

**Form 5 Chapter 1**  
**Circular Measure**  
**Fully-Worked Solutions**

**UPSKILL 1.1**

1 (a)  $53.6^\circ = 53.6 \times \frac{3.142}{180} = 0.9356 \text{ rad}$   
 (b)  $126.4^\circ = 126.4 \times \frac{3.142}{180} = 2.2064 \text{ rad}$   
 (c)  $37^\circ 38' = 37 \frac{38}{60} \times \frac{3.142}{180} = 0.6569 \text{ rad}$   
 (d)  $93^\circ 20' = 93 \frac{20}{60} \times \frac{3.142}{180} = 1.6292 \text{ rad}$   
 (e)  $176^\circ 30' = 176 \frac{30}{60} \times \frac{3.142}{180} = 3.0809 \text{ rad}$

2 (a)  $30^\circ = 30 \times \frac{\pi}{180} = \frac{\pi}{6} \text{ rad}$   
 (b)  $45^\circ = 45 \times \frac{\pi}{180} = \frac{\pi}{4} \text{ rad}$   
 (c)  $60^\circ = 60 \times \frac{\pi}{180} = \frac{\pi}{3} \text{ rad}$   
 (d)  $120^\circ = 120 \times \frac{\pi}{180} = \frac{2}{3} \pi \text{ rad}$   
 (e)  $300^\circ = 300 \times \frac{\pi}{180} = \frac{5}{3} \pi \text{ rad}$

3 (a)  $0.3574 \text{ rad} = 0.3574 \times \frac{180}{3.142} = 20.47^\circ$   
 (b)  $0.8121 \text{ rad} = 0.8121 \times \frac{180}{3.142} = 46.52^\circ$   
 (c)  $1.0438 \text{ rad} = 1.0438 \times \frac{180}{3.142} = 59.80^\circ$   
 (d)  $1.1693 \text{ rad} = 1.1693 \times \frac{180}{3.142} = 66.99^\circ$   
 (e)  $1.2325 \text{ rad} = 1.2325 \times \frac{180}{3.142} = 70.61^\circ$

4 (a)  $\frac{5}{12} \pi = \frac{5}{12} \pi \times \frac{180}{\pi} = 75^\circ$   
 (b)  $\frac{7}{4} \pi = \frac{7}{4} \pi \times \frac{180}{\pi} = 315^\circ$   
 (c)  $\frac{28}{9} \pi = \frac{28}{9} \pi \times \frac{180}{\pi} = 560^\circ$   
 (d)  $\frac{13}{4} \pi = \frac{13}{4} \pi \times \frac{180}{\pi} = 585^\circ$   
 (e)  $\frac{18}{5} \pi = \frac{18}{5} \pi \times \frac{180}{\pi} = 648^\circ$

**UPSKILL 1.2a**

1 (a)  $s = 11 \times 2 = 22 \text{ cm}$

(b)  $s = 7 \times \left( 160 \times \frac{3.142}{180} \right) = 19.55 \text{ cm}$

(c)  $\theta = 2 \times 3.142 - 1.2 = 5.084 \text{ rad}$   
 $s = 8 \times 5.084 = 40.672 \text{ cm}$

(d)  $\theta = 2 \times 3.142 - 4.5 = 1.784 \text{ rad}$   
 $s = 4 \times 1.784 = 7.136 \text{ cm}$

2 (a)  $\theta = \frac{15}{4} \text{ rad}$

$\alpha = 2 \times 3.142 - \frac{15}{4} = 2.534 \text{ rad}$

(b)  $\alpha = 2 \times 3.142 - 4.2 = 2.084 \text{ rad}$

$r = \frac{10.42}{2.084} = 5 \text{ cm}$

3 Perimeter of  $ABCD = 40$

$6 + 6 + \frac{2}{3}OA + \frac{2}{3}(OA + 6) = 40$

$12 + \frac{4}{3}OA + 4 = 40$

$\frac{4}{3}OA = 40 - 16$

$\frac{4}{3}OA = 24$

$OA = 24 \times \frac{3}{4}$

$OA = 18 \text{ cm}$

4 (a)  $r = \frac{s}{\theta}$

$r = \frac{12}{1.2}$

$r = 10 \text{ cm}$

(b)  $\angle HOK = \frac{s}{r}$

$\angle HOK = \frac{36}{10} = 3.6 \text{ rad}$

5 (a) Let  $PS = x$  cm

$$\frac{\text{Arc length } SR}{\text{Arc length } PQ} = \frac{3}{2}$$

$$\frac{\frac{\pi}{3}(14+x)}{\frac{\pi}{3}(14)} = \frac{3}{2}$$

$$14+x = \frac{3}{2} \times 14$$

$$x = 21 - 14$$

$$RS = 7 \text{ cm}$$

(b) Arc length  $RS$

$$= 21 \times \frac{3.142}{3}$$

$$= 21.994 \text{ cm}$$

6 (a)  $\angle AOM = \frac{12}{18} = \frac{2}{3}$  rad

(b) Perimeter of the shaded region

$$= 18 + 18 + \left(\frac{\pi}{2} - \frac{2}{3}\right)(18)$$

$$= (24 + 9\pi) \text{ cm}$$

7 Perimeter of the minor sector  $POQ$  = Major arc length  $PQ$

$$r + r + r\alpha = (2\pi - \alpha)r$$

$$2r + r\alpha = (2\pi r - \alpha r)$$

$$2 + \alpha = 2\pi - \alpha$$

$$2\alpha = 2\pi - 2$$

$$\alpha = \pi - 1 \text{ [Shown]}$$

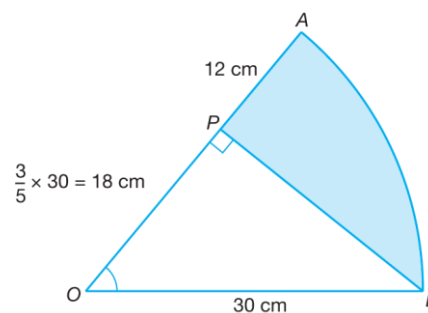
$$\angle POQ = 3.142 - 1 = 2.142 \text{ rad}$$

$$= 2.142 \times \frac{180}{3.142}$$

$$= 122.71^\circ$$

$$= 122^\circ 43'$$

8



$$\cos \theta = \frac{18}{30}$$

$$\theta = 53.13^\circ = 53.13 \times \frac{3.142}{180}$$

$$= 0.9274 \text{ rad}$$

$$\text{Arc length } AB = 30 (0.9274)$$

$$= 27.822 \text{ cm}$$

$$PB = \sqrt{30^2 - 18^2} = 24 \text{ cm}$$

Perimeter of the shaded region

$$= PB + AP + \text{Arc length } AB$$

$$= 24 + 12 + 27.822$$

$$= 63.82 \text{ cm}$$

9  $PQ = \sqrt{14^2 + 14^2} = 19.7990 \text{ cm}$

Arc length  $QS$

$$= \frac{3.142}{4} \times 19.7990$$

$$= 15.5521 \text{ cm}$$

Length of  $OS$

$$= 19.7990 - 14$$

$$= 5.799 \text{ cm}$$

Perimeter of the shaded region

$$= OQ + OS + \text{Arc length } QS$$

$$= 14 + 5.799 + 15.5521$$

$$= 35.35 \text{ cm}$$

$$10 \text{ (a) } \cos \angle PBA = \frac{24}{30}$$

$$\angle PBA = 36.87^\circ = 36.87 \times \frac{3.142}{180}$$

$$= 0.6436 \text{ rad}$$

$$\angle PBQ = 2 \times 0.6436 = 1.2872 \text{ rad}$$

$$(b) AP = AQ = \sqrt{30^2 - 24^2} = 18 \text{ cm}$$

$$\text{Arc length } PQ = 24 \times 1.2872$$

$$= 30.8928 \text{ cm}$$

Perimeter of the shaded region

$$= 18 + 18 + 30.8928$$

$$= 66.90 \text{ cm}$$

### UPSKILL 1.2b

1 (a) Length of the chord  $PR = 12 \text{ cm}$

$$2r \sin \frac{\theta}{2} = 12$$

$$2(10) \sin \frac{\theta}{2} = 12$$

$$\sin \frac{\theta}{2} = \frac{3}{5}$$

$$\frac{\theta}{2} = 36.87^\circ$$

$$\theta = 73.74^\circ$$

$$\theta = 73.74 \times \frac{3.142}{180} = 1.2872 \text{ rad}$$

(b) Perimeter of the shaded segment

$$= 10 \times 1.2872 + 12 = 24.87 \text{ cm}$$

2 (a) Length of chord  $PQ = 10 \text{ cm}$

$$2r \sin \frac{\theta}{2} = 10$$

$$2r \sin \frac{120^\circ}{2} = 10$$

$$2r \times 0.8660 = 10$$

$$r = 5.774 \text{ cm}$$

(b) Major arc length  $PRQ$

$$= 5.774 \times \left( 240 \times \frac{3.142}{180} \right) = 24.19 \text{ cm}$$

$$3 \text{ (a) (i) } \angle POT = 2(3.142) - 5 = 1.284 \text{ rad}$$

$$(ii) 1.284 \text{ rad} = 1.284 \times \frac{180}{3.142} = 73.56^\circ$$

(b) Perimeter of the shaded segment

$$= 2(10) \sin \left( \frac{73.56}{2} \right) + (10 \times 1.284)$$

$$= 24.81 \text{ cm}$$

### UPSKILL 1.2c

1 (a) Using the cosine rule,

$$4^2 = 7^2 + 5^2 - 2(7)(5) \cos \angle PRQ$$

$$\cos \angle PRQ = \frac{7^2 + 5^2 - 4^2}{2(7)(5)}$$

$$\cos \angle PRQ = \frac{7^2 + 5^2 - 4^2}{2(7)(5)}$$

$$\cos \angle PRQ = 0.82857$$

$$\angle PRQ = 34.05^\circ$$

$$\angle PRQ = 34.05 \times \frac{3.142}{180}$$

$$\angle PRQ = 0.5944 \text{ rad}$$

(b) Perimeter of the shaded region

$$= PS + PQ + \text{Arc length } SQ$$

$$= 2 + 4 + 5(0.5944)$$

$$= 8.972 \text{ cm}$$

**UPSKILL 1.3a**

1 (a)  $A = \frac{1}{2}(12)^2(3) = 216 \text{ cm}^2$

(b)  $A = \frac{1}{2}(10)^2\left(120 \times \frac{3.142}{180}\right)$

$A = \frac{1}{2}(100)(2.09467)$

$A = 104.73 \text{ cm}^2$

(c)  $\theta = 2 \times 3.142 - 5 = 1.284 \text{ rad}$

$A = \frac{1}{2}(12)^2(1.284)$

$A = 92.448 \text{ cm}^2$

2 (a)  $A = \frac{1}{2}r^2\theta$

$48 = \frac{1}{2}(12)^2\theta$

$48 = 72\theta$

$\theta = \frac{2}{3} \text{ rad}$

(b)  $A = \frac{1}{2}r^2\theta$

$\frac{27}{2}\pi = \frac{1}{2}r^2\left(\frac{\pi}{3}\right)$

$\frac{27}{2} = \frac{1}{2}r^2\left(\frac{1}{3}\right)$

$r^2 = 81$

$r = 9 \text{ cm}$

3  $\angle POQ = \frac{7}{10} = 0.7 \text{ rad}$

$\angle QOR = 3.142 - 0.7 = 2.442 \text{ rad}$

Area of the shaded region

$= \frac{1}{2}(10)^2(2.442)$

$= 122.1 \text{ cm}^2$

4 (a)  $\frac{\text{Arc length } BC}{\text{Arc length } AD} = \frac{3}{2}$

$\frac{9}{6\theta} = \frac{3}{2}$

$\theta = 1 \text{ rad}$

(b)  $OB = \frac{9}{1} = 9 \text{ cm}$

Area of the shaded region

$= \frac{1}{2}(9)^2(1) - \frac{1}{2}(6)^2(1)$

$= 22.5 \text{ cm}^2$

5 Area of sector  $ORS$  – Area of sector  $OPQ = 24$

$\frac{1}{2}(5k)^2\left(\frac{3}{4}\right) - \frac{1}{2}(3k)^2\left(\frac{3}{4}\right) = 24$

$75k^2 - 27k^2 = 192$

$48k^2 = 192$

$k^2 = 4$

$k = 2$

6  $\frac{\frac{1}{2}(18)^2\left(\frac{2}{3}\right)}{\frac{1}{2}r^2\left(\frac{2}{3}\right) - \frac{1}{2}(18)^2\left(\frac{2}{3}\right)} = \frac{9}{7}$

$\frac{(18)^2(2)}{r^2(2) - (18)^2(2)} = \frac{9}{7}$

$\frac{648}{2r^2 - 648} = \frac{9}{7}$

$9(2r^2 - 648) = 4536$

$18r^2 - 5832 = 4536$

$r^2 = 576$

$r = 24$

7 (a)  $\tan \angle XOT = \frac{14}{4}$

$\angle XOT = 74.05^\circ$

$\angle XOT = 74.05 \times \frac{3.142}{180}$

$\angle XOT = 1.29266$

$\angle XOT = 1.2927$

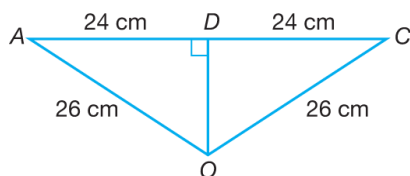
$\therefore \angle XOY = 2 \times 1.29270 = 2.5853 \text{ rad}$

(b) Area of the shaded region

$= 2 \times \left(\frac{1}{2} \times 4 \times 14\right) - \frac{1}{2}(4)^2(2.5853)$

$= 35.3176 \text{ cm}^2$

8 (a)



$$\sin \angle AOD = \frac{24}{26}$$

$$\angle AOD = 67.38^\circ$$

$$\angle AOC = 67.38^\circ \times 2$$

$$\angle AOC = 134.76^\circ$$

$$\angle AOC = 134.76 \times \frac{3.142}{180}$$

$$\angle AOC = 2.3523 \text{ rad}$$

(b) Area of the shaded region

$$= \frac{1}{2}(26)^2(2 \times 3.142 - 2.3523)$$

$$= 1\,328.91 \text{ cm}^2$$

### UPSKILL 1.3b

1 (a) Area of the shaded segment

$$= \frac{1}{2}r^2(\theta - \sin \theta)$$

$$= \frac{1}{2}(20)^2(1.5 - \sin 85.93^\circ)$$

$$= 100.5 \text{ cm}^2$$

(b)  $\theta = 2 \times 3.142 - 4.9 = 1.384 \text{ rad}$

$$\theta = 1.384 \times \frac{180}{3.142} = 79.29^\circ$$

Area of the shaded segment

$$= \frac{1}{2}r^2(\theta - \sin \theta)$$

$$= \frac{1}{2}(12)^2(1.384 - \sin 79.29^\circ)$$

$$= 28.90 \text{ cm}^2$$

### UPSKILL 1.3c

1 (a)  $\frac{\pi}{3} = \frac{3.142}{180} = 1.04733 \text{ rad} = 60^\circ$

$$\cos 60^\circ = \frac{r}{20}$$

$$r = 10$$

(b) Major arc length  $PAT$

$$= 10 \left( 3.142 - \frac{3.142}{3} \right)$$

$$= 20.9467 \text{ cm}$$

(c)  $\sin 60^\circ = \frac{PR}{20}$

$$PR = 20 \times \sin 60^\circ = 17.3205 \text{ cm}$$

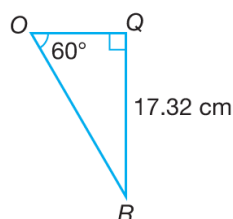
Area of the shaded region

$$= \frac{1}{2} \times 20^2 \times 1.04733 - \frac{1}{2} \times 10 \times 17.3205$$

$$= 209.466 - 86.6025$$

$$= 122.86 \text{ cm}^2$$

2 (a)



$$PR = 2(20) \sin \frac{120^\circ}{2} = 34.6410 \text{ cm}$$

$$QR = \frac{34.6410}{2} = 17.32 \text{ cm}$$

$$\tan 60^\circ = \frac{17.32}{r}$$

$$r = \frac{17.32}{\tan 60^\circ}$$

$$r = 10$$

(b)  $\angle PQR = 120 \times \frac{3.142}{180} = 2.094667 \text{ rad}$

Area of the shaded region

$$= \frac{1}{2}(20)^2 \left( 120 \times \frac{3.142}{180} - \sin 120^\circ \right)$$

$$= 245.73 \text{ cm}^2$$

(c) Perimeter of the shaded region

$$= 2 \times 20 \times \sin \frac{120}{2} + 20 \left( 120 \times \frac{3.142}{180} \right)$$

$$= 34.6410 + 41.8933$$

$$= 76.53 \text{ cm}$$

$$\begin{aligned} 3 \text{ (a) } 0.9 \text{ rad} &= 0.9 \times \frac{180}{3.142} \\ &= 51.56^\circ \end{aligned}$$

$$\begin{aligned} \cos 51.56^\circ &= \frac{OD}{16} \\ OD &= 9.9471 \text{ cm} \end{aligned}$$

$$\begin{aligned} \sin 51.56^\circ &= \frac{AD}{16} \\ AD &= 12.5322 \text{ cm} \end{aligned}$$

$$\begin{aligned} \text{Perimeter of the region } X \\ &= (16 - 9.9471) + 12.5322 + 16(0.9) \\ &= 32.9851 \text{ cm} \end{aligned}$$

$$\begin{aligned} \text{(b) } \tan 51.56^\circ &= \frac{AC}{16} \\ AC &= 20.1581 \text{ cm} \end{aligned}$$

$$\begin{aligned} \text{Area of the region } Y \\ &= \frac{1}{2} \times 16 \times 20.1581 - \frac{1}{2} \times 16^2 \times 0.9 \\ &= 46.0648 \text{ cm}^2 \end{aligned}$$

$$\begin{aligned} 4 \text{ (a) Area of the sector } KOL \\ &= \frac{1}{2} \times (2 \times \sqrt{3})^2 \times \frac{3.142}{3} \\ &= 6.284 \text{ cm}^2 \end{aligned}$$

$$\begin{aligned} \text{(b) Area of the sector } PAQB \\ &= \frac{1}{2} \times (\sqrt{3})^2 \times \left( \frac{2}{3} \times 3.142 \right) \\ &= 3.142 \text{ cm}^2 \end{aligned}$$

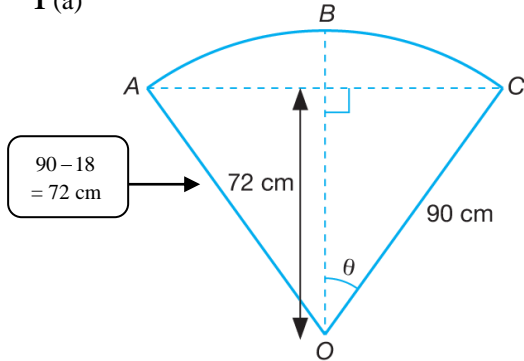
$$\begin{aligned} \text{(c) Area of the quadrilateral } OAPB \\ &= \text{Area of } \triangle OPA \times 2 \\ &= 2 \left( \frac{1}{2} \times \sqrt{3} \times \sqrt{3} \times \sin 120^\circ \right) \times 2 \\ &= 2.59808 \text{ cm}^2 \end{aligned}$$

$$\begin{aligned} \text{(d) Area of the sector } OKL \\ &= \frac{1}{2} \times (2\sqrt{3})^2 \times \frac{3.142}{3} \\ &= 6.284 \text{ cm}^2 \end{aligned}$$

$$\begin{aligned} \text{Area of the shaded region} \\ &= \text{Area of the sector } OKL - \text{Area of the} \\ &\quad \text{quadrilateral } OAPB - \text{Area of the} \\ &\quad \text{sector } PAQB \\ &= 6.284 - 2.59808 - 3.142 \\ &= 0.5439 \text{ cm}^2 \end{aligned}$$

**UPSKILL 1.4**

1 (a)



$$\cos \theta = \frac{72}{90} = 0.8$$

$$\theta = 36.87^\circ$$

$$\theta = 36.87 \times \frac{3.142}{180}$$

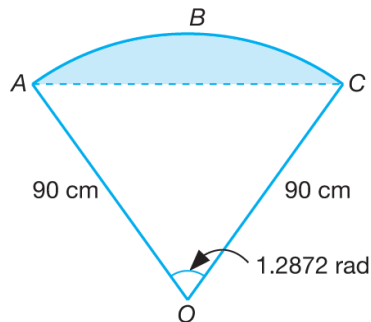
$$\theta = 0.6436 \text{ rad}$$

$$\angle AOC = 2\theta = 2 \times 0.6436 = 1.2872 \text{ rad}$$

$$= 1.2872 \times \frac{180}{3.142}$$

$$= 73.74^\circ$$

(b)



Cross-sectional area above the water surface

$$= \frac{1}{2}(90)^2(1.2872 - \sin 73.74^\circ)$$

$$= 1\,325.16 \text{ cm}^2$$

2 (a) Radius of the sector  $OAPB$   
= Slant height of the cone

$$= \sqrt{6^2 + 5^2}$$

$$= \sqrt{61}$$

$$= 7.8102 \text{ cm}$$

(b) Arc length  $APB$

= Circumference of the circular base of the cone

$$= 2 \times 3.142 \times 5$$

$$= 31.42 \text{ cm}$$

$$(c) \theta = \frac{s}{r} = \frac{31.42}{7.8102} = 4.0229 \text{ rad}$$

(d) Area of the sector  $OAPB$

$$= \frac{1}{2}(7.8102)^2(4.0229)$$

$$= 122.7 \text{ cm}^2$$

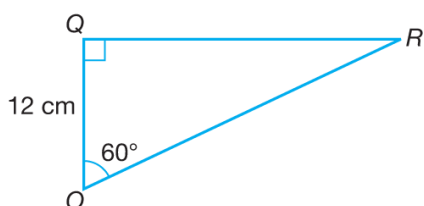
### Summative Practice 1

- 1 (a) Since the radii of a circle are always the same,  $OP = OQ = OR = 12$  cm  
 Since the opposite side of a rhombus are the same, then  $PQ = OR$  and  $RQ = OP$ .  
 Therefore,  $\triangle OPQ$  and  $\triangle OQR$  are equilateral triangles such that  
 $\angle POR = \angle POQ + \angle QOR = 60^\circ + 60^\circ = 120^\circ$

$$\text{Hence, } \angle POR = 120 \times \frac{\pi}{180} = \frac{2}{3}\pi \text{ rad}$$

$$\angle POR = \frac{2}{3} \times 3.142 = 2.09467 \text{ rad}$$

(b)



$$\cos 60^\circ = \frac{12}{OR}$$

$$\frac{1}{2} = \frac{12}{OR}$$

$$OR = 12 \times 2$$

$$OR = 24 \text{ cm}$$

Hence, arc length  $JKL$

$$= 24 \times \frac{2}{3} (3.142)$$

$$= 50.27 \text{ cm}$$

(c) Area of the shaded segment

$$= \frac{1}{2} (24)^2 (2.09467 - \sin 120^\circ)$$

$$= 353.8 \text{ cm}^2$$

$$2 \text{ (a) } 1 \text{ rad} = \frac{180}{3.142} = 57.29^\circ$$

In  $\triangle OQR$ ,

$$\cos 57.29^\circ = \frac{10}{OR}$$

$$OR = \frac{10}{\cos 57.29^\circ} \\ = 18.50525 \text{ cm}$$

$$\sin 57.29^\circ = \frac{RQ}{18.50525}$$

$$RQ = 18.52525 \times \sin 57.29^\circ \\ = 15.5875 \text{ cm}$$

Perimeter of the shaded region A

$$= \text{Arc length } PQ + PR + QR$$

$$= 10(1) + (18.50525 - 10) + 15.5872$$

$$= 10 + 8.50525 + 15.5872$$

$$= 34.09 \text{ cm}$$

(b) Area of the shaded region B

$$= \text{Area of the sector } ORS - \text{Area of } \triangle OQR$$

$$= \frac{1}{2} \times (18.50535)^2 \times 1 - \frac{1}{2} \times 10 \times 15.57062$$

$$= 93.37 \text{ cm}^2$$

$$3 \text{ (a) } \angle COQ = 1.982 \text{ rad} = 1.982 \times \frac{180}{3.142}$$

$$= 113.55^\circ$$

$$\angle CAO = \angle COA = 3.142 - 1.982$$

$$= 1.16 \text{ rad}$$

$$(b) \angle CAO = \angle COA = 180^\circ - 113.55^\circ \\ = 66.45^\circ$$

$$\angle ACO = 180^\circ - 66.45^\circ - 66.45^\circ = 47.10^\circ$$

In  $\triangle ACO$ , using the sine rule,

$$\frac{AO}{\sin 47.10^\circ} = \frac{5}{\sin 66.45^\circ}$$

$$AO = \frac{5}{\sin 66.45^\circ} \times \sin 47.10^\circ$$

$$AO = 3.9955 \text{ cm}$$

$$AR = AQ = 5 + 3.9955 = 8.996 \text{ cm}$$

(c)  $CR = AR - AC = 8.996 - 5 = 3.996 \text{ cm}$

$$\text{Arc length } CQ = 1.982 \times 5$$

$$= 9.91 \text{ cm}$$

$$\text{Arc length } RQ = 8.996 \times 1.16$$

$$= 10.4354 \text{ cm}$$

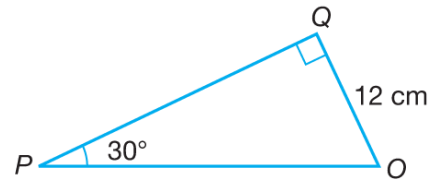


$$\begin{aligned}
 &\text{Perimeter of the shaded region} \\
 &= CR + \text{Arc length } CQ + \text{Arc length } RQ \\
 &= 3.996 + 9.91 + 10.4354 \\
 &= 24.34 \text{ cm}
 \end{aligned}$$

(d) Thus, the area of the shaded region

$$\begin{aligned}
 &= \text{Area of the sector } ARQ - \text{Area of the sector } OCQ - \text{Area of } \triangle ACO \\
 &= \frac{1}{2}(8.996)^2(1.16) - \frac{1}{2}(5)^2(1.982) - \\
 &\quad \frac{1}{2}(5)(5)\sin 47.10^\circ \\
 &= 46.9385 - 24.775 - 9.1568 \\
 &= 13.01 \text{ cm}^2
 \end{aligned}$$

4 (a)



$$\sin 30^\circ = \frac{12}{PO}$$

$$\begin{aligned}
 PO &= \frac{12}{\sin 30^\circ} \\
 &= 24 \text{ cm}
 \end{aligned}$$

$$\begin{aligned}
 \text{Length of } POM &= 24 + 12 \\
 &= 36 \text{ cm}
 \end{aligned}$$

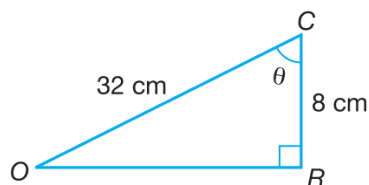
$$\begin{aligned}
 &\text{Arc length } AMB \\
 &= 36 \times \frac{3.142}{3} \\
 &= 37.70 \text{ cm}
 \end{aligned}$$

(b) In  $\triangle POQ$ ,  $\tan 30^\circ = \frac{12}{PQ}$

$$\begin{aligned}
 PQ &= \frac{12}{\tan 30^\circ} \\
 &= 20.78 \text{ cm}
 \end{aligned}$$

$$\begin{aligned}
 &\text{Area of the shaded region} \\
 &= \text{Area of the sector } PAMB - 2 \times \text{Area of } \triangle POQ - \text{Area the major sector } QMR \\
 &= \frac{1}{2}(36)^2\left(\frac{3.142}{3}\right) - 2\left(\frac{1}{2} \times 12 \times 20.7846\right) \\
 &\quad - \frac{1}{2}(12)^2\left(240 \times \frac{3.142}{180}\right) \\
 &= 678.672 - 249.4152 - 301.632 \\
 &= 127.6248 \text{ cm}^2
 \end{aligned}$$

5 (a)



$$\cos \theta = \frac{8}{32}$$

$$\theta = 75.5225^\circ$$

$$\theta = 75.5225 \times \frac{3.142}{180}$$

$$\theta = 1.318 \text{ [Shown]}$$

(b)  $OR = \sqrt{32^2 - 8^2} = 30.9839 \text{ cm}$

$$\sin \angle COR = \frac{8}{32}$$

$$\angle COR = 14.48^\circ$$

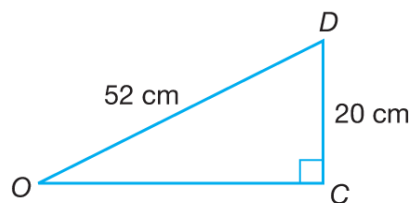
Thus,  $\angle ROP = 90^\circ + 14.48^\circ = 104.48^\circ$

$$104.48^\circ = 104.48 \times \frac{3.142}{180} = 1.8238 \text{ rad}$$

Therefore, arc length  $PR$   
 $= 12(1.8238) = 21.89 \text{ cm}$

(c) Area of the shaded region  
 $= \text{Area of the trapezium } POCQ - \text{Area of the sector } POR - \text{Area of the sector } CRQ$   
 $= \frac{1}{2}(12 + 20)(30.9839) - \frac{1}{2}(12)^2(1.8238) -$   
 $\frac{1}{2}(20)^2(1.318)$   
 $= 495.7424 - 131.3136 - 263.6$   
 $= 100.83 \text{ cm}^2$

6 (a)



$$\sin \angle AOB = \sin \angle DCO = \frac{20}{52}$$

$$\angle AOB = 22.62^\circ$$

$$\angle AOB = 22.62 \times \frac{3.142}{180}$$

$$\angle AOB = 0.3948 \text{ rad}$$

(b)  $\tan \angle DCO = \frac{20}{OC}$

$$OC = \frac{20}{\tan 22.62^\circ}$$

$$OC = \frac{20}{0.41667}$$

$$OC = 47.9996$$

Perimeter of the shaded region  
 $= \text{Arc length } DT + DA + TB + \text{Arc length } AB$   
 $= 20 \left( 90 \times \frac{3.142}{180} \right) + 52 + (104 - 20 - 48) +$   
 $104(0.3948)$   
 $= 31.42 + 52 + 36 + 41.06$   
 $= 160.48 \text{ cm}$

(c) Area of the shaded region  
 $= \text{Area of the sector } OAB - \text{Area of } \triangle OCD - \text{Area of quadrant } CDT$   
 $= \frac{1}{2}(104)^2(0.3948) - \frac{1}{2}(48)(20) -$   
 $\frac{1}{2}(20)^2 \left( 90 \times \frac{3.142}{180} \right)$   
 $= 2135.08 - 480 - 314.2$   
 $= 1340.88 \text{ cm}^2$

7 (a)  $\theta = 360^\circ - 80^\circ = 280^\circ$

$$280^\circ = 280 \times \frac{3.142}{180}$$

$$= 4.888 \text{ rad}$$

(b) Area of the shaded major sector

$$OAC = 244 \frac{17}{45} \text{ cm}^2.$$

$$\frac{1}{2} r^2 (4.88755) = 244 \frac{17}{45}$$

$$r^2 = 100$$

$$r = 10$$

Hence, the radius of the circle is 10 cm.

(c) Perimeter of the shaded segment

$$= 10 + 10 + 10(4.888)$$

$$= 68.88 \text{ cm}$$

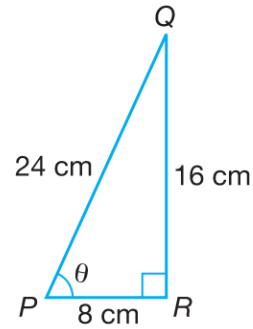
(d) Acute  $\angle AOC = 80^\circ \times \frac{3.142}{180} = 1.3964 \text{ rad}$

Area of the shaded segment  $ABC$

$$= \frac{1}{2} (10)^2 (1.3964 - \sin 80^\circ)$$

$$= 20.58 \text{ cm}^2$$

8



(a)  $\tan \theta = \frac{16}{8}$

$$\theta = 63.43^\circ$$

$$\theta = 63.43 \times \frac{3.142}{180}$$

$$\theta = 1.1072 \text{ rad}$$

(b)  $\sin 63.43^\circ = \frac{16}{PQ}$

$$PQ = \frac{16}{\sin 63.43^\circ} = 17.8893 \text{ cm}$$

Perimeter of the shaded region

$$= \text{Arc length } CB + \text{Arc length } QB + QC$$

$$= 24(1.1072) + 16 \left( \frac{3.142}{2} \right) + (24 -$$

$$17.8893)$$

$$= 26.5728 + 25.136 + 6.1107$$

$$= 57.82 \text{ cm}$$

(c) Area of the shaded region

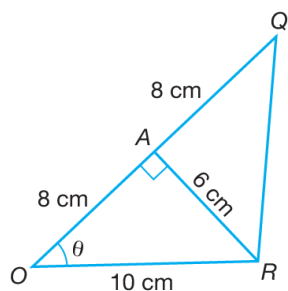
$$= \text{Area of the sector } PCB - \text{Area of}$$

$$\Delta POQ - \text{Area of the quadrant } QOB$$

$$= \frac{1}{2} (24)^2 (1.1072) - \frac{1}{2} (8)(16) - \frac{1}{2} \times 16^2 \times \left( \frac{3.142}{2} \right)$$

$$= 318.87 - 64 - 201.088 = 53.78 \text{ cm}^2$$

9 (a)



$$\cos \theta = \frac{8}{10}$$

$$\theta = 36.87^\circ$$

$$\theta = 36.87 \times \frac{3.142}{180}$$

$$\theta = 36.87 \times \frac{3.142}{180}$$

$$\theta = 0.6436 \text{ rad}$$

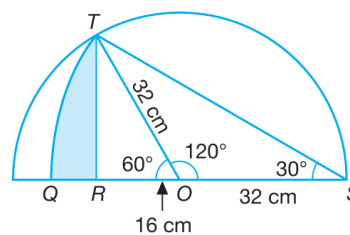
(b) Perimeter of the shaded region

$$\begin{aligned} &= QR + RP + \text{Arc length } PQ \\ &= 10 + 6 + 16(0.6436) \\ &= 26.30 \text{ cm} \end{aligned}$$

(c) Area of the shaded region

$$\begin{aligned} &= \text{Area of the sector } OPQ - \text{Area of } \triangle ORQ \\ &= \frac{1}{2}(16)^2(0.6436) - \frac{1}{2} \times 16 \times 6 \\ &= 34.38 \text{ cm}^2 \end{aligned}$$

10 (a)



$$\cos \angle TOR = \frac{16}{32}$$

$$\angle TOR = 60^\circ$$

$$\angle TOR = \frac{3.142}{3} = 1.0473 \text{ rad}$$

$$(b) TS^2 = 32^2 + 32^2 - 2(32)(32) \cos 120^\circ$$

$$TS^2 = 3072$$

$$TS = 55.4256 \text{ cm}$$

Arc length  $TQ$

$$= 55.4256 \left( \frac{3.142}{6} \right)$$

$$= 29.02455$$

$$= 29.02 \text{ cm}$$

(c) Area of the shaded region

$$= \text{Area of the sector } STR - \text{Area of } \triangle TOS - \text{Area of } \triangle TRO$$

$$= \frac{1}{2}(55.4256)^2 \left( \frac{3.142}{6} \right) -$$

$$\frac{1}{2}(32)(32) \sin 120^\circ - \frac{1}{2}(32)(16) \sin 60^\circ$$

$$= 804.3512 - 443.4050 - 221.7025$$

$$= 139.2 \text{ cm}^2$$

11 (a) Length of the arc of  $BE = \frac{4}{3}\pi$

$$2 \times \angle BAE = \frac{4}{3} \times 3.142$$

$$\angle BAE = \frac{1}{2} \times \frac{4}{3} \times \pi$$

$$= \frac{2}{3}\pi \text{ rad}$$

$$= \frac{2}{3} \times 3.142$$

$$= 2.0947 \text{ rad}$$

(b)  $\angle BAE = 120^\circ$   
 $\angle OAE = 180^\circ - 120^\circ$   
 $= 60^\circ$

$$\angle AOE = \angle AEO = 60^\circ = 1.0473 \text{ rad}$$

Area of the sector  $BOD$

$$= \frac{1}{2} \times 8^2 \times 1.0473$$

$$= 33.5136 \text{ cm}^2$$

(c) Area of the sector  $BAE$

$$= \frac{1}{2} \times 4^2 \times 2.0947$$

$$= 16.7576 \text{ cm}^2$$

Area of  $\triangle AOE$

$$= \frac{1}{2} \times 4(4) \times \sin 60^\circ$$

$$= 6.9282 \text{ cm}^2$$

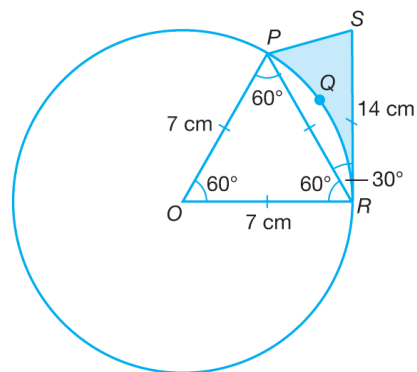
Area of the shaded region

$$= \text{Area of the sector } BOD - \text{Area of the sector } BAE - \text{Area of } \triangle AOE$$

$$= 33.5136 - 16.7576 - 6.9282$$

$$= 9.8278 \text{ cm}^2$$

12 (a)



$$\angle POR = \frac{3.142}{3} = 1.0473 \text{ rad}$$

Area of the segment  $PQR$

$$= \frac{1}{2}(7)^2(1.0473 - \sin 60^\circ)$$

$$= 4.4412 \text{ cm}^2$$

(b) Area of  $\triangle PRS$

$$= \frac{1}{2} \times 7 \times 7 \times \sin 30^\circ$$

$$= 12.25 \text{ cm}^2$$

Area of the shaded region

$$= \text{Area of } \triangle PRS - \text{Area of the segment } PQR$$

$$= 12.25 - 4.4412$$

$$= 7.8088 \text{ cm}^2$$