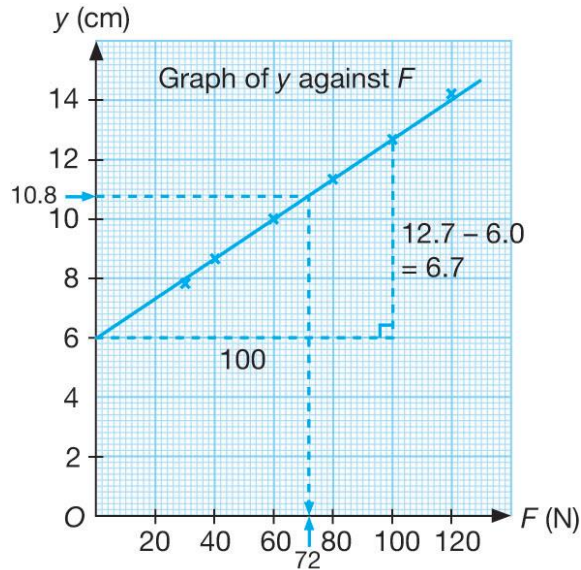


Form 4 Chapter 6
Linear Law
Fully-Worked Solutions

UPSKILL 6.1

1 (a)



(b) (i) $y = hF + k$

$h = \text{Gradient}$

$$h = \frac{6.7}{100}$$

$$h = 0.067$$

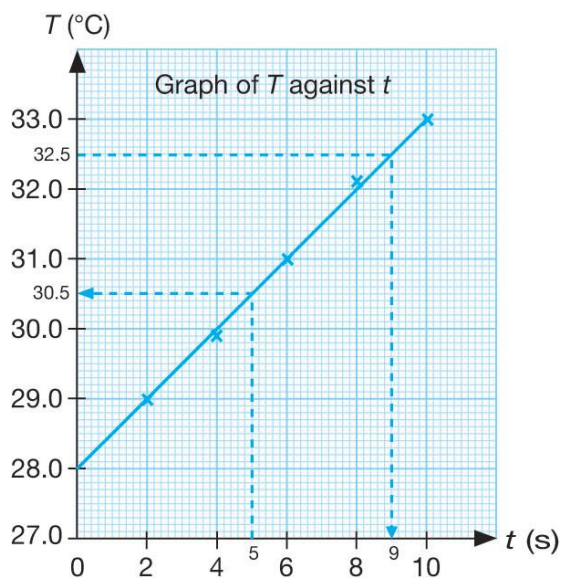
$k = \text{y-intercept}$

$$k = 6$$

(ii) When $F = 0$, $y = 6$ cm

(iii) When $y = 10.8$, $F = 72$ N

2 (a)



- (b) (i) When $t = 0$, $T = 28^\circ\text{C}$
(ii) When $t = 5$, $T = 30.5^\circ\text{C}$
(iii) When $T = 32.5$, $t = 9$ s

UPSKILL 6.2

1 (a) $y = hx^3 + kx^2$

$$\frac{y}{x^2} = hx + k$$

$$Y = \frac{y}{x^2}, X = x$$

$$\text{Gradient} = h$$

$$Y\text{-intercept} = k$$

(b) $y = hx + \frac{k}{x}$

$$xy = hx^2 + k$$

$$Y = xy, X = x^2$$

$$\text{Gradient} = h$$

$$Y\text{-intercept} = k$$

(c) $y = hx^k$

$$\log_{10} y = \log_{10} h + k \log_{10} x$$

$$\log_{10} y = k \log_{10} x + \log_{10} h$$

$$Y = \log_{10} y, X = \log_{10} x$$

$$\text{Gradient} = k$$

$$Y\text{-intercept} = \log_{10} h$$

(d) $\frac{h}{y} = \frac{k}{x} + 1$

$$\frac{1}{y} = \frac{k}{h} \left(\frac{1}{x} \right) + \frac{1}{h}$$

$$Y = \frac{1}{y}, X = \frac{1}{x}$$

$$\text{Gradient} = \frac{k}{h}$$

$$Y\text{-intercept} = \frac{1}{h}$$

2 (a) $16p^2x = (y - q)^2$

$$4p\sqrt{x} = y - q$$

$$y = 4p\sqrt{x} + q$$

Graph of y against \sqrt{x} is a straight line.

$$\text{Gradient} = 4p$$

Y-intercept = q

(b) $yx^q = p$

$$\log_{10} y + q \log_{10} x = \log_{10} p$$

$$\log_{10} y = -q \log_{10} x + \log_{10} p$$

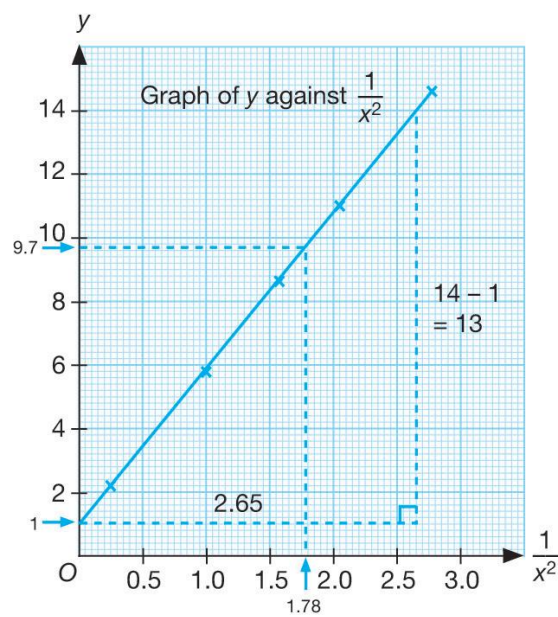
Graph of $\log_{10} y$ against $\log_{10} x$ is a straight line.

Gradient = $-q$

Y-intercept = $\log_{10} p$

3 (a)

x	0.6	0.7	0.8	1.0	2.0
y	14.6	10.8	8.6	5.8	2.2
$\frac{1}{x^2}$	2.78	2.04	1.56	1.00	0.25



(b) (i) $y = \frac{h}{x^2} + k$

$$h = \text{Gradient} = \frac{13}{2.65} = 4.9$$

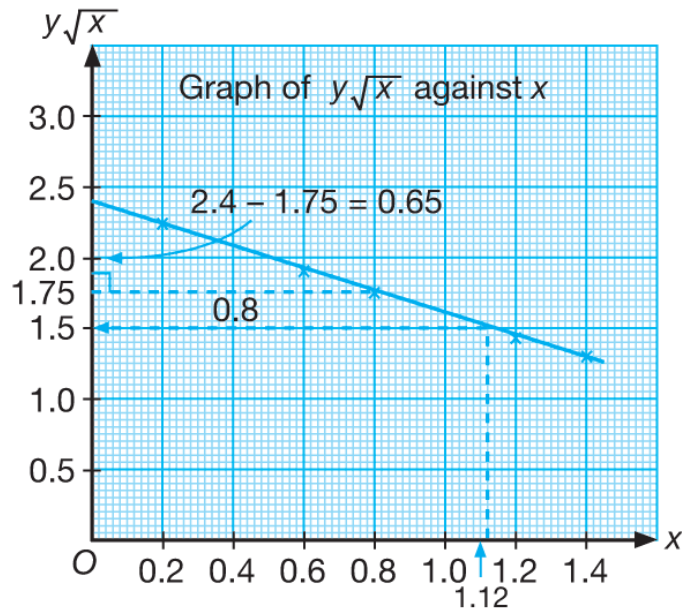
$$k = \text{Y-intercept} = 1$$

(ii) When $x = 0.75$, $\frac{1}{x^2} = 1.78$

From the graph, $y = 9.7$

4 (a)

x	0.2	0.6	0.8	1.2	1.4
y	5.03	2.45	1.96	1.32	1.10
$y\sqrt{x}$	2.25	1.90	1.75	1.45	1.30



(b) (i) $y = \frac{px}{\sqrt{x}} + \frac{q}{\sqrt{x}}$
 $y\sqrt{x} = px + q$

$q = Y\text{-intercept}$
 $= 2.4$

$p = \text{Gradient}$
 $= -\frac{0.65}{0.8}$
 $= -0.8$

(ii) When $x = 1.12$,

$$y\sqrt{x} = 1.5$$

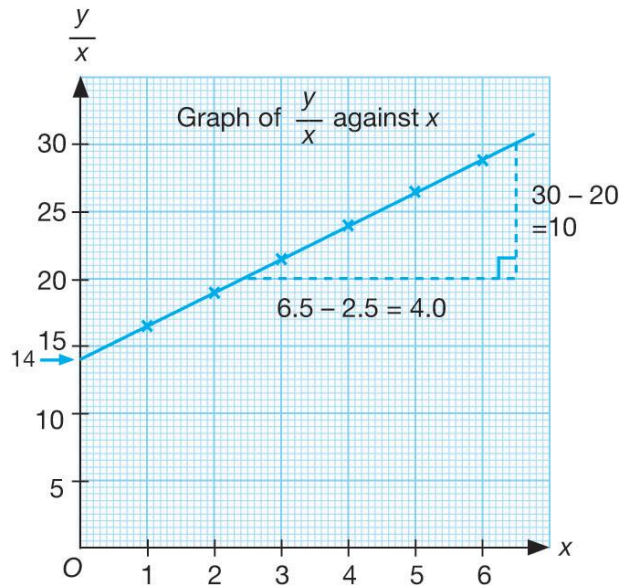
$$y\sqrt{1.12} = 1.5$$

$$y = \frac{1.5}{\sqrt{1.12}}$$

$$y = 1.4$$

5 (a)

x	1	2	3	4	5	6
y	16.3	37.5	63.7	95.0	131.3	172.5
$\frac{y}{x}$	16.3	18.75	21.23	23.75	26.26	28.75



(b) $y = px(x + q)$

$$y = px^2 + pqx$$

$$\frac{y}{x} = px + pq$$

$p = \text{Gradient}$

$$p = \frac{10}{4} = 2.5$$

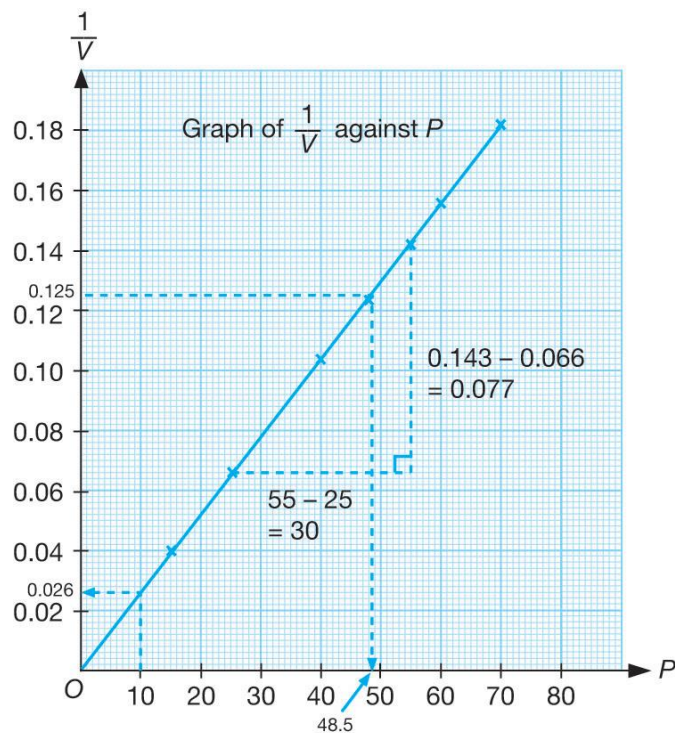
$pq = Y\text{-intercept}$

$$2.5q = 14$$

$$q = 5.6$$

6 (a)

P	15	25	40	48	55	60	70
V	25.0	15.1	9.6	8.1	7.0	6.4	5.5
$\frac{1}{V}$	0.040	0.066	0.104	0.123	0.143	0.156	0.182



(b) (i) $PV = d$

$$\frac{1}{PV} = \frac{1}{d}$$

$$\frac{1}{V} = \frac{1}{d}P$$

$$\frac{1}{d} = \text{Gradient}$$

$$\frac{1}{d} = \frac{0.077}{30}$$

$$d = \frac{30}{0.077} = 390$$

(ii) When $P = 10$, $\frac{1}{V} = 0.026$

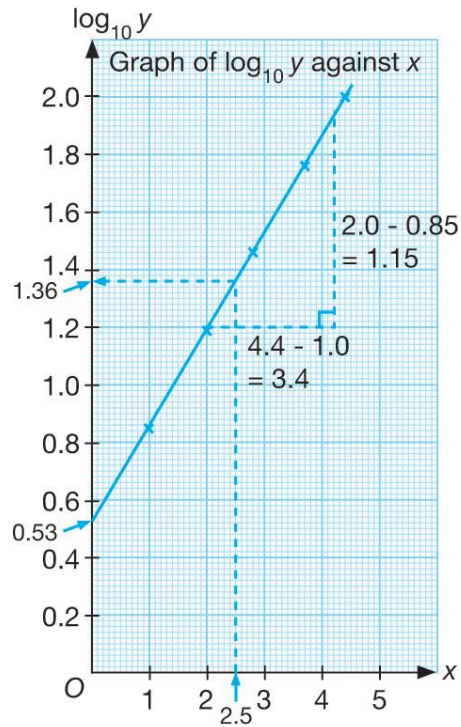
$$V = 38.5 \text{ cm}^3$$

(iii) When $V = 8$, $\frac{1}{V} = 0.125$

From the graph, $P = 48.5 \text{ cm Hg}$

7 (a)

x	1.0	2.0	2.8	3.7	4.4
y	7.2	15.8	28.8	57.5	100
$\frac{y}{x}$	0.86	1.20	1.46	1.76	2.00



(b) (i) $y = hk^x$
 $\log_{10} y = x \log_{10} k + \log_{10} h$

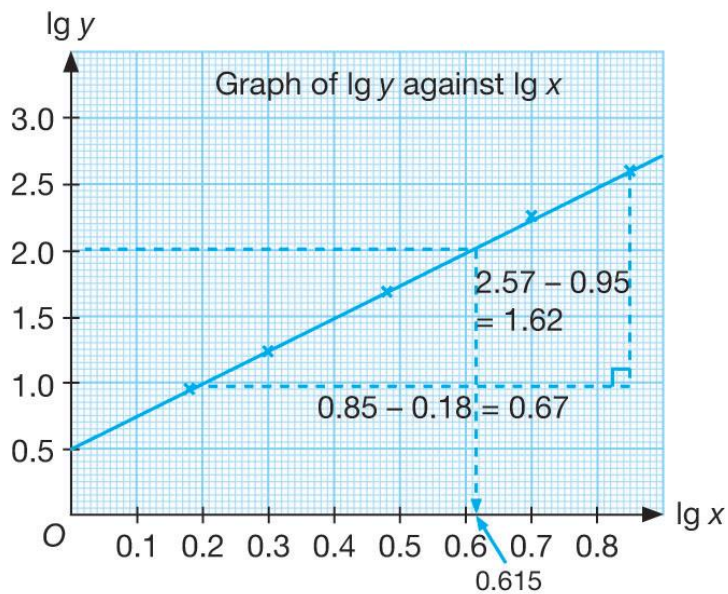
Y-intercept = 0.53
 $\log_{10} h = 0.53$
 $h = 3.4$

$\log_{10} k = \frac{1.15}{3.4} = 0.3382$
 $k = 2.2$

(ii) When $x = 2.5$, from the graph
 $\log_{10} y = 1.36$
 $y = 22.9$

8 (a)

x	1.5	2.0	3.0	5.0	7.0
y	9	17	47	170	370
$\log_{10} x$	0.18	0.30	0.48	0.70	0.85
$\log_{10} y$	0.95	1.23	1.67	2.23	2.57



(b) (i) $y = px^n$
 $\log_{10} y = n \log_{10} x + \log_{10} p$

$\log_{10} p = Y\text{-intercept}$

$\log_{10} p = 0.5$

$p = 3.16$

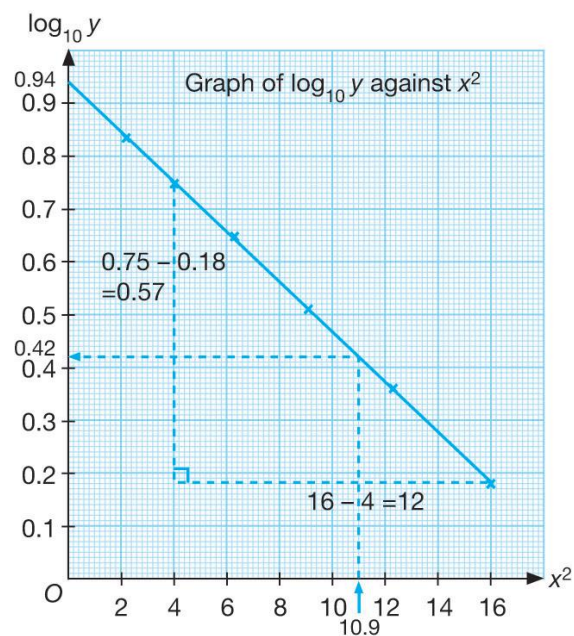
$n = \text{Gradient}$

$n = \frac{1.62}{0.67} = 2.42$

(ii) When $y = 100$, $\log_{10} y = 2$
 From the graph, $\log_{10} x = 0.615$
 $x = 4.12$

9 (a)

x	1.5	2.0	2.5	3.0	3.5	4.0
y	6.8	5.6	4.4	3.2	2.3	1.5
x^2	2.25	4.00	5.06	9.00	12.25	16.00
$\log_{10} y$	0.83	0.75	0.64	0.51	0.36	0.18



(b) (i) $y = \frac{t}{kx^2}$

$$\log_{10} y = \log_{10} t - x^2 \log_{10} k$$

$$\log_{10} y = -x^2 \log_{10} k + \log_{10} t$$

$$-\log_{10} k = \text{Gradient}$$

$$-\log_{10} k = -\frac{0.57}{12}$$

$$\log_{10} k = \frac{0.57}{12} = 0.0475$$

$$k = 1.12$$

$$\log_{10} t = Y\text{-intercept}$$

$$\log_{10} t = 0.94$$

$$t = 8.71$$

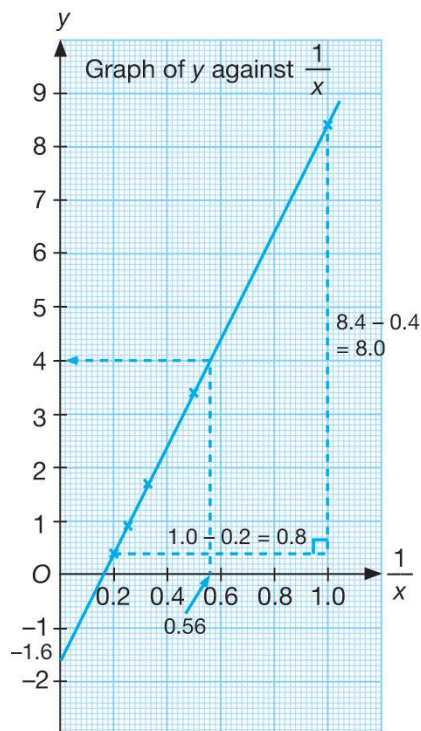
(ii) When $x = 3.3$, $x^2 = 10.89$

From the graph, $\log_{10} y = 0.42$

$$y = 2.63$$

10 (a)

x	1	2	3	4	5
y	8.4	3.4	1.7	0.9	0.4
$\frac{1}{x}$	1.00	0.50	0.33	0.25	0.20



(b) (i) $hy = t + \frac{2}{x}$

$$y = \frac{t}{h} + \frac{2}{h} \left(\frac{1}{x} \right)$$

$$y = \frac{2}{h} \left(\frac{1}{x} \right) + \frac{t}{h}$$

$$\frac{2}{h} = \text{Gradient}$$

$$\frac{2}{h} = \frac{8}{0.8}$$

$$h = 2 \times \frac{0.8}{8}$$

$$h = 0.2$$

$$\frac{t}{h} = Y\text{-intercept}$$

$$\frac{t}{h} = -1.6$$

$$\frac{t}{0.2} = -1.6$$

$$t = -0.32$$

(ii) When $x = 1.8$, $\frac{1}{x} = 0.56$

From the graph, $y = 4$

11 $\lg y = m \lg x + c$

$$\lg y = m \lg x + 3$$

$$m = \frac{11-3}{4} = 2$$

$$\lg y = 2 \lg x + 3$$

$$\lg y - 2 \lg x = 3$$

$$\lg y - \lg x^2 = 3$$

$$\log_{10} \left(\frac{y}{x^2} \right) = 3$$

$$\frac{y}{x^2} = 10^3$$

$$y = 1\,000x^2$$

12 $x - y = m(xy) + c$

For the point (2, 2), $xy = 2$ and $x - y = 2$,

$$2 = m(2) + c \dots (1)$$

For the point (8, 5), $xy = 8$ and $x - y = 5$,

$$5 = m(8) + c \dots (2)$$

$$(2) - (1) : 3 = 6m$$

$$m = \frac{1}{2}$$

$$\text{From (1) : } 2 = \frac{1}{2}(2) + c$$

$$c = 1$$

$$x - y = \frac{1}{2}(xy) + 1$$

$$2x - 2y = xy + 2$$

$$xy + 2y = 2x - 2$$

$$(x + 2)y = 2x - 2$$

$$y = \frac{2x - 2}{x + 2}$$

13 $\frac{1}{y} = m \left(\frac{1}{x} \right) + c$

For the point (2, 7), $\frac{1}{x} = 2$ and $\frac{1}{y} = 7$

$$7 = 2m + c \dots (1)$$

For the point (5, 1), $\frac{1}{x} = 5$ and $\frac{1}{y} = 1$

$$1 = 5m + c \dots (2)$$

$$(2) - (1): -6 = 3m \Rightarrow m = -2$$

$$7 = 2(-2) + c \dots (1)$$

$$c = 11$$

$$\frac{1}{y} = -2\left(\frac{1}{x}\right) + 11$$

$$\frac{1}{y} = \left(\frac{-2 + 11x}{x}\right)$$

$$y = \frac{x}{11x - 2}$$

14 $y = ab^{\sqrt{x}}$

$$\log_{10} y = \log_{10} ab^{\sqrt{x}}$$

$$\log_{10} y = \log_{10} a + \log_{10} b^{\sqrt{x}}$$

$$\log_{10} y = \log_{10} a + \sqrt{x} \log_{10} b$$

$$\log_{10} y = \sqrt{x} \log_{10} b + \log_{10} a$$

For the point (1, 5), $\sqrt{x} = 1$ and $\log_{10} y = 5$,

$$5 = \log_{10} b + \log_{10} a \dots (1)$$

For the point (3, 1), $\sqrt{x} = 3$ and $\log_{10} y = 1$,

$$1 = 3 \log_{10} b + \log_{10} a \dots (2)$$

$$(1) - (2):$$

$$4 = -2 \log_{10} b$$

$$\log_{10} b = -2$$

$$b = \frac{1}{100}$$

From (2):

$$1 = 3 \log_{10} b + \log_{10} a$$

$$1 = 3(-2) + \log_{10} a$$

$$\log_{10} a = 7$$

$$a = 10^7 = 10\,000\,000$$

15 $y = \frac{a}{b^x}$

$$\log_{10} y = \log_{10} a - x \log_{10} b$$

$$\log_{10} y = -x \log_{10} b + \log_{10} a$$

For the point (-1, 8), $x = -1$ and $\log_{10} y = 8$,

$$8 = -(-1) \log_{10} b + \log_{10} a$$

$$8 = \log_{10} b + \log_{10} a \dots (1)$$

For the point (3, 0), $x = 3$ and $\log_{10} y = 0$,
 $0 = -3 \log_{10} b + \log_{10} a \dots (2)$

$$(1) - (2) : 8 = 4 \log_{10} b$$

$$2 = \log_{10} b$$

$$b = 10^2 = 100$$

From (1) :

$$8 = 2 + \log_{10} a$$

$$\log_{10} a = 6$$

$$a = 10^4 = 1\,000\,000$$

16 $\frac{1}{2}y = x + \frac{q}{x}$

$$\frac{1}{2}xy = x^2 + q$$

$$xy = 2x^2 + 2q$$

For the point (7, 2), $x^2 = 7$ and $xy = 7$,
 $2 = 2(7) + 2q$
 $2q = -12$
 $q = -6$

For the point (0, h), $x^2 = 0$ and $xy = h$,
 $xy = 2x^2 + 2q$
 $h = 2(0) + 2(-6)$
 $h = -12$

17 $y = \frac{10^4}{x^2}$

$$\log_{10} y = \log_{10} 10^4 - \log_{10} x^2$$

$$\log_{10} y = 4 - 2 \log_{10} x$$

For the point (-1, p), $\log_{10} x = -1$ and $\log_{10} y = p$,
 $p = 4 - 2(-1) = 6$

For the point (r, -2), $\log_{10} x = r$ and $\log_{10} y = -2$,
 $-2 = 4 - 2r$
 $r = 3$

18 $yx^n = d$

$$\log_{10} y + n \log_{10} x = \log_{10} d$$

$$\log_{10} y = -n \log_{10} x + \log_{10} d$$

$$\text{Gradient} = -\frac{2}{5}$$

$$-n = -\frac{2}{5}$$

$$n = \frac{2}{5}$$

For the point (5, 0), $\log_{10} x = 5$ and $\log_{10} y = 0$

$$\log_{10} y = -n \log_{10} x + \log_{10} d$$

$$0 = -\frac{2}{5}(5) + \log_{10} d$$

$$\log_{10} d = 2$$

$$d = 100$$

19 $y = hk^{x-2}$

$$\log_{10} y = \log_{10} (hk^{x-2})$$

$$\log_{10} y = \log_{10} h + (x-2) \log_{10} k$$

$$\log_{10} y = (x-2) \log_{10} k + \log_{10} h$$

Gradient = 2

$$\log_{10} k = 2$$

$$k = 10^2 = 100$$

For the point (3, 0), $(x-2) = 3$ and $\log_{10} y = 0$

$$\log_{10} y = (x-2) \log_{10} k + \log_{10} h$$

$$0 = 3(2) + \log_{10} h$$

$$\log_{10} h = -6$$

$$h = 10^{-6} = \frac{1}{1\,000\,000}$$

20 $y = \frac{h}{x+k}$

$$xy + ky = h$$

$$ky = -xy + h$$

$$y = -\frac{1}{k}xy + \frac{h}{k}$$

Gradient = $-\frac{1}{2}$

$$-\frac{1}{k} = -\frac{1}{2}$$

$$k = 2$$

For the point (4, 4), $xy = 4$ and $y = 4$,

$$y = -\frac{1}{k}xy + \frac{h}{k}$$

$$4 = -\frac{1}{2}(4) + \frac{h}{2}$$

$$8 = -4 + h$$

$$h = 12$$

UPSKILL 6.3

1 (a) $10^M = a(T+1)^b$

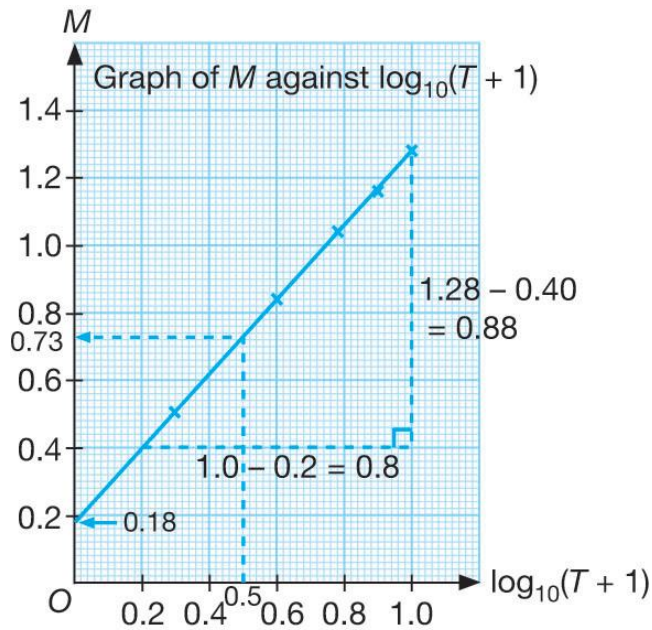
$$M = \log_{10} a (T+1)^b$$

$$M = \log_{10} a + b \log_{10} (T+1)$$

$$M = b \log_{10} (T+1) + \log_{10} a$$

(b)

T	1	3	5	7	9
M	0.51	0.84	1.04	1.18	1.28
$\log_{10}(T+1)$	0.300	0.600	0.780	0.90	1.00



(c) (i) $M = b \log_{10} (T+1) + \log_{10} a$

$$b = \text{Gradient}$$

$$b = \frac{0.88}{0.8} = 1.1$$

$$\log_{10} a = \text{Y-intercept}$$

$$\log_{10} a = 0.18$$

$$a = 1.51$$

(ii) When $T = 2.16$, $\log_{10} (T+1) = 0.5$

From the graph, $M = 0.73$

Summative Practice 6

1 $x^2y = a + bx$

$$xy = \frac{1}{x}a + b$$

For the point (3, 1), $\frac{1}{x} = \frac{1}{3}$ and $xy = 1$,

$$1 = 3a + b \dots (1)$$

For the point (7, 9), $\frac{1}{x} = \frac{1}{7}$ and $xy = 9$,

$$9 = 7a + b \dots (2)$$

$$(2) - (1): \quad 4a = 8$$

$$a = 2$$

From (1):

$$1 = 3(2) + b$$

$$b = -5$$

2 $y = \frac{x}{p + nx}$

$$\frac{1}{y} = \frac{p + nx}{x}$$

$$\frac{1}{y} = p\left(\frac{1}{x}\right) + n$$

$$p = \text{Gradient}$$

$$= \frac{0.2 - 0}{0 - 0.4}$$

$$= -0.5$$

$n = Y\text{-intercept}$

$$= 0.2$$

3 $ay = x + \frac{b}{x}$

$$axy = x^2 + b$$

$$xy = \left(\frac{1}{a}\right)x^2 + \frac{b}{a}$$

For the point (3, 0), $x^2 = 3$ and $xy = 0$,

$$0 = \frac{3}{a} + \frac{b}{a}$$

$$3 + b = 0$$

$$b = -3$$

For the point (7, 2), $x^2 = 7$ and $xy = 2$,

$$2 = \left(\frac{1}{a}\right)(7) + \frac{b}{a}$$

$$2a = 7 + b$$

$$2a = 7 - 3$$

$$a = 2$$

4 $y = px^{n-1}$
 $\log_{10} y = (n-1)\log_{10} x + \log_{10} p,$

$$n-1 = \text{Gradient}$$

$$n-1 = -2$$

$$n = -1$$

For the point (2, 4), $\log_{10} x = 2$ and $\log_{10} y = 4,$
 $\log_{10} y = -2\log_{10} x + \log_{10} p$
 $4 = -2(2) + \log_{10} p$
 $\log_{10} p = 8$
 $p = 10^8 = 100\,000\,000$

5 $(x+y) = mx^2 + c$

For the point (4, 1), $x^2 = 4$ and $(x+y) = 1,$
 $1 = 4m + c \dots (1)$

For the point (7, 2), $x^2 = 7$ and $(x+y) = 2,$
 $2 = 7m + c \dots (2)$

$$(2) - (1) : \quad 3m = 1$$

$$m = \frac{1}{3}$$

From (1): $1 = \frac{4}{3} + c$
 $c = -\frac{1}{3}$

Hence, $(x+y) = \frac{1}{3}x^2 - \frac{1}{3}$
 $y = \frac{1}{3}x^2 - x - \frac{1}{3}$

6 $m = \frac{9}{3} = 3$

$$\log_3 y = 3x$$

$$y = 3^{3x}$$

$$y = 27^x$$

7 $\log_2 y = m\log_2 x + c$

For the point (-3, 4): $\log_2 x = -3$ and $\log_2 y = 4,$
 $4 = -3m + c \dots (1)$

For the point (-2, 1): $\log_2 x = -2$ and $\log_2 y = 1,$
 $1 = -2m + c \dots (2)$

$$(1) - (2) : \quad -m = 3$$

$$m = -3$$

From (1): $4 = -3(-3) + c$

$$c = -5$$

Hence, $\log_2 y = -3 \log_2 x - 5$

$$\log_2 y + 3 \log_2 x = -5$$

$$\log_2 y + \log_2 x^3 = -5$$

$$\log_2 \left(\frac{y}{x^3} \right) = -5$$

$$\frac{y}{x^3} = 2^{-5}$$

$$\frac{y}{x^3} = \frac{1}{32}$$

$$y = \frac{x^3}{32}$$

8 $y = \frac{2}{x} + qx$

$$\frac{y}{x} = 2 \left(\frac{1}{x^2} \right) + q$$

For the point (2, 7), $\frac{1}{x^2} = 2$ and $\frac{y}{x} = 7$,

$$7 = 2(2) + q$$

$$q = 3$$

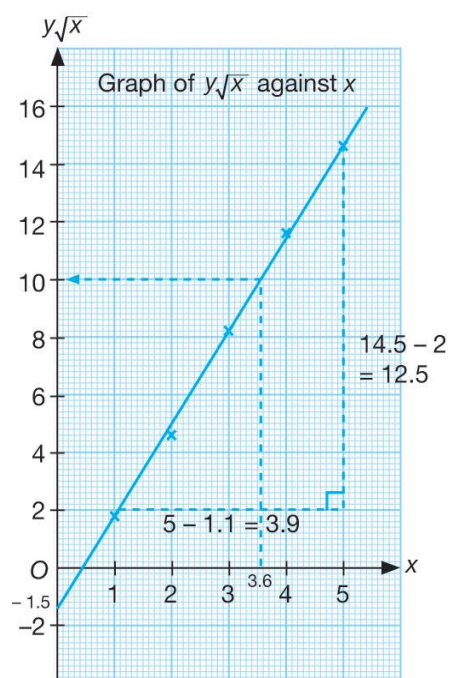
For the point (4, p), $\frac{1}{x^2} = 4$ and $\frac{y}{x} = p$,

$$p = 2(4) + q$$

$$p = 2(4) + 3 = 11$$

9 (a)

x	1	2	3	4	5
y	1.7	3.5	4.7	5.6	6.5
$y\sqrt{x}$	1.7	4.9	8.1	11.2	14.5



$$(b) (i) y = a\sqrt{x} + \frac{d}{\sqrt{x}}$$

$$y\sqrt{x} = ax + d$$

$$a = \text{Gradient}$$

$$= \frac{9.5}{3}$$

$$= 3.2$$

$$d = Y\text{-intercept}$$

$$= -1.5$$

(ii) When $x = 3.6$, from the graph,

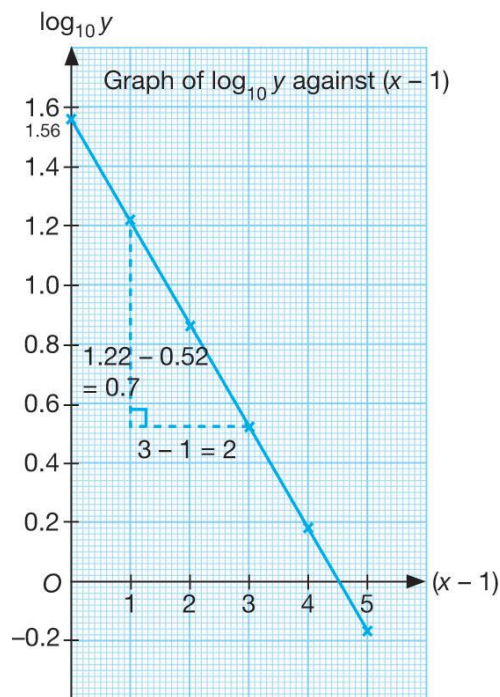
$$y\sqrt{x} = 10$$

$$y\sqrt{3.6} = 10$$

$$y = \frac{10}{\sqrt{3.6}} = 5.27$$

10 (a)

x	1	2	3	4	5	6
y	36.0	16.7	7.29	3.28	1.51	0.67
$x-1$	0	1	2	3	4	5
$\log_{10} y$	1.56	1.22	0.86	0.52	0.18	-0.17



$$(b) (i) y = db^{x-1}$$

$$\log_{10} y = \log_{10} d + (x-1) \log_{10} b$$

$$\log_{10} y = (x-1) \log_{10} b + \log_{10} d$$

$$\text{Gradient} = -\frac{0.7}{2}$$

$$\log_{10} b = -0.35$$

$$b = 0.45$$

$$Y\text{-intercept} = 1.56$$

$$\log_{10} d = 1.56$$

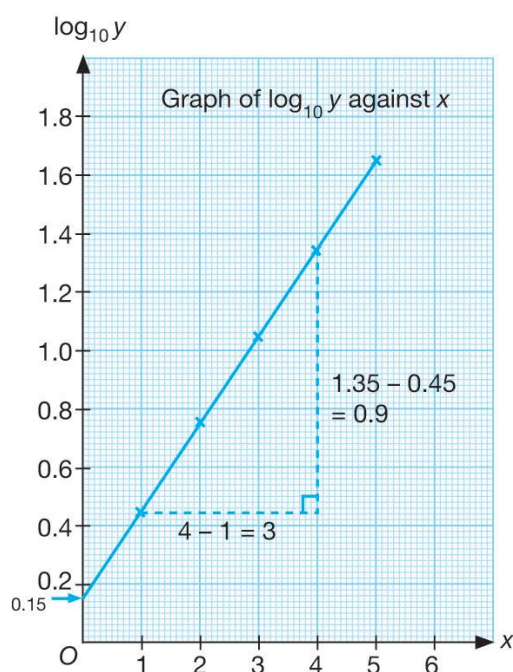
$$d = 36.31$$

- (ii) When $x = 5.5$, $x - 1 = 4.5$.
 From the graph $\log_{10} y = 0$
 $y = 1$

11 (a) $y = p^{b+x}$
 $\log_{10} y = (b+x)\log_{10} p$
 $\log_{10} y = x\log_{10} p + b\log_{10} p$

(b)

x	1	2	3	4	5
y	2.83	5.66	11.31	22.63	45.25
$\log_{10} y$	0.45	0.75	1.05	1.35	1.66



(c) $\log_{10} p = \text{Gradient}$

$$\log_{10} p = \frac{0.9}{3}$$

$$\log_{10} p = 0.3$$

$$p = 2$$

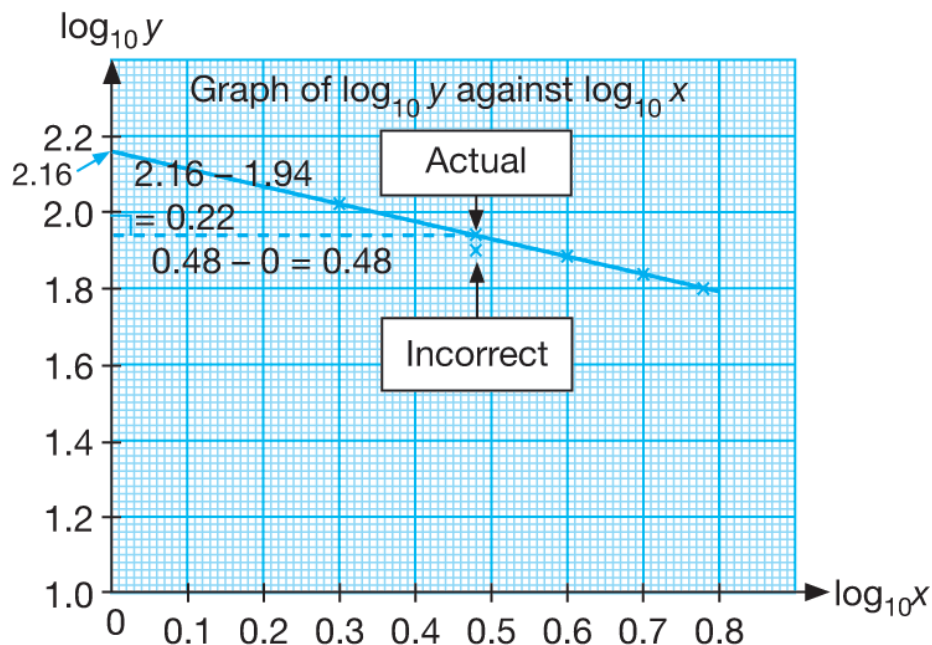
$$b\log_{10} p = 0.15$$

$$b(0.3) = 0.15$$

$$b = \frac{0.15}{0.3} = 0.5$$

12 (a)

x	2	3	4	5	6
y	106.0	78.5	75.0	67.0	61.0
$\log_{10} y$	2.03	1.89	1.88	1.83	1.79
$\log_{10} x$	0.30	0.48	0.60	0.70	0.78



(b) (i) The value of y which is incorrectly recorded is 78.5.

Actual:

$$\log_{10} y = 1.94$$

$$y = 87.1$$

(ii) $yx^n = k$

$$\log_{10} y + n \log_{10} x = \log_{10} k$$

$$\log_{10} y = -n \log_{10} x + \log_{10} k$$

$$-n = \text{Gradient}$$

$$-n = -\frac{0.22}{0.48}$$

$$n = 0.46$$

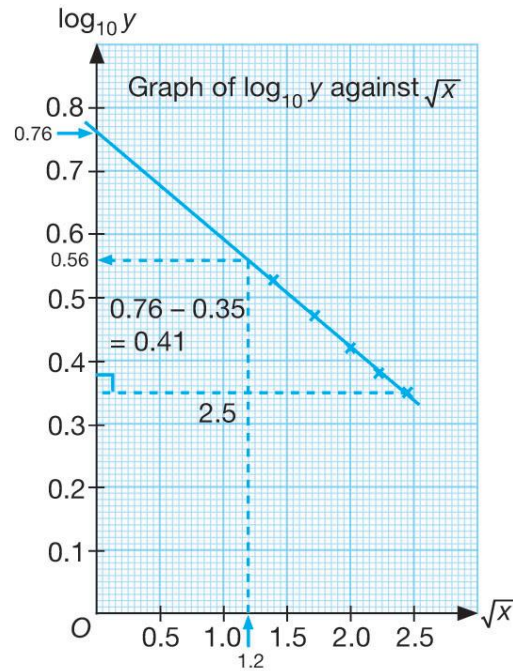
$$\log_{10} k = Y\text{-intercept}$$

$$\log_{10} k = 2.16$$

$$k = 145$$

13 (a)

x	2	3	4	5	6
y	3.39	2.92	2.63	2.39	2.23
\sqrt{x}	1.41	1.73	2.00	2.24	2.45
$\log_{10} y$	0.53	0.47	0.42	0.38	0.35



(b) (i) $y = qp^{\sqrt{x}}$

$$\log_{10} y = \log_{10} (qp^{\sqrt{x}})$$

$$\log_{10} y = \log_{10} q + \sqrt{x} \log_{10} p$$

$$\log_{10} y = \sqrt{x} \log_{10} p + \log_{10} q$$

$$\log_{10} q = Y\text{-intercept}$$

$$\log_{10} q = 0.76$$

$$q = 5.75$$

$$\log_{10} p = \text{Gradient}$$

$$\log_{10} p = -\frac{0.41}{2.5}$$

$$\log_{10} p = -0.164$$

$$p = 0.69$$

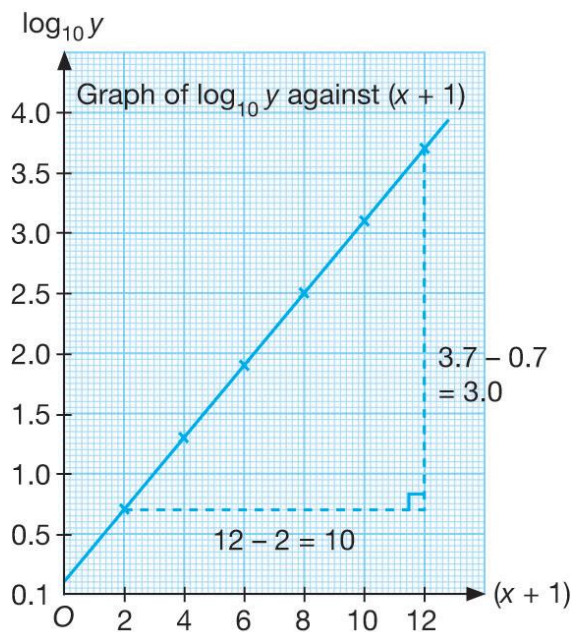
(ii) When $x = 1.44$, $\sqrt{x} = 1.2$.

From the graph, $\log_{10} y = 0.56$

$$y = 3.63$$

14 (a)

x	1	3	5	7	9	11
y	5	20	80	318	1 270	5 050
$\log_{10} y$	0.70	1.30	1.90	2.50	3.10	3.70
$(x + 1)$	2	4	6	8	10	12



(b) $y = hk^{x+1}$

$$\log_{10} y = \log_{10} h + (x+1) \log_{10} k$$

$$\log_{10} y = (x+1) \log_{10} k + \log_{10} h$$

$$\log_{10} k = \text{Gradient}$$

$$\log_{10} k = \frac{3}{10} = 0.3$$

$$k = 2$$

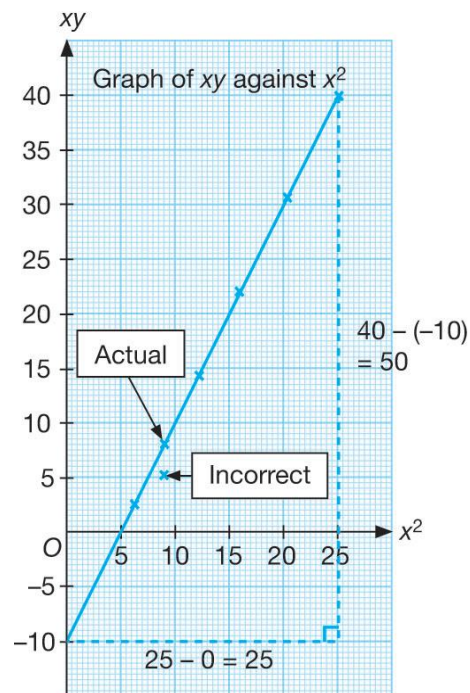
$$\log_{10} h = \text{Y-intercept}$$

$$\log_{10} h = 0.1$$

$$h = 1.26$$

15 (a)

x	2.5	3.0	3.5	4.0	4.5	5.0
y	1.0	1.7	4.1	5.5	6.8	8.0
xy	2.5	5.1	14.4	22.0	30.5	40.0
x^2	6.25	9.00	12.25	16.00	20.25	25.00



(b) (i) The value of y which is incorrectly recorded is 1.7.

Actual:

$$xy = 8.1$$

$$3y = 8.1$$

$$y = 2.7$$

$$(ii) \quad y = qx + \frac{p}{qx}$$

$$xy = qx^2 + \frac{p}{q}$$

$q = \text{Gradient}$

$$q = \frac{50}{25} = 2$$

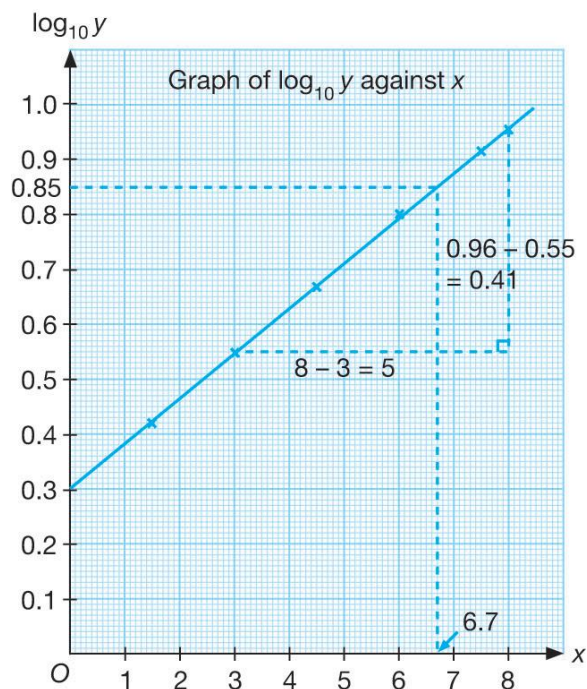
$$\frac{p}{q} = Y\text{-intercept}$$

$$\frac{p}{2} = -10$$

$$p = -20$$

16 (a)

x	1.5	3.0	4.5	6.0	7.5	8.0
y	2.66	3.54	4.72	6.28	8.35	9.19
$\log_{10} y$	0.42	0.55	0.67	0.80	0.92	0.96



(b) (i) When $y = 7.1$, $\log_{10} y = 0.85$
From the graph, $x = 6.7$

(ii) $y = bd^{2x}$

$$\log_{10} y = \log_{10} bd^{2x}$$

$$\log_{10} y = \log_{10} b + \log_{10} d^{2x}$$

$$\log_{10} y = \log_{10} b + 2x \log_{10} d$$

$$\log_{10} y = 2x \log_{10} d + \log_{10} b$$

$$\log_{10} b = Y\text{-intercept}$$

$$\log_{10} b = 0.3$$

$$b = 2.0$$

(iii) $2 \log_{10} d = \text{Gradient}$

$$2 \log_{10} d = \frac{0.41}{5}$$

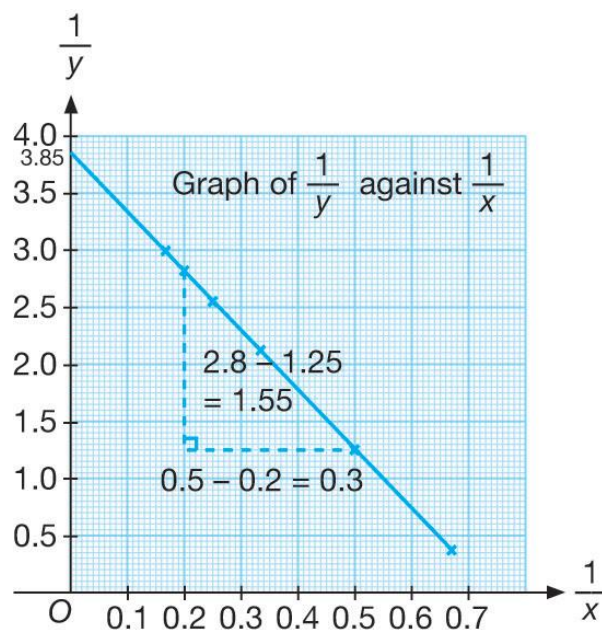
$$\log_{10} d = 0.041$$

$$d = 1.1$$

17 (a)

x	1.5	2.0	3.0	4.0	5.0	6.0
y	2.564	0.797	0.472	0.392	0.356	0.335
$\frac{1}{x}$	0.67	0.50	0.33	0.25	0.20	0.17
$\frac{1}{y}$	0.39	1.25	2.12	2.55	2.81	2.99

(b)



$$(c) \frac{c}{y} = \frac{d}{x} + 1$$

$$\frac{c}{y} = \frac{d+x}{x}$$

$$\frac{1}{y} = \frac{d+x}{cx}$$

$$\frac{1}{y} = \frac{d}{c} \left(\frac{1}{x} \right) + \frac{1}{c}$$

$$\frac{1}{c} = Y\text{-intercept}$$

$$\frac{1}{c} = 3.85$$

$$c = 0.26$$

$$\frac{d}{c} = \text{Gradient}$$

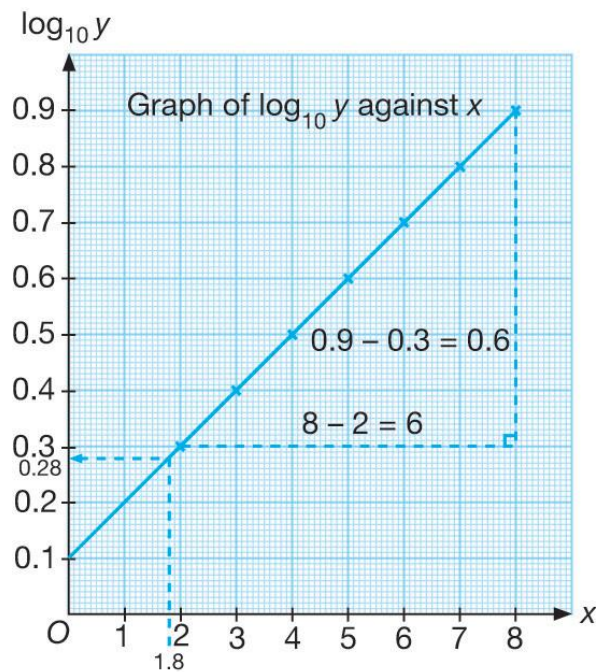
$$\frac{d}{0.26} = -\frac{1.55}{0.3}$$

$$d = -\frac{1.55}{0.3} \times 0.26$$

$$d = -1.34$$

18 (a)

x	3	4	5	6	7	8
y	2.49	3.15	3.96	5.01	6.29	7.95
$\log_{10} y$	0.4	0.5	0.6	0.7	0.8	0.9



(b) (i) $y = \frac{p^x}{b}$
 $\log_{10} y = x \log_{10} p - \log_{10} b$

$\log_{10} p = \text{Gradient}$

$\log_{10} p = \frac{0.6}{6} = 0.1$

$p = 1.26$

(ii) $-\log_{10} b = Y\text{-intercept}$

$-\log_{10} b = 0.1$

$\log_{10} b = -0.1$

$b = 0.79$

(iii) When $x = 1.8$

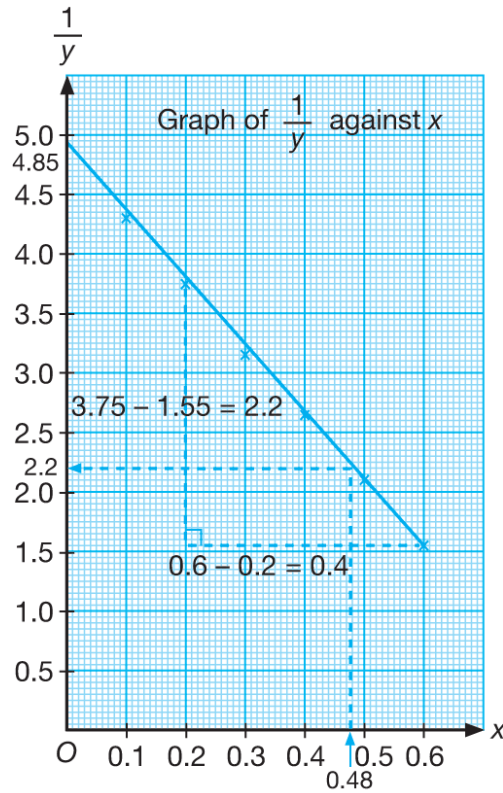
$\log_{10} y = 0.28$

$y = 1.91$

19 (a)

x	0.1	0.2	0.3	0.4	0.5	0.6
y	0.233	0.267	0.317	0.377	0.476	0.645
$\frac{1}{y}$	4.29	3.75	3.15	2.65	2.10	1.55

(b)



(c) $\frac{q}{y} = px + 1$

$$\frac{1}{y} = \frac{p}{q}x + \frac{1}{q}$$

(i) $\frac{1}{q} = Y\text{-intercept}$

$$\frac{1}{q} = 4.85$$

$$q = 0.206$$

(ii) $\frac{p}{q} = \text{Gradient}$

$$\frac{p}{0.206} = -\frac{2.2}{0.4}$$

$$p = -1.13$$

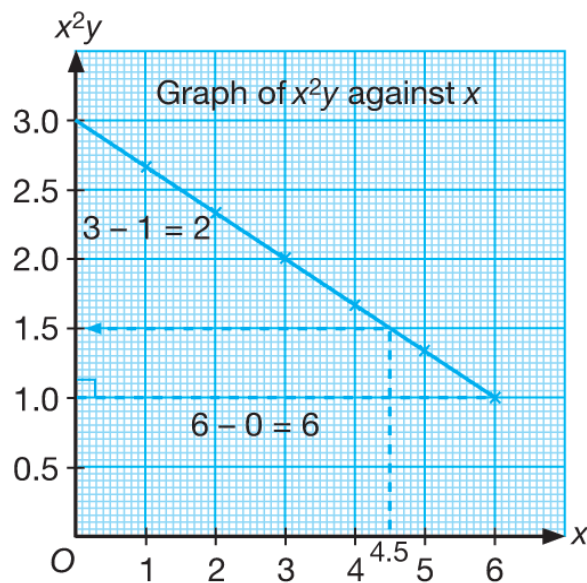
(iii) When $x = 0.48$, from the graph,

$$\frac{1}{y} = 2.2 \Rightarrow y = 0.45$$

20 (a)

x	1	2	3	4	5	6
y	2.667	0.583	0.222	0.104	0.053	0.028
x^2y	2.67	2.33	2.00	1.66	1.33	1.00

(b)



(c) (i) $y = \frac{a}{kx} + \frac{1}{kx^2}$
 $x^2y = \left(\frac{a}{k}\right)x + \frac{1}{k}$

$$\frac{1}{k} = Y\text{-intercept}$$

$$\frac{1}{k} = 3.0$$

$$k = \frac{1}{3}$$

(ii) $\frac{a}{k} = \text{Gradient}$

$$\frac{a}{\frac{1}{3}} = -\frac{2}{6}$$

$$a = -\frac{2}{6} \times \frac{1}{3}$$

$$a = -\frac{1}{9}$$

(iii) When $x = 4.5$, from the graph,

$$x^2y = 1.5$$

$$(4.5)^2y = 1.5$$

$$y = \frac{1.5}{(4.5)^2}$$

$$y = \frac{1.5}{(4.5)^2}$$

$$y = 0.074$$

$$21 \text{ (a)} \quad R = k \left(2.718^{-\frac{a}{T}} \right)$$

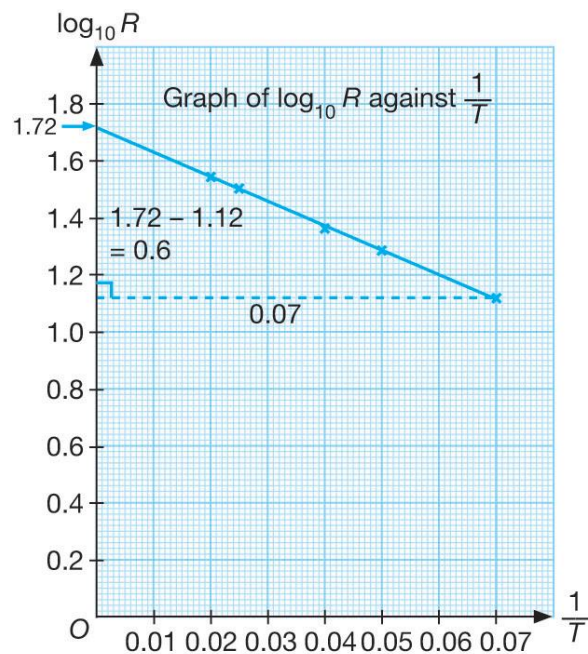
$$\log_{10} R = \log_{10} k - \frac{a}{T} \log_{10} 2.718$$

$$\log_{10} R = -\frac{a}{T} \log_{10} 2.718 + \log_{10} k$$

$$\log_{10} R = -0.4342 \frac{a}{T} + \log_{10} k$$

(b)

T	15	20	25	40	50
R	13.2	19.0	22.9	31.6	34.6
$\log_{10} R$	1.12	1.28	1.40	1.50	1.54
$\frac{1}{T}$	0.07	0.05	0.04	0.03	0.02



(c) $\log_{10} k = Y\text{-intercept}$

$$\log_{10} k = 1.72$$

$$k = 52.5$$

$$-0.4342a = \text{Gradient}$$

$$-0.4342a = -\frac{0.6}{0.07}$$

$$a = \frac{0.6}{0.07} \times \frac{1}{0.4342}$$

$$a = 19.7$$