Form 4 Chapter 5
Progressions
Fully-Worked Solutions

UPSKILL 5.1a

1 (a) $T_2 - T_1 = -13-(-17) = 4$

$T_3 - T_2 = -9-(-13) = 4$

Since $T_2 - T_1 = T_3 - T_2 = 4$ (a constant), then the number sequence is an arithmetic progression.

(b) $T_2 - T_1 = \frac{3}{4} - 1 = -\frac{1}{4}$

$T_3 - T_2 = \frac{1}{2} - \frac{3}{4} = -\frac{1}{4}$

$T_4 - T_3 = \frac{1}{4} - \frac{1}{2} = -\frac{1}{4}$

Since $T_2 - T_1 = T_3 - T_2 = T_4 - T_3$ (a constant), then the number sequence is an arithmetic progression.

(c) $T_2 - T_1 = \frac{7}{5} - \frac{3}{5} = \frac{2}{5}$

$T_3 - T_2 = 2 - \frac{7}{5} = \frac{3}{5}$

Since $T_2 - T_1 \neq T_3 - T_2$, then the number sequence is not an arithmetic progression.

2 (a) $T_2 - T_1 = 50-42=8$

$T_3 - T_2 = 58-50=8$

Since $T_2 - T_1 = T_3 - T_2 = 8$ (a constant), then the number sequence is an arithmetic progression. Common difference = 8.

(b) $T_2 - T_1 = \frac{17}{12} - \frac{7}{4} = -\frac{1}{3}$

$T_3 - T_2 = \frac{13}{12} - \frac{17}{12} = -\frac{1}{3}$

Since $T_2 - T_1 = T_3 - T_2 = -\frac{1}{3}$ (a constant), then the number sequence is an arithmetic progression. Common difference = $-\frac{1}{3}$

3 $T_1 = p(p+1) = p^2 + p$

$T_2 = p(p+3) = p^2 + 3p$

$T_3 = p(p+5) = p^2 + 5p$

$T_2 - T_1 = p^2 + 3p - (p^2 + p) = 2p$

$T_3 - T_2 = p^2 + 5p - (p^2 + 3p) = 2p$

Since $T_2 - T_1 = T_3 - T_2 = 2p$ (a constant), then the number sequence is an arithmetic progression.
Common difference = $2p$

UPSKILL 5.1b

1 (a) 4, 12, 20, …

$a = 4$

$d = 12 - 4 = 8$

$T_6 = a + 5d = 4 + 5(8) = 44$

(b) 2, $3\frac{1}{2}$, 5, …

$a = 2$

$d = 3\frac{1}{2} - 2 = 1\frac{1}{2}$

$T_{11} = a + 10d = 2 + 10\left(\frac{3}{2}\right) = 17$

(c) -9, -3, 3, …

$a = -9$

$d = -3-(-9) = 6$

$$\begin{aligned} T_n &= a + (n-1)(d) \\ &= -9 + (n-1)(6) \\ &= -9 + 6n - 6 \\ &= 6n - 15 \end{aligned}$$

2
$$\begin{aligned} T_n &= 23 \\ a + (n-1)d &= 23 \\ 2 + (n-1)(3) &= 23 \\ 2 + 3n - 3 &= 23 \\ 3n &= 24 \\ n &= 8 \end{aligned}$$

Hence, T_8 is 23.

3 (a) 9, 3, -3, …, -45

$$\begin{aligned} T_n &= -45 \\ a + (n-1)d &= -45 \\ 9 + (n-1)(-6) &= -45 \\ n-1 &= \frac{-45-9}{-6} \\ n-1 &= 9 \\ n &= 10 \end{aligned}$$

Number of terms is 10.

(b) $5, 6\frac{1}{2}, 8, \ldots, 26$

$$T_n = 26$$
$$a+(n-1)d = 26$$
$$5+(n-1)\left(\frac{3}{2}\right) = 26$$
$$\frac{3}{2}(n-1) = 21$$
$$n-1 = 21\times\frac{2}{3}$$
$$n-1 = 14$$
$$n = 15$$

Number of terms is 15.

4 $154, 161, \ldots, 294$

$$T_n = 294$$
$$154+(n-1)(7) = 294$$
$$7(n-1) = 294-154$$
$$7(n-1) = 140$$
$$n-1 = 20$$
$$n = 21$$

Number of multiples if 7 is 21.

5 $x^2-2, 3x+2, 11, \ldots$

$$3x+2-(x^2-2) = 11-(3x+2)$$
$$-x^2+3x+4 = -3x+9$$
$$x^2-6x+5 = 0$$
$$(x-5)(x-1) = 0$$
$$x = 5 \text{ or } 1$$

6 $4, p, q, \frac{35}{2}, \ldots$

$a = 4$

$$T_4 = \frac{35}{2}$$
$$a+3d = \frac{35}{2}$$
$$4+3d = \frac{35}{2}$$
$$8+6d = 35$$
$$6d = 27$$
$$d = 4.5$$

$$p = 4+4.5 = 8.5$$
$$q = 8.5+4.5 = 13$$

7 (a) $T_4 = 13$

$$a+3d = 13 \;\ldots (1)$$

$$T_{10} = 31$$
$$a+9d = 31 \;\ldots (2)$$

(2) – (1) : $6d = 18$

$$d = 3$$

From (1) : $a+3(3) = 13$

$$a = 13-9$$
$$a = 4$$

(b) $T_{25} = 4+24(3)$

$$= 76$$

8 $T_5 = 14$

$$a+4d = 14 \;\ldots (1)$$

$$T_{13} = -18$$
$$a+12d = -18 \;\ldots (2)$$

(2) – (1) : $8d = -32$

$$d = -4$$

From (1) : $a+4d = 14$

$$a+4(-4) = 14$$
$$a = 30$$

$$T_9 = a+8d = 30+8(-4) = -2$$

9 (a) $T_3 = 9$

$$a+2d = 9 \;\ldots (1)$$

$$T_7 = 49$$
$$a+6d = 49 \;\ldots (2)$$

(2) – (1) : $4d = 40$

$$d = 10$$

From (1) :

$$a+2(10) = 9$$
$$a = -11$$

$$T_{13} = a+12d = -11+12(10) = 109$$

(b) $T_n = 79$

$$a+(n-1)d = 79$$
$$-11+(n-1)(10) = 79$$
$$10(n-1) = 90$$
$$n-1 = 9$$
$$n = 10$$

The value of the 10th term is 79.

10 $T_4 = 36$

$a + 3d = 36 \quad \ldots (1)$

$T_{10} = 78$

$a + 9d = 78 \quad \ldots (2)$

$(2) - (1): \ 6d = 42$

$d = 7$

From (1):

$a + 3(7) = 36$

$a = 15$

$T_n > 1\ 000$

$a + (n-1)d > 1\ 000$

$15 + (n-1)(7) > 1\ 000$

$7(n-1) > 985$

$n - 1 > 140.7$

$n > 141.7$

The smallest integer value of n is 142.

$T_{142} = a + 141d = 15 + 141(7) = 1\ 002$

11 If x, y and z are three consecutive term of an arithmetic progression, hence

$y - x = z - y$

or $x - y = y - z \quad \ldots (1)$

$y + z, \ z + x, \ x + y.$

$\uparrow \quad \uparrow \quad \uparrow$

$T_1 \quad T_2 \quad T_3$

$T_2 - T_1 = z + x - (y + z) = x - y$

$T_3 - T_2 = x + y - (z + x) = y - z$

From (1), $x - y = y - z$.

Hence, $T_2 - T_1 = T_3 - T_2$.

Hence, $y + z$, $z + x$ and $x + y$ are three consecutive terms of an arithmetic progression.

12 (a) $a = 3\ 000$, $d = 250$

$T_7 = a + 6d = 3\ 000 + 6(250) = 4\ 500$

Encik Sulaiman's monthly salary in the 7th year is RM4 500.

(b) $T_n = 5\ 250$

$a + (n-1)d = 5\ 250$

$3\ 000 + (n-1)(250) = 5\ 250$

$250(n-1) = 2\ 250$

$n - 1 = 9$

$n = 10$

Hence, Encik Sulaiman's monthly salary is RM2 250 in the 10th year of his service.

13 (a) 60, 57, 54, …

$a = 60$, $d = -3$

$T_8 = a + 7d = 60 + 7(-3) = 39$

The distance of walk at the 8th minute is 39 m.

(b) $T_n = 30$

$a + (n-1)d = 30$

$60 + (n-1)(-3) = 30$

$(n-1)(-3) = -30$

$n = 11$

The distance of walk is 30 m at the 11th minute.

UPSKILL 5.1c

1 (a) $a = -11.5$, $d = -9 - (-11.5) = 2.5$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{10} = \frac{10}{2}[\ 2(-11.5) + (9)(2.5)\ ]$

$S_{10} = -2.5$

(b) $a = -1$, $d = 1 - (-1) = 2$

$S_n = \frac{n}{2}[\ 2(-1) + (n-1)(2)\ ]$

$S_n = \frac{n}{2}(-2 + 2n - 2)$

$S_n = \frac{n}{2}(2n - 4)$

$S_n = n^2 - 2n$

(c) $a = -1$, $d = 3 - (-1) = 4$

$T_n = 43$

$-1 + (n-1)(4) = 43$

$(n-1)(4) = 44$

$(n-1) = 11$

$n = 12$

$$S_n = \frac{n}{2}(a+l)$$
$$S_{12} = \frac{12}{2}(-1+43)$$
$$S_{12} = \frac{12}{2}(-1+43)$$
$$S_{12} = 252$$

(d) $a = \frac{3}{2}, d = \frac{5}{4} - \frac{3}{2} = -\frac{1}{4}$

$$T_n = -1$$
$$\frac{3}{2} + (n-1)\left(-\frac{1}{4}\right) = -1$$
$$(n-1)\left(-\frac{1}{4}\right) = -1 - \frac{3}{2}$$
$$(n-1)\left(-\frac{1}{4}\right) = -\frac{5}{2}$$
$$n-1 = \frac{-\frac{5}{2}}{-\frac{1}{4}}$$
$$n-1 = 10$$
$$n = 11$$

$$Sn = \frac{n}{2}(a+l)$$
$$S_{11} = \frac{11}{2}\left(\frac{3}{2} + (-1)\right)$$
$$S_{11} = 2\frac{3}{4}$$

2 301, 308, …, 994

$a = 301, d = 7$

$$T_n = 994$$
$$301 + (n-1)(7) = 994$$
$$(n-1)(7) = 693$$
$$n-1 = 99$$
$$n = 100$$

$$S_n = \frac{n}{2}(a+l)$$
$$S_{100} = \frac{100}{2}(301+994) = 64\ 750$$

3 $-100, -95, \ldots, 155$

$a = -100, d = 5$

$$T_n = 155$$
$$a + (n-1)d = 155$$
$$-100 + (n-1)(5) = 155$$
$$5(n-1) = 255$$
$$n-1 = 51$$
$$n = 52$$

$$T_n = \frac{n}{2}(a+l)$$
$$T_{52} = \frac{52}{2}(-100+155)$$
$$T_{52} = 1\ 430$$

4 27, 24, 21, …

$a = 27, d = -3$

$$S_n = 0$$
$$\frac{n}{2}[2a + (n-1)d] = 0$$
$$\frac{n}{2}[2(27) - 3(n-1)] = 0$$
$$n(54 - 3n + 3) = 0$$
$$n(57 - 3n) = 0$$
$$3n - 57 = 0$$
$$n = 19$$

5 10, 7, 4,. …

$a = 10, d = -3$

$$S_n = -35$$
$$\frac{n}{2}[2a + (n-1)d] = -35$$
$$\frac{n}{2}[2(10) + (n-1)(-3)] = -35$$
$$n(20 - 3n + 3) = -70$$
$$n(23 - 3n) = -70$$
$$23n - 3n^2 = -70$$
$$3n^2 - 23n - 70 = 0$$
$$(n-10)(3n+7) = 0$$
$$n = 10 \text{ or } n = -\frac{7}{3}$$

$n = -\frac{7}{3}$ is not accepted.

$\therefore n = 10$

6 2, 5, 8, …

$a = 2, d = 3$

$$S_n = 301$$
$$\frac{n}{2}[2a + (n-1)d] = 301$$
$$\frac{n}{2}[2(2) + (n-1)(3)] = 301$$
$$n(4 + 3n - 3) = 602$$

$$n(3n+1)=602$$
$$3n^2+n-602=0$$
$$(n-14)(3n-43)=0$$
$$n=14 \text{ or } n=\frac{43}{3}$$
$n=\frac{43}{3}$ is not accepted.
$$\therefore n=14$$

7 $T_4=13$
$a+3d=13 \quad \ldots (1)$

$T_{10}=31$
$a+9d=31 \quad \ldots (2)$

$(2)-(1): \quad 6d=18$
$d=3$

From (1) :
$a+3(3)=13$
$a=4$

$$S_{14}=\frac{14}{2}[2(4)+13(3)]=329$$

8 $S_8=8$
$$\frac{8}{2}(2a+7d)=8$$
$2a+7d=2 \quad \ldots (1)$

$S_{16}=144$
$$\frac{16}{2}(2a+15d)=144$$
$2a+15d=18 \quad \ldots (2)$

$(2)-(1): \ 8d=16$
$d=2$

From (1) : $2a+7(2)=2$
$a=-6$

$$T_{16}=a+15d=-6+15(2)=24$$

9 $T_4=7$
$a+3d=7 \quad \ldots (1)$
$2a+6d=14 \quad \ldots (1)\times 2$

$S_{10}=145$
$$\frac{10}{2}(2a+9d)=145$$
$2a+9d=29 \quad \ldots (2)$

$(2)-(1): \ 3d=15$
$d=5$

From (1) : $a+3(5)=7$
$a=-8$

$$S_{11}=\frac{11}{2}[2(-8)+10(5)]=187$$

10 4, 7, 10, …
$a=4, d=3$

$S_{12}-S_3$
$$=\frac{12}{2}[2(4)+11(3)]-\frac{3}{2}[2(4)+2(3)]$$
$=246-21$
$=225$

11 (a) $T_2=15$
$a+d=15 \quad \ldots (1)$

$T_4=23$
$a+3d=23 \quad \ldots (2)$

$2d=8$
$d=4$

From (1) :
$a+4=15$
$a=11$

(b) $S_{10}-S_4$
$$=\frac{10}{2}[2(11)+9(4)]-\frac{4}{2}[2(11)+3(4)]$$
$=290-68$
$=222$

12 $S_n=\frac{n}{2}(a+l)=360$
$$\frac{n}{2}(10+80)=360$$
$45n=360$
$n=8$

$$\frac{n}{2}[2(10)+(n-1)d]=360$$
$$\frac{8}{2}[20+(8-1)d]=360$$
$4(20+7d)=360$
$20+7d=90$
$7d=70$
$d=10$

Alternative Method

$T_n = 10 + (n-1)d = 80$

$10 + (8-1)d = 80$

$d = 10$

13 (a) $T_1 = \frac{1}{3}\pi h^2 k$

$T_2 = \frac{1}{3}\pi h^2(k+2) = \frac{1}{3}\pi h^2 k + \frac{2}{3}\pi h^2$

$T_3 = \frac{1}{3}\pi h^2(k+4) = \frac{1}{3}\pi h^2 k + \frac{4}{3}\pi h^2$

$T_2 - T_1 = \frac{2}{3}\pi h^2$

$T_3 - T_2 = \frac{2}{3}\pi h^2$

Since $T_2 - T_1 = T_3 - T_2 =$ $\frac{2}{3}\pi h^2$ (constant), hence the volumes of cones form an arithmetic progression.

Common difference = $\frac{2}{3}\pi h^2$

(b) $T_4 = 30\pi$

$a + 3d = 30\pi$

$\frac{1}{3}\pi h^2 k + 3\left(\frac{2}{3}\pi h^2\right) = 30\pi$

$h^2 k + 6h^2 = 90 \quad \ldots (1)$

$S_5 = 120\pi$

$\frac{5}{2}[2a + 4d] = 120\pi$

$5a + 10d = 120\pi$

$a + 2d = 24\pi$

$\frac{1}{3}\pi h^2 k + 2\left(\frac{2}{3}\pi h^2\right) = 24\pi$

$h^2 k + 4h^2 = 72 \quad \ldots (2)$

$2h^2 = 18$

$h^2 = 9$

$h = 3$

Radius of base = 3 cm

$3^2 k + 6(3)^2 = 90$

$9k + 54 = 90$

$k = 4$

Height = 4 cm

14 $S_n = 2n^2 + 3n$

(a) $T_3 = S_3 - S_2$

$= 2(3)^2 + 3(3) - [2(2)^2 + 3(2)]$

$= 27 - 14$

$= 13$

(b) $T_{10} = S_{10} - S_9$

$= 2(10)^2 + 3(10) - [2(9)^2 + 3(9)]$

$= 230 - 189$

$= 41$

15 $S_n = 6n - 3n^2$

(a) $T_1 = S_1$

$= 6 - 3$

$= 3$

(b) $T_2 = S_2 - S_1$

$= 6(2) - 3(2)^2 - (6-3)$

$= -3$

$d = T_2 - T_1$

$d = -3 - 3$

$d = -6$

16 $S_m : S_n = m^2 : n^2$

$$\frac{S_m}{S_n} = \frac{m^2}{n^2}$$

$$\frac{\frac{m}{2}\left[2a+(m-1)d\right]}{\frac{n}{2}\left[2a+(n-1)d\right]} = \frac{m^2}{n^2}$$

$$\frac{2a+(m-1)d}{2a+(n-1)d} = \frac{m}{n}$$

$$2an+(m-1)nd = 2am+(n-1)md$$

$$a(2n-2m) = (n-1)md-(m-1)nd$$

$$a = \frac{(n-1)md-(m-1)nd}{2n-2m}$$

$$a = \frac{-md+nd}{2n-2m}$$

$$a = \frac{d(-m+n)}{2(n-m)}$$

$$a = \frac{d}{2}$$

$$T_m : T_n = \frac{T_m}{T_n}$$

$$= \frac{a+(m-1)d}{a+(n-1)d}$$

$$= \frac{\frac{d}{2}+(m-1)d}{\frac{d}{2}+(n-1)d}$$

$$= \frac{d+2d(m-1)}{d+2d(n-1)}$$

$$= \frac{1+2(m-1)}{1+2(n-1)}$$

$$= \frac{2m-1}{2n-1} \text{ (Shown)}$$

UPSKILL 5.2a

1 (a) $\frac{1}{4}, 1, 4, \ldots$

$$\frac{T_2}{T_1} = \frac{1}{\frac{1}{4}} = 4$$

$$\frac{T_3}{T_2} = \frac{4}{1} = 4$$

Since $\frac{T_2}{T_1} = \frac{T_3}{T_2}$ = 4 (a constant), hence
The number sequence is a geometric progression.

(b) $1, -\frac{1}{2}, \frac{1}{4}, -\frac{1}{8}, \ldots$

$$\frac{T_2}{T_1} = \frac{-\frac{1}{2}}{1} = -\frac{1}{2}$$

$$\frac{T_3}{T_2} = \frac{\frac{1}{4}}{-\frac{1}{2}} = -\frac{1}{2}$$

$$\frac{T_4}{T_3} = \frac{-\frac{1}{8}}{\frac{1}{4}} = -\frac{1}{2}$$

Since $\frac{T_2}{T_1} = \frac{T_3}{T_2} = \frac{T_4}{T_3} = -\frac{1}{2}$ (constant),
hence the number sequence is a geometric progression.

(c) $\frac{2}{5}, \frac{3}{5}, \frac{4}{5}, \ldots$

$$\frac{T_2}{T_1} = \frac{\frac{3}{5}}{\frac{2}{5}} = \frac{3}{2}$$

$$\frac{T_3}{T_2} = \frac{\frac{4}{5}}{\frac{3}{5}} = \frac{4}{3}$$

Since $\frac{T_2}{T_1} \neq \frac{T_3}{T_2}$, hence the number sequence is not a geometric progression.

2 (a) $\frac{3}{16}, \frac{3}{4}, 3, \ldots$

$$\frac{T_2}{T_1} = \frac{\frac{3}{4}}{\frac{3}{16}} = 4$$

$$\frac{T_3}{T_2} = \frac{3}{\frac{3}{4}} = 4$$

Since $\frac{T_2}{T_1} = \frac{T_3}{T_2}$ = 4 (a constant), hence
the number sequence is a geometric progression.
Common ratio = 4

(b) $\frac{1}{3}, -\frac{1}{30}, \frac{1}{300}, -\frac{1}{3\,000}, \ldots$

$$\frac{T_2}{T_1} = \frac{-\frac{1}{30}}{\frac{1}{3}} = -\frac{1}{10}$$

$$\frac{T_3}{T_2} = \frac{\frac{1}{300}}{-\frac{1}{30}} = -\frac{1}{10}$$

$$\frac{T_4}{T_3} = \frac{-\frac{1}{3\,000}}{\frac{1}{300}} = -\frac{1}{10}$$

Since $\frac{T_2}{T_1} = \frac{T_3}{T_2} = \frac{T_4}{T_3} = -\frac{1}{10}$ (a constant),
hence the number sequence is a geometric progression. Common ratio = $-\frac{1}{10}$.

(c) $\frac{p}{p+2}, p, p^2 + 2p, \ldots$

$$\frac{T_2}{T_1} = \frac{p}{\frac{p}{p+2}} = p + 2$$

$$\frac{T_3}{T_2} = \frac{p(p+2)}{p} = p + 2$$

Since $\frac{T_2}{T_1} = \frac{T_3}{T_2}$ = p + 2 (a constant), hence
the number sequence is a geometric progression. Common ratio = p + 2.

3 T_1 = Area of $ABCD = bh$

T_2 = Area of $ABPQ = b\left(\frac{h}{2}\right) = \frac{bh}{2}$

T_3 = Area of $ABKL = b\left(\frac{h}{4}\right) = \frac{bh}{4}$

T_4 = Area of $ABTU = b\left(\frac{h}{8}\right) = \frac{bh}{8}$

$$\frac{T_2}{T_1} = \frac{\frac{bh}{2}}{bh} = \frac{1}{2}$$

$$\frac{T_3}{T_2} = \frac{\frac{bh}{4}}{\frac{bh}{2}} = \frac{1}{2}$$

$$\frac{T_4}{T_3} = \frac{\frac{bh}{8}}{\frac{bh}{4}} = \frac{1}{2}$$

Since $\frac{T_2}{T_1} = \frac{T_3}{T_2} = \frac{T_4}{T_3} = \frac{1}{2}$, hence the areas of *ABCD*, *ABPQ*, *ABKL* and *ABTU* form a geometric progression.

Common ratio = $\frac{1}{2}$.

UPSKILL 5.2b

1 (a) 8, 4, 2, …

$a = 1, r = \frac{1}{2}$

$T_{11} = ar^{n-1} = (8)\left(\frac{1}{2}\right)^{10} = \frac{1}{128}$

(b) $\frac{16}{27}, \frac{8}{9}, \frac{4}{3}, \ldots$

$a = \frac{16}{27},\ r = \frac{\frac{8}{9}}{\frac{16}{27}} = \frac{3}{2}$

$T_5 = ar^4 = \left(\frac{16}{27}\right)\left(\frac{3}{2}\right)^4 = 3$

(c) $3, 1, \frac{1}{3}, \ldots$

$a = 3, r = \frac{1}{3}$

$T_n = 3\left(\frac{1}{3}\right)^{n-1} = \frac{3}{3^{n-1}} = 3^{1-(n-1)} = 3^{2-n}$

2 (a) $\frac{1}{4}, \frac{1}{6}, \frac{1}{9}$

$a = \frac{1}{4}, r = \frac{\frac{1}{6}}{\frac{1}{4}} = \frac{2}{3}$

$T_n = \frac{4}{81}$

$\left(\frac{1}{4}\right)\left(\frac{2}{3}\right)^{n-1} = \frac{4}{81}$

$\left(\frac{2}{3}\right)^{n-1} = \frac{16}{81}$

$\left(\frac{2}{3}\right)^{n-1} = \left(\frac{2}{3}\right)^4$

$n - 1 = 4$

$n = 5$

Number of terms = 5

(b) $2, \frac{2}{3}, \frac{1}{3}, \ldots$

$a = 2, r = \frac{\frac{2}{3}}{2} = \frac{1}{3}$

$T_n = \frac{2}{729}$

$2\left(\frac{1}{3}\right)^{n-1} = \frac{2}{729}$

$\left(\frac{1}{3}\right)^{n-1} = \frac{1}{729}$

$\left(\frac{1}{3}\right)^{n-1} = \left(\frac{1}{3}\right)^6$

$n - 1 = 6$

$n = 7$

Number of terms = 7

(c) 1 000, –200, 40, …

$a = 1000,\ r = \frac{-200}{1\,000} = -\frac{1}{5}$

$T_n = -\frac{8}{625}$

$ar^{n-1} = -\frac{8}{625}$

$(1\,000)\left(-\frac{1}{5}\right)^{n-1} = -\frac{8}{625}$

$\left(-\frac{1}{5}\right)^{n-1} = -\frac{8}{625\ 000}$

$$\left(-\frac{1}{5}\right)^{n-1} = -\frac{1}{78\ 125}$$
$$\left(-\frac{1}{5}\right)^{n-1} = \left(-\frac{1}{5}\right)^{7}$$
$$n-1=7$$
$$n=8$$
Number of terms = 8

3 $x-2,\ x+4,\ 4x+7,\ \ldots$

$$\frac{x+4}{x-2} = \frac{4x+7}{x+4}$$
$$(x+4)^2 = (4x+7)(x-2)$$
$$x^2+8x+16 = 4x^2-x-14$$
$$3x^2-9x-30=0$$
$$x^2-3x-10=0$$
$$(x-5)(x+2)=0$$
$x=5$ or $x=-2$

When $x=5$, $r=\dfrac{x+4}{x-2}=\dfrac{9}{3}=3$

When $x=-2$, $r=\dfrac{-2+4}{-2-2}=-\dfrac{1}{2}$

4 (a) $x,\ x+4,\ 2x+2,\ \ldots$
$$\frac{x+4}{x} = \frac{2x+2}{x+4}$$
$$(x+4)^2 = x(2x+2)$$
$$x^2+8x+16 = 2x^2+2x$$
$$x^2-6x-16=0$$
$$(x-8)(x+2)=0$$
$x=8$ or $x=-2$

$x=-2$ is not accepted because the question states that all the terms are positive.

$\therefore x=8$

(b) $8, 12, 18, \ldots$
$$T_2 = x$$
$$ar=8$$
$$a\left(\frac{12}{8}\right)=8$$
$$a=\frac{16}{3}$$
$$T_6 = ar^5 = \left(\frac{16}{3}\right)\left(\frac{12}{8}\right)^5 = \frac{81}{2}$$

5 (a) $T_2 = \dfrac{1}{2}$
$$ar = \frac{1}{2}\ \ldots (1)$$
$$T_4 = \frac{1}{128}$$
$$ar^3 = \frac{1}{128}\ \ldots (2)$$

$$\frac{(2)}{(1)}: \quad \frac{ar^3}{ar} = \frac{\frac{1}{128}}{\frac{1}{2}} = \frac{1}{64}$$
$$r^2 = \frac{1}{64}$$
$$r = \frac{1}{8}$$
From (1) : $a\left(\dfrac{1}{8}\right)=\dfrac{1}{2}$
$$a=4$$

(b) $T_n = ar^{n-1}$
$$T_n = 4\left(\frac{1}{8}\right)^{n-1}$$
$$T_n = 2^2\left(\frac{1}{2^{3(n-1)}}\right)$$
$$T_n = 2^{2-3(n-1)}$$
$$T_n = 2^{5-3n}$$

6 $T_2 + T_3 = 12$
$$ar+ar^2=12$$
$$ar(1+r)=12\ \ldots (1)$$

$$T_3+T_4=4$$
$$ar^2+ar^3=4$$
$$ar^2(1+r)=4\ \ldots (2)$$

$$\frac{(2)}{(1)}: \frac{ar^2(1+r)}{ar(1+r)} = \frac{4}{12}$$
$$r=\frac{1}{3}$$
From (1) :
$$ar(1+r)=12$$
$$\frac{1}{3}\left(1+\frac{1}{3}\right)a=12$$
$$\frac{4}{9}a=12$$
$$a=27$$

7 $3, 5, 8\frac{1}{3}, \ldots$

$a = 3,\ r = \frac{5}{3}$

$T_n > 60$

$3\left(\frac{5}{3}\right)^{n-1} > 60$

$\left(\frac{5}{3}\right)^{n-1} > 20$

$(n-1)\lg\left(\frac{5}{3}\right) > \lg 20$

$n-1 > \frac{\lg 20}{\lg\left(\frac{5}{3}\right)}$

$n-1 > 5.86$

$n > 6.86$

The smallest term which exceeds 60 is the 7th term.

8 27, 18, 12, …

$a = 27,\ r = \frac{18}{27} = \frac{2}{3}$

$T_n < 1$

$27\left(\frac{2}{3}\right)^{n-1} < 1$

$\left(\frac{2}{3}\right)^{n-1} < \frac{1}{27}$

$(n-1)\lg\left(\frac{2}{3}\right) < \lg\frac{1}{27}$

$-0.1761\ (n-1) < -1.4314$

$n-1 > \frac{-1.4314}{-0.1761}$

$n-1 > 8.03$

$n > 9.03$

The smallest term which is less than 1 is the 10th term.

9 a = 40 000, r = 1.05

$T_{11} = ar^t = 40\,000(1.05)^{10}$

$= \text{RM}65\ 155.79$

10 a = 80 000

r = 0.9

The value of the car after 5 years:

$T_6 = ar^5 = 80\,000\,(0.9)^5 = \text{RM}47\ 239.20$

11 $T_p = ar^{p-1}$ $T_q = ar^{q-1}$ $T_u = ar^{u-1}$

$P = ar^{p-1}$ $Q = ar^{q-1}$ $U = ar^{u-1}$

$$\begin{aligned} P^{q-u}Q^{u-p}U^{p-q} &= \left(ar^{p-1}\right)^{q-u}\left(ar^{q-1}\right)^{u-p}\left(ar^{u-1}\right)^{p-q} \\ &= a^{q-u+u-p+p-q}\ r^{(p-1)(q-u)+(q-1)(u-p)+(u-1)(p-q)} \\ &= a^0 r^{pq-pu-q+u+qu-pq-u+p+pu-qu-p+q} \\ &= (1)\left(r^0\right) \\ &= \mathbf{1}\ \text{[Shown]} \end{aligned}$$

UPSKILL 5.2c

1 (a) 1, 2, 4,

$a = 1,\ r = 2$

$S_{10} = \dfrac{a(r^n - 1)}{r - 1} = \dfrac{1(2^{10} - 1)}{2 - 1} = 1\ 023$

(b) 4, –6, 9, …

$a = 4,\ r = -\dfrac{3}{2}$

$S_5 = \dfrac{4\left[\left(-\dfrac{3}{2}\right)^5 - 1\right]}{-\dfrac{3}{2} - 1}$

$S_5 = \dfrac{4\left(-\dfrac{275}{32}\right)}{\left(-\dfrac{5}{2}\right)} = \dfrac{-\dfrac{275}{8}}{-\dfrac{5}{2}} = 13\dfrac{3}{4}$

(c) 48, 12, 3, …, $\dfrac{3}{16}$

$a = 48,\ r = \left(\dfrac{1}{4}\right)$

$T_n = \dfrac{3}{16}$

$ar^{n-1} = \dfrac{3}{16}$

$48\left(\dfrac{1}{4}\right)^{n-1} = \dfrac{3}{16}$

$\left(\dfrac{1}{4}\right)^{n-1} = \dfrac{1}{256}$

$\left(\dfrac{1}{4}\right)^{n-1} = \left(\dfrac{1}{4}\right)^4$

$n - 1 = 4$

$n = 5$

$S_5 = \dfrac{48\left[1 - \left(\dfrac{1}{4}\right)^5\right]}{1 - \dfrac{1}{4}}$

$S_5 = \dfrac{48\left[1 - \left(\dfrac{1}{1\ 024}\right)\right]}{\dfrac{3}{4}}$

$S_5 = 48\left(\dfrac{1\ 023}{1\ 024}\right) \times \dfrac{4}{3}$

$S_5 = 64\left(\dfrac{1\ 023}{1\ 024}\right)$

$S_5 = \dfrac{1\ 023}{16} = 63\dfrac{15}{16}$

2 $r = \dfrac{1}{2}$

$S_4 = 11\dfrac{1}{4}$

$\dfrac{a\left[1 - \left(\dfrac{1}{2}\right)^4\right]}{1 - \dfrac{1}{2}} = \dfrac{45}{4}$

$\dfrac{15}{16}a = \dfrac{45}{4} \times \dfrac{1}{2}$

$a = \dfrac{\dfrac{45}{8}}{\dfrac{15}{16}} = 6$

3 $a = 8$

$T_2 = 4$

$r = \dfrac{4}{8} = \dfrac{1}{2}$

$S_n = 15\dfrac{1}{2}$

$\dfrac{8\left[1 - \left(\dfrac{1}{2}\right)^n\right]}{1 - \dfrac{1}{2}} = \dfrac{31}{2}$

$16\left[1 - \left(\dfrac{1}{2}\right)^n\right] = \dfrac{31}{2}$

$1 - \left(\dfrac{1}{2}\right)^n = \dfrac{31}{2(16)}$

$\left(\dfrac{1}{2}\right)^n = \dfrac{1}{32}$

$n = 5$

4 $a = 5,\ r = 2$

$S_{11} - S_5$

$= \dfrac{5(2^{11} - 1)}{2 - 1} - \dfrac{5(2^5 - 1)}{2 - 1}$

= 10 235 – 155

= 10 080

5 $T_4 = 54$

$ar^3 = 54 \ \ldots (1)$

$T_7 = 1\ 458$

$ar^6 = 1\ 458 \ \ldots (2)$

$\frac{(2)}{(1)}: \frac{ar^6}{ar^3} = \frac{1\ 458}{54}$

$r^3 = 27$

$r = 3$

From (1) :

$a(3)^3 = 54$

$a = \frac{54}{27}$

$a = 2$

$S_8 - S_3$

$= \frac{2(3^8 - 1)}{3-1} - \frac{2(3^3 - 1)}{3-1}$

$= 6\ 560 - 26$

$= 6\ 534$

6 $S_2 = 25$

$\frac{a(r^2 - 1)}{r - 1} = 25$

$\frac{a(r+1)(r-1)}{r-1} = 25$

$a(r+1) = 25 \ \ldots (1)$

$T_3 = 80$

$ar^2 = 80 \ \ldots (2)$

$\frac{(2)}{(1)}: \frac{ar^2}{a(r+1)} = \frac{80}{25}$

$\frac{r^2}{r+1} = \frac{16}{5}$

$5r^2 = 16r + 16$

$5r^2 - 16r - 16 = 0$

$(r-4)(5r+4) = 0$

$r = 4 \text{ or } r = -\frac{4}{5}$

From (1) : when $r = 4$,

$a(4+1) = 25$

$a = 5$

From (1) : when $r = -\frac{4}{5}$,

$a\left(-\frac{4}{5} + 1\right) = 25$

$\frac{1}{5}a = 25$

$a = 125$

7 2, 3, $\frac{9}{2}$, …

$S_n > 60$

$\frac{2\left[\left(\frac{3}{2}\right)^n - 1\right]}{\frac{3}{2} - 1} > 60$

$4\left[\left(\frac{3}{2}\right)^n - 1\right] > 60$

$\left(\frac{3}{2}\right)^n - 1 > 15$

$\left(\frac{3}{2}\right)^n > 16$

$n \lg 1.5 > \lg 16$

$n > \frac{\lg 16}{\lg 1.5}$

$n > 6.84$

Hence, the 7th term exceeds 60.

8 $a = 1$, $r = 2$

$S_{18} = \frac{1(2^{18} - 1)}{2 - 1}$ = RM262 143

9 $a = 4$, $r = 3$

$S_9 = \frac{4(3^9 - 1)}{3 - 1} = 39\ 364$

10 (a) $a = 600$, $r = 0.95$

$T_7 = 600(0.95)^{7-1}$ = RM441 (correct to the nearest RM)

(b) $T_n < 370$

$ar^{n-1} < 370$

$600(0.95)^{n-1} < 370$

$0.95^{n-1} < \frac{37}{60}$

$(n-1)\lg 0.95 < \lg \frac{37}{60}$

$-0.0223\,(n-1) < -0.2099$

$n-1 > \frac{-0.2099}{-0.0223}$

$n-1 > 9.41$

$n > 10.41$

Minimum value of $n = 11$

(c) $S_8 = \frac{600\left(1-0.95^8\right)}{1-0.95}$ = RM4 039 (correct to the nearest RM)

11 $S_n = 3\left(2^n - 1\right)$

(a) $T_3 = S_3 - S_2$

$= 3\left(2^3-1\right) - 3\left(2^2-1\right)$

$= 21 - 9$

$= 12$

(b) $T_6 = S_6 - S_5$

$T_6 = 3\left(2^6-1\right) - 3\left(2^5-1\right)$

$= 189 - 93$

$= 96$

12 $S_n = 200\left(1-0.5^n\right)$

(a) $T_1 = S_1 = 200\left(1-0.5^1\right) = 100$

(b) $T_2 = S_2 - S_1$

$= 200\left(1-0.5^2\right) - 100$

$= 50$

$r = \frac{T_2}{T_1} = \frac{50}{100} = \frac{1}{2}$

13 $S_{2n} = pS_n$

$\frac{a\left(r^{2n}-1\right)}{r-1} = p\left[\frac{a\left(r^n-1\right)}{r-1}\right]$

$\left(r^{2n}-1\right) = p\left(r^n-1\right)$

$\left(r^n\right)^2 - 1 = p\left(r^n-1\right)$

$\left(r^n+1\right)\left(r^n-1\right) = p\left(r^n-1\right)$

$p = \left(r^n+1\right)$

$S_{4n} = qS_{2n}$

$\frac{a\left(r^{4n}-1\right)}{r-1} = q\left[\frac{a\left(r^{2n}-1\right)}{r-1}\right]$

$\left(r^{4n}-1\right) = q\left(r^{2n}-1\right)$

$\left(r^{2n}\right)^2 - 1 = q\left(r^{2n}-1\right)$

$\left(r^{2n}+1\right)\left(r^{2n}-1\right) = q\left(r^{2n}-1\right)$

$q = \left(r^{2n}+1\right)$

$(p-1)^2 = \left[\left(r^n+1\right)-1\right]^2$

$= r^{2n}$

$= q-1$ [Shown]

UPSKILL 5.2d

1 (a) 4, 2, 1, …

$$S_\infty = \frac{4}{1-\frac{1}{2}} = 8$$

(b) $6, 2, \frac{2}{3}, \ldots$

$$S_\infty = \frac{6}{1-\frac{1}{3}} = 9$$

(c) $-\frac{2}{3}, \frac{4}{9}, -\frac{8}{27}, \ldots$

$$r = \frac{\frac{4}{9}}{-\frac{2}{3}} = -\frac{2}{3}$$

$$S_\infty = \frac{-\frac{2}{3}}{1-\left(-\frac{2}{3}\right)} = -\frac{2}{5}$$

2 $S_\infty = 16\frac{2}{3}$

$$\frac{10}{1-r} = \frac{50}{3}$$

$$50(1-r) = 30$$

$$1-r = \frac{3}{5}$$

$$r = \frac{2}{5}$$

3 (a) $k+26, k+2, k-6, \ldots$

$$\frac{k+2}{k+26} = \frac{k-6}{k+2}$$

$$(k+2)^2 = (k+26)(k-6)$$

$$k^2+4k+4 = k^2-6k+26k-156$$

$$16k-160 = 0$$

$$k = \frac{160}{16} = 10$$

(b) $10+26, 10+2, 10-6, \ldots$

36, 12, 4, …

$$r = \frac{12}{36} = \frac{1}{3}$$

(c) $S_\infty = \dfrac{36}{1-\left(\frac{1}{3}\right)} = 54$

4 $T_3 = 2$

$ar^2 = 2 \quad \ldots (1)$

$T_5 = \frac{1}{8}$

$ar^4 = \frac{1}{8} \quad \ldots (2)$

$$\frac{(2)}{(1)} : r^2 = \frac{1}{16}$$

$$r = \frac{1}{4}$$

From (1) : $a\left(\frac{1}{4}\right)^2 = 2$

$a = 32$

$$S_\infty = \frac{32}{1-\left(\frac{1}{4}\right)} = 42\frac{2}{3}$$

5 $T_2 = \frac{3}{2}$

$ar = \frac{3}{2} \quad \ldots (1)$

$S_\infty = 6$

$\frac{a}{1-r} = 6 \quad \ldots (2)$

$$\frac{(2)}{(1)} : \frac{\frac{a}{1-r}}{ar} = \frac{6}{\frac{3}{2}}$$

$$\frac{a}{1-r} \times \frac{1}{ar} = 4$$

$$\frac{1}{r(1-r)} = 4$$

$$1 = 4r(1-r)$$

$$1 = 4r-4r^2$$

$$4r^2-4r+1 = 0$$

$$(2r-1)^2 = 0$$

$$r = \frac{1}{2}$$

From (1) :

$$a\left(\frac{1}{2}\right) = \frac{3}{2}$$

$$a = 3$$

6 $S_\infty = \frac{27}{8}$

$\frac{a}{1-r} = \frac{27}{8}$ … (1)

$S_2 = \frac{15}{8}$

$T_1 + T_2 = \frac{15}{8}$

$a + ar = \frac{15}{8}$

$a(1+r) = \frac{15}{8}$ … (2)

$\frac{(2)}{(1)} : \frac{a(1+r)}{\frac{a}{1-r}} = \frac{\frac{15}{8}}{\frac{27}{8}}$

$(1+r)(1-r) = \frac{5}{9}$

$1 - r^2 = \frac{5}{9}$

$r^2 = 1 - \frac{5}{9}$

$r^2 = \frac{4}{9}$

$r = \pm\frac{2}{3}$

From (2) :

When $r = \frac{2}{3}$, $a\left(1+\frac{2}{3}\right) = \frac{15}{8}$

$a = \frac{\frac{15}{8}}{\frac{5}{3}}$

$a = \frac{9}{8}$

When $r = -\frac{2}{3}$, $a\left(1-\frac{2}{3}\right) = \frac{15}{8}$

$a = \frac{\frac{15}{8}}{\frac{1}{3}}$

$a = \frac{45}{8}$

7 (a) 0.888…

$= 0.8 + 0.08 + 0.0008 + ...$

$= \frac{0.8}{1-0.1}$

$= \frac{0.8}{1-0.1}$

$= \frac{0.8}{0.9}$

$= \frac{8}{9}$

(b) 0.454545…

$= 0.45 + 0.0045 + 0.00000045$

$= \frac{0.45}{1-0.01}$

$= \frac{0.45}{0.99}$

$= \frac{45}{99}$

$= \frac{5}{11}$

(c) 0.228228228…

$= 0.228 + 0.000000228 + 0.000000000228$

$= \frac{0.228}{1-0.999}$

$= \frac{228}{999}$

$= \frac{76}{333}$

8 (a) T_1 = Area of $PQR = \frac{1}{2}x^2 \sin\theta$

T_2 = Area of $PQ_1R_1 = \frac{1}{2}\left(\frac{x}{2}\right)^2 \sin\theta$

$= \frac{1}{8}x^2 \sin\theta$

T_3 = Area of $PQ_2R_2 = \frac{1}{2}\left(\frac{x}{4}\right)^2 \sin\theta$

$= \frac{1}{32}x^2 \sin\theta$

T_4 = Area of $PQ_3R_3 = \frac{1}{2}\left(\frac{x}{8}\right)^2 \sin\theta$

$= \frac{1}{128}x^2 \sin\theta$

$$\frac{T_2}{T_1} = \frac{\frac{1}{8}x^2 \sin\theta}{\frac{1}{2}x^2 \sin\theta} = \frac{1}{4}$$

$$\frac{T_3}{T_2} = \frac{\frac{1}{32}x^2 \sin\theta}{\frac{1}{8}x^2 \sin\theta} = \frac{1}{4}$$

$$\frac{T_4}{T_3} = \frac{\frac{1}{128}x^2 \sin\theta}{\frac{1}{32}x^2 \sin\theta} = \frac{1}{4}$$

Since $\frac{T_2}{T_1} = \frac{T_3}{T_2} = \frac{T_4}{T_3} = \frac{1}{4}$ (a constant),
hence the areas of the triangles form a geometric progression.

Common ratio = $\frac{1}{4}$

(b) (i) $a = T_1$ = Area of PQR

$$= \frac{1}{2}(24)^2 \sin 30^{\circ}$$
$$= 144$$

$$T_5 = ar^4 = 144\left(\frac{1}{4}\right)^4 = \frac{9}{16} \text{ cm}^2$$

(ii) $$T_\infty = \frac{a}{1-r} = \frac{\frac{1}{2}(24)^2 \sin 30^{\circ}}{1-\frac{1}{4}}$$
$$= \frac{144}{\frac{3}{4}}$$
$$= 192 \text{ cm}^2$$

Summative Practice 5

1 (a) $k-7,\ k-1,\ 2k-2,\ \ldots$

$$k-1-(k-7)=2k-2-(k-1)$$
$$6=k-1$$
$$k=7$$

(b) 0, 6, 12, …

S_9-S_3

$=\frac{9}{2}(2a+8d)-\frac{3}{2}(2a+2d)$

$=\frac{9}{2}[2(0)+8(6)]-\frac{3}{2}[2(0)+2(6)]$

$=216-18$

$=198$

2 4, 7, 10, …

$a=4,\ d=3$

$S_{23}-S_3$

$=\frac{23}{2}(2a+22d)-\frac{3}{2}(2a+2d)$

$=\frac{23}{2}[2(4)+22(3)]-\frac{3}{2}[2(4)+2(3)]$

$=851-21$

$=830$

3 $S_3=117$

$\frac{3}{2}[2a+(3-1)(7)]=117$

$2a+14=117\times\frac{2}{3}$

$2a+14=78$

$a=32$

The required three consecutive terms are 32, 39 and 46.

4 50, 46, 42, …

$T_n<0$

$50+(n-1)(-4)<0$

$-4(n-1)<-50$

$n-1>\frac{-50}{-4}$

$n-1>12.5$

$n>13.5$

Hence, the smallest value of n is 14.

5 a = 1 200, d = 80

$T_{15}=a+14d=1\ 200+14(80)=\text{RM}2\ 320$

6 a = 150, d = 5

$S_{24}=\frac{24}{2}(2a+23d)$

$=12[2(150)+23(5)]$

$=\text{RM}4\ 980$

7 1, 2, 3, …

$S_{20}=\frac{20}{2}[2(1)+19(1)]=210$ bricks

8 ***Condensed milk***

$T_n=a+(n-1)d$

$T_n=65+(n-1)(-5)$

$T_n=65-5n+5$

$T_n=70-5n\ \ldots(1)$

Evaporated milk

$T_n=a+(n-1)d$

$T_n=45+(n-1)(-3)$

$T_n=45-3n+3$

$T_n=48-3n\ \ldots(2)$

Substitute (1) into (2):

$70-5n=48-3n$

$22=2n$

$n=11$

Hence, the balance of the number of cans of the condensed milk and evaporated milk are the same after 10 days.

At the 0th day (T_1), the stall has 65 cans of condensed milk and 45 cans of evaporate milk. Hence, T_{11} is the 10 day.

9 $4,\ 4\frac{1}{7},\ 4\frac{2}{7},\ \ldots$

$S_{15}=\frac{15}{2}(2a+14d)$

$=\frac{15}{2}\left[2(4)+14\left(\frac{1}{7}\right)\right]$

$=75$ minutes

= 1 hour 15 minutes [less than 1.5 hours]

Hence, Nathan is qualified for the state level run.

10 $T_1 = \pi r^2 h$

$T_2 = \pi r^2(h+1)$

$T_3 = \pi r^2(h+2)$

$T_2 - T_1 = \pi r^2(h+1) - \pi r^2 h = \pi r^2$

$T_3 - T_2 = \pi r^2(h+2) - \pi r^2(h+1) = \pi r^2$

Since $T_2 - T_1 = T_3 - T_2 = \pi r^2$ (a constant), hence the volumes of cylinders form an arithmetic progression.

Common difference = πr^2

11 (a) Let the length of the side of the smallest equilateral triangle = x cm

$3x, 3(x+2), 3(x+4), ...$

$a = 3x, d = 6$

$S_5 = 90$

$\frac{5}{2}[2(3x)+4(6)] = 90$

$6x + 24 = 90 \times \frac{2}{5}$

$6x = 12$

$x = 2$

Hence, the length of each side of the smallest equilateral triangle is 2 cm.

(b) $6, 12, 18, ...$

$S_n = 350$

$\frac{n}{2}[2(6)+(n-1)(6)] = 350$

$n(12+6n-6) = 700$

$n(6n-4) = 700$

$6n^2 - 4n - 700 = 0$

$3n^2 - 2n - 350 = 0$

$n = \frac{2 \pm \sqrt{(-2)^2 - 4(3)(-350)}}{2(3)}$

$n = \frac{2 \pm \sqrt{4\ 202}}{6}$

$n = 11.14$ or $n = -10.47$

$n = 10.47$ is not accepted.

Hence, the number of the complete triangles that can be formed is 10.

12 (a) $S_n = 360^{\circ}$

$\frac{n}{2}(a+l) = 360$

$\frac{n}{2}(20+100) = 360$

$60n = 360$

$n = 6$

(b) $T_n = 100$

$a + (n-1)d = 100$

$20 + (6-1)d = 100$

$5d = 80$

$d = 16^{\circ}$

(c) $\pi r^2 = 25\pi$

$r = 5$

Angle of the 2nd sector

= 20° + 16°

= 36°

Area of the 2nd sector

= $\frac{36}{360} \times \pi(5)^2$

= $\frac{5}{2}\pi$ cm²

13 ***Particle A***

$120, 116, 112, \ldots$

$a = 120, d = -4$

Particle B

$x, x-5, x-10, \ldots$

$a = x, d = -5$

(a) $T_n = 0$

$120 + (n-1)(-4) = 0$

$-4(n-1) = -120$

$n - 1 = 30$

$n = 31$

(b) $T_{31} = 0$

$x + (n-1)(-5) = 0$

$x + (31-1)(-5) = 0$

$x = 150$

(c) S_{31} (particle A)

= $\frac{31}{2}[2(120)+30(-4)]$

= 1 860 cm

S_{31} (particle B)

$= \frac{31}{2}[2(150)+30(-5)]$

= 2 325 cm

Difference in distance travelled
= 2 325 – 1 860
= 465 cm

14 $T_1 = \pi(3)^2(5) = 45\pi$

$T_2 = \pi(3)^2(7) = 63\pi$

$T_3 = \pi(3)^2(9) = 81\pi$

$a = 45\pi,\ r = 63\pi - 45\pi = 18\pi$

(a) $T_{15} = a + 14d$

$= 45\pi + 14(18\pi)$

$= 297\pi$ cm³

(b) $S_{10} = 1260\pi$

$\frac{n}{2}[2(45\pi + (n-1)(18\pi)] = 1260\pi$

$\frac{n}{2}(90 + 18n - 18) = 1260$

$\frac{n}{2}(72 + 18n) = 1260$

$n(36 + 9n) = 1260$

$36n + 9n^2 = 1260$

$9n^2 + 36n - 1260 = 0$

$n^2 + 4n - 140 = 0$

$(n-10)(n+14) = 0$

$n = 10$ or $n = -14$

$n = -14$ is not accepted.
Hence, $n = 10$.

15 $T_n = 500$

$a + (n-1)d = 500$

$4000 + (n-1)(-250) = 500$

$(n-1)(-250) = -3500$

$n - 1 = 14$

$n = 15$

$S_{15} = \frac{n}{2}[2a + (n-1)d]$

$S_{15} = \frac{15}{2}[2(4000) + (15-1)(-250)]$

$S_{15} = 33\ 750$

Total cost
$= 33\,750 \times 0.50 = \text{RM}16\,875$

16 $\frac{1}{3}, 1, 3, \ldots$

$a = \frac{1}{3},\ r = 3$

$S_n = 364\frac{1}{3}$

$\frac{\frac{1}{3}(3^n - 1)}{3-1} = \frac{1\ 093}{3}$

$\frac{3^n - 1}{6} = \frac{1\ 093}{3}$

$3^n - 1 = 2\ 186$

$3^n = 2\ 187$

$3^n = 3^7$

$n = 7$

17 $T_3 - T_2 = 20a$

$ar^2 - ar = 20a$

$r^2 - r - 20 = 0$

$(r-5)(r+4) = 0$

$r = 5$ or -4

18 $S_n = \frac{5}{2}(3^n - 1)$

$T_3 = S_3 - S_2$

$= \frac{5}{2}(3^3 - 1) - \frac{5}{2}(3^2 - 1)$

$= 65 - 20$

$= 45$

19 $a = 3$, $r = 3$

$T_7 = ar^6 = (3)(3)^6 = 2\ 187$ cells

20 $a = 2\ 000$, $r = 1.05$

$S_8 = \frac{2\ 000(1.05^8 - 1)}{1.05 - 1}$ = RM19 098

21 $A_1 = \frac{1}{2}k^2$

$A_2 = \frac{1}{2}\left(\frac{k}{2}\right)^2 = \frac{1}{8}k^2$

$A_3 = \frac{1}{2}\left(\frac{k}{4}\right)^2 = \frac{1}{32}k^2$

$\frac{A_2}{A_1} = \frac{\frac{1}{8}k^2}{\frac{1}{2}k^2} = \frac{1}{4}$

$\frac{A_3}{A_2} = \frac{\frac{1}{32}k^2}{\frac{1}{8}k^2} = \frac{1}{4}$

Since $\frac{A_2}{A_1} = \frac{A_3}{A_2} = \frac{1}{4}$ (a constant), hence the areas of the right-angled triangles form a geometric progression.

Common ratio = $\frac{1}{4}$

22 (a) 50, 40, 32, …

$a = 50,\ r = \frac{4}{5}$

$T_8 = ar^7\ 50\left(\frac{4}{5}\right)^7 = 10.49$ cm

(b)

$$T_n < 5$$
$$50\left(\frac{4}{5}\right)^{n-1} < 5$$
$$\left(\frac{4}{5}\right)^{n-1} < \frac{5}{50}$$
$$(n-1)\ \lg\left(\frac{4}{5}\right) < \lg\frac{1}{10}$$
$$-0.097\ (n-1) < -1$$
$$n-1 > \frac{-1}{-0.097}$$
$$n-1 > 10.31$$
$$n > 11.31$$

Hence, height of the 12th bounce is less than 5 cm.

23 (a) 3, 6, 12, …

$T_6 = 3(2)^5 = 96$ members

(b) $S_9 = \frac{3\left[2^9-1\right]}{2-1} = 1\ 533$ members

24 (a) $T_1 = \frac{1}{2}bt$

$T_2 = \frac{1}{2}(2b)(2t) = 2bt$

$T_3 = \frac{1}{2}(4b)(4t) = 8bt$

$\frac{T_2}{T_1} = \frac{2bt}{\frac{1}{2}bt} = 4$

$\frac{T_3}{T_2} = \frac{8bt}{2bt} = 4$

Since $\frac{T_2}{T_1} = \frac{T_3}{T_2} = 4$ (a constant), the areas of the right-angled triangles form a geometric progression.
Common ratio = 4

(b) $b = 6$, $t = 3$

$a = T_1 = \frac{1}{2}(6)(3) = 9$

(i) $T_8 = ar^7 = 9(4)^7 = 147\ 456\ \text{cm}^2$

(ii) $S_4 = \frac{9\left(4^4-1\right)}{4-1} = 765\ \text{cm}^2$

25 (a)

×1.05 ×1.05

24 000, 25 200, 26 460, … → J.G.

$T_6 = 24\ 000(1.05)^{6-1} = \text{RM}30\ 631$

(b)

$$T_n > 35\ 000$$
$$24\ 000\left(1.05^{n-1}\right) > 35\ 000$$
$$1.05^{n-1} > \frac{35}{24}$$
$$(n-1)\lg\ 1.05 > \lg\left(\frac{35}{24}\right)$$
$$n-1 > \frac{\lg\left(\frac{35}{24}\right)}{\lg\ 1.05}$$
$$n-1 > 7.73$$
$$n > 8.73$$

Minimum value of n = 9

(c) $S_6 = \frac{24\ 000\left(1.05^6-1\right)}{1.05-1}$
$= \text{RM}163\ 246$

26 (a) $T_7 = 8T_4$

$ar^6 = 8ar^3$

$\frac{r^6}{r^3} = 8$

$r^3 = 8$

$r = 2$

(b) (i) $S_n = 3069$

$\frac{3(2^n - 1)}{2-1} = 3069$

$2^n - 1 = 1023$

$2^n = 1024$

$2^n = 2^{10}$

$n = 10$

(ii) $T_{10} = ar^9 = 3(2)^9 = 1\ 536$ cm

27 $T_2 = 48$

$ar = 48$ … (1)

$T_4 = 27$

$ar^3 = 27$ … (2)

$\frac{(2)}{(1)} : \frac{ar^3}{ar} = \frac{27}{48}$

$r^2 = \frac{9}{16}$

$r = \frac{3}{4}$

From (1) :

$a\left(\frac{3}{4}\right) = 48$

$a = 48 \times \frac{4}{3}$

$a = 64$

$S_\infty = \frac{64}{1 - \frac{3}{4}} = 256$

28 (a) $T_3 = 10$

$ar^2 = 10$ … (1)

$T_3 + T_4 = 15$

$10 + ar^3 = 15$

$ar^3 = 5$ … (2)

$\frac{(2)}{(1)} : \frac{ar^3}{ar^2} = \frac{5}{10} = \frac{1}{2}$

$r = \frac{1}{2}$

From (1) :

$a\left(\frac{1}{2}\right)^2 = 10$

$a = 10(4)$

$a = 40$

(b) $S_\infty = \frac{40}{1 - \frac{1}{2}} = 80$

29 (a) Let $AB = r$

$T_1 = \frac{1}{4}\pi r^2$

$T_2 = \frac{1}{4}\pi\left(\frac{r}{2}\right)^2 = \frac{1}{16}\pi r^2$

$T_3 = \frac{1}{4}\pi\left(\frac{r}{4}\right)^2 = \frac{1}{64}\pi r^2$

$\frac{T_2}{T_1} = \frac{\frac{1}{16}\pi r^2}{\frac{1}{4}\pi r^2} = \frac{1}{4}$

$\frac{T_3}{T_2} = \frac{\frac{1}{64}\pi r^2}{\frac{1}{16}\pi r^2} = \frac{1}{4}$

Since $\frac{T_2}{T_1} = \frac{T_3}{T_2} = \frac{1}{4}$ (a constant), hence the areas of the quadrants form a geometric progression.

Common ratio = $\frac{1}{4}$

(b) $AB =$ 6 cm

$a = \frac{1}{4}\pi(6)^2 = 9\pi$

$S_\infty = \frac{9\pi}{1 - \frac{1}{4}} = 12\pi$ cm^2

30 (a) $T_1 = \frac{1}{2}(50 + 40)(2) = 90$

$T_2 = \frac{1}{2}(40 + 32)(2) = 72$

$a = 90,\ r = \frac{72}{90} = \frac{4}{5}$

$T_5 = ar^4 = 90\left(\frac{4}{5}\right)^4 = 36.864\ \text{cm}^2$

(b) $T_\infty = \dfrac{90}{1 - \frac{4}{5}} = 450\ \text{cm}^2$