$x=-1$ is not accepted.
$\therefore x=2$

When $x=2$,

$$y=\frac{2^3}{3}-\frac{2^2}{2}-2(2)+\frac{1}{6}=-3\frac{1}{6}$$

Hence, Q is point $\left(2, -3\frac{1}{6}\right)$.

$$\frac{d^2y}{dx^2}=2x-1$$

When $x=2$,

$$\frac{d^2y}{dx^2}=2(2)-1=3 \text{ (positive)}$$

Hence, $\left(2, -3\frac{1}{6}\right)$ is a minimum point.

17 $y=12-x^3-\frac{48}{x}=12-x^3-48x^{-1}$

$$\frac{dy}{dx}=-3x^2+48x^{-2}=-3x^2+\frac{48}{x^2}$$
$$\frac{d^2y}{dx^2}=-6x-96x^{-3}=-6x-\frac{96}{x^3}$$

At the turning points,

$$\frac{dy}{dx}=0$$
$$-3x^2+\frac{48}{x^2}=0$$
$$\frac{48}{x^2}=3x^2$$
$$x^4=\frac{48}{3}$$
$$x^4=16$$
$$x=\pm 2$$

When $x=2$,

$$y=12-2^3-\frac{48}{2}=-20$$

(2, –20) is a turning point.

$$\frac{d^2y}{dx^2}=-6(2)-\frac{96}{2^3}=-24 \text{ (negative)}$$

Hence, (2, –20) is a maximum point.

When $x=-2$,

$$y=12-(-2)^3-\frac{48}{(-2)}=44$$

(–2, 44) is a turning point.

$$\frac{d^2y}{dx^2}=-6(-2)-\frac{96}{(-2)^3}=24 \text{ (positive)}$$

Hence, (–2, 44) is a minimum point.

18 (a) A = Area of rectangle – Area of triangle

$$A=60\times 40-\frac{1}{2}(3x)(40-x)$$
$$A=2\ 400-60x+\frac{3}{2}x^2 \text{ [Shown]}$$

(b) $\frac{dA}{dx}=-60+3x$

When A has a stationary value,

$$\frac{dA}{dx}=0$$
$$-60+3x=0$$
$$x=20$$

$$\frac{d^2A}{dx^2}=3 \text{ (positive)}$$

Hence, the minimum value of A

$$=2\ 400-60(20)+\frac{3}{2}(20)^2$$

= 1 800 cm^2

19 (a) $PR^2=x^2+10^2$

$$PR=\sqrt{x^2+100}$$

Radius of circle

$$=\frac{\sqrt{x^2+100}}{2}$$

A = Area of circle – Area of triangle

$$A=\pi\left(\frac{\sqrt{x^2+100}}{2}\right)^2-\frac{1}{2}x(10)$$
$$A=\pi\left(\frac{x^2+100}{4}\right)-5x$$
$$A=25\pi+\frac{\pi}{4}x^2-5x \text{ [Shown]}$$

(b) When A has a stationary value,

$$\frac{dA}{dx}=0$$

$$\frac{1}{2}\pi x-5=0$$

$$\pi x=10$$

$$x=\frac{10}{\pi}$$

$$\frac{d^2A}{dx^2}=\frac{1}{2}\pi \text{ (positive)}$$

Hence, the minimum value of A

$$=25\pi+\frac{\pi}{4}\left(\frac{10}{\pi}\right)^2-5\left(\frac{10}{\pi}\right)$$

$$=25\pi+\frac{25}{\pi}-\frac{50}{\pi}$$

$$=\left(25\pi-\frac{25}{\pi}\right)\text{cm}^2$$

20 (a) Perimeter of the rectangle

$$2x+2y=72$$

$$x+y=36$$

$$y=36-x$$

$$V=\pi r^2h$$

$$V=\pi\left(\frac{x}{2}\right)^2(y)$$

$$V=\pi\left(\frac{x^2}{4}\right)(36-x)$$

$$V=\frac{\pi x^2}{4}(36-x) \text{ [Shown]}$$

$$V=9\pi x^2-\frac{\pi}{4}x^3$$

$$\frac{dV}{dx}=18\pi x-\frac{\pi}{4}\left(3x^2\right)$$

$$\frac{dV}{dx}=18\pi x-\frac{3\pi}{4}x^2$$

When V has a stationary value

$$\frac{dV}{dx}=0$$

$$18\pi x-\frac{3\pi}{4}x^2=0$$

$$18\pi x=\frac{3\pi}{4}x^2$$

$$6x=\frac{1}{4}x^2$$

$$24x=x^2$$

$$x^2-24x=0$$

$$x(x-24)=0$$

$x=0$ or $x=24$

$x=0$ is not accepted.

$\therefore x=24$

$$\frac{d^2V}{dx^2}=18\pi-\frac{3\pi}{2}x$$

When x = 24, $\frac{d^2V}{dx^2}=18\pi-\frac{3\pi}{2}(24)$

$= -18\pi$ (negative)

Hence, the maximum value of V

$$= 9\pi(24)^2-\frac{\pi}{4}(24)^3$$

$$= 5\ 184\pi-3\ 456\pi$$

$$= 1\ 782\ \pi \text{ cm}^3$$

(b) $A=2\pi rh$

$$A=2\pi\left(\frac{x}{2}\right)(y)$$

$$A=\pi x(36-x)$$

$$A=36\pi x-\pi x^2$$

$$\frac{dA}{dx}=36\pi-2\pi x$$

When A has a stationary value,

$$\frac{dA}{dx}=0$$

$$36\pi-2\pi x=0$$

$$18-x=0$$

$$x=18$$

$$\frac{d^2A}{dx^2}=-2\pi \text{ (negative)}$$

Hence, the maximum value of A

$$= 36\pi(18)-\pi(18)^2$$

$$= 648\ \pi - 324\ \pi$$

$$= 324\ \pi \text{ cm}^2$$

21

$$V=\pi r^2h$$

$$\pi r^2h=686\pi$$

$$r^2h=686$$

$$h=\frac{686}{r^2}$$

$$A=2\pi r^2+2\pi rh$$

$$A=2\pi r^2+2\pi r\left(\frac{686}{r^2}\right)$$

$A = 2\pi r^2 + \frac{1\ 372\pi}{r}$ [Shown]

$\frac{dA}{dr} = 4\pi r - \frac{1\ 372\pi}{r^2}$

When A has a stationary value,

$\frac{dA}{dr} = 0$

$4\pi r - \frac{1\ 372\pi}{r^2} = 0$

$4r = \frac{1\ 372}{r^2}$

$r^3 = 343$

$r = 7$

$\frac{d^2A}{dr^2} = 4\pi + \frac{2\ 744}{r^3}$ (positive)

Hence, the minimum value of A

$= 2\pi(7)^2 + \frac{1\ 372\pi}{7}$

$= 294\pi\ \text{cm}^2$

22 (a) Perimeter = 100

$2x + 2y = 100$

$x + y = 50$

$x = 50 - y$

A = Area of rectangle – Area of the two semicircles

$A = xy - \pi r^2$

$A = y(50 - y) - \pi\left(\frac{y}{2}\right)^2$

$A = 50y - y^2 - \frac{\pi y^2}{4}$

$A = 50y - \frac{4y^2 + \pi y^2}{4}$

$A = 50y - \left(\frac{4+\pi}{4}\right)y^2$ [Shown]

(b) $\frac{dA}{dy} = 50 - 2y\left(\frac{4+\pi}{4}\right) = 50 - y\left(\frac{4+\pi}{2}\right)$

When A has a stationary value,

$\frac{dA}{dy} = 0$

$50 - y\left(\frac{4+\pi}{2}\right) = 0$

$y\left(\frac{4+\pi}{2}\right) = 50$

$y = \frac{100}{4+\pi}$

Width = $y = \frac{100}{4+\pi}$ cm

$x = 50 - y$

$x = 50 - \left(\frac{100}{4+\pi}\right)$

$x = \frac{50(4+\pi) - 100}{4+\pi}$

$x = \frac{200 + 50\pi - 100}{4+\pi}$

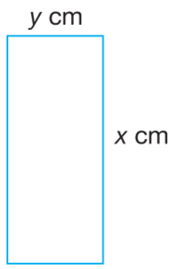
$x = \frac{50\pi + 100}{4+\pi}$

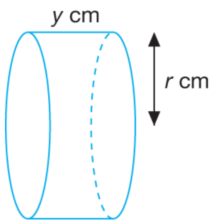
Length = $x = \frac{50\pi + 100}{4+\pi}$ cm

$\frac{d^2A}{dy^2} = -\left(\frac{4+\pi}{2}\right)$ [negative]

Hence, the value of A is a maximum.

23





Perimeter of the rectangle = 50 cm

$2x + 2y = 50$

$x + y = 25$

$y = 25 - x$

Perimeter of the right part of the cylinder is equal to the length of the rectangle

$2\pi r = x$

$r = \frac{x}{2\pi}$

Volume of the cylinder

$V = \pi r^2 h$

$V = \pi \left(\frac{x}{2\pi}\right)^2 y$

$V = \pi \left(\frac{x^2}{4\pi^2}\right)(25 - x)$

$V = \frac{1}{4\pi}\left(25x^2 - x^3\right)$

$\frac{dV}{dx} = \frac{1}{4\pi}\left(50x - 3x^2\right)$

$\frac{dV}{dx} = \frac{1}{4\pi}x(50 - 3x)$

When V has a stationary value,

$\frac{dV}{dx} = 0$

$\frac{1}{4\pi}x(50 - 3x) = 0$

$x = \frac{50}{3}$

$y = 25 - \frac{50}{3} = \frac{25}{3}$

$\frac{d^2V}{dx^2} = \frac{1}{4\pi}(50 - 6x)$

When $x = \frac{50}{3}$,

$\frac{d^2V}{dx^2} = \frac{1}{4\pi}\left[50 - 6\left(\frac{50}{3}\right)\right] = -\frac{25}{2\pi}\ (< 0)$

Hence, the volume of the cylinder is a maximum.

Length = $16\frac{2}{3}$ cm

Width = $8\frac{1}{3}$ cm

24 V = Area of triangle $ABC \times CD$

$V = \frac{1}{2}(x)(10) \times 5x$

$V = 5x(5x)$

$V = 25x^2$

$\frac{dV}{dx} = 50x$

$\frac{dV}{dt} = \frac{dV}{dx} \times \frac{dx}{dt}$

$= 50x \times 0.02$

$= 50(5) \times 0.02$

$= 5$ cm^{3}s^{-1}

25 $A = 2\pi r^2 + 2\pi r(12)$

$A = 2\pi r^2 + 24\pi r$

$\frac{dA}{dr} = 4\pi r + 24\pi$

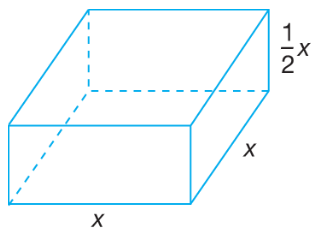
$\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dt}$

$\frac{dA}{dt} = (4\pi r + 24\pi) \times 0.1$

$\frac{dA}{dt} = [4\pi(4) + 24\pi] \times 0.1$

$\frac{dA}{dt} = 4\pi$ cm^{2}s^{-1}

26 $V = x^2\left(\frac{1}{2}x\right)$



$V = \frac{1}{2}x^3$

$\frac{dV}{dx} = \frac{3}{2}x^2$

When $A = 2x^2 \times 4\left(\frac{1}{2}x\right)(x) = 4x^2$

When $A = 1\,600$,

$4x^2 = 1\,600$

$x^2 = 400$

$x = 20$

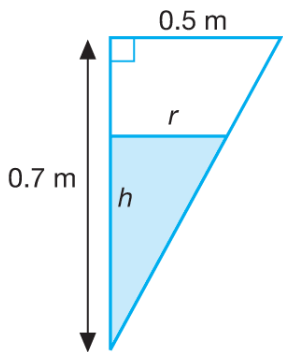
$\frac{dV}{dt} = \frac{dV}{dx} \times \frac{dx}{dt}$

$= \frac{3}{2}x^2 \times 0.02$

$= \frac{3}{2}(20)^2 \times 0.02$

$= 12$ cm^{3}s^{-1}

27



Using the ratios of similar triangles,

$$\frac{r}{0.5} = \frac{h}{0.7}$$

$$r = \frac{h}{0.7} \times 0.5$$

$$r = \frac{5}{7}h$$

$$V = \frac{1}{3}\pi r^2 h$$

$$V = \frac{1}{3}\pi\left(\frac{5}{7}h\right)^2 h$$

$$V = \frac{1}{3}\pi\left(\frac{25}{49}h^2\right)h$$

$$V = \frac{25}{147}\pi h^3$$

$$\frac{dV}{dh} = \frac{25}{49}\pi h^2$$

$$\frac{dh}{dt} = \frac{dh}{dV} \times \frac{dV}{dt}$$

$$= \frac{49}{25\pi h^2} \times 0.1$$

$$= \frac{49}{25(3.142)(0.3)^2} \times 0.1$$

$$= 0.693 \text{ cms}^{-1}$$

28 (a) $m = 5x - 2$

$$\frac{dm}{dx} = 5$$

$$\frac{dx}{dt} = \frac{dx}{dm} \times \frac{dm}{dt}$$

$$= \frac{1}{5} \times 2$$

$$= \frac{2}{5} \text{ unit s}^{-1}$$

(b) $m = 5x - 2$ $\quad y = -\frac{4}{m^2}$

$$\frac{dm}{dx} = 5 \qquad \frac{dy}{dm} = \frac{8}{m^3}$$

$$x = \frac{m+2}{5}$$

$$\frac{dy}{dx} = \frac{\frac{dy}{dm}}{\frac{dx}{dm}} = \frac{\frac{8}{m^3}}{\frac{1}{5}} = \frac{40}{m^3} = \frac{40}{(5x-2)^3}$$

(c) $\frac{\delta y}{\delta x} \approx \frac{dy}{dx}$

$$\delta y \approx \frac{dy}{dx} \times \delta x$$

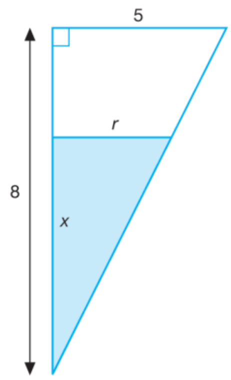
$$= \frac{40}{(5x-2)^3} \times \left(\frac{4}{5} - 1\right)$$

$$= \frac{40}{(5 \times 1 - 2)^3} \times \left(-\frac{1}{5}\right)$$

$$= -\frac{40}{27} \times \frac{1}{5}$$

$$= -\frac{8}{27}$$

29 (a)



Using the ratios of similar triangles,

$$\frac{r}{5} = \frac{x}{8}$$

$$r = \frac{5x}{8}$$

$$V = \frac{1}{3}\pi r^2 h$$

$$V = \frac{1}{3}\pi\left(\frac{5x}{8}\right)^2 (x)$$

$V = \frac{25}{192}\pi x^3$ [Shown]

(b) $\frac{\delta V}{\delta x} \approx \frac{dV}{dx}$

$\delta V \approx \frac{dV}{dx} \times \delta x$

$= \frac{25}{64}\pi x^2 \times (4.08 - 4)$

$= \frac{25}{64}\pi (4)^2 (0.08)$

$= 0.5\pi$ cm^3

30 ` $y = \frac{27}{x^4}$

$\frac{dy}{dx} = \frac{-108}{x^5}$

When $x = 3$

$\frac{dy}{dx} = \frac{-108}{3^5} = -\frac{4}{9}$

(a) $y_{\text{new}} = y_{\text{originall}} + \frac{dy}{dx} \times \delta x$

$\frac{27}{2.99^4} = \frac{27}{3^4} + \left(-\frac{4}{9}\right)(2.99 - 3)$

= 0.3378

(b) $\frac{27}{3.01^4} = \frac{27}{3^4} + \left(-\frac{4}{9}\right)(3.01 - 3)$

= 0.3289