Form 4 Chapter 4
Indices, Surds and Logarithms
Fully-Worked Solutions

UPSKILL 4.1a

1 (a) $2^{3d+1}\times 2^{-3d}\times 8$
$= 2^{3d+1-3d+3}$
$= 2^{4}$
$= 16$

(b) $3^{1-n}\times 27^{n+1}\div 9^{n+1}$
$= 3^{1-n+3n+3-2n-2}$
$= 3^{2}$
$= 9$

(c) $5^{3}\times 25^{2n-1}\div 5^{4n+1}$
$= 5^{3+4n-2-4n-1}$
$= 5^{0}$
$= 1$

2 (a) $\dfrac{2x^{-\frac{1}{3}}y^{-\frac{2}{3}}}{\left(x^{2}y^{4}\right)^{-\frac{1}{6}}}$
$= \dfrac{2x^{-\frac{1}{3}}y^{-\frac{2}{3}}}{x^{-\frac{1}{3}}y^{-\frac{2}{3}}}$
$= 2$

(b) $\dfrac{3^{3n}\times 3^{2-n}}{3^{4}\times 3^{2n+1}}$
$= 3^{3n+2-n-4-2n-1}$
$= 3^{-3}$
$= \dfrac{1}{27}$

(c) $\left(\dfrac{2x^{2}}{3y}\right)^{3}\left(\dfrac{9y^{2}}{4x^{3}}\right)^{2}$
$= \dfrac{8x^{6}}{27y^{3}}\times\dfrac{81y^{4}}{16x^{6}}$
$= \dfrac{3}{2}y$

(d) $\dfrac{25^{m}\times 2^{m+1}\times 5^{m+1}}{2^{m-1}\times 5^{3m+2}}$
$= 5^{2m+m+1-3m-2}2^{m+1-m+1}$
$= 5^{-1}\times 2^{2}$
$= \dfrac{4}{5}$

3 $2^{n+4}-2^{n+2}-2^{2n}$
$= 2^{n}2^{4}-2^{n}2^{2}-2^{n}$
$= (16-4-1)\,2^{n}$
$= 11\left(2^{n}\right)$
$k = 11$

4 $3^{n+1}+3^{n}+9\left(3^{n-2}\right)$
$= 3^{n}3+3^{n}+9\left(\dfrac{3^{n}}{9}\right)$
$= (3+1+1)\left(3^{n}\right)$
$= 5\left(3^{2}\right)$
$h = 5$

5 $5^{n+2}-7\left(5^{n}\right)-25\left(5^{n-1}\right)$
$= 5^{n}5^{2}-7\left(5^{n}\right)-25\left(\dfrac{5^{n}}{5}\right)$
$= (25-7-5)\left(5^{n}\right)$
$= 13\left(5^{n}\right)$
$q = 13$

UPSKILL 4.1b

1 (a) $8\left(4^{x}\right)=\dfrac{32}{16^{1-2x}}$
$2^{3+2x}=2^{5-4(1-2x)}$
$3+2x=5-4(1-2x)$
$3+2x=5-4+8x$
$6x=2$
$x=\dfrac{1}{3}$

(b) $\dfrac{3^{y}}{81^{y-1}}=\dfrac{1}{9^{y}}$
$\dfrac{3^{y}}{3^{4(y-1)}}=3^{-2y}$
$3^{y-4(y-1)}=3^{-2y}$
$y-4(y-1)=-2y$
$y-4y+4=-2y$
$-y=-4$
$y=4$

(c) $\dfrac{25^{3t-1}}{5^{2-t}} = 125^{t+3}$

$\dfrac{5^{2(3t-1)}}{5^{(2-t)}} = 5^{3(t+3)}$

$5^{6t-2-(2-t)} = 5^{3t+9}$

$6t-2-2+t = 3t+9$

$4t = 13$

$t = \dfrac{13}{4}$

2 $\dfrac{3}{x^{-\frac{4}{5}}} = 12x^{\frac{2}{5}}$

$3 = 12x^{\frac{2}{5}} \times x^{-\frac{4}{5}}$

$3 = 12x^{\frac{2}{5}-\frac{4}{5}}$

$\dfrac{1}{4} = x^{-\frac{2}{5}}$

$2^{-2} = x^{-\frac{2}{5}}$

$x = 2^{-2\times -\frac{5}{2}}$

$x = 2^5 = 32$

3 (a) $\dfrac{125^m}{25^n} = 625$

$\dfrac{5^{3m}}{5^{2n}} = 5^4$

$5^{3m-2n} = 5^4$

$3m-2n = 4 \;\ldots (1)$

$2\times 4^m = 32^n$

$2^{1+2m} = 2^{5n}$

$1+2m = 5n$

$2m-5n = -1 \;\ldots (2)$

$6m-4n = 8 \quad \ldots (1) \times 2$

$(-)\;\; 6m-15n = -3 \quad \ldots (2) \times 3$

$11n = 11$

$n = 1$

From (1) :

$3m-2(1) = 4$

$m = 2$

(b) $3^p \times 9^{2q} = 27$

$3^{p+4q} = 3^3$

$p+4q = 3 \;\ldots (1)$

$\dfrac{5^p}{25^q} = \dfrac{1}{125}$

$5^{p-2q} = 5^{-3}$

$p-2q = -3 \;\ldots (2)$

(1) – (2):

$6q = 6$

$q = 1$

From (1) :

$p+4(1) = 3$

$p = -1$

4 (a) $5^{n+2} - 5^{n+1} - 5^n = 19$

$5^n 5^2 - 5^n 5 - 5^n = 19$

$25(5^n) - 5(5^n) - 5^n = 19$

$19(5^n) = 19$

$5^n = 1$

$n = 0$

(b) $2(3^{r+2}) - 3^{r+1} = 45$

$2(3^r)(3^2) - 3^r(3) = 45$

$(18-3)(3^r) = 45$

$15(3^r) = 45$

$3^r = 3$

$r = 1$

5 $5^{m+1} = 7^{n-1} = 35^k = p$

$5 = p^{\frac{1}{m+1}} \qquad 7 = p^{\frac{1}{n-1}} \qquad 35 = p^{\frac{1}{k}}$

$5\times 7 = 35$

$p^{\frac{1}{m+1}} \times p^{\frac{1}{n-1}} = p^{\frac{1}{k}}$

$\dfrac{1}{m+1} + \dfrac{1}{n-1} = \dfrac{1}{k}$

$\dfrac{(n-1)+(m+1)}{(m+1)(n-1)} = \dfrac{1}{k}$

$\dfrac{m+n}{(m+1)(n-1)} = \dfrac{1}{k}$

$k(m+n) = (m+1)(n-1)$ [Shown]

6 $3^h = 11^k = 33^m = p$

$3 = p^{\frac{1}{h}} \qquad 11 = p^{\frac{1}{k}} \qquad 33 = p^{\frac{1}{m}}$

$3 \times 11 = 33$

$p^{\frac{1}{h}} \times p^{\frac{1}{k}} = p^{\frac{1}{m}}$

$\frac{1}{h} + \frac{1}{k} = \frac{1}{m}$

$\frac{h+k}{hk} = \frac{1}{m}$

$m = \frac{hk}{h+k}$

7 $120\,000(0.95)^n = 102\,885$

$0.95^n = 0.857375$

$0.95^n = 0.95^3$

$n = 3$ years

UPSKILL 4.2a

1 (a) $\sqrt{50} = \sqrt{25(2)} = 5\sqrt{2}$

(b) $\sqrt{147} = \sqrt{49 \times 3} = 7\sqrt{3}$

(c) $\sqrt{288} = \sqrt{144(2)} = 12\sqrt{2}$

(d) $\sqrt{450} = \sqrt{225 \times 2} = 15\sqrt{2}$

(e) $\sqrt{800} = \sqrt{400 \times 2} = 20\sqrt{2}$

2 (a) $\sqrt{3} \times \sqrt{27} = \sqrt{81} = 9$

(b) $6\sqrt{6} \times 3\sqrt{24} = 18\sqrt{144} = 18(12) = 216$

(c) $\sqrt{18} \times \sqrt{125} \div \sqrt{2}$

$= \frac{\sqrt{9 \times 2} \times \sqrt{25 \times 5}}{\sqrt{2}}$

$= \frac{3\sqrt{2} \times 5\sqrt{5}}{\sqrt{2}}$

$= 15\sqrt{5}$

3 (a) $\sqrt{12} - 3\sqrt{27} + \sqrt{48}$

$= 2\sqrt{3} - 3 \times 3\sqrt{3} + 4\sqrt{3}$

$= -3\sqrt{3}$

(b) $\sqrt{45} + \sqrt{125} - 2\sqrt{5}$

$= 3\sqrt{5} + 5\sqrt{5} - 2\sqrt{5}$

$= 6\sqrt{5}$

(c) $\sqrt{27} - \sqrt{12} + 2\sqrt{75}$

$= 3\sqrt{3} - 2\sqrt{3} + 2 \times 5\sqrt{3}$

$= 11\sqrt{3}$

(d) $\frac{3\sqrt{50}}{5} \times \frac{5\sqrt{48}}{4} \div \sqrt{24}$

$= \frac{\frac{3}{5} \times 5\sqrt{2} \times \frac{5}{4} \times 4\sqrt{3}}{2\sqrt{6}}$

$= \frac{3\sqrt{2} \times 3\sqrt{3}}{2\sqrt{6}}$

$= \frac{15}{2}$

4 (a) $\frac{5}{\sqrt{3}} = 5\frac{\sqrt{3}}{3}$

(b) $\frac{7}{2\sqrt{2}} = \frac{7}{2} \times \frac{\sqrt{2}}{2} = \frac{7}{4}\sqrt{2}$

(c) $\sqrt{\frac{3}{5}} = \frac{\sqrt{3}}{\sqrt{5}} = \frac{\sqrt{3} \times \sqrt{5}}{5} = \frac{\sqrt{15}}{5}$

5 (a) $\frac{1}{3-\sqrt{7}} + \frac{1}{3+\sqrt{7}}$

$= \frac{3+\sqrt{7}+3-\sqrt{7}}{(3-\sqrt{7})(3+\sqrt{7})}$

$= \frac{6}{9-7}$

$= 3$

(b) $\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$

$= \frac{(\sqrt{3}-\sqrt{2})(\sqrt{3}-\sqrt{2})}{(\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2})}$

$= \frac{3-\sqrt{6}-\sqrt{6}+2}{3-2}$

$= 5 - 2\sqrt{6}$

(c) $\dfrac{3\sqrt{5}-\sqrt{2}}{2\sqrt{5}+3\sqrt{2}}$

$= \dfrac{(3\sqrt{5}-\sqrt{2})(2\sqrt{5}-3\sqrt{2})}{(2\sqrt{5}+3\sqrt{2})(2\sqrt{5}-3\sqrt{2})}$

$= \dfrac{6(5)-9\sqrt{10}-2\sqrt{10}+3(2)}{20-18}$

$= \dfrac{36-11\sqrt{10}}{2}$

(d) $\dfrac{3\sqrt{2}-2\sqrt{3}}{3\sqrt{2}+2\sqrt{3}}$

$= \dfrac{(3\sqrt{2}-2\sqrt{3})(3\sqrt{2}-2\sqrt{3})}{(3\sqrt{2}+2\sqrt{3})(3\sqrt{2}-2\sqrt{3})}$

$= \dfrac{9(2)-6\sqrt{6}-6\sqrt{6}+4(3)}{18-12}$

$= \dfrac{30-12\sqrt{6}}{6}$

$= \dfrac{5-2\sqrt{6}}{1}$

$= 5-2\sqrt{6}$

UPSKILL 4.2b

1 $\sqrt{[2-(-4)]^2+[-5-(-3)]^2}$

$= \sqrt{36+4}$

$= \sqrt{40}$

$= 2\sqrt{10}$ units

2 $\tan 60^\circ \times \sin 45^\circ$

$= \sqrt{3}\times\dfrac{1}{\sqrt{2}}$

$= \sqrt{3}\times\dfrac{1}{\sqrt{2}}\times\dfrac{\sqrt{2}}{\sqrt{2}}$

$= \sqrt{3}\times\dfrac{\sqrt{2}}{2}$

$= \dfrac{\sqrt{6}}{2}$

3 $\dfrac{x^2}{45}+\dfrac{y^2}{50}=1$

x-intercept $= \pm\sqrt{45}=\pm 3\sqrt{5}$

y-intercept $= \pm\sqrt{50}=\pm 5\sqrt{2}$

4 (a) $\sqrt{2x-1}-\sqrt{x+3}=1$

$\left(\sqrt{2x-1}-\sqrt{x+3}\right)^2=1^2$

$2x-1-2\sqrt{2x-1}\sqrt{x+3}+x+3=1$

$3x+1=2\sqrt{2x-1}\sqrt{x+3}$

$(3x+1)^2=\left(2\sqrt{2x-1}\sqrt{x+3}\right)^2$

$9x^2+6x+1=4(2x-1)(x+3)$

$9x^2+6x+1=4\left(2x^2+5x-3\right)$

$9x^2+6x+1=8x^2+20x-12$

$x^2-14x+13=0$

$(x-1)(x-13)=0$

$x=1$ or $x=13$

$x=1$ is not accepted because it does not satisfy the original equation.

$\therefore x=13$

(b) $\sqrt{3x+4}-\sqrt{x+2}=\sqrt{x-3}$

$\left(\sqrt{3x+4}-\sqrt{x+2}\right)^2=\left(\sqrt{x-3}\right)^2$

$3x+4-2\sqrt{3x+4}\sqrt{x+2}+x+2=x-3$

$3x+9=2\sqrt{3x+4}\sqrt{x+2}$

$(3x+9)^2=\left(2\sqrt{3x+4}\sqrt{x+2}\right)^2$

$9x^2+54x+81=4(3x+4)(x+2)$

$9x^2+54x+81=4(3x^2+10x+8)$

$9x^2+54x+81=12x^2+40x+32$

$3x^2-14x-49=0$

$(x-7)(3x+7)=0$

$x=7$ or $x=-\dfrac{7}{3}$

$x=-\dfrac{7}{3}$ is not accepted it does not satisfy the original equation.

$\therefore x=7$

(c) $\sqrt{2p-1}+\sqrt{p-1}=5$

$\left(\sqrt{2p-1}+\sqrt{p-1}\right)^2=5^2$

$2p-1+2\sqrt{2p-1}\sqrt{p-1}+p-1=25$

$2\sqrt{2p-1}\sqrt{p-1}=27-3p$

$\left(2\sqrt{2p-1}\sqrt{p-1}\right)^2=(27-3p)^2$

$4(2p-1)(p-1)=729-162p+9p^2$

$4\left(2p^2-3p+1\right)=729-162p+9p^2$

$8p^2-12p+4=729-162p+9p^2$

$p^2-150p+725=0$

$(p-5)(p-145)=0$

$p=5$ or $p=145$

$p=145$ is not accepted it does not satisfy the original equation.

$\therefore p=5$

UPSKILL 4.3a

1 (a) $11^2=121$

$2=\log_{11}121$

(b) $7^0=1$

$0=\log_7 1$

(c) $5^{-3}=\frac{1}{125}$

$-3=\log_5\frac{1}{125}$

2 (a) $\log_2 x=3$

$x=2^3=8$

(b) $\log_3 x=-4$

$x=3^{-4}=\frac{1}{81}$

(c) $\log_x 64=3$

$64=x^3$

$x=4$

(d) $\log_x 5=\frac{1}{2}$

$5=x^{\frac{1}{2}}$

$x=5^2=25$

3 (a) $\log_{10}0.1945$

$=-0.7111$

(b) $\log_{10}0.7261$

$=-0.1390$

(c) $\log_{10}7.314$

$=0.8642$

(d) lg 335.7

$=2.5260$

4 (a) $\log_{10}x=0.9566$

$x=9.049$

(b) $\log_{10}x=0.7443$

$x=5.550$

(c) $\lg x=1.8151$

$x=65.33$

(d) $\lg x=2.0986$

$x=125.49$

5 (a) $\log_3 81$

$=\log_3 3^4$

$=4$

(b) $\log_5 125$

$=\log_5 5^3$

$=3$

(c) $\log_{64}8$

$=\log_{64}64^{\frac{1}{2}}$

$=\frac{1}{2}$

6 (a) $\log_a x^3yz^5$

$=\log_a x^3+\log_a y+\log_a z^5$

$=3\log_a x+\log_a y+5\log_a z$

$=3p+q+5r$

(b) $\log_a\frac{x^2\sqrt{y}}{z^3}$

$=\log_a x^2+\log_a y^{\frac{1}{2}}-\log_a z^3$

$=2\log_a x+\frac{1}{2}\log_a y-3\log_a z$

$=2p+\frac{1}{2}q-3r$

(c) $\log_a\frac{x^4}{y^3z^2}$

$=\log_a x^4-(\log_a y^3+\log_a z^2)$

$=4\log_a x-(3\log_a y+2\log_a z)$

$=4p-3q-2r$

7 (a) $\log_2\left(\frac{25}{9}\right)$

$= \log_2\left(\frac{5^2}{3^2}\right)$

$= \log_2 5^2 - \log_2 3^2$

$= 2k - 2m$

(b) $\log_2 360$

$= \log_2\left(2^3 \times 3^2 \times 5\right)$

$= \log_2 2^3 + \log_2 3^2 + \log_2 5$

$= 3\log_2 2 + 2\log_2 3 + \log_2 5$

$= 3 + 2m + k$

2	360
2	180
2	90
3	45
3	15
5	5
	1

(c) $\log_2 \sqrt{60}$

$= \log_2(2\times2\times3\times5)^{\frac{1}{2}}$

$= \frac{1}{2}\log_2(2\times2\times3\times5)$

$= \frac{1}{2}(2+m+k)$

$= 1+\frac{1}{2}m+\frac{1}{2}k$

(d) $\log_2 7.5$

$= \log_2\left(\frac{15}{2}\right)$

$= \log_2\left(\frac{3\times5}{2}\right)$

$= \log_2 3 + \log_2 5 - \log_2 2$

$= m + k - 1$

8 (a) $\log_a 200$

$= \log_a\left(2^3\times5^2\right)$

$= 3\log_a 2 + 2\log_a 5$

$= 3h + 2k$

2	200
2	100
2	50
5	25
5	5
	1

(b) $\log_a\left(\frac{4}{25}\right)$

$= \log_a\left(\frac{2^2}{5^2}\right)$

$= 2\log_a 2 - 2\log_a 5$

$= 2h - 2k$

(c) $\log_a \frac{64\sqrt{a}}{125}$

$= \log_a \frac{2^6 a^{\frac{1}{2}}}{5^3}$

$= 6\log_a 2 + \frac{1}{2}\log_a a - 3\log_a 5$

$= 6h + \frac{1}{2} - 3k$

9 (a) $\log_5 360$

$= \log_5\left(2^3\times3^2\times5\right)$

$= 3\log_5 2 + 2\log_5 3 + \log_5 5$

$= 3m + 2n + 1$

2	360
2	180
2	90
3	45
3	15
5	5
	1

(b) $\log_5 0.96$

$= \log_5\left(\frac{24}{25}\right)$

$= \log_5\left(\frac{2^3\times3}{5^2}\right)$

$= 3\log_5 2 + \log_5 3 - 2\log_5 5$

$= 3m + n - 2$

10 (a) $2\lg 4 + 2\lg 5 - \lg 4$

$= \lg\left(\frac{4^2\times5^2}{4}\right)$

$= \log_{10}(100)$

$= 2$

(b) $3\log_2 5 - 6\log_2\left(\frac{1}{2}\right) - 3\log_2 10$

$= \log_2\left(\frac{5^3}{\frac{1}{2^6}\times10^3}\right)$

$= \log_2\left(\frac{125\times64}{1000}\right)$

$= \log_2 8$

$= \log_2 2^3$

$= 3$

(c) $\log_2\left[\frac{75}{16}\times\left(\frac{9}{5}\right)^2\times\frac{32}{243}\right]$

$= \log_2 2$

$= 1$

11 (a) $11^{\log_{11}13} = 13$

(b) $7^{3\log_7 5} = 7^{\log_7 5^3} = 125$

(c) $5^{\frac{1}{2}\log_5 49} = 5^{\log_5\sqrt{49}} = 7$

UPSKILL 4.3b

1 (a) $\log_3 8 = \frac{\log_{10} 8}{\log_{10} 3} = 1.893$

(b) $\log_5 137 = \frac{\log_{10} 137}{\log_{10} 5} = 3.057$

(c) $\log_{0.3} 0.25 = \frac{\log_{10} 0.25}{\log_{10} 0.3} = 1.151$

2 $\log_3 125\times\log_5 81$

$= \frac{\log_{10}125}{\log_{10}3}\times\frac{\log_{10}81}{\log_{10}5}$

$= \frac{\log_{10}5^3}{\log_{10}3}\times\frac{\log_{10}3^4}{\log_{10}5}$

$= \frac{3\log_{10}5}{\log_{10}3}\times\frac{4\log_{10}3}{\log_{10}5}$

$= 3\times 4$

$= 12$

3 (a) $\log_2 30$

$= \log_2(2\times3\times5)$

$= \log_2 2+\log_2 3+\log_2 5$

$= 1+1.585+2.322$

$= 4.907$

(b) $\log_2\left(\frac{5}{6}\right)$

$= \log_2 5-\log_2 6$

$= \log_2 5-\log_2(2\times3)$

$= \log_2 5-\log_2 2-\log_2 3$

$= 2.322-1-1.585$

$= -0.263$

(c) $\log_{15} 6$

$= \frac{\log_2 6}{\log_2 15}$

$= \frac{\log_2(2\times3)}{\log_2(3\times5)}$

$= \frac{\log_2 2+\log_2 3}{\log_2 3+\log_2 5}$

$= \frac{1+1.585}{1.585+2.322}$

$= \frac{2.585}{3.907}$

$= 0.662$

4 (a) $\log_5 42$

$= \log_5(2\times3\times7)$

$= \log_5 2+\log_5 3+\log_5 7$

$= a+b+c$

(b) $\log_5 2.1$

$= \log_5\left(\frac{21}{10}\right)$

$= \log_5\left(\frac{3\times7}{2\times5}\right)$

$= \log_5 3+\log_5 7-\log_5 2-\log_5 5$

$= b+c-a-1$

(c) $\log_{25}\frac{6}{49} = \frac{\log_5\frac{6}{49}}{\log_5 25}$

$= \frac{\log_5 6-\log_5 49}{\log_5 5^2}$

$= \frac{\log_5(2\times3)-\log_5 7^2}{\log_5 5^2}$

$= \frac{\log_5 2+\log_5 3-2\log_5 7}{2}$

$= \frac{a+b-2c}{2}$

5 $x = 3^r$ | $y = 3^t$

$\log_3 x = r$ | $\log_3 y = t$

(a) $\log_3\left(\frac{xy^3}{81}\right)$

$= \log_3 x+3\log_3 y-\log_3 3^4$

$= r+3t-4$

(b) $\log_9\left(\dfrac{x^3}{y^5}\right)$

$$= \frac{\log_3 x^3 - \log_3 y^5}{\log_3 9}$$

$$= \frac{3\log_3 x - 5\log_3 y}{\log_3 3^2}$$

$$= \frac{3r - 5t}{2}$$

6 $\log_{11} a = r$ | $\log_7 a = t$

$\log_a 11 = \dfrac{1}{r}$ | $\log_a 7 = \dfrac{1}{t}$

$$\log_a 77 = \log_a (11 \times 7)$$
$$= \log_a 11 + \log_a 7$$
$$= \frac{1}{r} + \frac{1}{t}$$
$$= \frac{t + r}{rt}$$

7 $\log_{xy} a = \dfrac{\log_a a}{\log_a xy} = \dfrac{1}{\log_a xy}$

$$= \frac{1}{\log_a x + \log_a y}$$
$$= \frac{1}{0.26 + 0.24}$$
$$= \frac{1}{0.50}$$
$$= 2$$

UPSKILL 4.3c

1 (a) $5^x = 7$

$$x \lg 5 = \lg 7$$
$$x = \frac{\lg 7}{\lg 5}$$
$$x = 1.209$$

(b) $5^{x+3} = 9$

$$(x+3)\lg 5 = \lg 9$$
$$x\lg 5 + 3\lg 5 = \lg 9$$
$$x = \frac{\lg 9 - 3\lg 5}{\lg 5}$$
$$x = -1.635$$

(c) $0.2^x = 0.7^{x+2}$

$$x\lg 0.2 = (x+2)\lg 0.7$$
$$x\lg 0.2 = x\lg 0.7 + 2\lg 0.7$$
$$x(\lg 0.2 - \lg 0.7) = 2\lg 0.7$$
$$x = \frac{2\lg 0.7}{\lg 0.2 - \lg 0.7}$$
$$x = \frac{2\lg 0.7}{\lg 0.2 - \lg 0.7}$$
$$x = 0.5694$$

(d) $7^{2x+1} = 5^{x-3}$

$$(2x+1)\lg 7 = (x-3)\lg 5$$
$$2x\lg 7 - x\lg 5 = -\lg 7 - 3\lg 5$$
$$x(2\lg 7 - \lg 5) = -\lg 7 - 3\lg 5$$
$$x = \frac{-\lg 7 - 3\lg 5}{2\lg 7 - \lg 5}$$
$$x = \frac{-2.9420}{0.9912}$$
$$x = -2.968$$

(e) $(2^x)(5^{x+2}) = 13$

$$\lg 2^x + \lg 5^{x+2} = \lg 13$$
$$x\lg 2 + (x+2)\lg 5 = \lg 13$$
$$x(\lg 2 + \lg 5) = \lg 13 - 2\lg 5$$
$$x = \frac{\lg 13 - 2\lg 5}{\lg 2 + \lg 5}$$
$$x = -0.2840$$

2 (a) $\log_7 6x = 2\log_7 3$

$$\log_7 6x = \log_7 3^2$$
$$6x = 9$$
$$x = \frac{3}{2}$$

(b) $\log_3 (x-2) = \dfrac{1}{2}\log_3 25$

$$\log_3 (x-2) = \log_3 25^{\frac{1}{2}}$$
$$x - 2 = 5$$
$$x = 7$$

(c) $\log_3 x - \log_3 5 = 4$

$$\log_3\left(\frac{x}{5}\right) = 4$$
$$\frac{x}{5} = 3^4$$
$$x = 405$$

(d) $\log_x 2 + \log_x 16 = 5$

$$\log_x 32 = 5$$
$$32 = x^5$$
$$x = 2$$

(e) $2\log_x 3 + \frac{1}{2}\log_x 16 = 2$

$$\log_x 3^2 + \log_x 16^{\frac{1}{2}} = 2$$
$$\log_x (9 \times 4) = 2$$
$$36 = x^2$$
$$x = 6$$

(f) $\log_x 10 + 2\log_x 2 = 3 + 3\log_x 3 + \log_x 5$

$$\log_x \left(\frac{10 \times 4}{27 \times 5}\right) = 3$$
$$\frac{8}{27} = x^3$$
$$x = \frac{2}{3}$$

3 (a) $\log_2 (3x-1) = 1 + \log_2 (x+1)$

$$\log_2 (3x-1) - \log_2 (x+1) = 1$$
$$\log_2 \left(\frac{3x-1}{x+1}\right) = 1$$
$$\frac{3x-1}{x+1} = 2^1$$
$$3x - 1 = 2x + 2$$
$$x = 3$$

(b) $\log_2 5 + \log_2 (2x-1) = 1 + \log_2 (3x+1)$

$$\log_2 5 + \log_2 (2x-1) - \log_2 (3x+1) = 1$$
$$\log_2 \left[\frac{5(2x-1)}{3x+1}\right] = 1$$
$$\frac{10x-5}{3x+1} = 2^1$$
$$10x - 5 = 6x + 2$$
$$4x = 7$$
$$x = \frac{7}{4}$$

(c) $\log_3 (x+3) = 2 - \log_3 (x-5)$

$$\log_3 (x+3) + \log_3 (x-5) = 2$$
$$\log_3 (x+3)(x-5) = 2$$
$$x^2 - 2x - 15 = 3^2$$
$$x^2 - 2x - 24 = 0$$
$$(x+4)(x-6) = 0$$

$x = 4$ or $x = 6$

$x = 4$ is not accepted.

$\therefore\ x = 6$

4 (a) $\log_9 y + 4\log_9 x = 2$

$$\log_9 y + \log_9 x^4 = 2$$
$$\log_9 x^4 y = 2$$
$$x^4 y = 81$$
$$y = \frac{81}{x^4}$$

(b) $\log_3 y + 1 = 3\log_3 x$

$$\log_3 y - \log_3 x^3 = -1$$
$$\log_3 \frac{y}{x^3} = -1$$
$$\frac{y}{x^3} = 3^{-1}$$
$$y = \frac{x^3}{3}$$

5 (a) $\log_{25}(x-1) = \log_5 (x-3)$

$$\frac{\log_5 (x-1)}{\log_5 25} = \log_5 (x-3)$$
$$\frac{\log_5 (x-1)}{\log_5 5^2} = \log_5 (x-3)$$
$$\log_5 (x-1) = 2\log_5 (x-3)$$
$$\log_5 (x-1) = \log_5 (x-3)^2$$
$$x - 1 = (x-3)^2$$
$$x - 1 = x^2 - 6x + 9$$
$$x^2 - 7x + 10 = 0$$
$$(x-2)(x-5) = 10$$

$x = 2$ is not accepted.

$\therefore x = 5$

(b) $2 + 2\log_4 (x-6) = \log_2 x$

$$2 + \frac{2\log_2 (x-6)}{\log_2 4} = \log_2 x$$
$$2 + \frac{2\log_2 (x-6)}{\log_2 2^2} = \log_2 x$$
$$2 + \log_2 (x-6) = \log_2 x$$
$$\log_2 (x-6) - \log_2 x = -2$$
$$\log_2 \frac{x-6}{x} = -2$$
$$\frac{x-6}{x} = \frac{1}{4}$$
$$4x - 24 = x$$
$$3x = 24$$
$$x = 8$$

6 (a) $\log_{32} y = \log_2 x$

$\frac{\log_2 y}{\log_2 32} = \log_2 x$

$\frac{\log_2 y}{5} = \log_2 x$

$\log_2 y = 5\log_2 x$

$\log_2 y = \log_2 x^5$

$y = x^5$

(b) $\log_9 y - \log_3 x = \frac{1}{2}$

$\frac{\log_3 y}{\log_3 9} - \log_3 x = \frac{1}{2}$

$\frac{\log_3 y}{2} - \log_3 x = \frac{1}{2}$

$\log_3 y - 2\log_3 x = 1$

$\log_3 \frac{y}{x^2} = 1$

$\frac{y}{x^2} = 3$

$y = 3x^2$

7 $400\,000\,(1.02)^t > 500\,000$

$1.02^t > \frac{5}{4}$

$t\lg 1.02 > \lg\left(\frac{5}{4}\right)$

$t > \frac{\lg 1.25}{\lg 1.02}$

$t > 11.27$

$t_{\text{minimum}} = 12$ years

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1 $450\,000(1.03)^t > 600\,000$

$1.03^t > \frac{60}{45}$

$t\lg 1.03 > \lg\left(\frac{4}{3}\right)$

$t > \frac{\lg\left(\frac{4}{3}\right)}{\lg 1.03}$

$t > 9.73$

$t_{\text{minimum}} = 10$ years

2 $2\,000(1.05)^t > 4\,000$

$1.05^t > 2$

$t\lg 1.05 > \lg 2$

$t > \frac{\lg 2}{\lg 1.05}$

$t > 14.21$

$t_{\text{minimum}} = 15$ years

3 $5000(0.92)^n < 3000$

$0.92^n < \frac{3}{5}$

$n\lg 0.92 < \lg 0.6$

$n > \frac{\lg 0.6}{\lg 0.92}$

$n > 6.13$

$n_{\text{minimum}} = 7$ years

4 $j_o e^{-0.2t} = \frac{j_o}{2}$

$e^{-0.2t} = \frac{1}{2}$

$-0.2t = \log_e(0.5)$

$-0.2t = \ln 0.5$

$-0.2t = -0.6931$

$t = 3.466$ years

5 $\sqrt{3x+4} - \sqrt{x+2} = \sqrt{x-3}$

$\left(\sqrt{3x+4} - \sqrt{x+2}\right)^2 = \left(\sqrt{x-3}\right)^2$

$3x+4-2\sqrt{3x+4}\sqrt{x+2}+x+2 = x-3$

$3x+9 = 2\sqrt{3x+4}\sqrt{x+2}$

$(3x+9)^2 = 4(3x+4)\,(x+2)$

$9x^2+54x+81 = 4\left(3x^2+10x+8\right)$

$3x^2-14x-49 = 0$

$(x-7)(3x+7) = 0$

$x = 7$ or $x = -\frac{7}{3}$

$x = -\frac{7}{3}$ is not accepted.

$\therefore x = 7$

6 $y = ax^n + 2$

The curve passes through the point (3, 7).
Thus, $x = 3$ and $y = 7$.

$7 = a(3)^n + 2$

$5 = a(3)^n \quad \ldots (1)$

The curve passes through the point (9, 52),
Thus, $x = 9$ and $y = 52$.

$52 = a(9)^n + 2$

$50 = a\left(3^2\right)^n$

$50 = a\left(3^2\right)^n \quad \ldots (2)$

$\frac{(2)}{(1)} : \frac{a\left(3^2\right)^n}{a(3)^n} = \frac{50}{5}$

$3^n = 10$

$n \log_{10} 3 = \log_{10} 10$

$n = \frac{1}{\log_{10} 3}$

$n = 2.096$

Substitute $3^n = 10$ into (1).

$5 = a(3)^n$

$5 = a(10)$

$a = \frac{1}{2}$

Summative Practice 4

1 $\frac{2^{3(2n-4)} \times 2^{2(n+1)}}{2^{-6n} \times 2^{14n-7}}$

$= 2^{6n-12+2n+2+6n-14n+7}$

$= 2^{-3}$

$= \frac{1}{8}$

2 $\frac{(2\times 3)^{\frac{n}{2}} \times (2^2 \times 3)^{n+1} \times 3^{-\frac{3n}{2}}}{2^{\frac{5n}{2}}}$

$= 2^{\frac{n}{2}+2n+2-\frac{5n}{2}} \times 3^{\frac{n}{2}+n+1-\frac{3n}{2}}$

$= 2^2 \times 3^1$

$= 12$

3 $3^m 3^3 - 3^m 3^2 - 3^3\left(\frac{3^m}{3}\right) - 2\left(3^m\right)$

$= (27-9-9-2)\left(3^m\right)$

$= 7\left(3^m\right)$

$\therefore k = 7$

4 $3^{y^2+3} = 9^{2y}$

$3^{y^2+3} = \left(3^2\right)^{2y}$

$3^{y^2+3} = 3^{4y}$

Equating the indices :

$y^2 + 3 = 4y$

$y^2 - 4y + 3 = 0$

$(y-1)(y-3) = 0$

$y = 1$ or 3

5 $3^{x-1} = 36 - 3^x$

$\frac{3^x}{3} = 36 - 3^x$

$\frac{3^x}{3} + 3^x = 36$

$\frac{4}{3}\left(3^x\right) = 36$

$3^x = \frac{36 \times 3}{4} = 27$

$x = 3$

6 $5^{x+2} + 125\left(5^{x-1}\right) = 1\ 250$

$\left(5^x\right)\left(5^2\right) + 125\left(\frac{5^x}{5}\right) = 1\ 250$

$(25+25)\left(5^x\right) = 1\ 250$

$5^x = \frac{1\ 250}{50}$

$5^x = 25$

$x = 2$

7 $27^{2x-5} = \frac{1}{\sqrt{9^{x+1}}}$

$\left(3^3\right)^{(2x-5)} = \left[3^{2(x+1)}\right]^{-\frac{1}{2}}$

Equating the indices:

$6x - 15 = -(x+1)$

$7x = 14$

$x = 2$

8 $2^a = 5^b = 20^c = k$

$2^a = k$	$5^b = k$	$20^c = k$
$2 = k^{\frac{1}{a}}$	$5 = k^{\frac{1}{b}}$	$20 = k^{\frac{1}{c}}$

$2^2 \times 5 = 20$

$k^{\frac{2}{a}} \times k^{\frac{1}{b}} = k^{\frac{1}{4}}$

$k^{\frac{2}{a}+\frac{1}{b}} = k^{\frac{1}{c}}$

$\frac{2}{a} + \frac{1}{b} = \frac{1}{c}$

$\frac{2b+a}{ab} = \frac{1}{c}$

$c = \frac{ab}{a+2b}$

9 (a) $\dfrac{1}{3+\sqrt{2}}$

$= \left(\dfrac{1}{3+\sqrt{2}}\right)\left(\dfrac{3-\sqrt{2}}{3-\sqrt{2}}\right)$

$= \dfrac{3-\sqrt{2}}{9-2}$

$= \dfrac{3-\sqrt{2}}{7}$

(b) $\left(\dfrac{2\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}\right)\left(\dfrac{\sqrt{5}-\sqrt{3}}{\sqrt{5}-\sqrt{3}}\right)$

$= \dfrac{2(5)-2\sqrt{15}-\sqrt{15}+3}{5-3}$

$= \dfrac{13-3\sqrt{15}}{2}$

(c) $\left(\dfrac{3\sqrt{2}-2\sqrt{3}}{3\sqrt{2}+2\sqrt{3}}\right)\left(\dfrac{3\sqrt{2}-2\sqrt{3}}{3\sqrt{2}-2\sqrt{3}}\right)$

$= \dfrac{9(2)-6\sqrt{6}-6\sqrt{6}+4(3)}{9(2)-4(3)}$

$= \dfrac{30-12\sqrt{6}}{6}$

$= 5-2\sqrt{6}$

(d) $\dfrac{1}{(1+\sqrt{2})^2}+\dfrac{1}{(1-\sqrt{2})^2}$

$= \dfrac{1}{1+2+2\sqrt{2}}+\dfrac{1}{1+2-2\sqrt{2}}$

$= \dfrac{1}{3+2\sqrt{2}}+\dfrac{1}{3-2\sqrt{2}}$

$= \dfrac{3-2\sqrt{2}+3+2\sqrt{2}}{9-4(2)}$

$= \dfrac{6}{1}$

$= 6$

10 $\log_2 70 = \log_2 (2\times5\times7)$

$= \log_2 2+\log_2 5+\log_2 7$

$= 1+p+q$

11 $\log_3\left(2\dfrac{1}{7}\right) = \log_3\left(\dfrac{15}{7}\right)$

$= \log_3\left(\dfrac{3\times5}{7}\right)$

$= \log_3 3+\log_3 5-\log_3 7$

$= 1+m-q$

12

$m = 2^r$	$n = 2^t$
$\log_2 m = r$	$\log_2 n = t$

$\log_2\left(\dfrac{\sqrt{mn^2}}{16}\right)$

$= \log_2 \dfrac{m^{\frac{1}{2}}n}{16}$

$= \dfrac{1}{2}\log_2 m+\log_2 n-\log_2 2^4$

$= \dfrac{1}{2}r+t-4$

13 $\log_8 45$

$= \dfrac{\log_2 45}{\log_2 8}$

$= \dfrac{\log_2(3^2\times5)}{\log_2 2^3}$

$= \dfrac{2\log_2 3+\log_2 5}{3\log_2 2}$

$= \dfrac{2h+k}{3}$

14 $\log_3 ab = \log_3 a+\log_3 b$

$= \dfrac{1}{\log_a 3}+\dfrac{1}{\log_b 3}$

$= \dfrac{1}{m}+\dfrac{1}{n}$

$= \dfrac{m+n}{mn}$

15 $\log_{21} a = \dfrac{1}{\log_a 21}$

$= \dfrac{1}{\log_a (3\times7)}$

$= \dfrac{1}{\log_a 3+\log_a 7}$

$= \dfrac{1}{p+q}$

16 $\log_{\sqrt{x}} 9 = u$

$\log_9 \sqrt{x} = \dfrac{1}{u}$

$\dfrac{1}{2}\log_9 x = \dfrac{1}{u}$

$\log_9 x = \dfrac{2}{u}$

$\log_9 x^3 = 3\log_9 x = \dfrac{6}{u}$

17 $3 = a^p$ $\quad\quad 5 = a^q$

$\log_a 3 = p$ $\quad\quad \log_a 5 = q$

$$\log_3 1\frac{2}{3} = \log_3 \frac{5}{3}$$
$$= \log_3 5 - \log_3 3$$
$$= \frac{\log_a 5}{\log_a 3} - 1$$
$$= \frac{q}{p} - 1$$
$$= \frac{q-p}{p}$$

18 $\log_{16} 75 = \frac{\log_2 75}{\log_2 16}$

$$= \frac{\log_2 (3 \times 5^2)}{\log_2 2^4}$$
$$= \frac{\log_2 3 + 2\log_2 5}{4}$$
$$= \frac{p+2q}{4}$$

19 $3^{2x-1} = 4^x$

$$(2x-1)\lg 3 = x \lg 4$$
$$2x \lg 3 - x \lg 4 = \lg 3$$
$$x(2\lg 3 - \lg 4) = \lg 3$$
$$x = \frac{\lg 3}{2\lg 3 - \lg 4}$$
$$x = 1.355$$

20 $3^{\log_3 (x+1)} = 5$

$$\log_3 (x+1) = \log_3 5$$
$$x + 1 = 5$$
$$x = 4$$

21 $7^{\log_{(x-1)} 11} = 11$

$$\log_{(x-1)} 11 = \log_7 11$$
$$x - 1 = 7$$
$$x = 8$$

22 ` $\log_5 9 + \log_5 2x - \log_5 (3x+1) = 0$

$$\log_5 \left[\frac{9(2x)}{3x+1}\right] = 0$$
$$\frac{9(2x)}{3x+1} = 5^0 = 1$$
$$18x = 3x + 1$$
$$15x = 1$$
$$x = \frac{1}{15}$$

23 $\log_{10}(k^2 + 6k + 28) = 2$

$$k^2 + 6k + 28 = 10^2$$
$$k^2 + 6k - 72 = 0$$
$$(k-6)(k+12) = 0$$
$$k = 6 \text{ or } -12$$

24 $2 + 2\log_4 (p-6) = \log_2 p$

$$2 + 2\left[\frac{\log_2 (p-6)}{\log_2 4}\right] = \log_2 p$$
$$2 + 2\left[\frac{\log_2 (p-6)}{2}\right] = \log_2 p$$
$$2 + \log_2 (p-6) = \log_2 p$$
$$\log_2 (p-6) - \log_2 p = -2$$
$$\log_2 \left(\frac{p-6}{p}\right) = -2$$
$$\frac{p-6}{p} = 2^{-2}$$
$$\frac{p-6}{p} = \frac{1}{4}$$
$$4p - 24 = p$$
$$3p = 24$$
$$p = 8$$

25 $\log_2 m - \log_8 2m = 3$

$$\log_2 m - \frac{\log_2 2m}{\log_2 8} = 3$$
$$\log_2 m - \frac{\log_2 2m}{\log_2 2^3} = 3$$
$$\log_2 m - \frac{\log_2 2m}{3} = 3$$
$$3\log_2 m - \log_2 2m = 9$$
$$\log_2 m^3 - \log_2 2m = 9$$
$$\log_2 \left(\frac{m^3}{2m}\right) = 9$$
$$\frac{m^2}{2} = 2^9$$
$$m^2 = 2^{10}$$
$$m = (2^{10})^{\frac{1}{2}}$$
$$m = 32$$

26 $\log_3 2A = \log_9 B + 2$

$$\log_3 2A = \frac{\log_3 B}{\log_3 3^2} + 2$$

$$\log_3 2A = \frac{\log_3 B}{2} + 2$$

$$2\log_3 2A = \log_3 B + 4$$

$$2\log_3 2A - \log_3 B = 4$$

$$\log_3 \frac{(2A)^2}{B} = 4$$

$$\frac{4A^2}{B} = 3^4$$

$$\frac{4A^2}{B} = 81$$

$$A^2 = \frac{81B}{4}$$

$$A = \frac{9\sqrt{B}}{2}$$

27 $\log_4 x = \log_2 7$

$$\frac{\log_2 x}{\log_2 4} = \log_2 7$$

$$\frac{\log_2 x}{\log_2 2^2} = \log_2 7$$

$$\frac{\log_2 x}{2} = \log_2 7$$

$$\log_2 x = 2\log_2 7$$

$$\log_2 x = \log_2 7^2$$

$$x = 49$$

28 $\log_9 y - \log_3 x = 0$

$$\frac{\log_3 y}{\log_3 9} = \log_3 x$$

$$\frac{\log_3 y}{\log_3 3^2} = \log_3 x$$

$$\frac{\log_3 y}{2} = \log_3 x$$

$$\log_3 y = 2\log_3 x$$

$$\log_3 y = \log_3 x^2$$

$$y = x^2$$

29 $\log_n 1\ 024 - \log_{\sqrt{n}} 2n = 2$

$$\log_n 1\ 024 - \frac{\log_n 2n}{\log_n n^{\frac{1}{2}}} = 2$$

$$\log_n 1\ 024 - 2\log_n 2n = 2$$

$$\log_n \left(\frac{1\ 024}{4n^2}\right) = 2$$

$$\frac{256}{n^2} = n^2$$

$$n^4 = 256$$

$$n = 4$$

30 (a) $\log_2 150$

$$= \log_2 \left(2 \times 3 \times 5^2\right)$$

$$= \log_2 2 + \log_2 3 + 2\log_2 5$$

$$= 1 + h + 2k$$

(b) $\log_4 \left(\frac{125}{9}\right)$

$$= \frac{\log_2 \left(\frac{5^3}{3^2}\right)}{\log_2 4}$$

$$= \frac{\log_2 5^3 + \log_2 3^2}{\log_2 2^2}$$

$$= \frac{3\log_2 5 + 2\log_2 3}{2}$$

$$= \frac{3k - 2h}{2}$$

31 (a) $\log_5 0.84$

$$= \log_5 \frac{21}{25}$$

$$= \log_5 \left(\frac{3 \times 7}{5^2}\right)$$

$$= \log_5 3 + \log_5 7 - \log_5 5^2$$

$$= p + q - 2$$

(b) $\log_7 315 = \frac{\log_5 315}{\log_5 7}$

$$= \frac{\log_5 \left(3^2 \times 5 \times 7\right)}{\log_5 7}$$

$$= \frac{2\log_5 3 + \log_5 5 + \log_5 7}{\log_5 7}$$

$$= \frac{2p + 1 + q}{q}$$

32 $27^{x+1} = 3^y$

$$3^{3(x+1)} = 3^y$$

$$3^{3x+3} = 3^y$$

Equating the indices:

$3x + 3 = y$ … (1)

$$\log_3 y = 2 + \log_3 (x-1)$$

$$\log_3 y - \log_3 (x-1) = 2$$

$$\log_3\left(\frac{y}{x-1}\right)=2$$
$$\frac{y}{x-1}=3^2$$
$$y=9(x-1)$$
$$y=9x-9 \ldots (2)$$

Substitute (1) into (2) :
$$9x-9=3x+3$$
$$6x=12$$
$$x=2$$

From (2) : $y=9(2)-9=9$

33 $\log_a xy^3=9$

$\log_a x+3\log_a y=9$

$\log_a x+3\log_a y=9$

Let $\log_a x=m$ and $\log_a y=k$

$m+3k=9$ …

$m=9-3k$ … (1)

$\log_a x^2y=8$

$\log_a x^2+\log_a y=8$

$2\log_a x+\log_a y=8$

Let $\log_a x=m$ and $\log_a y=k$

$2m+k=8$ … (2)

Substitute (1) into (2) :

$2(9-3k)+k=8$

$18-6k+k=8$

$18-5k=8$

$5k=18-8$

$5k=10$

$k=2$

From (1) :

$m=9-3k=9-3(2)=3$

$k=2$

$\log_a y=2$

$m=3$

$\log_a x=3$

$\log_a \sqrt{xy}$

$=\log_a (xy)^{\frac{1}{2}}$

$=\frac{1}{2}(\log_a x+\log_a y)$

$=\frac{1}{2}(3+2)$

$=\frac{5}{2}$

34 $3\log_8 (2x+14)-4\log_{16} (x+1)=3$

$$3\left[\frac{\log_2(2x+14)}{\log_2 8}\right]-4\left[\frac{\log_2(x+1)}{\log_2 16}\right]=3$$
$$\cancel{3}\left[\frac{\log_2(2x+14)}{\cancel{\log_2 2^3}}\right]-\cancel{4}\left[\frac{\log_2(x+1)}{\cancel{\log_2 2^4}}\right]=3$$
$$\log_2 (2x+14)-\log_2 (x+1)=3$$
$$\log_2\left(\frac{2x+14}{x+1}\right)=3$$
$$\frac{2x+14}{x+1}=2^3$$
$$2x+14=8x+8$$
$$6x=6$$
$$x=1$$

35 (a) $2\log_2(x+y)=3+\log_2 x+\log_2 y$

$\log_2(x+y)^2=\log_2 2^3+\log_2 x+\log_2 y$

$\log_2(x+y)^2=\log_2 (8xy)$

$(x+y)^2=8xy$

$x^2+y^2+2xy=8xy$

$x^2+y^2=6xy$ [Shown]

(b) $\log_9 [\log_3(3x-6)]=5^{\log_5\left(\frac{1}{2}\right)}$

$\log_9 [\log_3(3x-6)]=\frac{1}{2}$ ← $a^{\log_a x}=x$

$\log_3 (3x-6)=9^{\frac{1}{2}}$

$\log_3 (3x-6)=3$

$3x-6=3^3$

$3x-6=27$

$3x=33$

$x=11$

36 $\log_{\sqrt{x}} 9=a$

$\log_9 \sqrt{x}=\frac{1}{a}$

$\log_9 x^{\frac{1}{2}}=\frac{1}{a}$

$\frac{1}{2}\log_9 x=\frac{1}{a}$

$\log_9 x=\frac{2}{a}$

$\log_y 3=b$

$\frac{\log_9 3}{\log_9 y}=b$

$$\frac{\log_9 9^{\frac{1}{2}}}{\log_9 y} = b$$

$$\frac{1}{2\log_9 y} = b$$

$$\log_9 y = \frac{1}{2b}$$

$$\log_9 xy^2 = \log_9 x + 2\log_9 y$$
$$= \frac{2}{a} + 2\left(\frac{1}{2b}\right)$$
$$= \frac{2}{a} + \left(\frac{1}{b}\right)$$
$$= \frac{2b+a}{ab}$$

37

Smart Strategy

If RHS is more complicated than the LHS, it is more appropriate to proof that RHS is equal to LHS.

RHS

$= 2\log_4 x + 2\log_4 y$

$= \frac{2\log_2 x}{\log_2 4} + \frac{2\log_2 y}{\log_2 4}$

$= \frac{2\log_2 x}{\log_2 2^2} + \frac{2\log_2 y}{\log_2 2^2}$

$= \frac{2\log_2 x}{2} + \frac{2\log_2 y}{2}$

$= \log_2 x + \log_2 y$

$= \log_2 xy$

= LHS

Let $\log_4 x = f$ and $\log_4 y = g$

$\log_2 xy = 2\log_4 x + 2\log_4 y$

$= 2f + 2g$

$\log_2 xy = 10$

$2f + 2g = 10$

$f + g = 5 \quad \ldots (1)$

$\frac{\log_4 x}{\log_4 y} = \frac{3}{2}$

$\frac{f}{g} = \frac{3}{2}$

$f = \frac{3}{2}g \quad \ldots (2)$

Substitute (2) into (1) :

$\frac{3}{2}g + g = 5$

$\frac{5}{2}g = 5$

$g = 2$

$\log_4 y = 2$

$y = 4^2 = 16$

From (2) :

$f = \frac{3}{2}g = \frac{3}{2}(2) = 3$

$\log_4 x = 3$

$x = 4^3$

$x = 64$

38 $\log_3 (3x+1) - \log_3 x^2 + \log_9 x^2 = 2$

$\log_3 (3x+1) - \log_3 x^2 + \frac{\log_3 x^2}{\log_3 9} = 2$

$\log_3 (3x+1) - \log_3 x^2 + \frac{\log_3 x^2}{2} = 2$

$2\log_3 (3x+1) - 2\log_3 x^2 + \log_3 x^2 = 4$

$\log_3 (3x+1)^2 - \log_3 x^4 + \log_3 x^2 = 4$

$\log_3 \frac{(3x+1)^2 \left(x^2\right)}{x^4} = 4$

$\frac{(3x+1)^2 \left(x^2\right)}{x^4} = 3^4$

$(3x+1)^2 = 81x^2$

$9x^2 + 6x + 1 = 81x^2$

$72x^2 - 6x - 1 = 0$

$(6x-1)(12x+1) = 0$

$x = \frac{1}{6}$ or $x = -\frac{1}{12}$

$x = -\frac{1}{12}$ is not accepted.

$\therefore x = \frac{1}{6}$

39 $\sqrt{2x+13}-\sqrt{x+10}=1$

$2x+13-2\sqrt{2x+13}\sqrt{x+10}+x+10=1$

$3x+22-2\sqrt{2x+13}\sqrt{x+10}=0$

$3x+22=2\sqrt{2x+13}\sqrt{x+10}$

$9x^2+132x+484=4(2x+13)(x+10)$

$9x^2+132x+484=4(2x^2+33x+130)$

$9x^2+132x+484=8x^2+132x+520$

$x^2-36=0$

$x=6$

40 $MV=P\left(1+\frac{r}{n}\right)^{nt}=1\ 077\ 484$

$800\ 000\left(1+\frac{r}{4}\right)^{4(5)}=1\ 077\ 484$

$\left(1+\frac{r}{4}\right)^{20}=1.346855$

$1+\frac{r}{4}=1.346855^{\frac{1}{20}}$

$1+\frac{r}{4}=1.015$

$\frac{r}{4}=0.015$

$r=0.06$, i.e. 6%

41 $6000(0.95)^n<3000$

$0.95^n<0.5$

$n\lg 0.95<\lg 0.5$

$n>\frac{\lg 0.5}{\lg 0.95}$

$n>13.51$

$n_{\text{minimum}}=14$ years