

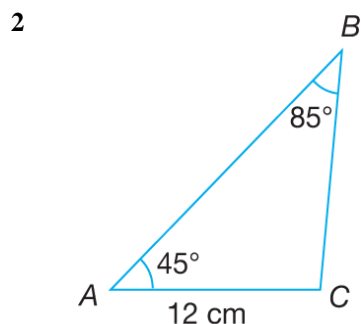
**Form 4 Chapter 9**  
**Solution of Triangles**  
**Fully-Worked Solutions**

**UPSKILL 9.1a**

$$1 \quad \frac{AC}{\sin 40^\circ} = \frac{13}{\sin 33^\circ}$$

$$AC = \frac{13}{\sin 33^\circ} \times \sin 40^\circ$$

$$AC = 15.34 \text{ cm}$$



$$(a) \quad \frac{BC}{\sin 40^\circ} = \frac{12}{\sin 85^\circ}$$

$$BC = \frac{12}{\sin 85^\circ} \times \sin 45^\circ$$

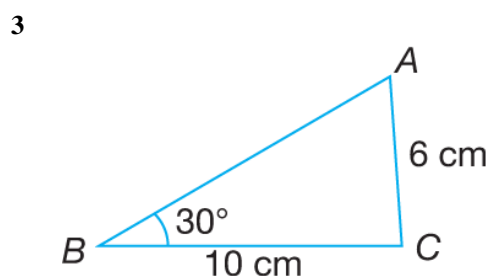
$$BC = 8.518 \text{ cm}$$

$$(b) \quad \angle ACB = 180^\circ - 45^\circ - 85^\circ = 50^\circ$$

$$\frac{AB}{\sin 50^\circ} = \frac{12}{\sin 85^\circ}$$

$$AB = \frac{12}{\sin 85^\circ} \times \sin 50^\circ$$

$$AB = 9.228 \text{ cm}$$

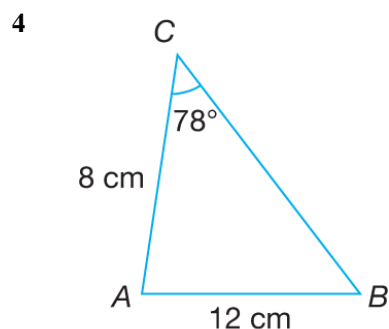


$$\frac{\sin \angle BAC}{10} = \frac{\sin 30^\circ}{6}$$

$$\sin \angle BAC = \frac{\sin 30^\circ}{6} \times 10$$

$$\sin \angle BAC = 0.83333$$

$$\angle BAC = 56.44^\circ$$



$$\frac{\sin \angle ABC}{8} = \frac{\sin 78^\circ}{12}$$

$$\sin \angle ABC = \frac{\sin 78^\circ}{12} \times 8$$

$$\sin \angle ABC = 0.6521$$

$$\angle ABC = 40.70$$

5  $\angle PQR = 180^\circ - 66^\circ - 72^\circ = 42^\circ$

$$\frac{PQ}{\sin 72^\circ} = \frac{7}{\sin 66^\circ}$$

$$PQ = \frac{7}{\sin 66^\circ} \times \sin 72^\circ$$

$$PQ = 7.287 \text{ cm}$$

$$\frac{PR}{\sin 42^\circ} = \frac{7}{\sin 66^\circ}$$

$$PR = \frac{7}{\sin 66^\circ} \times \sin 42^\circ$$

$$PR = 5.127 \text{ cm}$$

6  $\angle RPQ = 180^\circ - 42^\circ - 100^\circ = 38^\circ$

$$\frac{PR}{\sin 100^\circ} = \frac{5}{\sin 42^\circ}$$

$$PR = \frac{5}{\sin 42^\circ} \times \sin 100^\circ$$

$$PR = 7.359 \text{ cm}$$

$$\frac{QR}{\sin 38^\circ} = \frac{5}{\sin 42^\circ}$$

$$QR = \frac{5}{\sin 42^\circ} \times \sin 38^\circ$$

$$QR = 4.600 \text{ cm}$$

$$7 \quad \frac{\sin \angle ABC}{12} = \frac{\sin 65^\circ}{14}$$

$$\sin \angle ABC = \frac{\sin 65^\circ}{14} \times 12$$

$$\sin \angle ABC = 0.77684$$

$$\angle ABC = 50.97^\circ$$

$$\angle ACB = 180^\circ - 65^\circ - 50.97^\circ = 64.03^\circ$$

$$\frac{AB}{\sin 64.03^\circ} = \frac{14}{\sin 65^\circ}$$

$$AB = \frac{14}{\sin 65^\circ} \times \sin 64.03^\circ$$

$$AB = 13.89 \text{ cm}$$

$$8 \quad \frac{\sin \angle BAC}{2} = \frac{\sin 35^\circ}{8}$$

$$\sin \angle BAC = \frac{\sin 35^\circ}{8} \times 2$$

$$\sin \angle BAC = 0.14339$$

$$\angle BAC = 8.24^\circ$$

$$\angle ACB = 180^\circ - 8.24^\circ - 35^\circ = 136.76^\circ$$

$$\frac{AB}{\sin 136.76^\circ} = \frac{8}{\sin 35^\circ}$$

$$AB = \frac{8}{\sin 35^\circ} \times \sin 136.76^\circ$$

$$AB = 9.555 \text{ cm}$$

$$9 \quad \frac{\sin \angle PRQ}{13} = \frac{\sin 19^\circ}{5}$$

$$\sin \angle PRQ = \frac{\sin 19^\circ}{5} \times 13$$

$$\text{Basic } \angle = 57.83$$

$$\angle PRQ = 180^\circ - 57.83^\circ$$

$$\angle PRQ = 122.17^\circ$$

10 (a) In  $\triangle ACD$ ,

$$\angle ADC = 180^\circ - 70^\circ - 60^\circ = 50^\circ$$

$$\frac{AC}{\sin 50^\circ} = \frac{20}{\sin 60^\circ}$$

$$AC = \frac{20}{\sin 60^\circ} \times \sin 50^\circ$$

$$AC = 17.69 \text{ cm}$$

(b) In  $\triangle ABC$ ,

$$\frac{\sin \angle ABC}{17.69} = \frac{\sin 120^\circ}{19}$$

$$\sin \angle ABC = \frac{\sin 120^\circ}{19} \times 17.69$$

$$\sin \angle ABC = 0.80632$$

$$\angle ABC = 53.74^\circ$$

11 (a) In  $\triangle PQR$ ,

$$\frac{PQ}{\sin 60^\circ} = \frac{14}{\sin 50^\circ}$$

$$PQ = \frac{14}{\sin 50^\circ} \times \sin 60^\circ$$

$$PQ = 15.83 \text{ cm}$$

(b) In  $\triangle PRS$ ,

$$\frac{\sin \angle RSP}{14} = \frac{\sin 120^\circ}{20}$$

$$\sin \angle RSP = \frac{\sin 120^\circ}{20} \times 14$$

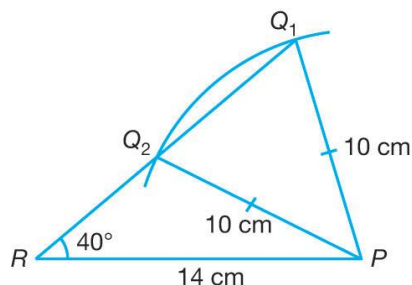
$$\sin \angle RSP = 0.60622$$

$$\angle RSP = 37.32^\circ$$

**UPSKILL 9.1b**

- 1 (a) Not ambiguous  
(b) Ambiguous

2 (a)



$$(b) \frac{\sin \angle Q}{14} = \frac{\sin 40^\circ}{10}$$

$$\sin \angle Q = \frac{\sin 40^\circ}{10} \times 14$$

$$\text{Basic } \angle = 64.15^\circ$$

$$\angle PQ_1R = 64.15^\circ$$

$$\angle PQ_2R = 180^\circ - 64.15^\circ = 115.85^\circ$$

In  $\triangle PQ_1R$ ,

$$\begin{aligned} \angle RPQ_1 &= 180^\circ - 40^\circ - 64.15^\circ \\ &= 75.85^\circ \end{aligned}$$

$$\frac{RQ_1}{\sin 75.85^\circ} = \frac{10}{\sin 40^\circ}$$

$$RQ_1 = \frac{10}{\sin 40^\circ} \times \sin 75.85^\circ$$

$$RQ_1 = 15.09 \text{ cm}$$

In  $\triangle PQ_2R$ ,

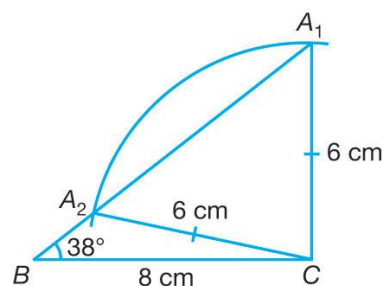
$$\begin{aligned} \angle RPQ_2 &= 180^\circ - 40^\circ - 115.85^\circ \\ &= 24.15^\circ \end{aligned}$$

$$\frac{RQ_2}{\sin 24.15^\circ} = \frac{10}{\sin 40^\circ}$$

$$RQ_2 = \frac{10}{\sin 40^\circ} \times \sin 24.15^\circ$$

$$RQ_2 = 6.365 \text{ cm}$$

3 (a)



$$(b) \frac{\sin \angle A}{8} = \frac{\sin 38^\circ}{6}$$

$$\sin \angle A = \frac{\sin 38^\circ}{6} \times 8$$

$$\sin \angle A = 0.82088$$

$$\text{Basic } \angle = 55.17^\circ$$

$$\angle BA_1C = 55.17^\circ$$

$$\angle BA_2C = 124.83^\circ$$

In  $\triangle BA_1C$ ,

$$\begin{aligned} \angle BCA_1 &= 180^\circ - 38^\circ - 55.17^\circ \\ &= 86.83^\circ \end{aligned}$$

$$\frac{BA_1}{\sin 86.83^\circ} = \frac{6}{\sin 38^\circ}$$

$$BA_1 = \frac{6}{\sin 38^\circ} \times \sin 86.83^\circ$$

$$BA_1 = 9.731 \text{ cm}$$

In  $\triangle BA_2C$ ,

$$\begin{aligned} \angle BCA_2 &= 180^\circ - 38^\circ - 124.83^\circ \\ &= 17.17^\circ \end{aligned}$$

$$\frac{BA_2}{\sin 17.17^\circ} = \frac{6}{\sin 38^\circ}$$

$$BA_2 = 2.877 \text{ cm}$$

$$4 \text{ (a) } \frac{\sin \angle ABC}{12} = \frac{\sin 35^\circ}{8}$$

$$\sin \angle ABC = \frac{\sin 35^\circ}{8} \times 12$$

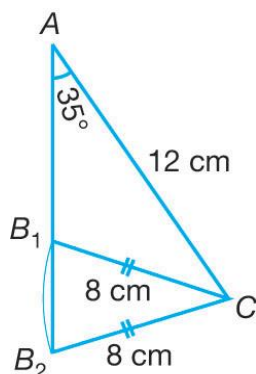
$$\sin \angle ABC = 0.86036$$

$$\text{Basic } \angle = 59.36^\circ$$

$$\angle AB_1C = 180^\circ - 29.36^\circ$$

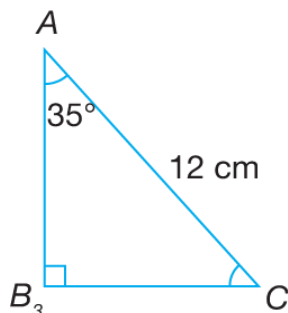
$$= 120.64^\circ$$

(b)



(c) Acute angle  $AB_2C = 59.36^\circ$

(d)



$$\sin 35^\circ = \frac{B_3C}{12}$$

$$B_3C = 6.883 \text{ cm}$$

### UPSKILL 9.1c

$$1 \text{ (a) } \angle BCD = 180^\circ - 65^\circ - 80^\circ = 35^\circ$$

In  $\triangle BCD$ ,

$$\frac{BD}{\sin 35^\circ} = \frac{11}{\sin 65^\circ}$$

$$BD = \frac{11}{\sin 65^\circ} \times \sin 35^\circ$$

$$BD = 6.962 \text{ cm}$$

(b) In  $\triangle ABD$ ,

$$\frac{\sin \angle DAB}{6.962} = \frac{\sin 115^\circ}{9}$$

$$\sin \angle DAB = \frac{\sin 115^\circ}{9} \times 6.962$$

$$\angle DAB = 44.51^\circ$$

### UPSKILL 9.2

$$1 \text{ } PR^2 = 12^2 + 15^2 - 2(12)(15) \cos 53^\circ$$

$$PR^2 = 152.3466$$

$$PR = 12.34 \text{ cm}$$

$$\frac{\sin \angle RPQ}{12} = \frac{\sin 53^\circ}{12.34}$$

$$\sin \angle RPQ = 0.77663$$

$$\angle RPQ = 50.95^\circ$$

$$\angle PRQ = 180^\circ - 53^\circ - 50.95^\circ = 76.05^\circ$$

$$2 \text{ } XY^2 = 9^2 + 6^2 - 2(9)(6) \cos 140^\circ$$

$$XY^2 = 199.7328$$

$$XY = 14.13 \text{ cm}$$

$$\frac{\sin \angle YXZ}{6} = \frac{\sin 140^\circ}{14.13}$$

$$\sin \angle YXZ = 0.27295$$

$$\angle YXZ = 15.84^\circ$$

$$\angle XYZ = 180^\circ - 140^\circ - 15.84^\circ = 24.16^\circ$$

$$3 \quad 10^2 = 15^2 + 18^2 - 2(15)(18) \cos \angle LKM$$

$$\cos \angle LKM = \frac{15^2 + 18^2 - 10^2}{2(15)(18)}$$

$$\cos \angle LKM = 0.83148$$

$$\angle LKM = 33.75^\circ$$

$$\frac{\sin \angle LMK}{15} = \frac{\sin 33.75^\circ}{10}$$

$$\sin \angle LMK = \frac{\sin 33.75^\circ}{10} \times 15$$

$$\angle LMK = 56.44^\circ$$

$$\begin{aligned} \angle KLM &= 180^\circ - 33.75^\circ - 56.44^\circ \\ &= 89.81^\circ \end{aligned}$$

$$4 \quad 14^2 = 11^2 + 8^2 - 2(11)(8) \cos \angle PQR$$

$$\cos \angle PQR = \frac{11^2 + 8^2 - 14^2}{2(11)(8)}$$

$$\cos \angle PQR = -0.0625$$

$$\angle PQR = 93.58^\circ$$

$$\frac{\sin \angle QPR}{8} = \frac{\sin 93.58^\circ}{14}$$

$$\sin \angle QPR = 0.5703$$

$$\angle QPR = 34.77^\circ$$

$$\angle PRQ = 180^\circ - 93.58^\circ - 34.77^\circ$$

$$\angle PRQ = 51.65^\circ$$

5 (a) In  $\triangle ABP$ ,

$$\angle ABP = 180^\circ - 38^\circ - 78^\circ = 64^\circ$$

$$\frac{BP}{\sin 38^\circ} = \frac{5}{\sin 64^\circ}$$

$$BP = 3.4249 \text{ cm}$$

(b) In  $\triangle CBP$ ,

$$\angle CBP = 180^\circ - 64^\circ = 116^\circ$$

$$CP^2 = 7^2 + 3.4249^2 - 2(7)(3.4249) \cos 116^\circ$$

$$CP = 9.0415 \text{ cm}$$

6 (a) In  $\triangle ABD$ ,

$$6^2 = 10^2 + 7^2 - 2(10)(7) \cos \angle ABD$$

$$\cos \angle ABD = \frac{10^2 + 7^2 - 6^2}{2(10)(7)}$$

$$\angle ABD = 36.18^\circ$$

(b) In  $\triangle DBP$ ,

$$\angle DBP = 180^\circ - 36.18^\circ = 143.82^\circ$$

$$\frac{\sin \angle DPB}{7} = \frac{\sin 143.82^\circ}{16}$$

$$\sin \angle DPB = 0.25827$$

$$\angle DPB = 14.97^\circ$$

### UPSKILL 9.3

$$1 \quad \text{Area of } \triangle ABC = \frac{1}{2} \times 9 \times 11 \times \sin 43^\circ = 33.76 \text{ cm}^2$$

$$2 \quad \angle ABC = 180^\circ - 48^\circ - 24^\circ = 108^\circ$$

$$\begin{aligned} \text{Area of } \triangle ABC &= \frac{1}{2} \times 12 \times 8 \times \sin 108^\circ \\ &= 45.65 \text{ cm}^2 \end{aligned}$$

$$3 \text{ (a)} \quad 3^2 = 2^2 + 4^2 - 2(2)(4) \cos \angle BAC$$

$$\cos \angle BAC = \frac{2^2 + 4^2 - 3^2}{2(2)(4)}$$

$$\cos \angle BAC = 0.6875$$

$$\angle BAC = 46.57^\circ$$

(b) Area of  $\triangle ABC$

$$\begin{aligned} &= \frac{1}{2} \times 2 \times 4 \times \sin 46.57^\circ \\ &= 2.905 \text{ cm}^2 \end{aligned}$$

$$4 \quad s = \frac{4+6+9}{2} = 9.5$$

Area of  $\triangle ABC$

$$= \sqrt{9.5(9.5-4)(9.5-6)(9.5-9)}$$

$$= \sqrt{9.5(5.5)(3.5)(0.5)}$$

$$= \sqrt{91.4375}$$

$$= 9.562 \text{ cm}^2$$

5 Area of  $\triangle ABC = 20 \text{ cm}^2$

$$\frac{1}{2} \times 7 \times 10 \times \sin \theta = 20$$

$$\sin \theta = 0.57143$$

$$\theta = 34.85^\circ$$

6 Area of  $\triangle ABC = 43 \text{ cm}^2$

$$\frac{1}{2} \times 8 \times 13 \times \sin \angle BAC = 43$$

$$\sin \angle BAC = 0.8269$$

$$\text{Basic } \angle = 55.78^\circ$$

$$\angle BAC = 124.22^\circ$$

### UPSKILL 9.4

1 (a) In  $\triangle PQS$ ,

$$QS^2 = 18^2 + 16^2 - 2(18)(16) \cos 43^\circ$$

$$QS^2 = 1587403$$

$$QS = 12.599 \text{ cm}$$

(b) In  $\triangle SQR$ ,

$$\frac{\sin \angle SQR}{20} = \frac{\sin 20^\circ}{12.599}$$

$$\sin \angle SQR = \frac{\sin 20^\circ}{12.599} \times 20$$

$$\sin \angle SQR = 0.5429$$

$$\text{Basic } \angle = 32.88^\circ$$

$$\angle SQR = 180^\circ - 32.88^\circ$$

$$= 147.12^\circ$$

(c) In  $\triangle QSR$ ,

$$\angle QSR = 180^\circ - 20^\circ - 147.12^\circ$$

$$= 12.88^\circ$$

Area of the whole diagram

$$= \frac{1}{2}(18)(16) \sin 43^\circ + \frac{1}{2}(12.599)(20) \sin 12.88^\circ$$

$$= 98.21 + 28.08$$

$$= 126.29 \text{ cm}^2$$

2 (a) In  $\triangle PQS$ ,

$$\frac{\sin \angle PSQ}{8} = \frac{\sin 35^\circ}{7}$$

$$\sin \angle PSQ = \frac{\sin 35^\circ}{7} \times 8$$

$$\sin \angle PSQ = 0.65552$$

$$\angle PSQ = 40.96^\circ$$

$$\angle PQS = 180^\circ - 40.96^\circ - 35^\circ$$

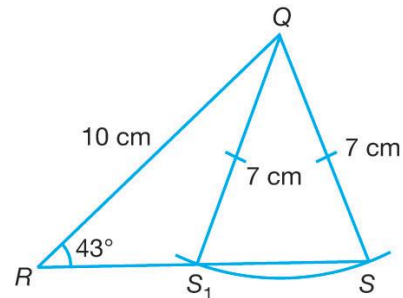
$$= 104.04^\circ$$

Area of  $\triangle PQS$

$$= \frac{1}{2} \times 8 \times 7 \times \sin 104.04^\circ$$

$$= 27.16 \text{ cm}^2$$

(b)



$$\frac{\sin \angle S}{10} = \frac{\sin 43^\circ}{7}$$

$$\sin \angle S = \frac{\sin 43^\circ}{7} \times 10$$

$$\sin \angle S = 0.97428$$

$$\angle S = 76.98^\circ \text{ or } 103.02^\circ$$

Thus,  $\angle RS_1Q = 103.02^\circ$

$$\angle RQS_1 = 180^\circ - 43^\circ - 103.02^\circ$$

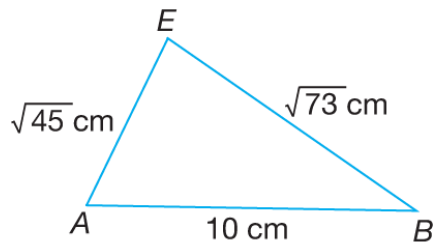
$$= 33.98^\circ$$

$$\frac{RS_1}{\sin 33.98^\circ} = \frac{7}{\sin 43^\circ}$$

$$RS_1 = \frac{7}{\sin 43^\circ} \times \sin 33.98^\circ$$

$$RS_1 = 5.737 \text{ cm}$$

3 (a)  $AE = \sqrt{6^2 + 3^2} = \sqrt{45}$  cm  
 $BE = \sqrt{8^2 + 3^2} = \sqrt{73}$  cm  
 $AB = DC = 10$



$$10^2 = 45 + 73 - 2\sqrt{45}\sqrt{73} \cos \angle AEB$$

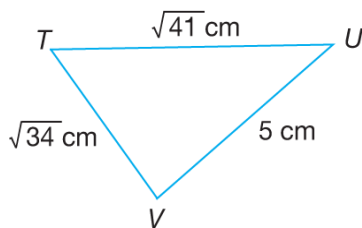
$$\cos \angle AEB = \frac{45 + 73 - 100}{2\sqrt{45}\sqrt{73}}$$

$$\cos \angle AEB = 0.15703$$

$$\angle AEB = 80.97^\circ$$

(b) Area of  $\triangle AEB$   
 $= \frac{1}{2} \times \sqrt{45} \times \sqrt{73} \sin 80.97^\circ$   
 $= 28.30 \text{ cm}^2$

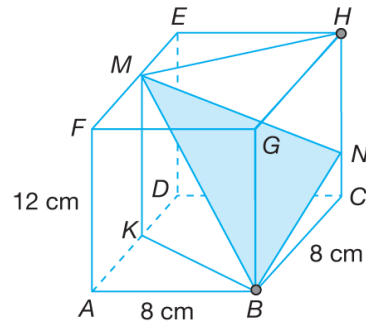
4  $TV = \sqrt{3^2 + 5^2} = \sqrt{34}$   
 $UV = \sqrt{3^2 + 4^2} = 5$   
 $TU = \sqrt{5^2 + 4^2} = \sqrt{41}$



(a)  $5^2 = 34 + 41 - 2\sqrt{34}\sqrt{41} \cos \angle UTV$   
 $\cos \angle UTV = \frac{34 + 41 - 25}{2\sqrt{34}\sqrt{41}}$   
 $\cos \angle UTV = 0.66959$   
 $\angle UTV = 47.96^\circ$

(b) Area of  $\triangle UTV$   
 $= \frac{1}{2} \sqrt{34}\sqrt{41} \times \sin 47.96^\circ$   
 $= 13.86 \text{ cm}^2$

5



$$KB^2 = 4^2 + 8^2 = 80$$

$$MB^2 = KB^2 + MK^2 = 80 + 12^2$$

$$MB = \sqrt{224} \text{ cm}$$

$$NC = \frac{1}{4}(12) = 3 \text{ cm}$$

$$NB^2 = NC^2 + BC^2 = 9 + 64 = 73$$

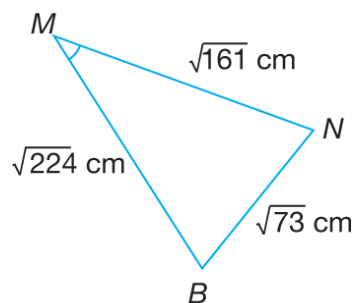
$$NB = \sqrt{73} \text{ cm}$$

$$MH^2 = 4^2 + 8^2 = 80$$

$$HN = \frac{3}{4}(12) = 9 \text{ cm}$$

$$MN^2 = MH^2 + HN^2 = 80 + 9^2 = 161$$

$$MN = \sqrt{161} \text{ cm}$$



$$73 = 224 + 161 - 2\sqrt{224}\sqrt{161} \cos \angle MBN$$

$$\cos \angle MBN = \frac{224 + 161 - 73}{2\sqrt{224}\sqrt{161}}$$

$$\cos \angle MBN = 0.821462$$

$$\angle MBN = 34.77^\circ$$

Area of  $\triangle MBN$   
 $= \frac{1}{2} \times \sqrt{224} \times \sqrt{161} \times \sin 34.77^\circ$   
 $= 54.15 \text{ cm}^2$

### Summative Practice 9

1 (a) Area of  $\triangle DBC = 29 \text{ cm}^2$

$$\frac{1}{2} \times 6 \times 10 \times \sin \angle DCB = 29$$

$$\sin \angle DCB = 0.96667$$

$$\text{Basic } \angle = 75.16^\circ$$

$$\angle DCB = 180^\circ - 75.16^\circ$$

$$\angle DCB = 104.84^\circ \text{ (obtuse)}$$

(b)  $BD^2 = 6^2 + 10^2 - 2(6)(10) \cos 104.84^\circ$

$$BD^2 = 166.73448$$

$$BD = 12.91 \text{ cm}$$

(c) In  $\triangle ABD$ ,

$$\frac{\sin \angle A}{12.91} = \frac{\sin 45^\circ}{9.5}$$

$$\sin \angle A = \frac{\sin 45^\circ}{9.5} \times 12.91$$

$$\sin \angle A = 0.96092$$

$$\angle A = 73.93^\circ$$

$$\therefore \angle DAB = 73.93^\circ$$

$$\text{and } \angle DA'B = 180^\circ - 73.93^\circ$$

$$= 106.07^\circ$$

Quadrant I

Quadrant II

$$\text{Thus, } \angle ABD = 180^\circ - 45^\circ - \angle DAB$$

$$= 180^\circ - 45^\circ - 73.93^\circ$$

$$= 61.07^\circ$$

Thus,

$$\angle A'BD = 180^\circ - 45^\circ - \angle DA'B$$

$$= 180^\circ - 45^\circ - 106.07^\circ$$

$$= 28.93^\circ$$

2 (a) (i) In  $\triangle ABC$ ,

$$BC^2 = 16^2 + 7^2 - 2(16)(7) \cos 55^\circ$$

$$= 176.5189$$

$$BC = \sqrt{176.5189} = 13.29 \text{ cm}$$

(ii) In  $\triangle CBD$ ,

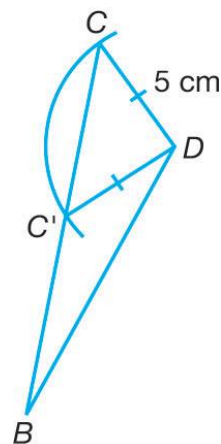
$$\frac{\sin \angle CBD}{5} = \frac{\sin 115^\circ}{13.29}$$

$$\sin \angle CBD = \frac{\sin 115^\circ}{13.29} \times 5$$

$$\sin \angle CBD = 0.34097$$

$$\angle CBD = 19.94^\circ$$

(b) (i)



(ii) In  $\triangle BCD$ ,

$$\angle BCD = 180^\circ - 115^\circ - 19.94^\circ = 45.06^\circ$$

In  $\triangle DCC'$ ,

$$\angle DC'C = \angle DCC' = 45.06^\circ$$

$$\therefore \angle DC'B = 180^\circ - 45.06^\circ = 134.94^\circ$$

In  $\triangle C'DB$ ,

$$\frac{BD}{\sin 134.94^\circ} = \frac{5}{\sin 19.94^\circ}$$

$$BD = \frac{5 \times \sin 134.94^\circ}{\sin 19.94^\circ}$$

$$BD = 10.38 \text{ cm}$$

$\therefore$  Area of  $\triangle BC'D$

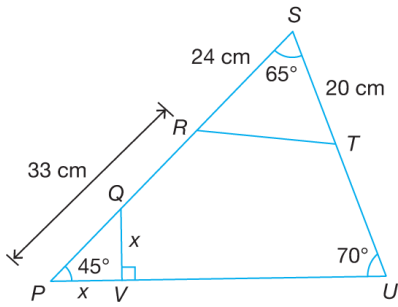
$$= \frac{1}{2} \times 5 \times 10.38 \times \sin 25.12^\circ$$

$$= 11.02 \text{ cm}^2$$

$$\angle C'DB = 180^\circ - 19.94^\circ - 134.94^\circ = 25.12^\circ$$



3



(a) (i) In  $\Delta RST$ ,

$$RT^2 = 24^2 + 20^2 - 2(24)(20)\cos 65^\circ$$

$$RT^2 = 570.2865$$

$$RT = 23.88 \text{ cm}$$

(ii)  $\angle SUP = 180^\circ - 45^\circ - 65^\circ = 70^\circ$

In  $\Delta PSU$ ,

$$\frac{SU}{\sin 45^\circ} = \frac{57}{\sin 70^\circ}$$

$$SU = \frac{57}{\sin 70^\circ} \times \sin 45^\circ$$

$$SU = 42.89 \text{ cm}$$

$$\therefore TU = 42.89 - 20 = 22.89 \text{ cm}$$

(b) Area of  $\Delta PVQ$

$$= \frac{1}{2} \times \text{Area of } \Delta RST$$

$$= \frac{1}{2} \times \left( \frac{1}{2} \times 24 \times 20 \times \sin 65^\circ \right)$$

$$= 108.7569 \text{ cm}^2$$

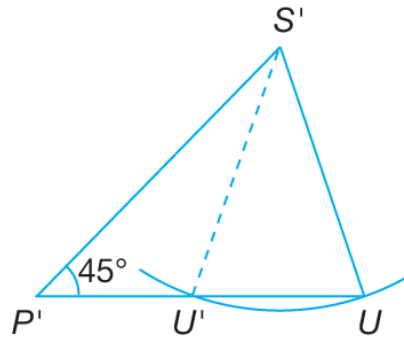
Hence,  $\frac{1}{2}x^2 = 108.7569$

$$x^2 = 217.5139$$

$$x = 14.75$$

$$PV = 14.75 \text{ cm}$$

(c)



4 (a) In  $\Delta EHN$ ,

$$NH^2 = 8^2 + 7^2 - 2(8)(7)\cos 67^\circ$$

$$NH^2 = 69.2381$$

$$NH = 8.321 \text{ cm}$$

(b) In  $\Delta EHN$ ,

$$\frac{\sin \angle ENH}{7} = \frac{\sin 67^\circ}{8.321}$$

$$\sin \angle ENH = \frac{\sin 67^\circ}{8.321} \times 7$$

$$\sin \angle ENH = 0.7744$$

$$\angle ENH = 50.75^\circ$$

In  $\Delta HGN$ ,

$$NG^2 = 3^2 + 8.321^2 - 2(3)(8.321)\cos 50.75^\circ$$

$$NG^2 = 46.6506$$

$$NG = 6.830 \text{ cm}$$

(c) In  $\Delta HGN$ ,

$$\frac{\sin \angle NGH}{8.321} = \frac{\sin 50.75^\circ}{6.830}$$

$$\sin \angle NGH = 0.9434$$

$$\text{Basic } \angle = 70.64^\circ$$

$$\angle NGH = 180^\circ - 70.64^\circ$$

$$= 109.36^\circ \text{ (obtuse)}$$

(d) Area of  $\Delta HGN$

$$= \frac{1}{2} \times 3 \times 8.321 \times \sin 50.75^\circ$$

$$= 9.666 \text{ cm}^2$$

5 (a) (i) In  $\triangle ABC$ ,

$$\frac{BC}{\sin 35^\circ} = \frac{7}{\sin 40^\circ}$$

$$BC = \frac{7}{\sin 40^\circ} \times \sin 35^\circ$$

$$= 6.246 \text{ cm}$$

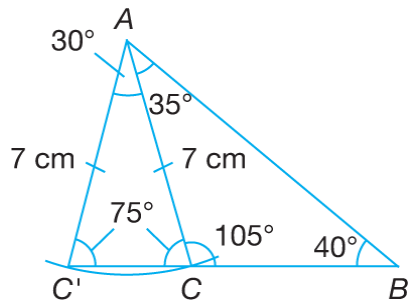
(ii)  $\angle ECD = \angle ACB$   
 $= 180^\circ - 35^\circ - 40^\circ$   
 $= 105^\circ$

In  $\triangle ECD$ ,

$$DE^2 = 11^2 + 4^2 - 2(11)(4)\cos 105^\circ$$

$$DE = 12.64 \text{ cm}$$

(b) (i)



(ii)  $\angle AC'B = \angle ACC'$   
 $= 180^\circ - 105^\circ$   
 $= 75^\circ$

(iii)  $\frac{C'B}{\sin 65^\circ} = \frac{7}{\sin 40^\circ}$   
 $C'B = \frac{7}{\sin 40^\circ} \times \sin 65^\circ$   
 $= 9.8698 \text{ cm}$

Area of  $\triangle AC'B$   
 $= \frac{1}{2} \times 7 \times 9.8698 \times \sin 75^\circ$   
 $= 33.37 \text{ cm}^2$

6 (a) (i) In  $\triangle PQR$ ,

$$\frac{\sin \angle PRQ}{9} = \frac{\sin 80.94^\circ}{10}$$

$$\sin \angle PRQ = 0.88877$$

$$\angle PRQ = 62.72^\circ$$

(ii)  $\angle TQR = 180^\circ - \angle TRQ - \angle QTR$   
 $= 180^\circ - 62.72^\circ - 62.72^\circ$   
 $= 54.56^\circ$

In  $\triangle QRT$ ,

$$TR^2 = 6^2 + 6^2 - 2(6)(6)\cos 54.56^\circ$$

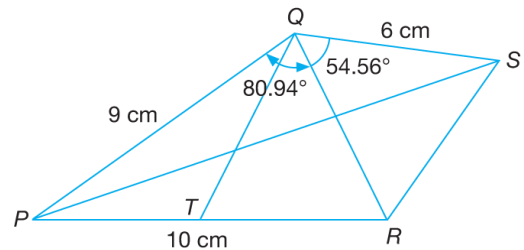
$$TR^2 = 30.25079$$

$$TR = 5.50 \text{ cm}$$

(iii)  $\angle PQT = 80.94^\circ - \angle TQR$   
 $= 80.94^\circ - 54.56^\circ$   
 $= 26.38^\circ$

Area of  $\triangle PQT$   
 $= \frac{1}{2} \times 9 \times 6 \times \sin 26.38^\circ$   
 $= 12.00 \text{ cm}^2$

(b)  $\angle SQR = \angle TQR = 54.56^\circ$   
 $\therefore \angle PQS = 80.94^\circ + 54.56^\circ = 135.5^\circ$



In  $\triangle PQS$ ,

$$PS^2 = 9^2 + 6^2 - 2(9)(6)\cos 135.5^\circ$$

$$PS^2 = 194.0310$$

$$PS = 13.93 \text{ cm}$$

7 (a) (i) Area of  $\triangle ABC = 112 \text{ cm}^2$

$$\frac{1}{2} \times 18 \times BC \times \sin 34^\circ = 112$$

$$BC = 22.25 \text{ cm}$$

(ii)

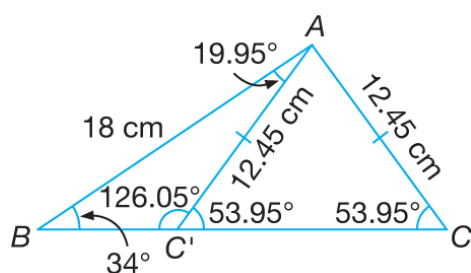
$$AC^2 = 18^2 + 22.25^2 - 2(18)(22.25) \cos 34^\circ$$

$$AC = 12.45 \text{ cm}$$

(iii)  $\frac{\sin \angle ACB}{18} = \frac{\sin 34^\circ}{12.45}$

$$\angle ACB = 53.95^\circ$$

(b) (i)



(ii) Area of  $\triangle ABC'$

$$= \frac{1}{2} \times 18 \times 12.45 \times \sin 19.95^\circ$$

$$= 38.23 \text{ cm}^2$$

8 (a) (i) In  $\triangle PQR$ ,

$$10^2 = 8^2 + 14^2 - 2(8)(14) \cos \angle QPR$$

$$\cos \angle QPR = \frac{8^2 + 14^2 - 10^2}{2(8)(14)}$$

$$\angle QPR = 44.42^\circ$$

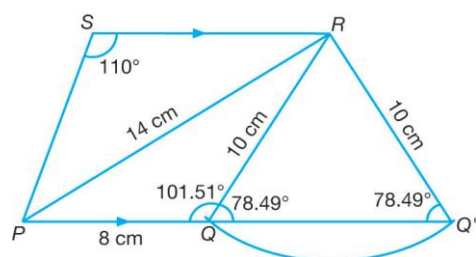
(ii) In  $\triangle PSR$ ,

$$\frac{PS}{\sin 44.42^\circ} = \frac{14}{\sin 110^\circ}$$

$$PS = \frac{14}{\sin 110^\circ} \times \sin 44.42^\circ$$

$$PS = 10.43 \text{ cm}$$

(b) (i)



In  $\triangle PQR$ ,

$$\frac{\sin \angle PQR}{14} = \frac{\sin 44.42^\circ}{10}$$

$$\sin \angle PQR = \frac{\sin 44.42^\circ}{10} \times 14$$

$$\text{Basic } \angle = 78.49^\circ$$

$$\angle PQR = 180^\circ - 78.49^\circ$$

$$= 101.51^\circ \text{ (obtuse)}$$

(ii)  $\angle RQ'Q = 180^\circ - 78.49^\circ - 78.49^\circ$

$$= 23.02^\circ$$

Area of  $\triangle RQ'Q$

$$= \frac{1}{2} \times 10 \times 10 \times \sin 23.02^\circ$$

$$= 19.55 \text{ cm}^2$$

9 (a) (i) In  $\triangle ABC$ ,

$$\frac{BC}{\sin 101^\circ} = \frac{11.6}{\sin 34^\circ}$$

$$BC = \frac{11.6}{\sin 34^\circ} \times \sin 101^\circ$$

$$BC = 20.36 \text{ cm}$$

(ii) In  $\triangle BED$ ,

$$BD^2 = 17^2 + 9.2^2 - 2(17)(9.2)\cos 140^\circ$$

$$BD^2 = 613.2587$$

$$BD = 24.76 \text{ cm}$$

$$\therefore CD = BD - BC$$

$$= 24.76 - 20.36$$

$$= 4.40 \text{ cm}$$

(iii)  $\angle ACB = 180^\circ - 34^\circ - 101^\circ$

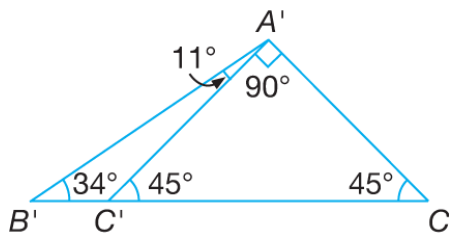
$$= 45^\circ$$

Area of  $\triangle ABC$

$$= \frac{1}{2}(11.6)(20.36)\sin 45^\circ$$

$$= 83.50 \text{ cm}^2$$

(b) (i)



(ii)  $\angle B'A'C' = 101^\circ - 90^\circ = 11^\circ$

10 (a) (i)  $\angle SPQ = 180^\circ - 78^\circ - 32^\circ = 70^\circ$

In  $\triangle PSQ$ ,

$$\frac{QS}{\sin 70^\circ} = \frac{16}{\sin 32^\circ}$$

$$QS = 28.37 \text{ cm}$$

(ii) In  $\triangle SRQ$ ,

$$28.37^2 = 12^2 + 18^2 - 2(12)(18)\cos \angle QRS$$

$$\cos \angle QRS = \frac{12^2 + 18^2 - 28.37^2}{2(12)(18)}$$

$$\cos \angle QRS = -0.7798$$

$$\angle QRS = 141.24^\circ$$

(iii) Area of  $PQRS$

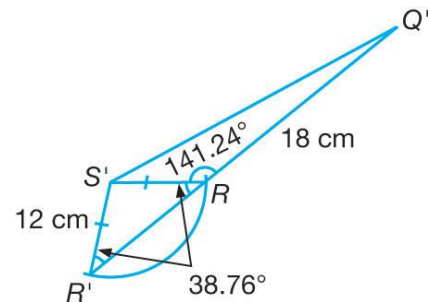
$$= \text{Area of } \triangle PSQ + \text{Area of } \triangle SRQ$$

$$= \frac{1}{2} \times 16 \times 28.37 \times \sin 78^\circ +$$

$$\frac{1}{2} \times 12 \times 18 \times \sin 141.24^\circ$$

$$= 289.61 \text{ cm}^2$$

(b) (i)



(ii)  $\angle S'R'Q' = \angle S'RR'$

$$= 180^\circ - 141.24^\circ$$

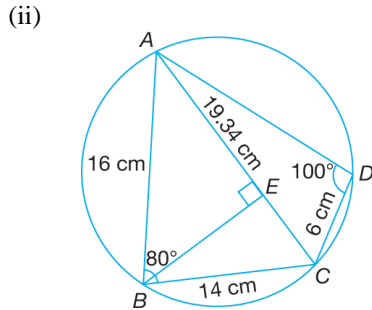
$$= 38.76^\circ$$

11 (a) (i) In  $\triangle ABC$ ,

$$AC^2 = 16^2 + 14^2 - 2(16)(14) \cos 80^\circ$$

$$AC^2 = 374.2056$$

$$AC = 19.34 \text{ cm}$$



$$\begin{aligned} \angle ADC &= 180^\circ - \angle ABC \\ &= 180^\circ - 80^\circ \\ &= 100^\circ \end{aligned}$$

In  $\triangle ACD$ ,

$$\frac{\sin \angle CAD}{6} = \frac{\sin 100^\circ}{19.34}$$

$$\sin \angle CAD = 0.30552$$

$$\angle CAD = 17.79^\circ$$

$$\begin{aligned} \therefore \angle ACD &= 180^\circ - 17.79^\circ - 100^\circ \\ &= 62.21^\circ \end{aligned}$$

(b) (i) Area of  $\triangle ABC$

$$\begin{aligned} &= \frac{1}{2} \times 16 \times 14 \times \sin 80^\circ \\ &= 110.30 \text{ cm}^2 \end{aligned}$$

(ii) Area of  $\triangle ABC = 110.30 \text{ cm}^2$

$$\frac{1}{2} \times AC \times BE = 110.30$$

$$\frac{1}{2} \times 19.34 \times BE = 110.30$$

$$BE = 11.41 \text{ cm}$$

Hence, the shortest distance from  $B$  to  $AC$  is 11.41 cm.

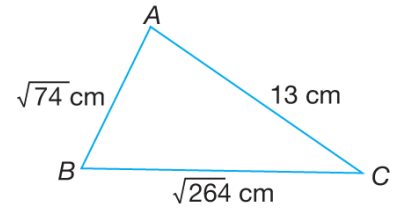
12 (a) (i)  $BC^2 = 7^2 + 12^2 - 2(7)(12) \cos 115^\circ$

$$BC^2 = 264.0$$

$$BC = \sqrt{264} \text{ cm}$$

$$BC = 16.25 \text{ cm}$$

(ii)  $AB = \sqrt{5^2 + 7^2} = \sqrt{74} \text{ cm}$   
 $AC = 13 \text{ cm}$



$$264 = 74 + 13^2 - 2\sqrt{74} \times 13 \cos \angle BAC$$

$$\cos \angle BAC = \frac{74 + 13^2 - 264}{2\sqrt{74} (13)}$$

$$\angle BAC = 95.39^\circ$$

(iii) Area of  $\triangle BAC$

$$\begin{aligned} &= \frac{1}{2} \sqrt{74} (13) \sin 95.39^\circ \\ &= 55.67 \text{ cm}^2 \end{aligned}$$

(b)  $\frac{\sin \angle T}{16.25} = \frac{\sin 30^\circ}{9}$

$$\sin \angle T = \frac{\sin 30^\circ}{9} \times 16.25$$

$$\sin \angle T = 0.9028$$

$$\angle T = 64.53^\circ \text{ or } 115.47^\circ$$