Form 4 Chapter 8
Vectors
Fully-Worked Solutions

UPSKILL 8.1a

1 $\left|\overrightarrow{AB}\right| = \sqrt{3^2+3^2} = \sqrt{18} = 2\sqrt{2} = 4.243$ units

The direction of $\overrightarrow{AB}$ is due southwest.

UPSKILL 8.1b

1 (i) $\overrightarrow{AB} = \underset{\sim}{a}$

(ii) $\overrightarrow{RS} = -\underset{\sim}{a}$

(iii) $\overrightarrow{XY} = \underset{\sim}{b}$

(iv) $\overrightarrow{KL} = \underset{\sim}{c}$

(v) $\overrightarrow{PQ} = \underset{\sim}{d}$

(vi) $\overrightarrow{MN} = -\underset{\sim}{b}$

(vii) $\overrightarrow{VW} = -\underset{\sim}{c}$

(viii) $\overrightarrow{CD} = -\underset{\sim}{d}$

(b) (i) $\left|\overrightarrow{RS}\right| = \sqrt{3^2+4^2} = 5$ units

(ii) $\left|\overrightarrow{XY}\right| = 3$ units

(iii) $\left|\overrightarrow{KL}\right| = 4$ units

(iv) $\left|\overrightarrow{PQ}\right| = \sqrt{2^2+3^2} = \sqrt{13} = 3.606$ units

UPSKILL 8.1c

1 (a) $\overrightarrow{CD} = \frac{1}{2}\underset{\sim}{b}$

(b) $\overrightarrow{EF} = 2\underset{\sim}{b}$

(c) $\overrightarrow{GH} = -\frac{3}{2}\underset{\sim}{b}$

2 (a) $(h-4)\underset{\sim}{v} = (5h-k)\underset{\sim}{w}$

$h-4=0 \Rightarrow h=4$

$5h-k=0 \Rightarrow 20-k=0 \Rightarrow k=20$

(b) $(2h-4)\underset{\sim}{v} = (k-6h+3)\underset{\sim}{w}$

$2h-4=0 \Rightarrow h=2$

$k-6h+3=0$

$k-6(2)+3=0$

$k=9$

UPSKILL 8.2a

1 (a) $\underset{\sim}{a}+\underset{\sim}{b} = \overrightarrow{PR}$

(b) $\underset{\sim}{b}+\underset{\sim}{c} = \overrightarrow{QS}$

(c) $\overrightarrow{PQ}+\overrightarrow{QS} = \overrightarrow{PS}$

2 (a) $\overrightarrow{EH}+\overrightarrow{EF} = \overrightarrow{EG}$

(b) $\overrightarrow{EH}+\overrightarrow{EF} = \overrightarrow{FH}$

3 (a) $\overrightarrow{PQ}+\overrightarrow{QR}+\overrightarrow{RS} = \overrightarrow{PS}$

(b) $\overrightarrow{PR}+\overrightarrow{RS}+\overrightarrow{ST} = \overrightarrow{PT}$

(c) $\overrightarrow{PQ}+\overrightarrow{QR}+\overrightarrow{RS}+\overrightarrow{ST} = \overrightarrow{PT}$

4 (a) $\overrightarrow{ON}-\overrightarrow{MN}-\overrightarrow{LM}$

$= \overrightarrow{ON}+\overrightarrow{NM}+\overrightarrow{ML}$

$= \overrightarrow{OL}$

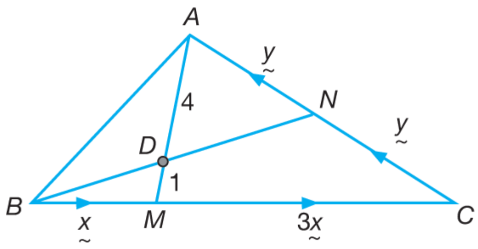
(b) $\overrightarrow{NJ}-\overrightarrow{KJ}-\overrightarrow{MK}$

$= \overrightarrow{NJ}+\overrightarrow{JK}+\overrightarrow{KM}$

$= \overrightarrow{NM}$

UPSKILL 8.2b

1



(a) $\overrightarrow{MA} = \overrightarrow{MC} + \overrightarrow{CA} = 3\underset{\sim}{x} + 2\underset{\sim}{y}$

(b) $\overrightarrow{MD} = \frac{1}{5}\overrightarrow{MA} = \frac{1}{5}\left(3\underset{\sim}{x} + 2\underset{\sim}{y}\right) = \frac{3}{5}\underset{\sim}{x} + \frac{2}{5}\underset{\sim}{y}$

(c) $\overrightarrow{BD} = \overrightarrow{BM} + \overrightarrow{MD}$

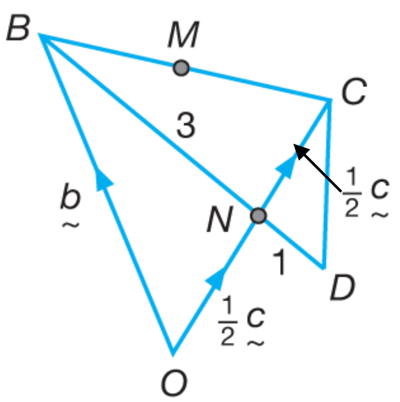
$= \underset{\sim}{x} + \frac{3}{5}\underset{\sim}{x} + \frac{2}{5}\underset{\sim}{y}$

$= \frac{8}{5}\underset{\sim}{x} + \frac{2}{5}\underset{\sim}{y}$

(d) $\overrightarrow{BN} = \overrightarrow{BC} + \overrightarrow{CN} = 4\underset{\sim}{x} + \underset{\sim}{y}$

UPSKILL 8.2c

1



(a) (i) $\overrightarrow{OM} = \overrightarrow{OC} + \overrightarrow{CM}$

$= c + \frac{1}{2}\overrightarrow{CB}$

$= c + \frac{1}{2}\left(\overrightarrow{CO} + \overrightarrow{OB}\right)$

$= c + \frac{1}{2}\left(-\underset{\sim}{c} + \underset{\sim}{b}\right)$

$= \frac{1}{2}\underset{\sim}{c} + \frac{1}{2}\underset{\sim}{b}$

$= \frac{1}{2}\left(\underset{\sim}{c} + \underset{\sim}{b}\right)$ … (1)

(ii) $\frac{NB}{DB} = \frac{3}{4}$

$\overrightarrow{DB} = \frac{4}{3}\overrightarrow{NB}$

$= \frac{4}{3}\left(\overrightarrow{NO} + \overrightarrow{OB}\right)$

$= \frac{4}{3}\left(-\frac{1}{2}\underset{\sim}{c} + \underset{\sim}{b}\right)$

$= -\frac{2}{3}\underset{\sim}{c} + \frac{4}{3}\underset{\sim}{b}$

(b) $\overrightarrow{DC} = \overrightarrow{DB} + \overrightarrow{BC}$

$= -\frac{2}{3}\underset{\sim}{c} + \frac{4}{3}\underset{\sim}{b} + \left(-\underset{\sim}{b} + \underset{\sim}{c}\right)$

$= \frac{1}{3}c + \frac{1}{3}b$

$= \frac{1}{3}(c + b)$

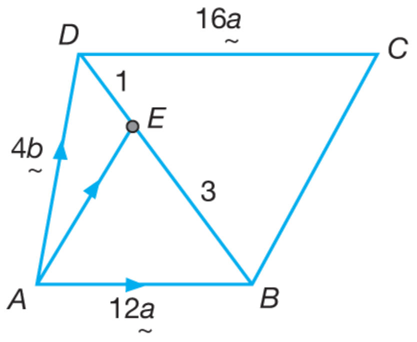
$= \frac{1}{3}\left(2\overrightarrow{OM}\right)$

From (1) : $\underset{\sim}{c} + \underset{\sim}{b} = 2\overrightarrow{OM}$

$= \frac{2}{3}\overrightarrow{OM}$

Since $\overrightarrow{DC} = \frac{2}{3}\overrightarrow{OM}$, thus $\overrightarrow{DC}$ can be expressed as a scalar multiple of $\overrightarrow{OM}$.
Hence, DC is parallel to OM.

2



(a) (i) $\overrightarrow{DB} = -4\underline{b} + 12\underline{a}$

(ii) $\overrightarrow{AE} = \overrightarrow{AD} + \frac{1}{4}\overrightarrow{DB}$

$= 4b + \frac{1}{4}(-4b + 12a)$

$= 4\underline{b} - \underline{b} + 3\underline{a}$

$= 3\underline{b} + 3\underline{a}$

$= 3(\underline{b} + \underline{a})$ … (1)

(b) $\overrightarrow{BC} = \overrightarrow{BD} + \overrightarrow{DC}$

$= 4\underline{b} - 12\underline{a} + \frac{4}{3}\left(\overrightarrow{AB}\right)$

$= 4\underline{b} - 12\underline{a} + \frac{4}{3}(12\underline{a})$

$= 4\underline{b} + 4\underline{a}$

$= 4(\underline{b} + \underline{a})$

$= 4\left(\frac{1}{3}\overrightarrow{AE}\right)$

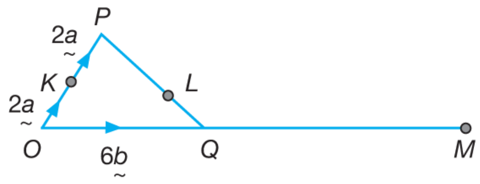
$= \frac{4}{3}\overrightarrow{AE}$

From (1) : $\underline{b} + \underline{c} = \frac{1}{3}\overrightarrow{AE}$

Since $\overrightarrow{BC} = \frac{4}{3}\overrightarrow{AE}$, thus $\overrightarrow{BC}$ can be expressed as a scalar multiple of $\overrightarrow{AE}$. Hence, *BC* is parallel to *AE*.

UPSKILL 8.2d

1



(a) $\overrightarrow{PQ} = \overrightarrow{PO} + \overrightarrow{OQ}$

$= -4\underline{a} + 6\underline{b}$

(b) $\overrightarrow{PL} = \frac{3}{5}\overrightarrow{PQ}$

$= \frac{3}{5}(-4\underline{a} + 6\underline{b})$

$= -\frac{12}{5}\underline{a} + \frac{18}{5}\underline{b}$

(c) $\overrightarrow{OM} = 3\overrightarrow{OQ}$

$= 3(6\underline{b})$

$= 18\underline{b}$

(d) $\overrightarrow{KL} = \overrightarrow{KP} + \overrightarrow{PL}$

$= 2\underline{a} - \frac{12}{5}\underline{a} + \frac{18}{5}\underline{b}$

$= -\frac{2}{5}\underline{a} + \frac{18}{5}\underline{b}$

(e) $\overrightarrow{KM} = \overrightarrow{KO} + \overrightarrow{OM}$

$= -2\underline{a} + 18\underline{b}$

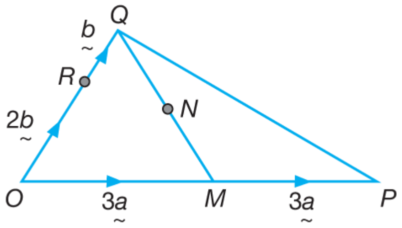
$\overrightarrow{KL} = -\frac{2}{5}\underline{a} + \frac{18}{b}\underline{b}$

$= \frac{1}{5}(-2\underline{a} + 18\underline{b})$

$= \frac{1}{5}\overrightarrow{KM}$

Since $\overrightarrow{KL} = \frac{1}{5}\overrightarrow{KM}$, $\overrightarrow{KL}$ can be expressed as a scalar multiple of $\overrightarrow{KM}$ and *K* is a common point. Thus, the points *K*, *L* and *M* are collinear.

2



(a) $\overrightarrow{PQ} = \overrightarrow{PO} + \overrightarrow{OQ}$

$= -6\underline{a} + 3\underline{b}$

(b) $\overrightarrow{QM} = \overrightarrow{QO} + \overrightarrow{OM}$

$= -3\underline{b} + 3\underline{a}$

(c) $\overrightarrow{ON} = \overrightarrow{OQ} + \overrightarrow{QN}$

$= 3\underline{b} + \frac{1}{2}\overrightarrow{QM}$

$= 3\underline{b} + \frac{1}{2}(-3\underline{b} + 3\underline{a})$

$= 3\underline{b} - \frac{3}{2}\underline{b} + \frac{3}{2}\underline{a}$

$= \frac{3}{2}\underline{b} + \frac{3}{2}\underline{a}$

(d) $\overrightarrow{RN} = \overrightarrow{RQ} + \overrightarrow{QN}$

$= \underline{b} - \frac{3}{2}\underline{b} + \frac{3}{2}\underline{a}$

$= -\frac{1}{2}\underline{b} + \frac{3}{2}\underline{a}$

(e) $\overrightarrow{RP} = \overrightarrow{RO} + \overrightarrow{OP}$

$= -2\underline{b} + 6\underline{a}$

$= 2(-\underline{b} + 3\underline{a}) \quad \ldots (1)$

$\overrightarrow{RN} = -\frac{1}{2}\underline{b} + \frac{3}{2}\underline{a}$

$= \frac{1}{2}(-\underline{b} + 3\underline{a})$

$= \frac{1}{2}\left(\frac{1}{2}\overrightarrow{RP}\right)$

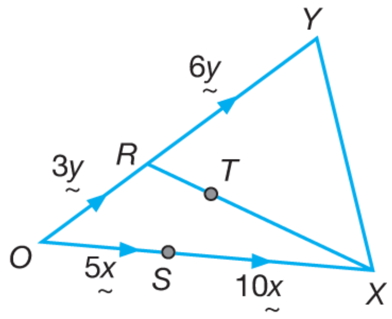
$= \frac{1}{4}\overrightarrow{RP}$

From (1) : $-\underline{b} + 2\underline{a} = \frac{1}{2}\overrightarrow{RP}$

Since $\overrightarrow{RN} = \frac{1}{4}\overrightarrow{RP}$, $\overrightarrow{RN}$ can be expressed as a scalar multiple of $\overrightarrow{RP}$ and R is a common point. Thus, the points R, N and P are collinear.

$RN : RP = 1 : 4$

3



(a) $\overrightarrow{RS} = \overrightarrow{RO} + \overrightarrow{OS}$

$= -3\underline{y} + 5\underline{x}$

(b) $\overrightarrow{RX} = \overrightarrow{RO} + \overrightarrow{OX}$

$= -3\underline{y} + 15\underline{x}$

(c) $\overrightarrow{SY} = \overrightarrow{SO} + \overrightarrow{OY}$

$= -5\underline{x} + 9\underline{y} \quad \ldots (1)$

(d) $\overrightarrow{OT} = \overrightarrow{OR} + \overrightarrow{RT}$

$= 3\underline{y} + \frac{1}{4}\overrightarrow{RX}$

$= 3\underline{y} + \frac{1}{4}(-3\underline{y} + 15\underline{x})$

$= \frac{9}{4}\underline{y} + \frac{15}{4}\underline{x}$

(e) $\overrightarrow{ST} = \overrightarrow{SO} + \overrightarrow{OT}$

$= -5\underline{x} + \frac{9}{4}\underline{y} + \frac{15}{4}\underline{x}$

$= -\frac{5}{4}\underline{x} + \frac{9}{4}\underline{y}$

$= \frac{1}{4}(-5\underline{x} + 9\underline{y})$

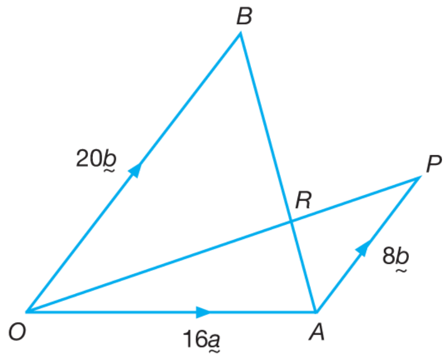
$= \frac{1}{4}\overrightarrow{SY}$

From (1) : $-5\underline{x} + 9\underline{y} = \overrightarrow{SY}$

Since $\overrightarrow{ST} = \frac{1}{4}\overrightarrow{SY}$, $\overrightarrow{ST}$ can be expressed as a scalar multiple of $\overrightarrow{SY}$ and S is a common point. Thus, the points S, T and Y are collinear.

UPSKILL 8.2e

1



(a) (i) $\overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{AP}$

$= 16\underset{\sim}{a} + \frac{2}{5}\overrightarrow{OB}$

$= 16\underset{\sim}{a} + \frac{2}{5}(20\underset{\sim}{b})$

$= 16\underset{\sim}{a} + 8\underset{\sim}{b}$

(ii) $\overrightarrow{BA} = -20b + 16a$

(b) (i) $\overrightarrow{OR} = m\overrightarrow{OP}$

$= m(16\underset{\sim}{a} + 8\underset{\sim}{b})$

$= 16m\underset{\sim}{a} + 8m\underset{\sim}{b}$

(ii) $\overrightarrow{OR} = \overrightarrow{OB} + \overrightarrow{BR}$

$= 20\underset{\sim}{b} + n\overrightarrow{BA}$

$= 20\underset{\sim}{b} + n(-20\underset{\sim}{b} + 16\underset{\sim}{a})$

$= 20\underset{\sim}{b} - 20n\underset{\sim}{b} + 16n\underset{\sim}{a}$

$= (20b - 20n)\underset{\sim}{b} + 16n\underset{\sim}{a}$

(c) $16m\underset{\sim}{a} + 8m\underset{\sim}{b} = (20 - 20n)\underset{\sim}{b} + 16n\underset{\sim}{a}$

Equating the coefficients of $\underset{\sim}{a}$,

$16m = 16n$

$m = n \quad \dots (1)$

Equating the coefficients of $\underset{\sim}{b}$,

$8m = 20 - 20n$

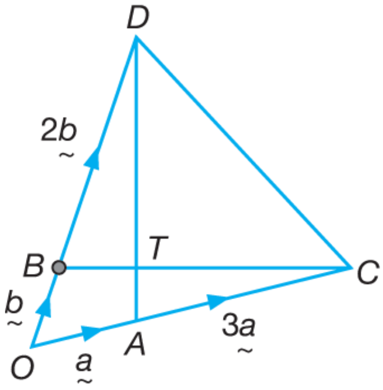
$8m = 20 - 20m$

$28m = 20$

$m = \frac{5}{7}$

From (1) : $n = m = \frac{5}{7}$

2



$\overrightarrow{AD} = -\underset{\sim}{a} + 3\underset{\sim}{b}$

$\overrightarrow{BC} = -\underset{\sim}{b} + 4\underset{\sim}{a}$

(a) $\overrightarrow{OT} = \overrightarrow{OA} + \overrightarrow{AT}$

$= \underset{\sim}{a} + k\,\overrightarrow{AD}$

$= \underset{\sim}{a} + k(-\underset{\sim}{a} + 3\underset{\sim}{b})$

$= (1-k)\underset{\sim}{a} + 3k\underset{\sim}{b}$

(b) $\overrightarrow{OT} = \overrightarrow{OB} + \overrightarrow{BT}$

$= \underset{\sim}{b} + t\,\overrightarrow{BC}$

$= \underset{\sim}{b} + t(-\underset{\sim}{b} + 4\underset{\sim}{a})$

$= (1-t)\underset{\sim}{b} + 4t\underset{\sim}{a}$ c

$(1-k)\underset{\sim}{a} + 3k\underset{\sim}{b} = (1-t)\underset{\sim}{b} + 4t\underset{\sim}{a}$

Equating the coefficients of $\underset{\sim}{a}$,

$1 - k = 4t$

$k = 1 - 4t \quad \dots (1)$

Equating the coefficients of $\underset{\sim}{b}$,

$3k = 1 - t \quad \dots (2)$

Substitute (1) into (2) :

$3(1 - 4t) = 1 - t$

$3 - 12t = 1 - t$

$11t = 2$

$t = \frac{2}{11}$

From (1) :

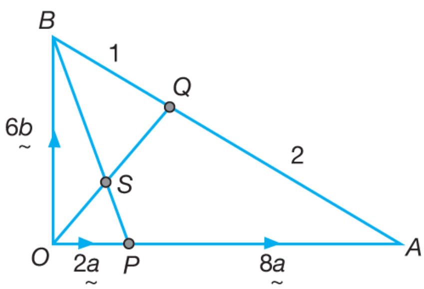
$k = 1 - 4\left(\frac{2}{11}\right) = \frac{3}{11}$

$$\overrightarrow{OT} = (1-k)\underline{a} + 3k\underline{b}$$

$$\overrightarrow{OT} = \left(1-\frac{3}{11}\right)\underline{a} + 3\left(\frac{3}{11}\right)\underline{b}$$

$$\overrightarrow{OT} = \frac{8}{11}\underline{a} + \frac{9}{11}\underline{b}$$

3



(a) (i) $\overrightarrow{BP} = -6\underline{b} + 2\underline{a}$

(ii) $\overrightarrow{OQ} = \overrightarrow{OB} + \overrightarrow{BQ}$

$$= 6\underline{b} + \frac{1}{3}\overrightarrow{BA}$$

$$= 6\underline{b} + \frac{1}{3}(-6\underline{b} + 10\underline{a})$$

$$= 4\underline{b} + \frac{10}{3}\underline{a}$$

(b) $\overrightarrow{OS} = \overrightarrow{OB} + \overrightarrow{BS}$

$$h\overrightarrow{OQ} = 6\underline{b} + k\,\overrightarrow{BP}$$

$$h\left(4\underline{b} + \frac{10}{3}\underline{a}\right) = 6\underline{b} + k(-6\underline{b} + 2\underline{a})$$

$$4h\underline{b} + \frac{10}{3}h\underline{a} = 6\underline{b} - 6k\underline{b} + 2k\underline{a}$$

$$4h\underline{b} + \frac{10}{3}h\underline{a} = (6-6k)\underline{b} + 2k\underline{a}$$

Equating the coefficients of $\underline{b}$,

$4h = 6 - 6k \quad \ldots (1)$

Equating the coefficients of $\underline{a}$,

$$\frac{10}{3}h = 2k$$

$10h = 6k$

$5h = 3k$

$6k = 10h \quad \ldots (2)$

Substitute (2) into (1) :

$4h = 6 - 10h$

$14h = 6$

$$h = \frac{3}{7}$$

From (2) :

$$6k = 10\left(\frac{3}{7}\right)$$

$$k = \frac{5}{7}$$

UPSKILL 8.3a

1 (a) $\overrightarrow{AB} = 5\underline{i}$

$$\left|\overrightarrow{AB}\right| = 5$$

(b) $\overrightarrow{CD} = 4\underline{i} + 3\underline{j}$

$$\left|\overrightarrow{CD}\right| = \sqrt{4^2 + 3^2} = 5$$

(c) $\overrightarrow{EF} = 5\underline{i} - 3\underline{j}$

$$\left|\overrightarrow{EF}\right| = \sqrt{5^5 + (-3)^2} = \sqrt{34} = 5.831$$

(d) $\overrightarrow{PQ} = -11\underline{i} - 5\underline{j}$

$$\left|\overrightarrow{PQ}\right| = \sqrt{(-11)^2 + (-5)^2} = \sqrt{146} = 12.08$$

UPSKILL 8.3b

1 (a) $|\underline{r}| = \sqrt{(-8)^2 + (-6)^2} = 10$

$$\hat{\underline{r}} = \frac{1}{10}(-8i - 6j) = -\frac{4}{5}\underline{i} + \frac{3}{5}\underline{j}$$

(b) $|\underline{s}| = \sqrt{(-8)^2 + 15^2} = 17$

$$\hat{s} = \frac{1}{17}\begin{pmatrix} -8 \\ 15 \end{pmatrix} = \begin{pmatrix} -\frac{8}{17} \\ \frac{15}{17} \end{pmatrix}$$

UPSKILL 8.3c

1 (a) $2\underset{\sim}{a} = 2(-3\underset{\sim}{i} + 4\underset{\sim}{j}) = -6\underset{\sim}{i} + 8\underset{\sim}{j}$

$|2\underset{\sim}{a}| = \sqrt{(-6)^2 + 8^2} = 10$

(b) $-3\underset{\sim}{b} = -3(\underset{\sim}{i} + 3\underset{\sim}{j}) = -3\underset{\sim}{i} - 9\underset{\sim}{j}$

$|-3\underset{\sim}{b}| = \sqrt{(-3)^2 + (-9)^2} = 3\sqrt{10} = 9.487$

2 (a) $2\underset{\sim}{a} + 3\underset{\sim}{b} = 2\begin{pmatrix}3\\4\end{pmatrix} + 3\begin{pmatrix}1\\-3\end{pmatrix} = \begin{pmatrix}9\\-1\end{pmatrix}$

(b) $3\underset{\sim}{a} + 2\underset{\sim}{c} = 3\begin{pmatrix}3\\4\end{pmatrix} + 2\begin{pmatrix}-2\\4\end{pmatrix} = \begin{pmatrix}5\\20\end{pmatrix}$

(c) $\underset{\sim}{a} - \underset{\sim}{b} + \underset{\sim}{c} = \begin{pmatrix}3\\4\end{pmatrix} - \begin{pmatrix}1\\-3\end{pmatrix} + \begin{pmatrix}-2\\4\end{pmatrix} = \begin{pmatrix}0\\11\end{pmatrix}$

(d) $2\underset{\sim}{a} - 3\underset{\sim}{b} + 3\underset{\sim}{c} = 2\begin{pmatrix}3\\4\end{pmatrix} - 3\begin{pmatrix}1\\-3\end{pmatrix} + 3\begin{pmatrix}-2\\4\end{pmatrix}$

$= \begin{pmatrix}-3\\29\end{pmatrix}$

3 $t\underset{\sim}{b} - k\underset{\sim}{a} = \underset{\sim}{c}$

$t(\underset{\sim}{i} + 2\underset{\sim}{j}) - k(3\underset{\sim}{i} + 4\underset{\sim}{j}) = 3\underset{\sim}{i} - 2\underset{\sim}{j}$

$(t - 3k)\underset{\sim}{i} + (2t - 4k)\underset{\sim}{j} = 3\underset{\sim}{i} - 2\underset{\sim}{j}$

Equating the coefficients of $\underset{\sim}{i}$,

$t - 3k = 3$

$2t - 6k = 6 \ \ldots (1)$

Equating the coefficients of $\underset{\sim}{j}$,

$2t - 4k = -2 \ \ldots (2)$

(1) – (2) : $-2k = 8$

$k = -4$

From (1) :

$t - 3(-4) = 3$

$t = -9$

4 $p\underset{\sim}{a} + k\underset{\sim}{b} = \underset{\sim}{c}$

$p\begin{pmatrix}2\\1\end{pmatrix} + k\begin{pmatrix}3\\-2\end{pmatrix} = \begin{pmatrix}13\\-4\end{pmatrix}$

$2p + 3k = 13 \ \ldots (1)$

$p - 2k = -4 \ \ldots (2)$

$p = 2k - 4 \ \ldots (3)$

Substitute (3) into (1) :

$2(2k - 4) + 3k = 13$

$4k - 8 + 3k = 13$

$7k = 21$

$k = 3$

From (3) :

$p = 2(3) - 4 = 2$

5 (a) $\overrightarrow{OP} = h\underset{\sim}{a} + k\underset{\sim}{b}$

$\begin{pmatrix}5\\-3\end{pmatrix} = h\begin{pmatrix}3\\0\end{pmatrix} + k\begin{pmatrix}1\\3\end{pmatrix}$

$3h + k = 5 \ \ldots (1)$

$3k = -3 \ \ldots (2)$

From (2) : $k = -1$

From (1) : $3h - 1 = 5$

$h = 2$

$\therefore \overrightarrow{OP} = 2\underset{\sim}{a} - \underset{\sim}{b}$

(b) $\overrightarrow{PQ} = p\underset{\sim}{a} + q\underset{\sim}{b}$

$\begin{pmatrix}-1\\6\end{pmatrix} = p\begin{pmatrix}3\\0\end{pmatrix} + q\begin{pmatrix}1\\3\end{pmatrix}$

$3p + q = -1 \ \ldots (1)$

$3q = 6 \ \ldots (2)$

From (2) : $q = 2$

From (1) : $3p + 2 = -1$

$p = -1$

$\therefore \overrightarrow{PQ} = -\underset{\sim}{a} + 2\underset{\sim}{b}$

6 Since $\underset{\sim}{u}$ and $\underset{\sim}{v}$ are parallel, thus

$\underset{\sim}{u} = m\underset{\sim}{v}$ [m is a constant.]

$3\underset{\sim}{i} + 6\underset{\sim}{j} = m(k\underset{\sim}{i} - 2\underset{\sim}{j})$

$3\underset{\sim}{i} + 6\underset{\sim}{j} = mk\underset{\sim}{i} - 2m\underset{\sim}{j}$

Equating the coefficients of $\underset{\sim}{j}$,

$-2m = 6$

$m = -3$

Equating the coefficients of $\underset{\sim}{i}$,

$mk = 3$

$-3k = 3$

$k = -1$

7 $\underset{\sim}{r} = k\underset{\sim}{s}$

$2\underset{\sim}{i} + (p + 3)\underset{\sim}{j} = k[(p - 5)\underset{\sim}{i} - 8\underset{\sim}{j}]$

$2\underset{\sim}{i} + (p + 3)\underset{\sim}{j} = k(p - 5)\underset{\sim}{i} - 8k\underset{\sim}{j}$

Equating the coefficients of $\underset{\sim}{i}$,

$2 = k(p-5)$

$2 = kp - 5k \quad \ldots (1)$

Equating the coefficients of $\underset{\sim}{j}$,

$p + 3 = -8k$

$p = -8k - 3 \quad \ldots (2)$

Substitute (2) into (1) :

$2 = k(-8k-3) - 5k$

$8k^2 + 8k + 2 = 0$

$4k^2 + 4k + 1 = 0$

$(2k+1)(2k+1) = 0$

$k = -\frac{1}{2}$

From (2) :

When $k = -\frac{1}{2}$,

$p = -8\left(-\frac{1}{2}\right) - 3 = 1$

8 $\overrightarrow{PQ} = m\overrightarrow{PR}$

$\overrightarrow{OQ} - \overrightarrow{OP} = m\left(\overrightarrow{OR} - \overrightarrow{OP}\right)$

$5i - 2j - (3\underset{\sim}{i} + \underset{\sim}{j}) = m\left[k\underset{\sim}{i} - 6\underset{\sim}{j} - (3\underset{\sim}{i} + \underset{\sim}{j})\right]$

$2\underset{\sim}{i} - 3\underset{\sim}{j} = m\left[(k-3)\underset{\sim}{i} - 7\underset{\sim}{j}\right]$

$2\underset{\sim}{i} - 3\underset{\sim}{j} = m(k-3)\underset{\sim}{i} - 7m\underset{\sim}{j}$

Equating the coefficients of- $\underset{\sim}{j}$,

$-7m = -3$

$m = \frac{3}{7}$

Equating the coefficients of- $\underset{\sim}{i}$,

$m(k-3) = 2$

$\frac{3}{7}(k-3) = 2$

$3(k-3) = 14$

$3k - 9 = 14$

$3k = 23$

$k = \frac{23}{3}$

9 $|\underset{\sim}{u}| = |\underset{\sim}{v}|$

$\sqrt{(k-2)^2 + 4^2} = \sqrt{(k-1)^2 + 3^2}$

$(k-2)^2 + 16 = (k-1)^2 + 9$

$k^2 - 4k + 4 + 16 = k^2 - 2k + 1 + 9$

$20 - 4k = 10 - 2k$

$2k = 10$

$k = 5$

10 $\underset{\sim}{x} - \underset{\sim}{y} = 3\underset{\sim}{i} + k\underset{\sim}{j} - (4\underset{\sim}{i} - 3\underset{\sim}{j})$

$= -\underset{\sim}{i} + (k+3)\underset{\sim}{j}$

$|x - y| = \sqrt{5}$

$\sqrt{(-1)^2 + (k+3)^2} = \sqrt{5}$

$1 + k^2 + 6k + 9 = 5$

$k^2 + 6k + 5 = 0$

$(k+1)(k+5) = 0$

$k = -1$ or $k = -5$

UPSKILL 8.3d

1 (a) Resultant vector

$= (16\underset{\sim}{i} + 12\underset{\sim}{j}) + (6\underset{\sim}{i} - 8\underset{\sim}{j})$

$= 22\underset{\sim}{i} + 4\underset{\sim}{j}$

(b) Magnitude

$= \sqrt{22^2 + 4^2}$

$= \sqrt{500}$

$= 22.36 \text{ km h}^{-1}$

2 (a) Resultant vector

$= 500\underset{\sim}{i} - 300\underset{\sim}{j} + (-60\underset{\sim}{i} - 80\underset{\sim}{j})$

$= 440\underset{\sim}{i} - 380\underset{\sim}{j}$

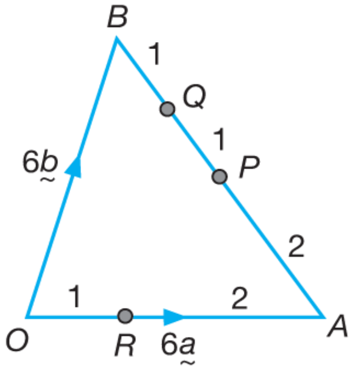
(b) Magnitude

$= \sqrt{440^2 + (-380)^2}$

$= 581.38 \text{ km h}^{-1}$

Summative Practice 8

1



(a) $\overrightarrow{OQ} = \overrightarrow{OB} + \overrightarrow{BQ}$

$= 6\underline{b} + \frac{1}{4}\overrightarrow{BA}$

$= 6\underline{b} + \frac{1}{4}(-6\underline{b} + 6\underline{a})$

$= 6\underline{b} - \frac{3}{2}\underline{b} + \frac{3}{2}a$

$= \frac{9}{2}\underline{b} + \frac{3}{2}\underline{a}$

$= \frac{3}{2}(3\underline{b} + \underline{a}) \quad \ldots (1)$

(b) $\overrightarrow{RP} = \overrightarrow{RA} + \overrightarrow{AP}$

$= \frac{2}{3}\overrightarrow{OA} + \frac{1}{2}\overrightarrow{AB}$

$= \frac{2}{3}(6\underline{a}) + \frac{1}{2}\left(\overrightarrow{AO} + OB\right)$

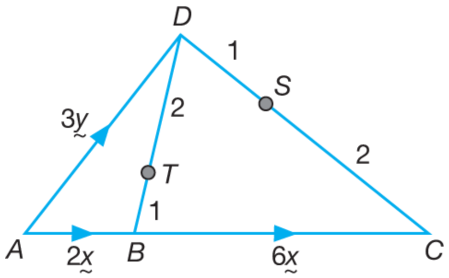
$= 4\underline{a} + \frac{1}{2}(-6a + 6b)$

$= \underline{a} + 3\underline{b}$

Since $\overrightarrow{OQ} = \frac{3}{2}\overrightarrow{RP}$, $\overrightarrow{OQ}$ can be expressed as a scalar multiple of $\overrightarrow{RP}$.

Thus, $\overrightarrow{OQ}$ is parallel to $\overrightarrow{RP}$.

2



(a) (i) $\overrightarrow{AS} = \overrightarrow{AD} + \overrightarrow{DS}$

$= 3\underline{y} + \frac{1}{3}\overrightarrow{DC}$

$= 3y + \frac{1}{3}(-3\underline{y} + 8\underline{x})$

$= 2\underline{y} + \frac{8}{3}\underline{x}$

(ii) $\overrightarrow{TC} = \overrightarrow{TD} + \overrightarrow{DC}$

$= \frac{2}{3}\left(\overrightarrow{BD}\right) + \overrightarrow{DA} + \overrightarrow{AC}$

$= \frac{2}{3}(-2\underline{x} + 3\underline{y}) + (-3\underline{y} + 8\underline{x})$

$= -\frac{4}{3}\underline{x} + 2\underline{y} - 3\underline{y} + 8\underline{x}$

$= \frac{20}{3}\underline{x} - \underline{y}$

(b) $\overrightarrow{AT} = \overrightarrow{AB} + \overrightarrow{BT}$

$= 2\underline{x} + \frac{1}{3}\overrightarrow{BD}$

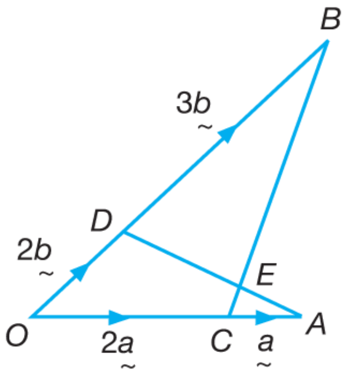
$= 2\underline{x} + \frac{1}{3}(-2\underline{x} + 3\underline{y})$

$= \frac{4}{3}\underline{x} + \underline{y}$

$= \frac{1}{2}\overrightarrow{AS}$

Hence, $\overrightarrow{AT}$ can be expressed as a scalar multiple of $\overrightarrow{AS}$ and A is a common point. Thus, the points A, T and S are collinear.

3



(a) (i) $\overrightarrow{OE} = \overrightarrow{OA} + \overrightarrow{AE}$

$= 3\underset{\sim}{a} + h\,\overrightarrow{AD}$
$= 3\underset{\sim}{a} + h(-3\underset{\sim}{a} + 2\underset{\sim}{b})$
$= (3 - 3h)\underset{\sim}{a} + 2h\underset{\sim}{b}\ \ldots (1)$

(ii) $\overrightarrow{OE} = \overrightarrow{OB} + \overrightarrow{BE}$

$= 5\underset{\sim}{b} + k\,\overrightarrow{BC}$
$= 5\underset{\sim}{b} + k(-5\underset{\sim}{b} + 2\underset{\sim}{a})$
$= (5 - 5k)\underset{\sim}{b} + 2k\underset{\sim}{a}\ \ldots (2)$

(b) $(3 - 3h)\underset{\sim}{a} + 2h\underset{\sim}{b} = (5 - 5k)\underset{\sim}{b} + 2k\underset{\sim}{a}$

Equating the coefficients of $\underset{\sim}{a}$,

$3 - 3h = 2k\ \ldots (1)$

Equating the coefficients of $\underset{\sim}{b}$,

$2h = 5 - 5k$

$h = \dfrac{5 - 5k}{2}\ \ldots (2)$

Substitute (2) into (1) :

$3 - 3\left(\dfrac{5 - 5k}{2}\right) = 2k$

$6 - 3(5 - 5k) = 4k$

$6 - 15 + 15k = 4k$

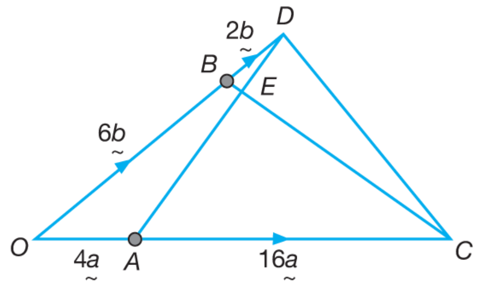
$11k = 9$

$k = \dfrac{9}{11}$

From (2) :

$h = \dfrac{5 - 5\left(\dfrac{9}{11}\right)}{2} = \dfrac{5}{11}$

4



(a) (i) $\overrightarrow{AD} = -4\underset{\sim}{a} + 8\underset{\sim}{b}$

(ii) $\overrightarrow{BC} = -6\underset{\sim}{b} + 20\underset{\sim}{a}$

(b) $\overrightarrow{AE} = \overrightarrow{AB} + \overrightarrow{BE}$

$h\,\overrightarrow{AD} = -4\underset{\sim}{a} + 6\underset{\sim}{b} + k\,\overrightarrow{BC}$

$h(-4\underset{\sim}{a} + 8\underset{\sim}{b}) = -4\underset{\sim}{a} + 6\underset{\sim}{b} + k(-6\underset{\sim}{b} + 20\underset{\sim}{a})$

$-4h\underset{\sim}{a} + 8h\underset{\sim}{b} = -4\underset{\sim}{a} + 6\underset{\sim}{b} - 6k\underset{\sim}{b} + 20k\underset{\sim}{a}$

$-4h\underset{\sim}{a} + 8h\underset{\sim}{b} = (-4 + 20k)\underset{\sim}{a} + (6\underset{\sim}{b} - 6k)\underset{\sim}{b}$

Equating the coefficients of $\underset{\sim}{a}$,

$-4h = -4 + 20k\ \ldots (1)$

Equating the coefficients of $\underset{\sim}{b}$,

$8h = 6 - 6k$

$h = \dfrac{6 - 6k}{8}\ \ldots (2)$

Substitute (2) into (1) :

$-4\left(\dfrac{6 - 6k}{8}\right) = -4 + 20k$

$-\left(\dfrac{6 - 6k}{2}\right) = -4 + 20k$

$-(3 - 3k) = -4 + 20k$

$-3 + 3k = -4 + 20k$

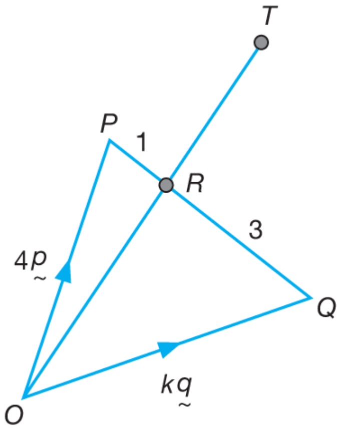
$17k = 1$

$k = \dfrac{1}{17}$

From (2) :

$h = \dfrac{6 - 6\left(\dfrac{1}{17}\right)}{8} = \dfrac{12}{17}$

5



(a) $\overrightarrow{OR} = \overrightarrow{OP} + \overrightarrow{PR}$

$= 4\underline{p} + \frac{1}{4}\overrightarrow{PQ}$

$= 4\underline{p} + \frac{1}{4}\left(\overrightarrow{PO} + \overrightarrow{OQ}\right)$

$= 4\underline{p} + \frac{1}{4}\left(-4\underline{p} + k\underline{q}\right)$

$= 4\underline{p} - \underline{p} + \frac{k}{4}\underline{q}$

$= 3\underline{p} + \frac{k}{4}\underline{q}$

(b) $\overrightarrow{OR} = m\,\overrightarrow{RT}$

$3\underline{p} + \frac{k}{4}\underline{q} = m\left(2\underline{p} + \frac{5}{3}\underline{q}\right)$

$3\underline{p} + \frac{k}{4}\underline{q} = 2m\underline{p} + \frac{5}{3}m\underline{q}$

Equating the coefficients of $\underline{p}$,

$2m = 3$

$m = \frac{3}{2}$

Equating the coefficients of $\underline{q}$,

$\frac{k}{4} = \frac{5}{3}m$

$\frac{k}{4} = \frac{5}{3}\left(\frac{3}{2}\right)$

$k = 10$

6 (a) $\overrightarrow{PR} = \overrightarrow{PQ} + \overrightarrow{QR}$

$= 4\underline{u} + \frac{3}{2}\overrightarrow{PS}$

$= 4\underline{u} + \frac{3}{2}(12\underline{v})$

$= 4\underline{u} + 18\underline{v}$

Given $\overrightarrow{PS} = \frac{2}{3}\overrightarrow{QR}$, thus $\overrightarrow{QR} = \frac{3}{2}\overrightarrow{PS}$.

(b) (i) $\overrightarrow{TX} = m\,\overrightarrow{PQ}$

$= m(4\underline{u})$

$= 4m\underline{u}$

(ii) P, X and R are collinear.

$\overrightarrow{PX} = k\,\overrightarrow{PR}$

$\overrightarrow{PT} + \overrightarrow{TX} = k(4\underline{u} + 18\underline{v})$

$\frac{4}{3}\overrightarrow{PS} + 4m\underline{u} = 4k\underline{u} + 18k\underline{v}$

Given | From (b) (i)

$\frac{3}{4}(12\underline{v}) + 4m\underline{u} = 4k\underline{u} + 18k\underline{v}$

$9\underline{v} + 4m\underline{u} = 4k\underline{u} + 18k\underline{v}$

Equating the coefficients of $\underline{v}$,

$9 = 18k$

$k = \frac{1}{2}$

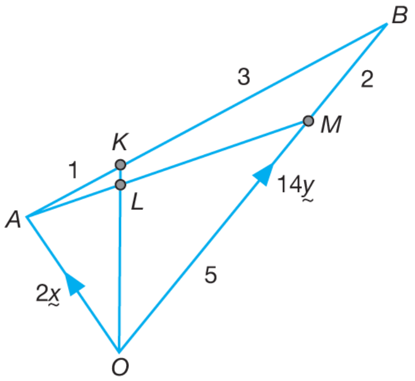
Equating the coefficients of $\underline{u}$,

$4m = 4k$

$4m = 4\left(\frac{1}{2}\right)$

$m = \frac{1}{2}$

7



(a) (i) $\overrightarrow{OM} = \frac{5}{7}\overrightarrow{OB}$

$= \frac{5}{7}(14\underline{y})$

$= 10\underline{y}$

(ii) $\overrightarrow{AK} = \frac{1}{4}\overrightarrow{AB}$

$= \frac{1}{4}(-2\underline{x} + 10\underline{y})$

$= -\frac{1}{2}\underline{x} + \frac{7}{2}\underline{y}$

(b) (i) $\overrightarrow{AL} = p\,\overrightarrow{AM}$

$= p(-2\underline{x} + 10\underline{y})$

$= -2p\underline{x} + 10p\underline{y}$

(ii) $\overrightarrow{KL} = q\,\overrightarrow{KO}$

$= q\left(\overrightarrow{KA} + \overrightarrow{AO}\right)$

$= q\left(\frac{1}{4}\overrightarrow{BA} - 2\underline{x}\right)$

$= q\left[\frac{1}{4}(-14y + 2x) - 2\underline{x}\right]$

$= q\left(-\frac{7}{2}y + \frac{1}{2}x - 2x\right)$

$= q\left(-\frac{7}{2}y - \frac{3}{2}x\right)$

$= -\frac{7}{2}q\underline{y} - \frac{3}{2}q\underline{x}$

(c) $\overrightarrow{AK} = \overrightarrow{AL} + \overrightarrow{LK}$

$\frac{7}{2}y - \frac{1}{2}x = -2px + 10py + \frac{7}{2}qy + \frac{3}{2}qx$

$\frac{7}{2}\underline{y} - \frac{1}{2}\underline{x} = \left(10p + \frac{7}{2}q\right)\underline{y} + \left(-2p + \frac{3}{2}q\right)\underline{x}$

Equating the coefficients of $\underline{x}$:

$-\frac{1}{2} = -2p + \frac{3}{2}q$

$-4p + 3q = -1 \quad \ldots (1)$

Equating the coefficients of $\underline{y}$:

$10p + \frac{7}{2}q = \frac{7}{2}$

$20p + 7q = 7 \quad \ldots (2)$

$-20p + 15q = -5 \quad \ldots (1) \times 5$

$(+) \quad 20p + 7q = 7 \quad \ldots (2)$

$22q = 2$

$q = \frac{1}{11}$

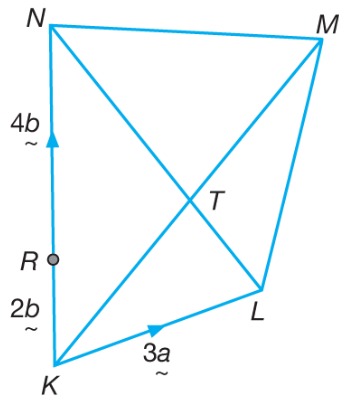
From (1) :

$-4p + 3\left(\frac{1}{11}\right) = -1$

$-4p = -\frac{14}{11}$

$p = \frac{7}{22}$

8



(a) (i) $\overrightarrow{KR} = \frac{1}{3}\overrightarrow{KN}$

$\overrightarrow{KN} = 3\overrightarrow{KR}$

$\overrightarrow{KN} = 3(2\underline{b})$

$\overrightarrow{KN} = 6\underline{b}$

$\overrightarrow{NL} = \overrightarrow{NK} + \overrightarrow{KL}$

$= -6\underline{b} + 3\underline{a}$

(ii) $\overrightarrow{KT} = \overrightarrow{KL} + \overrightarrow{LT}$

$= \overrightarrow{KL} + \frac{1}{3}\overrightarrow{LN}$

$= 3\underline{a} + \frac{1}{3}(6\underline{b} - 3\underline{a})$

$= 3\underline{a} + 2\underline{b} - \underline{a}$

$= 2\underline{a} + 2\underline{b}$

(b) (i) $\overrightarrow{NM} = \overrightarrow{NK} + \overrightarrow{KM}$

$= \overrightarrow{NK} + \frac{1}{q}\overrightarrow{KT}$

$= -6\underline{b} + \frac{1}{q}(2\underline{a} + 2\underline{b})$

$= -6\underline{b} + \frac{2}{q}\underline{a} + \frac{2}{q}\underline{b}$

$= \left(\frac{2}{q} - 6\right)\underline{b} + \frac{2}{q}\underline{a}$

(ii) But it is given that $\overrightarrow{NM} = 3p\underline{a} - 2\underline{b}$.

By comparison,

$\frac{2}{q} - 6 = -2$

$\frac{2}{q} = 4$

$q = \frac{1}{2}$

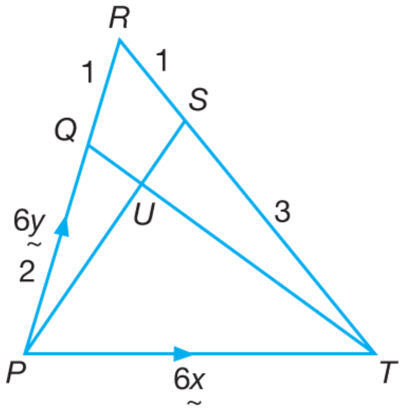
$3p = \frac{2}{q}$

$p = \frac{2}{3q}$

$p = \frac{2}{3\left(\frac{1}{2}\right)}$

$p = \frac{4}{3}$

9



(a) (i) $\overrightarrow{TR} = \overrightarrow{TP} + \overrightarrow{PR} = -6\underline{x} + 6\underline{y}$

(ii) $\overrightarrow{PS} = \overrightarrow{PT} + \overrightarrow{TS}$

$= \overrightarrow{PT} + \frac{3}{4}\overrightarrow{TR}$

$= 6\underline{x} + \frac{3}{4}(-6\underline{x} + 6\underline{y})$

$= 6\underline{x} - \frac{9}{2}\underline{x} + \frac{9}{2}\underline{y}$

$= \frac{3}{2}\underline{x} + \frac{9}{2}\underline{y}$

(b) $\overrightarrow{PU} = h\overrightarrow{PS}$

$= h\left(\frac{3}{2}\underline{x} + \frac{9}{2}\underline{y}\right)$

$= \frac{3}{2}h\underline{x} + \frac{9}{2}h\underline{y}$

$\overrightarrow{PU} = \overrightarrow{PT} + k\overrightarrow{TQ}$

$= 6\underline{x} + k(-6\underline{x} + 4\underline{y})$

$= (6 - 6k)\underline{x} + 4k\underline{y}$

Equating the coefficients of $\underline{x}$,

$$\frac{3}{2}h = 6 - 6k$$
$$3h = 12 - 12k$$
$$h = 4 - 4k \quad \ldots (1)$$

Equating the coefficients of $\underline{y}$,

$$4k = \frac{9}{2}h$$
$$8k = 9h \quad \ldots (2)$$

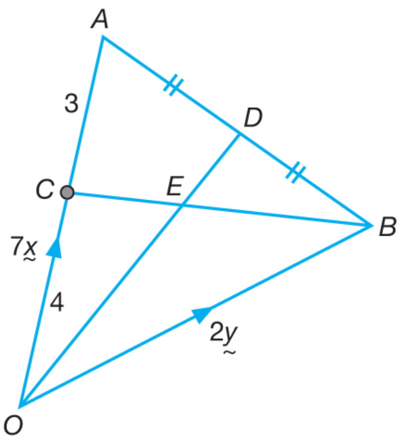
Substitute (1) into (2) :

$$8k = 9(4 - 4k)$$
$$8k = 36 - 36k$$
$$44k = 36$$
$$k = \frac{9}{11}$$

From (1) :

$$h = 4 - 4\left(\frac{9}{11}\right) = \frac{8}{11}$$

10



(a) (i) $\overrightarrow{BC} = \overrightarrow{BO} + \overrightarrow{OC}$

$$= -2\underline{y} + \frac{4}{7}\overrightarrow{OA}$$
$$= -2\underline{y} + \frac{4}{7}(7\underline{x})$$
$$= -2\underline{y} + 4\underline{x}$$

(ii) $\overrightarrow{OD} = \overrightarrow{OB} + \overrightarrow{BD}$

$$= 2\underline{y} + \frac{1}{2}\overrightarrow{BA}$$
$$= 2\underline{y} + \frac{1}{2}\left(-2\underline{y} + 7\underline{x}\right)$$
$$= 2\underline{y} - \underline{y} + \frac{7}{2}\underline{x}$$
$$= \underline{y} + \frac{7}{2}\underline{x}$$

(b) (i) $\overrightarrow{OE} = p\,\overrightarrow{OD}$

$$= p\left(\underline{y} + \frac{7}{2}\underline{x}\right)$$
$$= p\underline{y} + \frac{7}{2}p\underline{x} \quad \ldots (1)$$

(ii) $\overrightarrow{OE} = \overrightarrow{OB} + \overrightarrow{BE}$

$$= \overrightarrow{OB} + q\,\overrightarrow{BC}$$
$$= 2\underline{y} + q\left(-2\underline{y} + 4\underline{x}\right)$$
$$= (2 - 2q)\underline{y} + 4q\underline{x} \quad \ldots (2)$$

(c) Equating (1) and (2) ,

$$p\underline{y} + \frac{7}{2}p\underline{x} = (2 - 2q)\underline{y} + 4q\underline{x}$$

Equating the coefficients of $\underline{y}$,

$$p = 2 - 2q \quad \ldots (3)$$

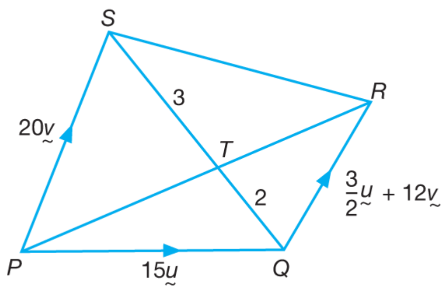
Equating the coefficients of $\underline{x}$,

$$\frac{7}{2}p = 4q$$
$$7p = 8q$$
$$7(2 - 2q) = 8q$$
$$14 - 14q = 8q$$
$$14 = 22q$$
$$q = \frac{7}{11}$$

From (3) ,

$$p = 2 - 2\left(\frac{7}{11}\right) = \frac{8}{11}$$

11



(a) (i) $\overrightarrow{QS} = \overrightarrow{QP} + \overrightarrow{PS}$

$= -15\underline{u} + 20\underline{v}$

(ii) $\overrightarrow{PT} = \overrightarrow{PQ} + \overrightarrow{QT}$

$= 15\underline{u} + \frac{2}{5}\overrightarrow{QS}$

$= 15\underline{u} + \frac{2}{5}(-15\underline{u} + 20\underline{v})$

$= 15\underline{u} - 6\underline{u} + 8\underline{v}$

$= 9\underline{u} + 8\underline{v}$

(b) $\overrightarrow{PT} = k\overrightarrow{TR}$

$9\underline{u} + 8\underline{v} = k\left(\overrightarrow{TQ} + \overrightarrow{QR}\right)$

$= k\left(6\underline{u} - 8\underline{v} - \frac{3}{2}\underline{u} + 12\underline{v}\right)$

$= k\left(\frac{9}{2}\underline{u} + 4\underline{v}\right)$

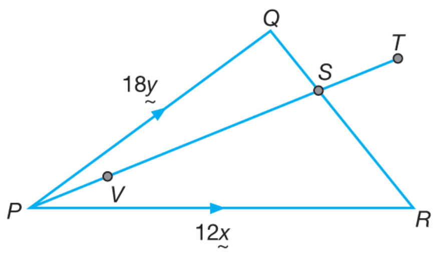
$= \frac{9}{2}k\underline{u} + 4k\underline{v}$

Equating the coefficients of $\underline{u}$,

$9 = \frac{9}{2}k$

$k = 2 \Rightarrow PT : TR = 2 : 1$

12



(a) (i) $\overrightarrow{QR} = \overrightarrow{QP} + \overrightarrow{PR}$

$= -18\underline{y} + 12\underline{x}$

(ii) $\overrightarrow{PS} = \overrightarrow{PQ} + \overrightarrow{QS}$

$= \overrightarrow{PQ} + \frac{1}{3}\overrightarrow{QR}$

$= 18\underline{y} + \frac{1}{3}(-18\underline{y} + 12\underline{x})$

$= 18\underline{y} - 6\underline{y} + 4\underline{x}$

$= 12\underline{y} + 4\underline{x}$

(b) Using the triangle addition rule,

$\overrightarrow{PV} + \overrightarrow{VQ} = \overrightarrow{PQ}$

$m\overrightarrow{PS} - n(2\underline{x} - 18\underline{y}) = 18\underline{y}$

$\overrightarrow{VQ} = -\overrightarrow{QV}$

$m(12\underline{y} + 4\underline{x}) - n(2\underline{x} - 18\underline{y}) = 18\underline{y}$

$12m\underline{y} + 4m\underline{x} - 2n\underline{x} + 18n\underline{y} = 18\underline{y}$

$(12m + 18n)\underline{y} + (4m - 2n)\underline{x} = 18\underline{y}$

Equating the coefficients of $\underline{y}$,

$12m + 18n = 18$

$2m + 3n = 3 \ \ldots (1)$

Equating the coefficients of $\underline{x}$,

$4m - 2n = 0$

$2m - n = 0 \ \ldots (2)$

$(1) - (2): \quad 4n = 3$

$n = \frac{3}{4}$

From (2), $2m - \frac{3}{4} = 0$

$m = \frac{3}{8}$

(c) Since the points P, S and T are collinear,

$\overrightarrow{PS} = k\overrightarrow{PT}$ (k is a constant.)

$12\underline{y} + 4\underline{x} = k(h\underline{x} + 18\underline{y})$

$12\underline{y} + 4\underline{x} = hk\underline{x} + 18k\underline{y}$

Equating the coefficients of $\underline{y}$,

$18k = 12$

$k = \frac{2}{3}$

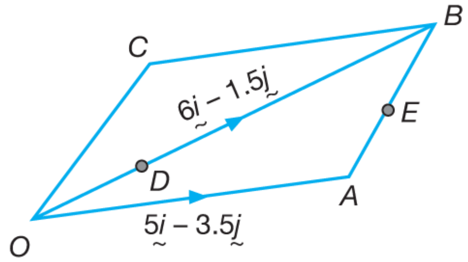
Equating the coefficients of $\underline{x}$,

$hk = 4$

$\frac{2}{3}h = 4$

$h = 6$

13 (a)



(i) $\overrightarrow{OD} = \frac{1}{3}\overrightarrow{OB}$

$= \frac{1}{3}(6\underset{\sim}{i} - 1.5\underset{\sim}{j})$

$= 2\underset{\sim}{i} - 0.5\underset{\sim}{j}$

(ii) $\overrightarrow{OE} = \overrightarrow{OA} + \overrightarrow{AE}$

$= \overrightarrow{OA} + \frac{1}{4}\overrightarrow{AB}$

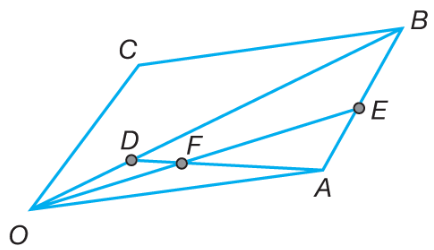
$= 5\underset{\sim}{i} + 3.5\underset{\sim}{j} + \frac{1}{4}\left(\overrightarrow{AO} + \overrightarrow{OB}\right)$

$= 5\underset{\sim}{i} + 3.5\underset{\sim}{j} + \frac{1}{4}(-5\underset{\sim}{i} - 3.5\underset{\sim}{j} + 6\underset{\sim}{i} - 1.5\underset{\sim}{j})$

$= 5\underset{\sim}{i} + 3.5\underset{\sim}{j} + \frac{1}{4}(\underset{\sim}{i} - 5\underset{\sim}{j})$

$= \frac{21}{4}\underset{\sim}{i} + \frac{9}{4}\underset{\sim}{j}$

(b)



(i) $\overrightarrow{OF} = k\overrightarrow{OE}$

$= k\left(\frac{21}{4}\underset{\sim}{i} + \frac{9}{4}\underset{\sim}{j}\right)$

$= \frac{21}{4}k\underset{\sim}{i} + \frac{9}{4}k\underset{\sim}{j}$ … (1)

(ii) $\overrightarrow{OF} = \overrightarrow{OD} + \overrightarrow{DF}$

$= 2\underset{\sim}{i} - 0.5\underset{\sim}{j} + t\overrightarrow{DA}$

$= 2\underset{\sim}{i} - 0.5\underset{\sim}{j} + t\left(\overrightarrow{DO} + \overrightarrow{OA}\right)$

$= 2\underset{\sim}{i} - 0.5\underset{\sim}{j} + t(-2\underset{\sim}{i} + 0.5\underset{\sim}{j} + 5\underset{\sim}{i} + 3.5\underset{\sim}{j})$

$= 2\underset{\sim}{i} - 0.5\underset{\sim}{j} + t(3\underset{\sim}{i} + 4\underset{\sim}{j})$

$= (2 + 3t)\underset{\sim}{i} + (4t - 0.5)\underset{\sim}{j}$ … (2)

(c) Equating (1) and (2) :

$\frac{21}{4}k\underset{\sim}{i} + \frac{9}{4}k\underset{\sim}{j} = (2 + 3t)\underset{\sim}{i} + (4t - 0.5)\underset{\sim}{j}$

Equating the coefficients of $\underset{\sim}{i}$,

$\frac{21}{4}k = 2 + 3t$

$21k = 8 + 12t$ …(3)

Equating the coefficients of $\underset{\sim}{j}$,

$\frac{9}{4}k = 4t - 0.5$

$9k = 16t - 2$ … (4)

(3) × 9 : $189k = 72 + 108t$ … (5)

(4) × 21 : $189k = -42 + 336t$ … (6)

(5) – (6) : $0 = 114 - 228t$

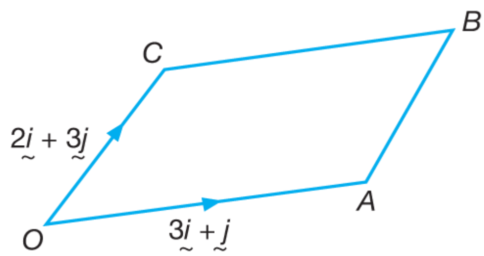
$t = \frac{1}{2}$

From (5) :

$189k = 72 + 108 \times \frac{1}{2}$

$k = \frac{126}{189} = \frac{2}{3}$

14



(a) (i) $\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB}$

$= 3\underset{\sim}{i} + \underset{\sim}{j} + 2\underset{\sim}{i} + 3\underset{\sim}{j}$

$= 5\underset{\sim}{i} + 4\underset{\sim}{j}$

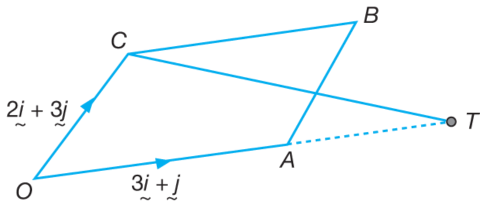
(ii) $\left|\overrightarrow{OB}\right| = \sqrt{5^2 + 4^2} = \sqrt{41}$

Unit vector in the direction of $\overrightarrow{AB}$

$$= \frac{1}{\sqrt{41}}\begin{pmatrix}5\\4\end{pmatrix}$$

$$= \begin{pmatrix}\frac{5}{\sqrt{41}}\\ \frac{4}{\sqrt{41}}\end{pmatrix}$$

(b)



(i) $\overrightarrow{AT} = \overrightarrow{AC} + \overrightarrow{CT}$

$= \overrightarrow{AO} + \overrightarrow{OC} + \overrightarrow{CT}$

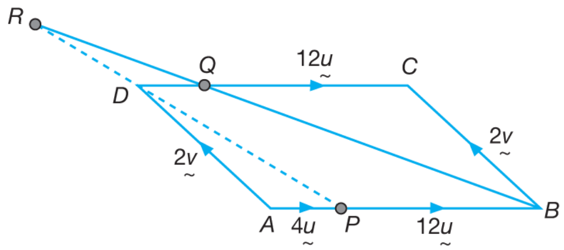
$= -3\underline{i} - \underline{j} + 2\underline{i} + 3\underline{j} + 16\underline{i} + 3\underline{j}$

$= 15\underline{i} + 5\underline{j}$

(ii) $\overrightarrow{AT} = 5(3\underline{i} + \underline{j}) = 5\overrightarrow{OA}$

Since $\overrightarrow{AT}$ can be expressed as a scalar multiple of $\overrightarrow{OA}$ and A is a common point, thus the points O, A dan T are collinear.

15



(a) (i) $\overrightarrow{BQ} = \overrightarrow{BC} + \overrightarrow{CQ}$

$= 2\underline{v} - 12\underline{u}$

(ii) $\overrightarrow{PD} = \overrightarrow{PA} + \overrightarrow{AD}$

$= -4\underline{u} + 2\underline{v}$ … (1)

$\overrightarrow{PR} = \overrightarrow{PB} + \overrightarrow{BR}$

$= \overrightarrow{PB} + \frac{3}{2}\overrightarrow{BQ}$

$= 12\underline{u} + \frac{3}{2}(2\underline{v} - 12\underline{u})$

$= -6\underline{u} + 3\underline{v}$

$= 3(-2\underline{u} + \underline{v})$ … (2)

From (1) : $\overrightarrow{PD} = -4\underline{u} + 2\underline{v}$,

$= 2(-2\underline{u} + \underline{v})$

$-2\underline{u} + \underline{v} = \frac{1}{2}\overrightarrow{PD}$

Hence, from (2) :

$\overrightarrow{PR} = 3(-2\underline{u} + \underline{v}) = 3\left(\frac{1}{2}\overrightarrow{PD}\right)$

$\overrightarrow{PR} = \frac{3}{2}\overrightarrow{PD}$

Since $\overrightarrow{PR}$ can be expressed as a scalar multiple of $\overrightarrow{PD}$ and P is a common point, thus the points P, D dan R are collinear.

(b) (i) $\overrightarrow{PD} = -4\underline{u} + 2\underline{v}$

$= -4(3\underline{i}) + 2(-\underline{i} + 6\underline{j})$

$= -14\underline{i} + 12\underline{j}$

(ii) $\left|\overrightarrow{PD}\right| = \sqrt{(-14)^2 + 12^2} = \sqrt{340}$

Unit vector in the direction of $\overrightarrow{PD}$

$= \frac{1}{\sqrt{340}}(-14\underline{i} + 12\underline{j})$

$$= \frac{2}{\sqrt{4 \times 85}}(-7\underset{\sim}{i} + 6\underset{\sim}{j})$$

$$= \frac{2}{2\sqrt{85}}(-7\underset{\sim}{i} + 6\underset{\sim}{j})$$

$$= -\frac{7}{\sqrt{85}}\underset{\sim}{i} + \frac{6}{\sqrt{85}}\underset{\sim}{j}$$

16 (a) (i) $\overrightarrow{BC} = \overrightarrow{BA} + \overrightarrow{AC}$

$$= 2\underset{\sim}{i} - 3\underset{\sim}{j} - 6\underset{\sim}{i} + 6\underset{\sim}{j}$$

$$= -4\underset{\sim}{i} + 3\underset{\sim}{j}$$

(ii) $\left|\overrightarrow{BC}\right| = \sqrt{(-4)^2 + 3^2} = 5$

Unit vector in the direction of $\overrightarrow{BC}$

$$= \frac{1}{5}\left(-4\underset{\sim}{i} + 3\underset{\sim}{j}\right)$$

$$= -\frac{4}{5}\underset{\sim}{i} + \frac{3}{5}\underset{\sim}{j}$$

(b) $\overrightarrow{AD} = k\,\overrightarrow{BC}$

$$p\underset{\sim}{i} - 12\underset{\sim}{j} = k\left(-4\underset{\sim}{i} + 3\underset{\sim}{j}\right)$$

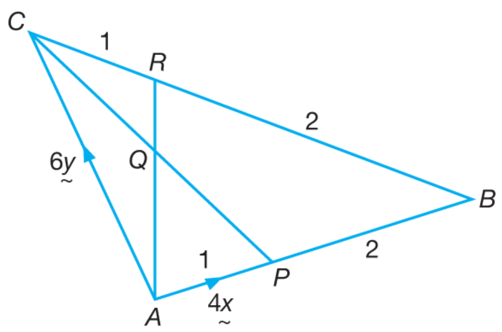
$$p\underset{\sim}{i} - 12\underset{\sim}{j} = -4k\underset{\sim}{i} + 3k\underset{\sim}{j}$$

$3k = -12$ $p = -4k$

$k = -4$ $= -4(-4)$

$= 16$

17



(a) (i) $\overrightarrow{CP} = \overrightarrow{CA} + \overrightarrow{AP}$

$$= -6\underline{y} + 4\underline{x}$$

(ii) $\overrightarrow{CR} = \frac{1}{3}\overrightarrow{CB}$.

$$= \frac{1}{3}\left(\overrightarrow{CA} + \overrightarrow{AB}\right)$$

$$= \frac{1}{3}\left(-6\underline{y} + 12\underline{x}\right)$$

$$= -2\underline{y} + 4\underline{x}$$

(b) $\overrightarrow{CR} = -2\underline{y} + 4\underline{x}$

$$\overrightarrow{CR} = -2\left(-3\underline{i} + 4\underline{j}\right) + 4\left(2\underline{i} + \underline{j}\right)$$

$$\overrightarrow{CR} = 14\underline{i} - 4\underline{j}$$

$$\left|\overrightarrow{CR}\right| = \sqrt{14^2 + (-4)^2} = \sqrt{212} = 14.56$$

(c) Using the triangle addition rule,

$$\overrightarrow{CQ} + \overrightarrow{QR} = \overrightarrow{CR}$$

$$m\overrightarrow{CP} + n\overrightarrow{AR} = \overrightarrow{CR}$$

$$m(-6\underline{y} + 4\underline{x}) + n\left(\overrightarrow{AC} + \overrightarrow{CR}\right) = \overrightarrow{CR}$$

$$m(-6\underline{y} + 4\underline{x}) + n\left[6\underline{y} + (-2\underline{y} + 4\underline{x})\right] = -2\underline{y} + 4\underline{x}$$

$$(4m + 4n)\underline{x} + (-6m + 4n)\underline{y} = -2\underline{y} + 4\underline{x}$$

Equating the coefficients of $\underline{x}$,

$4m + 4n = 4$

$2m + 2n = 2$ … (1)

Equating the coefficients of $\underline{y}$,

$-6m + 4n = -2$

$-3m + 2n = -1$ … (2)

(1) – (2) : $5m = 3$

$$m = \frac{3}{5}$$

From (2) :

$$-3\left(\frac{3}{5}\right) + 2n = -1$$

$$2n = \frac{4}{5}$$

$$n = \frac{2}{5}$$