Form 4 Chapter 2
Quadratic Functions
Fully-Worked Solutions

UPSKILL 2.1a

1 (a) $6x^2 - 7x - 3 = 0$

$$6x^2 - 7x = 3$$
$$x^2 - \frac{7}{6}x = \frac{3}{6}$$
$$x^2 - \frac{7}{6}x = \frac{1}{2}$$
$$x^2 - \frac{7}{6}x + \left(-\frac{7}{6} \times \frac{1}{2}\right)^2 = \frac{1}{2} + \left(-\frac{7}{6} \times \frac{1}{2}\right)^2$$
$$x^2 - \frac{7}{6}x + \frac{49}{144} = \frac{1}{2} + \frac{49}{144}$$
$$\left(x - \frac{7}{12}\right)^2 = \frac{121}{144}$$
$$x - \frac{7}{12} = \pm\frac{11}{12}$$
$$x = \frac{7}{12} + \frac{11}{12} \quad \text{or} \quad x = \frac{7}{12} - \frac{11}{12}$$
$$x = \frac{3}{2} \qquad\qquad x = -\frac{1}{3}$$

(b) $2p^2 - 10p + 3 = 0$

$$2p^2 - 10p = -3$$
$$p^2 - 5p = -\frac{3}{2}$$
$$p^2 - 5p + \left(-\frac{5}{2}\right)^2 = -\frac{3}{2} + \left(-\frac{5}{2}\right)^2$$
$$p^2 - 5p + \frac{25}{4} = -\frac{3}{2} + \frac{25}{4}$$
$$\left(p - \frac{5}{2}\right)^2 = \frac{19}{4}$$
$$p - \frac{5}{2} = \pm\sqrt{\frac{19}{4}}$$
$$p = \frac{5}{2} + \sqrt{\frac{19}{4}} \quad \text{or} \quad p = \frac{5}{2} - \sqrt{\frac{19}{4}}$$
$$p = 4.679 \qquad\qquad p = 0.321$$

2 (a) $s^2 + 1 = -\frac{10}{3}s$

$$3s^2 + 3 + 10s = 0$$
$$3s^2 + 10s + 3 = 0$$
$$s = \frac{-10 \pm \sqrt{10^2 - 4(3)(3)}}{2(3)}$$
$$s = \frac{-10 \pm \sqrt{64}}{6}$$
$$s = -\frac{1}{3} \text{ or } -3$$

(b) $\frac{11v - 2}{v + 3} = 2v$

$$11v - 2 = 2v^2 + 6v$$
$$2v^2 - 5v + 2 = 0$$
$$v = \frac{5 \pm \sqrt{(-5)^2 - (2)(2)}}{2(2)}$$
$$v = \frac{1}{2} \text{ or } 2$$

(c) $8 + x(2x + 35) = 10x(2x - 1)$

$$8 + 2x^2 + 35x = 20x^2 - 10x$$
$$18x^2 - 45x - 8 = 0$$
$$x = \frac{45 \pm \sqrt{(-45)^2 - 4(18)(-8)}}{2(18)}$$
$$x = \frac{45 \pm 51}{36}$$
$$x = \frac{8}{3} \text{ or } -\frac{1}{6}$$

3 (a) $(x - 1)(4x - 9) = 10x - 5$

$$4x^2 - 13x + 9 = 10x - 5$$
$$4x^2 - 13x - 10x + 9 + 5 = 0$$
$$4x^2 - 23x + 14 = 0$$
$$x = \frac{23 \pm \sqrt{(-23)^2 - 4(4)(14)}}{2(4)}$$
$$x = \frac{23 \pm \sqrt{305}}{8}$$
$$x = 5.058 \text{ or } 0.692$$

(b) $\frac{z}{3}+4=z^2$

$z+12=3z^2$

$3z^2-z-12=0$

$z=\frac{1\pm\sqrt{(-1)^2-4(3)(-12)}}{2(3)}$

$z=\frac{1\pm\sqrt{145}}{6}$

$z=2.174$ or -1.840

(c) $\frac{y^2+3y-1}{y^2-y-1}=2$

$y^2+3y-1=2y^2-2y-2$

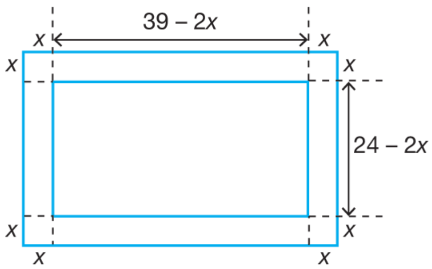
$y^2-5y-1=0$

$y=\frac{5\pm\sqrt{(-5)^2-4(-1)}}{2}$

$y=\frac{5\pm\sqrt{29}}{2}$

$y=-0.193$ or -0.193

4



Total area of the four pieces of wood

$= 39\times24 - (39-2x)(24-2x)$

$=936-(936-78x-48x+4x^2)$

$=126x-4x^2$

It is given that the total area of the four pieces of wood $=180$ cm^2

$-4x^2+126x=180$

$4x^2-126x+180=0$

$x=\frac{126\pm\sqrt{(-126)^2-4(4)(180)}}{2(4)}$

$x=\frac{126\pm\sqrt{12\,996}}{8}$

$x=1.5$ or 30

$x = 30$ is not accepted.

$x=1.5$

UPSKILL 2.1b

1 (a) S.O.R. = $\frac{2}{3}+\frac{1}{4}=\frac{11}{12}$

P.O.R. = $\frac{2}{3}\times\frac{1}{4}=\frac{1}{6}$

The quadratic equation is

$x^2-\frac{11}{12}x+\frac{1}{6}=0$

$12x^2-11x+2=0$

(b) S.O.R. = $-5+4=-1$

P.O.R. = $-5\times4=-20$

The quadratic equation is

$x^2+x-20=0$

(c) S.O.R. = $-3-3=-6$

P.O.R. = $(-3)(-3)=9$

The quadratic equation is

$x^2+6x+9=0$

(d) S.O.R. = $\frac{2}{3}-\frac{2}{5}=\frac{4}{15}$

P.O.R. = $\frac{2}{3}\times\left(-\frac{2}{5}\right)=-\frac{4}{15}$

The quadratic equation is

$x^2-\frac{4}{15}x-\frac{4}{15}=0$

$15x^2-4x-4=0$

(e) S.O.R. = $-3-\frac{1}{2}=-\frac{7}{2}$

P.O.R. = $-3\left(-\frac{1}{2}\right)=\frac{3}{2}$

The quadratic equation is

$x^2+\frac{7}{2}x+\frac{3}{2}=0$

$2x^2+7x+3=0$

2 (a) $2x^2+4x-7=0$

S.O.R. = $-\frac{b}{a}=-\frac{4}{2}=-2$

P.O.R. = $\frac{c}{a}=-\frac{7}{2}$

(b) $3h^2-10h+5=0$

S.O.R. = $\frac{10}{3}$, P.O.R. = $\frac{5}{3}$

3 (a) $2p^2+2p-3=0$

The roots are α and β.

Sum of roots = $-\frac{b}{a}$

$\alpha+\beta=-\frac{2}{2}=-1$

Product of roots = $\frac{c}{a}$

$\alpha\beta=-\frac{3}{2}$

The new roots are $\alpha+2$ and $\beta+2$.

S.O.R. = $(\alpha+2)+(\beta+2)$

$=\alpha+\beta+4$

$=-1+4$

$=3$

P.O.R. = $(\alpha+2)(\beta+2)$

$=\alpha\beta+2(\alpha+\beta)+4$

$=-\frac{3}{2}+2(-1)+4$

$=\frac{1}{2}$

The new quadratic equation is

$p^2-3p+\frac{1}{2}=0$

$2p^2-6p+1=0$

(b) The new roots are $\frac{2}{\alpha}$ and $\frac{2}{\beta}$.

S.O.R. = $\frac{2}{\alpha}+\frac{2}{\beta}$

$=\frac{2(\alpha+\beta)}{\alpha\beta}$

$=\frac{2(-1)}{-\frac{3}{2}}$

$=\frac{4}{3}$

P.O.R. = $\frac{2}{\alpha}\times\frac{2}{\beta}$

$=\frac{4}{\alpha\beta}$

$=\frac{4}{-\frac{3}{2}}$

$=-\frac{8}{3}$

The new quadratic equation is

$p^2-\frac{4}{3}p-\frac{8}{3}=0$

$3p^2-4p-8=0$

4 (a) $2t^2-5t+1=0$

The roots are α and β.

Sum of roots = $-\frac{b}{a}$

$\alpha+\beta=\frac{5}{2}$

Product of roots = $\frac{c}{a}$

$\alpha\beta=\frac{1}{2}$

The new roots are $\frac{\alpha}{3}$ and $\frac{\beta}{3}$.

S.O.R. = $\frac{\alpha}{3}+\frac{\beta}{3}$

$=\frac{\alpha+\beta}{3}$

$=\frac{\frac{5}{2}}{3}$

$=\frac{5}{6}$

P.O.R. = $\frac{\alpha}{3}\times\frac{\beta}{3}$

$=\frac{\alpha\beta}{9}$

$=\frac{\frac{1}{2}}{9}$

$=\frac{1}{18}$

The new quadratic equation is

$t^2-\frac{5}{6}t+\frac{1}{18}=0$

$18t^2-15t+1=0$

(b) The new roots are $3-\alpha$ and $3-\beta$.

S.O.R. = $3-\alpha+3-\beta$

$=6-(\alpha+\beta)$

$=6-\frac{5}{2}$

$=\frac{7}{2}$

$$\text{P.O.R.} = (3-\alpha)(3-\beta)$$
$$= 9-(\alpha+\beta)+\alpha\beta$$
$$= 9-\frac{5}{2}+\frac{1}{2}$$
$$= 7$$

The new quadratic equation is

$$t^2-\frac{7}{2}t+7=0$$
$$2t^2-7t+14=0$$

5 $x^2+9x+q=0$

The roots are α and 2α.

$$\text{S.O.R.} = -\frac{b}{a}$$
$$\alpha+2\alpha=-9$$
$$3\alpha=-9$$
$$\alpha=-3$$

$$\text{P.O.R.} = \frac{c}{a}$$
$$\alpha\times 2\alpha=q$$
$$q=2(-3)^2=18$$

6 $5x^2+px+1=0$

The roots are α and 5α.

$$\text{S.O.R.} = -\frac{b}{a}$$
$$\alpha+5\alpha=-\frac{p}{5}$$
$$6\alpha=-\frac{p}{5}$$
$$\alpha=-\frac{p}{30}$$

$$\text{P.O.R.} = \frac{c}{a}$$
$$\alpha\times 5\alpha=\frac{1}{5}$$
$$5\alpha^2=\frac{1}{5}$$
$$5\left(-\frac{p}{30}\right)^2=\frac{1}{5}$$
$$\frac{p^2}{900}=\frac{1}{25}$$
$$p^2=\frac{900}{25}$$
$$p^2=\frac{900}{25}$$
$$p^2=36$$
$$p=\pm 6$$

7 $2x^2-(d+3)x+d=0$

The roots are α and 4α.

$$\text{S.O.R.} = -\frac{b}{a}$$
$$\alpha+4\alpha=\frac{d+3}{2}$$
$$2(\alpha+4\alpha)=d+3$$
$$10\alpha=d+3 \quad \ldots (1)$$
$$\alpha=\frac{d+3}{10}$$

$$\text{P.O.R.} = \frac{c}{a}$$
$$\alpha\times 4\alpha=\frac{d}{2}$$
$$4\alpha^2=\frac{d}{2}$$
$$\alpha^2=\frac{d}{8}$$
$$\left(\frac{d+3}{10}\right)^2=\frac{d}{8}$$
$$\frac{(d+3)^2}{100}=\frac{d}{8}$$
$$\frac{(d+3)^2}{25}=\frac{d}{2}$$
$$2(d+3)^2=25d$$
$$2(d^2+6d+9)=25d$$
$$2d^2+12d+18-25d=0$$
$$2d^2-13d+18=0$$
$$(d-2)(2d-9)=0$$
$$d=2 \text{ or } \frac{9}{2}$$

8 $2x^2+hx-4=0$

The roots are 4 and k.

$$\text{S.O.R.} = -\frac{b}{a}$$
$$k+4=-\frac{h}{2}$$
$$2k+8=-h$$
$$h=-2k-8 \quad \ldots (1)$$

P.O.R. = $\frac{c}{a}$

$4k = -\frac{4}{2}$

$k = -\frac{1}{2}$

From (1) : $h = -2\left(-\frac{1}{2}\right) - 8 = -7$

9 $8x^2 + 26x + k = 0$

The roots are $-\frac{5}{2}$ and m.

S.O.R. = $-\frac{b}{a}$

$-\frac{5}{2} + m = -\frac{26}{8}$

$m = -\frac{26}{8} + \frac{5}{2}$

$m = -\frac{3}{4}$

P.O.R. = $\frac{c}{a}$

$-\frac{5}{2}m = \frac{k}{8}$

$-\frac{5}{2}\left(-\frac{3}{4}\right) = \frac{k}{8}$

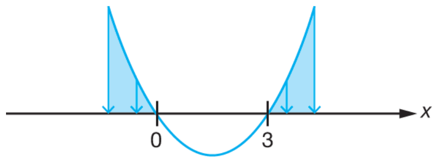
$\frac{15}{8} = \frac{k}{8}$

$k = 15$

UPSKILL 2.1c

1 (a) $x^2 - 3x \geq 0$

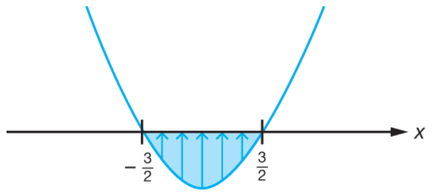
$x(x - 3) \geq 0$



The required range of values of x is $x \leq 0$ or $x \geq 3$.

(b) $4x^2 - 9 \leq 0$

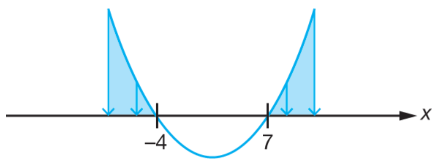
$(2x + 3)(2x - 3) \leq 0$



The required range of values of x is $-\frac{3}{2} \leq x \leq \frac{3}{2}$.

(c) $x^2 - 3x - 28 > 0$

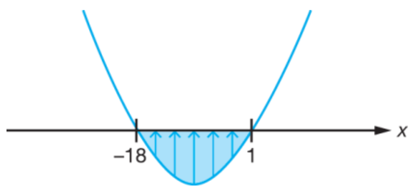
$(x + 4)(x - 7) > 0$



The required range of values of x is $x < -4$ or $x > 7$.

(d) $x^2 + 17x - 18 < 0$

$(x + 18)(x - 1) < 0$

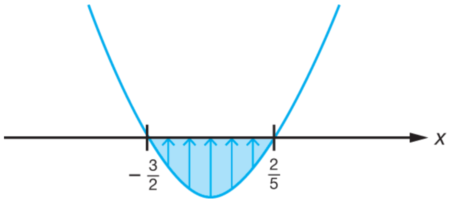


The required range of values of x is $-18 < x < 1$.

(e) $6 - 11x - 10x^2 \geq 0$

$10x^2 + 11x - 6 \leq 0$

$(2x+3)(5x-2) \leq 0$

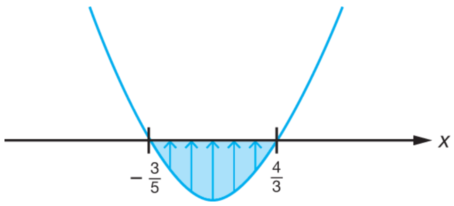


The required range of values of x is $-\frac{3}{2} \leq x \leq \frac{2}{5}$.

(f) $12 + 11x - 15x^2 \geq 0$

$15x^2 - 11x - 12 \leq 0$

$(5x+3)(3x-4) \leq 0$

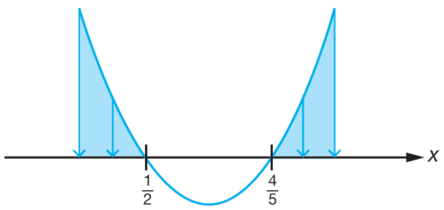


The range of values of x is $-\frac{3}{5} \leq x \leq \frac{4}{3}$.

(g) $10x^2 > 13x - 4$

$10x^2 - 13x + 4 > 0$

$(2x-1)(5x-4) > 0$

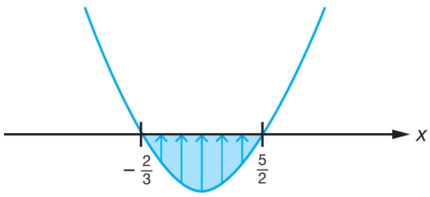


The required range of values of x is $x < \frac{1}{2}$ or $x > \frac{4}{5}$.

(h) $11x + 10 \geq 6x^2$

$6x^2 - 11x - 10 \leq 0$

$(3x+2)(2x-5) \leq 0$

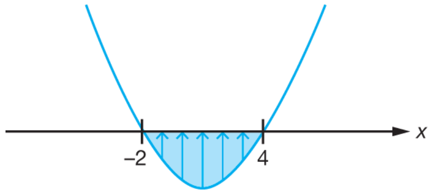


The required range of values of x is $-\frac{2}{3} \leq x \leq \frac{5}{2}$.

(i) $x(x-2) \leq 8$

$x^2 - 2x - 8 \leq 0$

$(x+2)(x-4) \leq 0$



The range of values of x is $-2 \leq x \leq 4$.

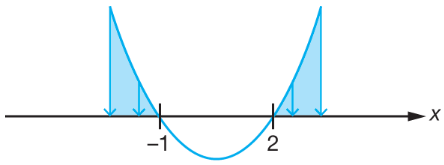
(j) $(2x-1)^2 > 9$

$4x^2 - 4x + 1 - 9 > 0$

$4x^2 - 4x - 8 > 0$

$x^2 - x - 2 > 0$

$(x+1)(x-2) > 0$



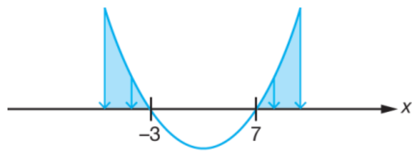
The required range of values of x is $x < -1$ or $x > 2$.

(k) $(x+1)(x-5) \geq 16$

$x^2 - 4x - 5 - 16 \geq 0$

$x^2 - 4x - 21 \geq 0$

$(x+3)(x-7) \geq 0$

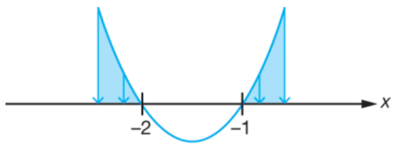


The required range of values of x is $x \leq -3$ or $x \geq 7$.

2 $-1 < x^2 + 3x + 1 \leq 1$.

The first inequality is

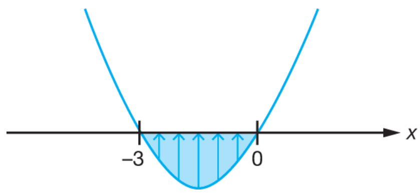
$$-1 < x^2 + 3x + 1$$
$$x^2 + 3x + 1 + 1 > 0$$
$$x^2 + 3x + 2 > 0$$
$$(x+1)(x+2) > 0$$



The range of values of x is
$x < -2$ or $x > -1$ ... (1)

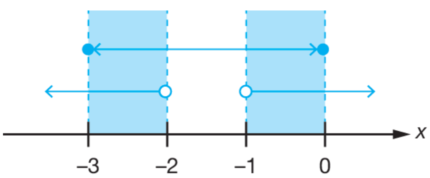
The second inequality is

$$x^2 + 3x + 1 \leq 1$$
$$x^2 + 3x \leq 0$$
$$x(x+3) \leq 0$$



The range of values of x is
$-3 \leq x \leq 0$ ... (2)

Combining (1) and (2) :



The required range of values of x is
$-3 \leq x < -2$ or $-1 < x \leq 0$.

UPSKILL 2.2a

1 (a) $2x^2 - 8x + 3 = 0$

$b^2 - 4ac = (-8)^2 - 4(2)(3) = 40$

Since $b^2 - 4ac > 0$, then the quadratic equation has real and distinct roots.

(b) $3x^2 - 2x + 9 = 0$

$b^2 - 4ac = (-2)^2 - 4(3)(9) = -104$

Since $b^2 - 4ac < 0$, then the quadratic equation does not have real roots.

(c) $x^2 + 10x + 25 = 0$

$b^2 - 4ac = 10^2 - 4(1)(25) = 0$

Since $b^2 - 4ac = 0$, then the quadratic equation has real and equal roots.

(d) $-2x^2 + 6x + 3 = 0$

$b^2 - 4ac = 6^2 - 4(-2)(3) = 60$

Since $b^2 - 4ac > 0$, then the quadratic equation has real and distinct roots.

(e) $3x^2 - 6x + 4 = 0$

$b^2 - 4ac = (-6)^2 - 4(3)(4) = -12$

Since $b^2 - 4ac < 0$, then the quadratic equation does not have real roots.

(f) $4x^2 - 12x + 9 = 0$

$b^2 - 4ac = (-12)^2 - 4(4)(9) = 0$

Since $b^2 - 4ac = 0$, then the quadratic equation has real and equal roots.

UPSKILL 2.2b

1 (a) $x^2 - 2hx + 3h + 4 = 0$

$a = 1,\ b = -2h,\ c = 3h + 4$

$$b^2 - 4ac = 0$$
$$(-2h)^2 - 4(1)(3h+4) = 0$$
$$4h^2 - 12h - 16 = 0$$
$$h^2 - 3h - 4 = 0$$
$$(h+1)(h-4) = 0$$
$$h = -1 \text{ or } 4$$

(b) $x^2 - 2(3+h)x - h - 1 = 0$

$a = 1,\ b = -2(3+h),\ c = -h-1$

$$b^2 - 4ac = 0$$
$$[(-2(3+h)]^2 - 4(1)(-h-1) = 0$$
$$4(3+h)^2 + 4h + 4 = 0$$
$$(3+h)^2 + h + 1 = 0$$
$$h^2 + 6h + 9 + h + 1 = 0$$
$$h^2 + 7h + 10 = 0$$
$$(h+2)(h+5) = 0$$
$$h = -2 \text{ or } -5$$

(c) $hx^2 + 8x = 8hx - 36$

$hx^2 + 8x - 8hx + 36 = 0$

$a = h,\ b = 8 - 8h,\ c = 36$

$$b^2 - 4ac = 0$$
$$(8-8h)^2 - 4h(36) = 0$$
$$64 - 128h + 64h^2 - 144 = 0$$
$$64h^2 - 128h - 80 = 0$$
$$4h^2 - 8h - 5 = 0$$
$$(2h+1)(2h-5) = 0$$
$$h = -\frac{1}{2} \text{ or } \frac{5}{2}$$

(d) $(3-h)x^2 + h + 1 = 2(h+1)x$

$(3-h)x^2 - 2(h+1)x + h + 1 = 0$

$a = 3-h,\ b = -2(h+1),\ c = h+1$

$$b^2 - 4ac = 0$$
$$[-2(h+1)]^2 - 4(3-h)(h+1) = 0$$
$$4(h+1)^2 - 4(3h + 3 - h^2 - h) = 0$$
$$(h+1)^2 - (-h^2 + 2h + 3) = 0$$
$$h^2 + 2h + 1 + h^2 - 2h - 3 = 0$$
$$2h^2 - 2 = 0$$
$$h^2 - 1 = 0$$
$$h = \pm 1$$

2 $x^2 + 2kx = 5p - 1$

$x^2 + 2kx + 1 - 5p = 0$

$a = 1,\ b = -2k,\ c = -5p + 1$

$$b^2 - 4ac = 0$$
$$(-2k)^2 - 4(1)(-5p+1) = 0$$
$$4k^2 + 20p - 4 = 0$$
$$k^2 + 5p - 1 = 0$$
$$5p = 1 - k^2$$
$$p = \frac{1-k^2}{5}$$

3 $3mx + q = 2 - 2x^2$

$2x^2 + 3mx + q - 2 = 0$

$a = 2,\ b = 3m,\ c = q - 2$

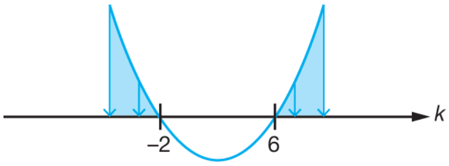
$$b^2 - 4ac = 0$$
$$(3m)^2 - 4(2)(q-2) = 0$$
$$9m^2 - 8q + 16 = 0$$
$$8q = 9m^2 + 16$$
$$q = \frac{9m^2 + 16}{8}$$

4 (a) $x^2 + k = kx - 3$

$x^2 - kx + k + 3 = 0$

$a = 1,\ b = -k,\ k + 3$

$$b^2 - 4ac > 0$$
$$(-k)^2 - 4(1)(k+3) > 0$$
$$k^2 - 4k - 12 > 0$$
$$(k+2)(k-6) > 0$$



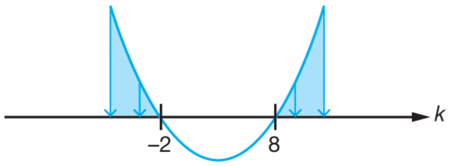
Hence, the required range of values of k is $k < -2$ or $k > 6$.

(b) $kx^2 + k = 6 - 8x$

$kx^2 + 8x + k - 6 = 0$

$a = k,\ b = 8,\ c = k - 6$

$$b^2 - 4ac > 0$$
$$8^2 - 4k(k-6) > 0$$
$$16 - k(k-6) > 0$$
$$16 - k^2 + 6k > 0$$
$$k^2 - 6k - 16 < 0$$
$$(k+2)(k-8) < 0$$



The required range of values of k is $-2 < k < 8$.

(c) $x^2 + 2k = (2k - 3)x$

$x^2 - (2k - 3)x + 2k = 0$

$a = 1,\ b = -(2k - 3),\ c = 2k$

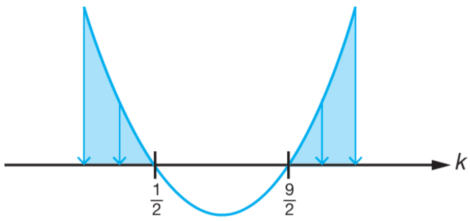
$b^2 - 4ac > 0$

$[-(2k - 3)]^2 - 4(1)(2k) > 0$

$4k^2 - 12k + 9 - 8k > 0$

$4k^2 - 20k + 9 > 0$

$(2k - 9)(2k - 1) > 0$



The required range of values of k is

$k < \frac{1}{2}$ or $k > \frac{9}{2}$.

5 (a) $x^2 - dx + d + 3 = 0$

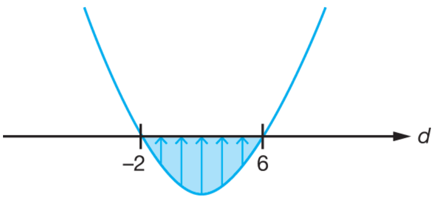
$a = 1,\ b = -d,\ c = d + 3$

$b^2 - 4ac < 0$

$(-d)^2 - 4(1)(d + 3) < 0$

$d^2 - 4d - 12 < 0$

$(d + 2)(d - 6) < 0$



The required range of values of d is

$-2 < d < 6$.

(b) $dx^2 + 4dx = -9 - x^2$

$(d + 1)x^2 - 4dx + 9 = 0$

$a = d + 1,\ b = -4d,\ c = 9$

$b^2 - 4ac < 0$

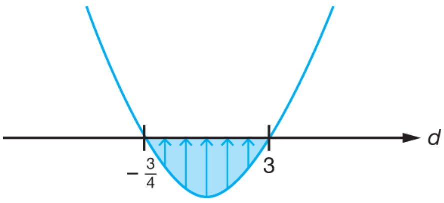
$(-4d)^2 - 4(d + 1)(9) < 0$

$16d^2 - 36(d + 1) < 0$

$4d^2 - 9(d + 1) < 0$

$4d^2 - 9d - 9 < 0$

$(d - 3)(4d + 3) < 0$



The required range of values of d is

$-\frac{3}{4} < d < 3$.

(c) $(2 - 3d)x^2 + 2 = (d - 4)x$

$a = (2 - 3d),\ b = -(d - 4),\ c = 2$

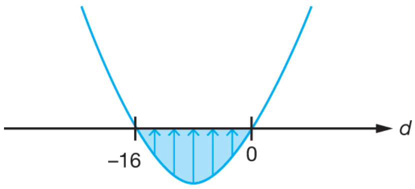
$b^2 - 4ac < 0$

$[-(d - 4)]^2 - 4(2 - 3d)(2) < 0$

$d^2 - 8d + 16 - 16 + 24d < 0$

$d^2 + 16d < 0$

$d(d + 16) < 0$



The required range of values of d is

$-16 < d < 0$.

6 $2x^2 - tx + 1 = 2x - 1$

$2x^2 - tx - 2x + 2 = 0$

$a = 2,\ b = -t - 2,\ c = 2$

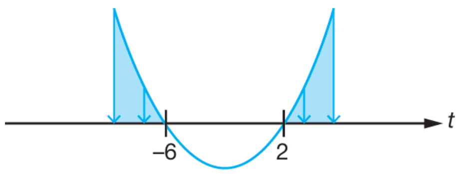
$b^2 - 4ac \geq 0$

$(-t - 2)^2 - 4(2)(2) \geq 0$

$t^2 + 4t + 4 - 16 \geq 0$

$t^2 + 4t - 12 \geq 0$

$(t + 6)(t - 2) \geq 0$



The required range of values of k is

$t \leq -6$ or $t \geq 2$.

UPSKILL 2.3a

1 (a) $f(x)=9x^2-12x+8$

Since $a>0$, then the graph has the shape of $\cup$.

(b) $g(x)=-2x^2-5x+3$

Since $a<0$, then the graph has the shape of $\cap$.

(c) $h(x)=(x+1)^2-4$

Since $a>0$, then the graph has the shape of $\cup$.

(d) $m(x)=1-(2-x)^2$

Since $a<0$, then the graph has the shape of $\cap$.

UPSKILL 2.3b

1 (a) The shape of the graph is $\cup$ and it touches the x-axis at only a point.

Its function is $d(x)=4x^2-20x+25$

because $(-20)^2-4(4)(25)=0$.

(b) The shape of the graph is $\cup$ and it intersects the x-axis at two different points.

Its function is $p(x)=x^2-6x+8$

because $(-6)^2-4(1)(8)=4$ (>0).

(c) The shape of the graph is $\cap$ and it does not intersect the x-axis.

Its function is $q(x)=-2x^2-3x-4$

because $(-3)^2-4(-2)(-4)=-23$ (<0).

(d) The shape of the graph is $\cap$ and it intersects the x-axis at two different points.

Its function is $h(x)=5x-6-x^2$ =

$-x^2+5x-6$ because

$5^2-4(-1)(-6)=1$ (>0).

(e) The shape of the graph is $\cup$ and it does not intersect the x-axis.

Its function is $k(x)=x^2-2x+4$

because $(-2)^2-4(1)(4)=-12$.

(f) The shape of the graph is $\cap$ and it touches the x-axis at only a point.
Its function is

$m(x)=8x-16-x^2=-x^2+8x-16$

because $(8)^2-4(-1)(-16)=0$.

2 (a) No real roots
(b) Real and distinct roots
(c) Real and equal roots
(d) No real roots
(e) Real and equal roots
(f) Real and distinct roots

3 (a) $f(x)=-2x^2+3x-4$

$b^2-4ac=3^2-4(-2)(-4)=-23$ (<0)

The graph of $f(x)$ will not intersect the x-axis.

(b) $g(x)=4x^2-3x-5$

$b^2-4ac=(-3)^2-4(4)(-5)=89$ (>0)

The graph of $f(x)$ will intersect the x-axis.

(c) $m(x)=(x-2)^2+3$

$=x^2-4x+4+3$

$=x^2-4x+7$

$b^2-4ac=(-4)^2-4(1)(7)=-12$ (<0)

The graph of $f(x)$ will not intersect the x-axis.

(d) $n(x)=5-(2x+1)^2$

$=5-(4x^2+4x+1)$

$=-4x^2-4x+4$

$b^2-4ac=(-4)^2-4(-4)(4)=80$ (>0)

The graph of $f(x)$ will intersect the x-axis.

4 (a) $f(x)=x^2-(w+4)x+1$

$b^2-4ac=0$

$[-(w+4)]^2-4(1)(1)=0$

$w^2+8w+16-4=0$

$w^2+8w+12=0$

$(w+2)(w+6)=0$

$w=-2$ or -6

(b) $g(x)=x^2-wx+w+3$

$b^2-4ac=0$

$(-w)^2-4(1)(w+3)=0$

$w^2-4w-12=0$

$(w+2)(w-6)=0$

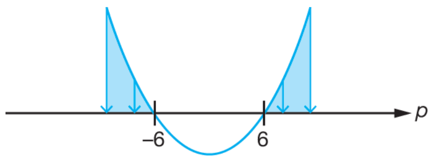
$w=-2$ or 6

(c) $h(x) = (4-2w)x^2 + 3wx - 2w - 1$

$$b^2 - 4ac = 0$$
$$(3w)^2 - 4(4-2w)(-2w-1) = 0$$
$$9w^2 - 4(-6w + 4w^2 - 4) = 0$$
$$9w^2 + 24w - 16w^2 + 16 = 0$$
$$-7w^2 + 24w + 16 = 0$$
$$7w^2 - 24w - 16 = 0$$
$$(w-4)(7w+4) = 0$$
$$w = 4 \text{ or } -\frac{4}{7}$$

5 (a) $f(x) = 3x^2 + px + 3$

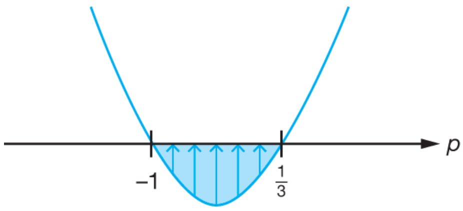
$$b^2 - 4ac > 0$$
$$p^2 - 4(3)(3) > 0$$
$$p^2 - 36 > 0$$
$$(p+6)(p-6) > 0$$



The required range of values of p is $p < -6$ or $p > 6$.

(b) $g(x) = px^2 + (p+1)x + p + 1$

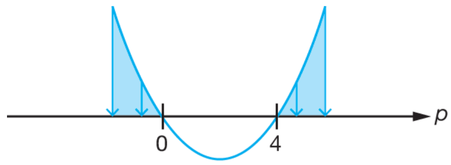
$$b^2 - 4ac > 0$$
$$(p+1)^2 - 4p(p+1) > 0$$
$$p^2 + 2p + 1 - 4p^2 - 4p > 0$$
$$-3p^2 - 2p + 1 > 0$$
$$3p^2 + 2p - 1 < 0$$
$$(p+1)(3p-1) < 0$$



The required range of values of p is $-1 < p < \frac{1}{3}$.

(c) $m(x) = x^2 + (2-2p)x + 2p + 1$

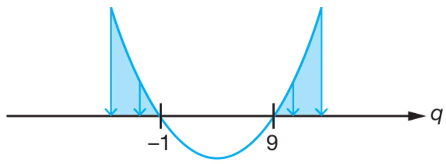
$$b^2 - 4ac > 0$$
$$(2-2p)^2 - 4(1)(2p+1) > 0$$
$$4 - 8p + 4p^2 - 8p - 4 > 0$$
$$4p^2 - 16p > 0$$
$$p^2 - 4p > 0$$
$$p(p-4) > 0$$



The required range of values of p is $p < 0$ or $p > 4$.

6 (a) $f(x) = qx^2 + 6x + q - 8$

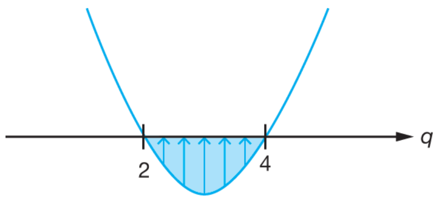
$$b^2 - 4ac < 0$$
$$6^2 - 4q(q-8) < 0$$
$$36 - 4q^2 + 32q < 0$$
$$-4q^2 + 32q + 36 < 0$$
$$q^2 - 8q - 9 > 0$$
$$(q-9)(q+1) > 0$$



The required range of values of q is $q < -1$ or $q > 9$.

(b) $g(x) = 4x^2 + 4(3-q)x + 1$

$$b^2 - 4ac < 0$$
$$[4(3-q)]^2 - 4(4)(1) < 0$$
$$16(3-q)^2 - 16 < 0$$
$$(3-q)^2 - 1 < 0$$
$$9 - 6q + q^2 - 1 < 0$$
$$q^2 - 6q + 8 < 0$$
$$(q-2)(q-4) < 0$$



The required range of values of q is $2 < q < 4$.

(c) $m(x) = x^2 + (q-1)x + q + 2$

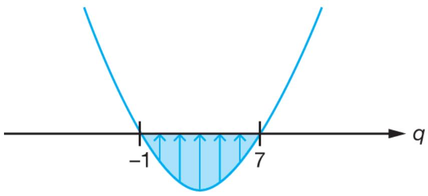
$b^2 - 4ac < 0$

$(q-1)^2 - 4(1)(q+2) < 0$

$q^2 - 2q + 1 - 4q - 8 < 0$

$q^2 - 6q - 7 < 0$

$(q+1)(q-7) < 0$



The required range of values of q is $-1 < q < 7$.

UPSKILL 2.3c

1 (a) $f(x) = x^2 - 2x + 3$

$= x^2 - 2x + \left[\frac{(-2)}{2}\right]^2 - \left[\frac{(-2)}{2}\right]^2 + 3$

$= x^2 - 2x + 1 - 1 + 3$

$= (x-1)^2 + 2$

Minimum value = 2 when x = 1.

(b) $g(x) = 2 + 6x - x^2$

$= -(x^2 - 6x - 2)$

$= -\left[x^2 - 6x + \left(\frac{-6}{2}\right)^2 - \left(\frac{-6}{2}\right)^2 - 2\right]$

$= -(x^2 - 6x + 9 - 9 - 2)$

$= -[(x-3)^2 - 11]$

$= -(x-3)^2 + 11$

Maximum value = 11 when x = 3.

(c) $q(x) = 2x^2 + 8x - 1$

$= 2\left(x^2 + 4x - \frac{1}{2}\right)$

$= 2\left[x^2 + 4x + \left(\frac{4}{2}\right)^2 - \left(\frac{4}{2}\right)^2 - \frac{1}{2}\right]$

$= 2\left(x^2 + 4x + 4 - 4 - \frac{1}{2}\right)$

$= 2\left[(x+2)^2 - \frac{9}{2}\right]$

$= 2(x+2)^2 - 9$

Minimum value = –9 when x = –2

(d) $m(x) = 5 - 4x - 2x^2$

$= -2\left(x^2 + 2x - \frac{5}{2}\right)$

$= -2\left[x^2 + 2x + \left(\frac{2}{2}\right)^2 - \left(\frac{2}{2}\right)^2 - \frac{5}{2}\right]$

$= -2\left(x^2 + 2x + 1 - 1 - \frac{5}{2}\right)$

$= -2\left[(x+1)^2 - \frac{7}{2}\right]$

$= -2(x+1)^2 + 7$

Maximum value = 7 when $x = -1$

(e) $n(x) = 5x^2 + 8x - 10$

$= 5\left(x^2 + \frac{8}{5}x - 2\right)$

$= 5\left[x^2 + \frac{8}{5}x + \left(\frac{1}{2} \times \frac{8}{5}\right)^2 - \left(\frac{1}{2} \times \frac{8}{5}\right)^2 - 2\right]$

$= 5\left(x^2 + \frac{8}{5}x + \frac{16}{25} - \frac{16}{25} - 2\right)$

$= 5\left[\left(x + \frac{4}{5}\right)^2 - \frac{66}{25}\right]$

$= 5\left[\left(x + \frac{4}{5}\right)^2 - \frac{66}{25}\right]$

$= 5\left(x + \frac{4}{5}\right)^2 - \frac{66}{5}$

Minimum value = $-\frac{66}{5}$ when $x = -\frac{4}{5}$.

(f) $p(x)=6x-9-4x^2$

$$=-4\left(x^2-\frac{6}{4}x+\frac{9}{4}\right)$$

$$=-4\left(x^2-\frac{3}{2}x+\frac{9}{4}\right)$$

$$=-4\left[x^2-\frac{3}{2}x+\left(-\frac{3}{4}\right)^2-\left(-\frac{3}{4}\right)^2+\frac{9}{4}\right]$$

$$=-4\left(x^2-\frac{3}{2}x+\frac{9}{16}-\frac{9}{16}+\frac{9}{4}\right)$$

$$=-4\left[\left(x-\frac{3}{4}\right)^2+\frac{27}{16}\right]$$

$$=-4\left(x-\frac{3}{4}\right)^2-\frac{27}{4}$$

Maximum value = $-\frac{27}{4}$ when

$x=\frac{3}{4}$.

2 $f(x)=x^2+6x+k$

$$=x^2+6x+\left(\frac{6}{2}\right)^2-\left(\frac{6}{2}\right)^2+k$$

$$=x^2+6x+9-9+k$$

$$=(x+3)^2-9+k$$

Minimum value = –2

$-9+k=-2$

$k=7$

3 $m(x)=3x(2-x)+q$

$$=6x-3x^2+q$$

$$=-3\left(x^2-2x-\frac{q}{3}\right)$$

$$=-3\left[x^2-2x+\left(\frac{-2}{2}\right)^2-\left(\frac{-2}{2}\right)^2-\frac{q}{3}\right]$$

$$=-3\left(x^2-2x+1-1-\frac{q}{3}\right)$$

$$=-3\left[(x-1)^2-1-\frac{q}{3}\right]$$

$$=-3(x-1)^2+3+q$$

Maximum value = 5

$3+q=5$

$q=2$

4 $g(x)=4-3x-dx^2$

$$=-d\left(x^2+\frac{3}{d}-\frac{4}{d}\right)$$

$$=-d\left[x^2+\frac{3}{d}+\left(\frac{3}{2d}\right)^2-\left(\frac{3}{2d}\right)^2-\frac{4}{d}\right]$$

$$=-d\left[x^2+\frac{3}{d}+\left(\frac{9}{4d^2}\right)-\left(\frac{9}{4d^2}\right)-\frac{4}{d}\right]$$

$$=-d\left(x+\frac{3}{2d}\right)^2+\left(\frac{9}{4d}\right)+4$$

Maximum value = $\frac{41}{8}$

$$\frac{9}{4d}+4=\frac{41}{8}$$

$$\frac{9}{4d}=\frac{9}{8}$$

$d=2$

5 $f(x)=-3x^2+px+18$

$$=-3\left(x^2-\frac{p}{3}x-6\right)$$

$$=-3\left[x^2-\frac{p}{3}x+\left(-\frac{p}{6}\right)^2-\left(-\frac{p}{6}\right)^2-6\right]$$

$$=-3\left[x^2-\frac{p}{3}x+\left(\frac{p^2}{36}\right)-\left(\frac{p^2}{36}\right)-6\right]$$

$$=-3\left[\left(x-\frac{p}{6}\right)^2-\left(\frac{p^2}{36}\right)-6\right]$$

$$=-3\left(x-\frac{p}{6}\right)^2+\left(\frac{p^2}{12}\right)+18$$

Maximum value = q when x = –2

Maximum value = $\left(\frac{p^2}{12}\right)+18$ when

$x=\frac{p}{6}$

By comparison, $\frac{p}{6}=-2$

$p=-12$

$$q=\left(\frac{p^2}{12}\right)+18$$

$$q=\left(\frac{144}{12}\right)+18=12+18=30$$

6 $f(x) = tx^2 - 12x + 20$

$$= t\left(x^2 - \frac{12}{t} + \frac{20}{t}\right)$$

$$= t\left[x^2 - \frac{12}{t} + \left(-\frac{12}{2t}\right)^2 - \left(-\frac{12}{2t}\right)^2 + \frac{20}{t}\right]$$

$$= t\left[x^2 - \frac{12}{t} + \left(\frac{36}{t^2}\right) - \left(\frac{36}{t^2}\right) + \frac{20}{t}\right]$$

$$= t\left[\left(x - \frac{6}{t}\right)^2 - \left(\frac{36}{t^2}\right) + \frac{20}{t}\right]$$

$$= t\left(x - \frac{6}{t}\right)^2 - \frac{36}{t} + 20$$

$f(x)$ has a maximum value when $x = \frac{6}{t}$.

But it is given that $f(x)$ has a maximum value when $x = -2$.

By comparison, $\frac{6}{t} = -2$

$t = -3$

Maximum value $= -\frac{36}{t} + 20$

$= -\frac{36}{-3} + 20$

$= 12 + 20$

$= 32$

UPSKILL 2.3d

1 (a) $g(x) = a(x-h)^2 + k$

From the graph,

$g(x) = a(x-3)^2 - 2$

By comparison, $h = 3$ and $k = -2$

(b) $g(x) = a(x-3)^2 - 2$

When $x = 0$, $y = -6$

$g(0) = a(0-3)^2 - 2 = -6$

$9a = -4$

$a = -\frac{4}{9}$

(c) The equation of the axis of symmetry is $x = 3$.

2 (a) Equation of axis of symmetry is

$x = \frac{-1+3}{2}$

$x = 1$

(b) $f(x) = a(x-h)^2 + k$

But it is given that

$f(x) = a(x-1)^2 - 5$

Hence, $h = 1$ and $k = -5$

(c) When $x = 3$, $y = 0$.

$f(3) = a(3-1)^2 - 5$

$0 = 4a - 5$

$a = \frac{5}{4}$

3 (a) The equation of the axis of symmetry is

$x = \frac{1+3}{2}$

$x = 2$

(b) $f(x) = a(x-k)^2 - 4$

$x = k$

It is found in (a) that $x = 2$.

By comparison, $k = 2$.

(c) $f(x) = a(x-2)^2 - 4$

The coordinates of the minimum point are (2, –4).

(d) When $x = 1$, $y = 0$

$f(x) = a(x-k)^2 - 4$

$0 = a(1-2)^2 - 4$

$a = 4$

(e) When the curve is reflected in the x-axis, each term will change, i.e.

$f(x) = -4(x-2)^2 + 4$

(f) When the curve is reflected in the y-axis, the value of h will change, i.e.

$f(x) = 4(x+2)^2 - 4$

UPSKILL 2.3e

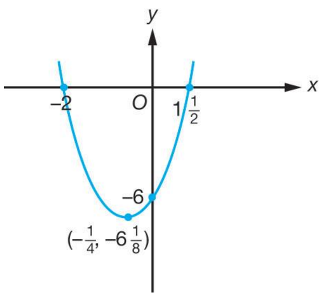
1 (a) $f(x) = (2x-3)(x+2)$

$$= 2x^2 + x - 6$$
$$= 2\left(x^2 + \frac{x}{2} - 3\right)$$
$$= 2\left[x^2 + \frac{x}{2} + \left(\frac{1}{2(2)}\right)^2 - \left(\frac{1}{2(2)}\right)^2 - 3\right]$$
$$= 2\left[x^2 + \frac{x}{2} + \left(\frac{1}{16}\right) - \left(\frac{1}{16}\right) - 3\right]$$
$$= 2\left[\left(x + \frac{1}{4}\right)^2 - \frac{49}{16}\right]$$
$$= 2\left(x + \frac{1}{4}\right)^2 - \frac{49}{8}$$

The minimum point is $\left(-\frac{1}{4}, -6\frac{1}{8}\right)$.

The y-intercept is –6.

The x-intercepts are $1\frac{1}{2}$ and –2.



The equation of the axis of symmetry is $x = -\frac{1}{4}$.

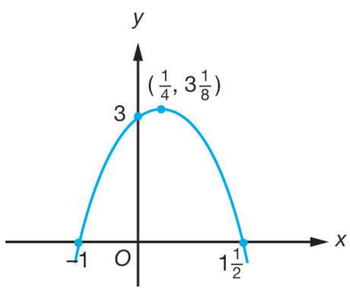
(b) $g(x) = (1+x)(3-2x)$

$$= -2x^2 + x + 3$$
$$= -2\left(x^2 - \frac{1}{2}x - \frac{3}{2}\right)$$
$$= -2\left[x^2 - \frac{1}{2}x + \left(\frac{-1}{2\times 2}\right)^2 - \left(\frac{-1}{2\times 2}\right)^2 - \frac{3}{2}\right]$$
$$= -2\left[x^2 - \frac{1}{2}x + \left(\frac{1}{16}\right) - \left(\frac{1}{16}\right) - \frac{3}{2}\right]$$
$$= -2\left[\left(x - \frac{1}{4}\right)^2 - \left(\frac{25}{16}\right)\right]$$
$$= -2\left(x - \frac{1}{4}\right)^2 + \frac{25}{8}$$

The maximum point is $\left(\frac{1}{4}, 3\frac{1}{8}\right)$.

The y-intercept is 3.

The x-intercepts are –1 and $1\frac{1}{2}$.



The equation of the axis of symmetry is $x = \frac{1}{4}$.

(c) $h(x) = 3x^2 + 12x - 4$

$$= 3\left(x^2 + 4x - \frac{4}{3}\right)$$
$$= 3\left(x^2 + 4x - \frac{4}{3}\right)$$
$$= 3\left[x^2 + 4x + \left(\frac{4}{2}\right)^2 - \left(\frac{4}{2}\right)^2 - \frac{4}{3}\right]$$
$$= 3\left(x^2 + 4x + 4 - 4 - \frac{4}{3}\right)$$
$$= 3\left[(x+2)^2 - 16\right]$$
$$= 3(x+2)^2 - 16$$

The minimum point is $(-2, -16)$.

y-intercept = –4

On the x-axis (y = 0),

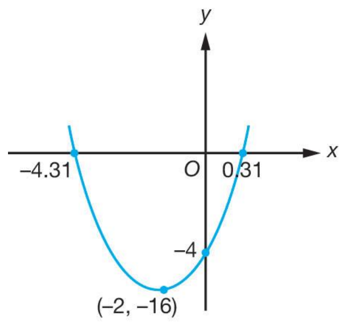
$3x^2 + 12x - 4 = 0$

$$x = \frac{-12 \pm \sqrt{12^2 - 4(3)(-4)}}{2(3)}$$

$$x = \frac{-12 \pm \sqrt{192}}{6}$$

$x = -4.31$ or 0.72

The curve intersects the x-axis at (–4.31, 0.72).



The equation of the axis of symmetry is $x = -2$.

(d) $m(x) = 2x^2 + 7x + 11$

$$= 2\left(x^2 + \frac{7}{2}x + \frac{11}{2}\right)$$

$$= 2\left(x^2 + \frac{7}{2}x + \left(\frac{1}{2} \times \frac{7}{2}\right)^2 - \left(\frac{1}{2} \times \frac{7}{2}\right)^2 + \frac{11}{2}\right)$$

$$= 2\left(x^2 + \frac{7}{2}x + \frac{49}{16} - \left(\frac{49}{16}\right) + \frac{11}{2}\right)$$

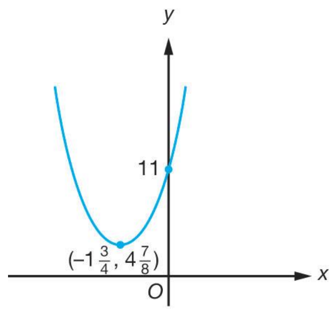
$$= 2\left[\left(x + \frac{7}{4}\right)^2 + \frac{39}{16}\right]$$

$$= 2\left(x + \frac{7}{2}\right)^2 + \frac{39}{8}$$

The minimum point is

$\left(-1\frac{3}{4}, 4\frac{7}{8}\right)$

y-intercept = 11



The equation of the axis of symmetry is $x = -\frac{7}{4}$.

(e) $n(x) = 1 - 2x - x^2$

$$= -x^2 - 2x + 1$$

$$= -\left(x^2 + 2x - 1\right)$$

$$= -\left[x^2 + 2x + \left(\frac{2}{2}\right)^2 - \left(\frac{2}{2}\right)^2 - 1\right]$$

$$= -\left(x^2 + 2x + 1 - 1 - 1\right)$$

$$= -\left[(x+1)^2 - 2\right]$$

$$= -(x+1)^2 + 2$$

The maximum point is $(-1, 2)$.

y-intercept = 1

On the x-axis, $y = 0$

$$-x^2 - 2x + 1 = 0$$

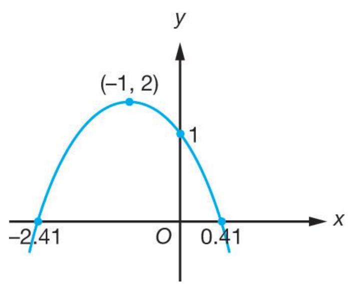
$$x^2 + 2x - 1 = 0$$

$$x = \frac{-2 \pm \sqrt{2^2 - 4(1)(-1)}}{2(1)}$$

$$x = \frac{-2 \pm \sqrt{8}}{2(1)}$$

$x = 0.21$ or 0.41

The curve intersects the x-axis at (–2.41, 0) and (0.41, 0).



The equation of the axis of symmetry is $x = -1$.

(f) $p(x) = 2x - 3 - 2x^2$

$$= -2x^2 + 2x - 3$$

$$= -2\left(x^2 - x + \frac{3}{2}\right)$$

$$= -2\left[x^2 - x + \left(-\frac{1}{2}\right)^2 - \left(-\frac{1}{2}\right)^2 + \frac{3}{2}\right]$$

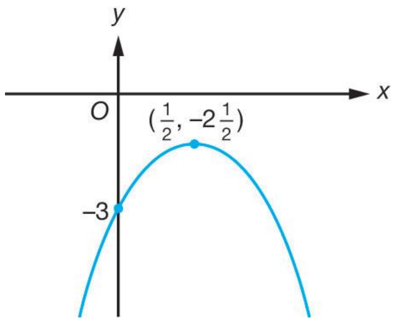
$$= -2\left(x^2 - x + \frac{1}{4} - \frac{1}{4} + \frac{3}{2}\right)$$

$$= -2\left[\left(x - \frac{1}{2}\right)^2 + \frac{5}{4}\right]$$

$$= -2\left(x - \frac{1}{2}\right)^2 - \frac{5}{2}$$

The maximum point is $\left(\frac{1}{2}, -\frac{5}{2}\right)$.

y-intercept = –3



The equation of the axis of symmetry is $x = \frac{1}{2}$.

2 (a) $f(x) = (x-2)^2 - (2x-3)^2$

$= x^2 - 4x + 4 - \left(4x^2 - 12x + 9\right)$

$= x^2 - 4x + 4 - 4x^2 + 12x - 9$

$= -3x^2 + 8x - 5$

$= -3\left(x^2 - \frac{8}{3}x + + \frac{5}{3}\right)$

$= -3\left(x^2 - \frac{8}{3}x + 16 - 16 - 5\right)$

$= -3\left[x^2 - \frac{8}{3}x + \left(-\frac{8}{(2)(3)}\right)^2 - \left(-\frac{8}{(2)(3)}\right)^2 + \frac{5}{3}\right]$

$= -3\left[x^2 - \frac{8}{3}x + \frac{16}{9} - \frac{16}{9} + \frac{5}{3}\right]$

$= -3\left[\left(x - \frac{4}{3}\right)^2 - \frac{1}{9}\right]$

$= -3\left(x - \frac{4}{3}\right)^2 + \frac{1}{3}$

The maximum point is $\left(\frac{4}{3}, \frac{1}{3}\right)$.

y-intercept = –5

At the x-axis ($y = 0$),

$-3x^2 + 8x - 5 = 0$

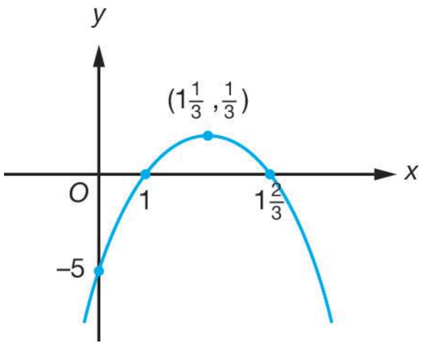
$3x^2 - 8x + 5 = 0$

$(3x-5)(x-1) = 0$

$x = \frac{5}{3}$ or 1

The curve will intersect the x-axis at (1, 0) and $\left(\frac{5}{3}, 0\right)$

(b)



3 (a) $g(x) = \frac{1}{2}\left[(x+5)^2 + (x-3)^2\right]$

$= \frac{1}{2}\left(x^2 + 10x + 25 + x^2 - 6x + 9\right)$

$= \frac{1}{2}\left(2x^2 + 4x + 34\right)$

$= x^2 + 2x + 17$

$= x^2 + 2x + \left(\frac{2}{2}\right)^2 - \left(\frac{2}{2}\right)^2 + 17$

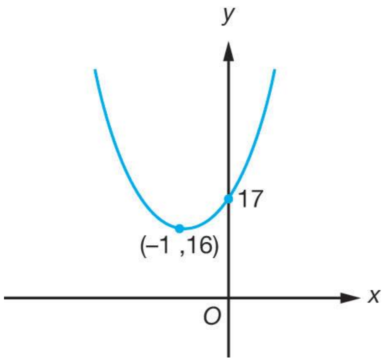
$= x^2 + 2x + 1 - 1 + 17$

$= (x+1)^2 + 16$

The minimum point of the curve is (–1, 16).

y-intercept = 17

(b)



4 (a) $f(x) = x^2 + px + 5$

$$= x^2 + px + \left(\frac{p}{2}\right)^2 - \left(\frac{p}{2}\right)^2 + 5$$

$$= x^2 + px + \frac{p^2}{4} - \frac{p^2}{4} + 5$$

$$= \left(x + \frac{p}{2}\right)^2 - \frac{p^2}{4} + 5$$

Minimum point is $\left(-\frac{p}{2}, -\frac{p^2}{4} + 5\right)$

Given minimum point is $\left(q, \frac{11}{4}\right)$

By comparison, $-\frac{p^2}{4} + 5 = \frac{11}{4}$

$$-p^2 + 20 = 11$$

$$p^2 = 9$$

$$p = 3$$

$$q = -\frac{p}{2} = -\frac{3}{2}$$

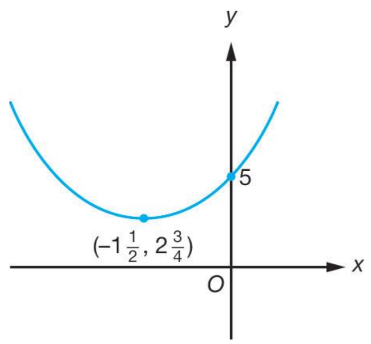
(b) $f(x) = \left(x + \frac{3}{2}\right)^2 - \frac{3^2}{4} + 5$

$$= \left(x + \frac{3}{2}\right)^2 + \frac{11}{4}$$

Minimum point is $\left(-\frac{3}{2}, \frac{11}{4}\right)$ i.e.

$\left(-1\frac{1}{2}, 2\frac{3}{4}\right)$.

y-intercept = 5



5 (a) $g(x) = -x^2 + hx - 4$

$$= -(x^2 - hx + 4)$$

$$= -\left[x^2 - hx + \left(\frac{h}{2}\right)^2 - \left(\frac{h}{2}\right)^2 + 4\right]$$

$$= -\left(x^2 - hx + \frac{h^2}{4} - \frac{h^2}{4} + 4\right)$$

$$= -\left[\left(x - \frac{h}{2}\right)^2 - \frac{h^2}{4} + 4\right]$$

$$= -\left(x - \frac{h}{2}\right)^2 + \frac{h^2}{4} - 4$$

Maximum point is $\left(\frac{h}{2}, \frac{h^2}{4} - 4\right)$.

Given maximum point is $(k, -3)$.

By comparison, $\frac{h^2}{4} - 4 = -3$

$$h^2 = 4$$

$$h = 2$$

$$k = \frac{h}{2} = \frac{2}{2} = 1$$

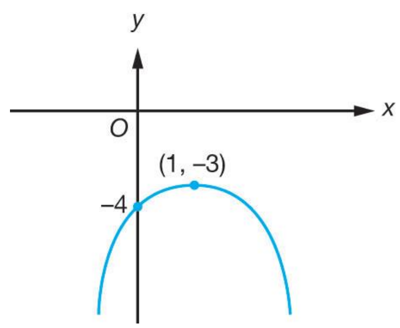
(b) $-\left(x - \frac{h}{2}\right)^2 + \frac{h^2}{4} - 4$

$$= -\left(x - \frac{2}{2}\right)^2 + \frac{2^2}{4} - 4$$

$$= -(x - 1)^2 - 3$$

Maximum point is $(1, -3)$.

y-intercept = –4



6 (a) $f(x) = 2x^2 - 7x + 5$

$$= 2\left(x^2 - \frac{7}{2}x + \frac{5}{2}\right)$$

$$= 2\left(x^2 - \frac{7}{2}x + \left(-\frac{7}{2\times 2}\right)^2 - \left(-\frac{7}{2\times 2}\right)^2 + \frac{5}{2}\right)$$

$$= 2\left(x^2 - \frac{7}{2}x + \frac{49}{16} - \frac{49}{16} + \frac{5}{2}\right)$$

$$= 2\left[\left(x - \frac{7}{4}\right)^2 - \frac{9}{16}\right]$$

$$= 2\left(x - \frac{7}{4}\right)^2 - \frac{9}{8}$$

Minimum point is $\left(\frac{7}{4}, -\frac{9}{8}\right)$, i.e.

$\left(1\frac{3}{4}, -1\frac{1}{8}\right)$.

y-intercept = 5

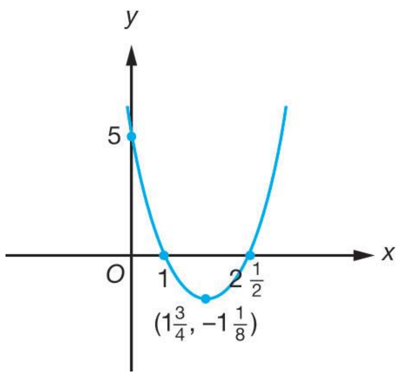
On the x-axis, $y = 0$

$$2x^2 - 7x + 5 = 0$$

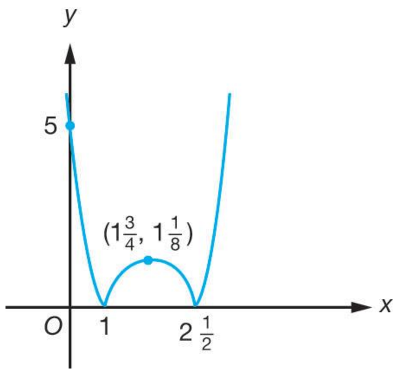
$$(x-1)(2x-5) = 0$$

$$x = 1 \text{ or } \frac{5}{2}$$

Hence, the curve intersects the x-axis at $(1, 0)$ and $\left(2\frac{1}{2}, 0\right)$



(b)



7 (a) $f(x) = -10 + 7x - x^2$

$$= -x^2 + 7x - 10$$
$$= -(x^2 - 7x + 10)$$
$$= -\left[x^2 - 7x + \left(-\frac{7}{2}\right)^2 - \left(-\frac{7}{2}\right)^2 + 10\right]$$
$$= -\left(x^2 - 7x + \frac{49}{4} - \frac{49}{4} + 10\right)$$
$$= -\left[\left(x - \frac{7}{4}\right)^2 - \frac{49}{4} + 10\right]$$
$$= -\left[\left(x - \frac{7}{4}\right)^2 - \frac{9}{4}\right]$$
$$= -\left(x - \frac{7}{4}\right)^2 + \frac{9}{4}$$

Maximum point is $\left(\frac{7}{4}, \frac{9}{4}\right)$, i.e.

$\left(3\frac{1}{2}, 2\frac{1}{4}\right)$.

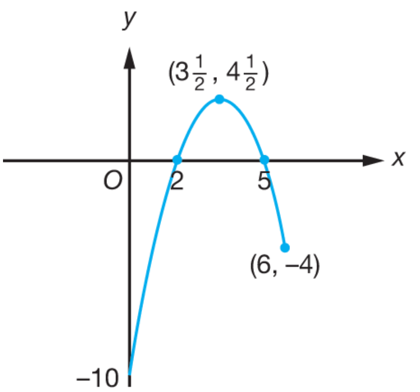
y-intercept = –10

At the x-axis, $y = 0$

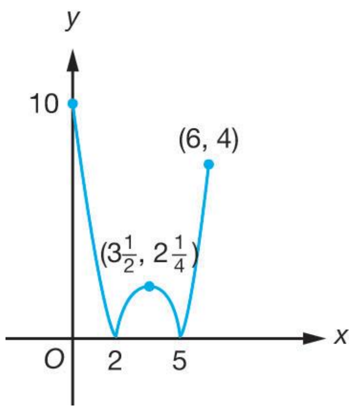
$$-x^2 + 7x - 10 = 0$$
$$x^2 - 7x + 10 = 0$$
$$(x - 2)(x - 5) = 0$$
$$x = 2 \text{ or } 5$$

Hence, the curve will intersect the x-axis at the points (2, 0) and (5, 0).

x	0	6
$f(x)$	–10	–4



(b)



UPSKILL 2.3f

1 $y = m(x-3)-1 \quad \ldots (1)$

$y = x^2 - 3x \quad \ldots (2)$

Substitute (2) into (1) :

$x^2 - 3x = m(x-3)-1$

$x^2 - 3x = mx - 3m - 1$

$x^2 - 3x - mx + 3m + 1 = 0$

$a = 1, b = -3 - m, c = 3m + 1$

$b^2 - 4ac = 0$

$(-3-m)^2 - 4(1)(3m+1) = 0$

$9 + 6m + m^2 - 12m - 4 = 0$

$m^2 - 6m + 5 = 0$

$(m-5)(m-1) = 0$

$m = 5$ or 1

2 $y = nx - 2 \quad \ldots (1)$

$y = 2x^2 - x \quad \ldots (2)$

Substitute (2) into (1) :

$2x^2 - x = nx - 2$

$2x^2 - x - nx + 2 = 0$

$a = 2, b = -1 - n, c = 2$

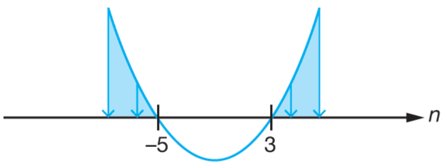
$(-1-n)^2 - 4(2)(2) > 0$

$b^2 - 16 > 0$

$1 + 2n + n^2 - 16 > 0$

$n^2 + 2n - 15 > 0$

$(n-3)(n+5) > 0$



The range of values of n is
$n < -5$ or $n > 3$.

3 $y = k(x-1)-1 \quad \ldots (1)$

$y = x^2 - kx + 1 \quad \ldots (2)$

Substitute (2) into (1) :

$x^2 - kx + 1 = k(x-1) - 1$

$x^2 - kx + 1 = kx - k - 1$

$x^2 - 2kx + k + 2 = 0$

$a = 1, b = -2k, k + 2$

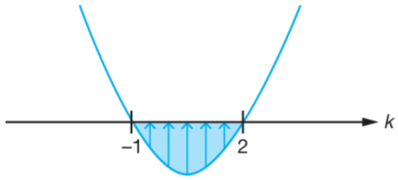
$b^2 - 4ac < 0$

$(-2k)^2 - 4(1)(k+2) < 0$

$4k^2 - 4k - 8 < 0$

$k^2 - k - 2 < 0$

$(k-2)(k+1) < 0$



The range of values of k is
$-1 < k < 2$.

4 $f(x) = 2x^2 - 2tx - 3t + 20$

$a = 2, b = -2t\,,\ c = -3t + 20$

$b^2 - 4ac < 0$

$(-2t)^2 - 4(2)(20 - 3t) < 0$

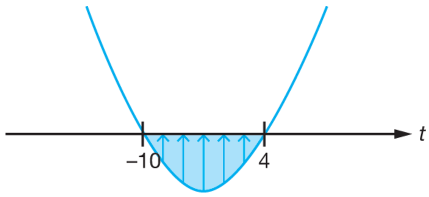
$4t^2 - 8(20 - 3t) < 0$

$t^2 - 2(20 - 3t) < 0$

$t^2 - 40 + 6t < 0$

$t^2 + 6t - 40 < 0$

$(t-4)(t+10) < 0$

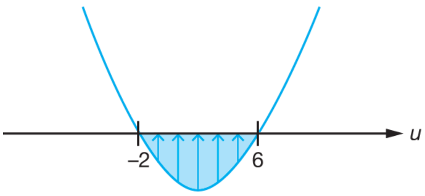


The range of values of t is
$-10 < t < 4$.

5 $g(x) = -2x^2 + (u+6)x - 2u - 6$

$a = -2,\ b = u + 6,\ c = -2u - 6$

$$b^2 - 4ac < 0$$
$$(u+6)^2 - 4(-2)(-2u) < 0$$
$$u^2 + 12u + 36 + 8(-2u - 6) < 0$$
$$u^2 + 12u + 36 - 16u - 48 < 0$$
$$u^2 - 4u - 12 < 0$$
$$(u+2)(u-6) < 0$$



The range of values of u is
$-2 < u < 6$.

6 (a) When $x - 0$, $h(x) = 0$, thus $c = 0$.

When $x = 120$, $h(120) = 0$.

Thus, $h(120) = a(120)^2 + b(120) = 0$

$$120a + b = 0 \quad \ldots (1)$$

When $x = 60$, $h(60) = 70$.

Thus, $h(60) = a(60)^2 + b(60) = 70$

$$3600a + 60b = 70$$
$$360a + 6b = 7 \quad \ldots (2)$$

$$\begin{array}{rl} & 360a + 3b = 0 \quad \ldots (1) \times 3 \\ (-) & \underline{360a + 6b = 7} \\ & -3b = -7 \end{array}$$

$$b = \frac{-7}{-3}$$

$$b = \frac{7}{3}$$

Substitute $b = \frac{7}{3}$ into (1) :

$$120a + \frac{7}{3} = 0$$

$$120a = -\frac{7}{3}$$

$$a = -\frac{7}{360}$$

Hence, $h(x) = -\frac{7}{360}x^2 + \frac{7}{3}x$

(b) When, $h(x) = 52\frac{1}{2}$,

$$52\frac{1}{2} = -\frac{7}{360}x^2 + \frac{7}{3}x$$

$$18\,900 = -7x^2 + 840x^2$$
$$7x^2 - 840x + 18\,900 = 0$$
$$x^2 - 120x + 2700 = 0$$
$$(x-3)(x-90) = 0$$
$$x = 30 \text{ or } x = 90$$

Hence, when the height of the parabolic curve is $52\frac{1}{2}$ m, the distance from P is 30 m or 90 m.

Summative Practice 2

1 $x(3x-2)=7-5x$

$3x^2-2x+5x-7=0$

$3x^2+3x-7=0$

$x=\dfrac{-3\pm\sqrt{3^2-4(3)(-7)}}{2(3)}$

$x=\dfrac{-3\pm\sqrt{93}}{6}$

$x=1.107$ or -2.107

2 $(k-30)(2k+50)-1400=1.61\times 10\ 000$

$2k^2-10k-1500-1400-16\ 100=0$

$2k^2-10k-19\ 000=0$

$k^2-5x-9\ 500=0$

$k=\dfrac{-(-5)\pm\sqrt{(-5)^2-4(1)(-9\ 500)}}{2(1)}$

$k=\dfrac{5\pm\sqrt{38\ 025}}{2}$

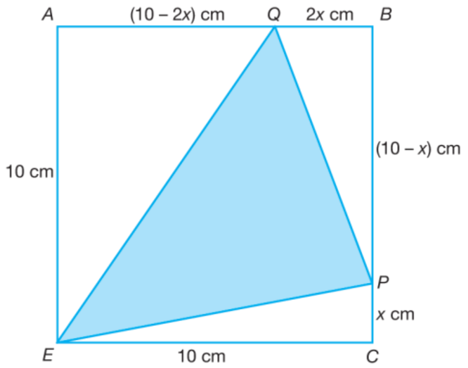
$k=\dfrac{5\pm 195}{2}$

$k=-95$ or 100

$k=-95$ is not accepted.

$k=100$

3



(a) $A(x)=10(10)-\frac{1}{2}(10)(x)-\frac{1}{2}(10)(10-2x)$

$-\frac{1}{2}(2x)(10-x)$

$=100-5x-50+10x-10x+x^2$

$=x^2-5x+50$ [Shown]

(b) $x^2-5x+50=44.75$

$4x^2-20x+200=179$

$4x^2-20x+21=0$

$(2x-7)(2x-3)=0$

$x=3.5$ or $x=1.5$

$x=3.5$ is not accepted because it does not satisfy $AQ > QB$.

Hence, $x=1.5$

4 $2x^2-8x-3=0$

The roots are α and β.

S.O.R. = $\alpha+\beta=\frac{8}{2}=4$

P.O.R. = $\alpha\beta=-\frac{3}{2}$

The new roots are $\alpha(1-\beta)$ and $p(1-\alpha)$.

S.O.R = $\alpha(1-\beta)+\beta(1-\alpha)$

$=\alpha-\alpha\beta+\beta-\beta\alpha$

$=\alpha+\beta-2\alpha\beta$

$=4-2\left(-\frac{3}{2}\right)$

$=4+3$

$=7$

P.O.R. = $\alpha(1-\beta)\times\beta(1-\alpha)$

$=\alpha\beta[(1-\beta)(1-\alpha)]$

$=\alpha\beta[1-(\alpha+\beta)+\alpha\beta]$

$=-\frac{3}{2}\left[1-4+\left(-\frac{3}{2}\right)\right]$

$=\frac{27}{4}$

The new quadratic equation is

$x^2-7x+\frac{27}{4}=0$

$4x^2-28x+27=0$

5 $x^2+k=15x$

$x^2-15x+k=0$

The roots are 2α and 3α.

S.O.R. = 15

$2\alpha+3\alpha=15$

$5\alpha=15$

$\alpha=3$

P.O.R. = k

$(2\alpha)(3\alpha)=k$

$k=6\alpha^2$

$k=6(3)^2$

$k=54$

6 (a) $hx^2 + kx + 2k = 8x + 4$

$hx^2 + kx - 8x + 2k - 4 = 0$

$hx^2 + (k - 8)x + 2k - 4 = 0$

$a = h, b = k - 8, c = 2k - 4$

The roots are k and $\frac{1}{h}$.

S.O.R. = $-\frac{b}{a}$

$k + \frac{1}{h} = -\frac{(k-8)}{h}$

$\frac{hk+1}{h} = -\frac{(k-8)}{h}$

$hk + 1 = -k + 8$

$hk + k = 7 \quad \ldots (1)$

P.O.R. = $\frac{c}{a}$

$\frac{k}{h} = \frac{2k-4}{h}$

$k = 2k - 4$

$k = 4$

From (1) :

$4h + 4 = 7$

$4h = 3$

$h = \frac{3}{4}$

(b) New roots are $2h = 2\left(\frac{3}{4}\right) = \frac{3}{2}$ and $-k = -4$.

S.O.R. = $\frac{3}{2} + (-4) = -\frac{5}{2}$

P.O.R. = $\frac{3}{2}(-4) = -6$

The new quadratic equation is

$x^2 + \frac{5}{2}x - 6 = 0$

$2x^2 + 5x - 12 = 0$

7 $x^2 + 2x - 5 = 0$

The roots are α and β.

$\alpha + \beta = -2$

$\alpha\beta = -5$

$x^2 + 4x + q = 0$

The roots are $\frac{p}{\alpha}$ and $\frac{p}{\beta}$.

S.O.R. = $\frac{p}{\alpha} + \frac{p}{\beta}$

$= \frac{p(\alpha+\beta)}{\alpha\beta}$

$= \frac{p(-2)}{-5}$

$= \frac{2}{5}p$

S.O.R. = $-\frac{b}{a} = -4$

By comparison, $\frac{2}{5}p = -4$

$p = \frac{5}{2}(-4)$

$p = -10$

P.O.R. = $\left(\frac{p}{\alpha}\right)\left(\frac{p}{\beta}\right)$

$= \frac{p^2}{\alpha\beta}$

$= \frac{p^2}{-5}$

P.O.R. = $\frac{c}{a} = q$

By comparison,

$q = \frac{p^2}{-5}$

$q = \frac{(-10)^2}{-5}$

$q = \frac{100}{-5}$

$q = -20$

8 $x^2 + 2mx + 1 = 0$
The roots are α and β.
$\alpha + \beta = -2m$
$\alpha\beta = 1$

$x^2 + 4x - n = 0$
The roots are 2α and 2β.
$2\alpha + 2\beta = -\frac{b}{a}$
$2(\alpha + \beta) = -4$
$2(-2m) = -4$
$-4m = -4$
$m = \frac{-4}{-4} = 1$

$(2\alpha)(2\beta) = \frac{c}{a}$
$4\alpha\beta = -n$
$4(1) = -n$
$n = -4$

9 $(x + m)^2 = kx$
$x^2 + 2mx + m^2 - kx = 0$
$x^2 + (2m - k)x + m^2 = 0$
The roots are 1 and 16.
S.O.R. = $-\frac{b}{a}$
$1 + 16 = -(2m - k)$
$17 = -2m + k$ … (1)

P.O.R. = $\frac{c}{a}$
$1 \times 16 = m^2$
$m = \pm 4$

From (1) :
When $m = 4$,
$17 = -2(4) + k$
$k = 17 + 8$
$k = 25$

From (1) :
When $m = -4$,
$17 = -2(-4) + k$
$17 = 8 + k$
$k = 9$

10 $x^2 + 15 = 8x$
$x^2 - 8x + 15 = 0$
The roots are $(h+1)$ and $(k-2)$.
S.O.R. = $-\frac{b}{a}$
$(h+1) + (k-2) = 8$
$h + k - 1 = 8$
$h + k = 9$
$h = 9 - k$ … (1)

P.O.R. = $\frac{c}{a}$
$(h+1)\ (k-2) = 15$
$hk - 2h + k - 2 = 15$
$hk - 2h + k = 17$ … (2)

Substitute (1) into (2) :
$k(9 - k) - 2(9 - k) + k = 17$
$9k - k^2 - 18 + 2k + k = 17$
$-k^2 + 12k - 35 = 0$
$k^2 - 12k + 35 = 0$
$(k - 7)(k - 5) = 0$
$k = 7$ or 5

From (1) :
When $k = 7$,
$h = 9 - k = 9 - 7 = 2$
When $k = 5$,
$h = 9 - 5 = 4$

Hence, $k = 7$, $h = 2$ or $k = 5$. $h = 4$

11 $y = 2(x - 2)^2 + 3q$
$y = 2(x^2 - 4x + 4) + 3q$
$y = 2x^2 - 8x + 8 + 3q$
S.O.R. = $-\frac{-8}{2} = 4$ … (1)
P.O.R. = $\frac{8 + 3q}{2}$ … (2)

$y = x^2 + x - px - 5$
$y = x^2 + (1 - p)x - 5$
S.O.R. = $-\frac{(1-p)}{1} = p - 1$ … (3)
P.O.R. = -5 … (4)

Equating (1) and (3) :
$p - 1 = 4$
$p = 5$

Equating (2) and (4) :

$\frac{8+3q}{2} = -5$

$3q+8=-10$

$3q=-10-8$

$3q=-18$

$q=-6$

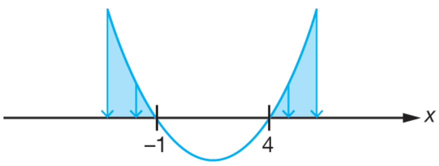
12 $-x(x-4) < x-4$

$-x^2+4x-x+4<0$

$-x^2+3x+4<0$

$x^2-3x-4>0$

$(x-4)(x+1)>0$



The range of values of x is $x<-1$ or $x>4$.

13 (a) $x(x-4)=2$

$x^2-4x-2=0$

(b) S.O.R. = $-\frac{b}{a} = -\left(\frac{-4}{1}\right) = 4$

(c) b^2-4ac

$=(-4)^2-4(-2)$

$=16+8$

$=24\ (>0)$

Hence, the roots are real and distinct.

14 $3x^2-2mx=5-4p$

$3x^2-2mx+4p-5=0$

$a=3,\ b=-2m,\ c=4p-5$

$b^2-4ac=0$

$(-2m)^2-4(3)(4p-5)=0$

$4m^2-48p+60=0$

$m^2-12p+15=0$

$12p=m^2+15$

$p=\frac{m^2+15}{12}$

15 $9x^2+qx+1=4x$

$9x^2+qx-4x+1=0$

$a=9,\ b=q-4,\ c=1$

$b^2-4ac=0$

$(q-4)^2-4(9)(1)=0$

$q^2-8q+16-36=0$

$q^2-8q-20=0$

$(q+2)(q-10)=0$

$q=-2$ or 10

16 $f(x)=2x^2-px+p+6$

$b^2-4ac=0$

$(-p)^2-4(2)(p+6)=0$

$p^2-8p-48=0$

$(p+4)(p-12)=0$

$p=-4$ or 12

17 $g(x)=x^2+2kx+2-k$

$a=1,\ b=2k,\ c=2-k$

$b^2-4ac=0$

$(2k)^2-4(1)(2-k)=0$

$4k^2-8+4k=0$

$k^2+k-2=0$

$(k+2)(k-1)=0$

$k=-2$ or 1

18 (a) $x^2-2px+2p+3=0$

$a=1,\ b=-2p,\ c=2p+3$

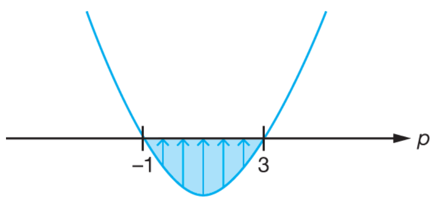
$b^2-4ac<0$

$(-2p)^2-4(1)(2p+3)<0$

$4p^2-8p-12<0$

$p^2-2p-3<0$

$(p-3)(p+1)<0$



The range of values of p is $-1<p<3$.

(b) $x^2 + 2p^2 + 3p + 2 = 2px + 4x$

$$x^2 - 2px - 4x + 2p^2 + 3p + 2 = 0$$

$$a = 1,\ b = -2p - 4,\ c = 2p^2 + 3p + 2$$

$$b^2 - 4ac > 0$$

$$(-2p - 4)^2 - 4(1)\left(2p^2 + 3p + 2\right) > 0$$

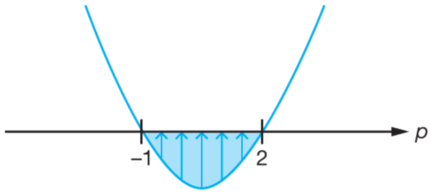
$$4p^2 + 16p + 16 - 8p^2 - 12p - 8 > 0$$

$$-4p^2 + 4p + 8 > 0$$

$$-p^2 + p + 2 > 0$$

$$p^2 - p - 2 < 0$$

$$(p - 2)(p + 1) < 0$$



The range of values of p is
$-1 < p < 2$.

19 (a) $f(x) = (1 - m)x^2 - 4mx + 9$

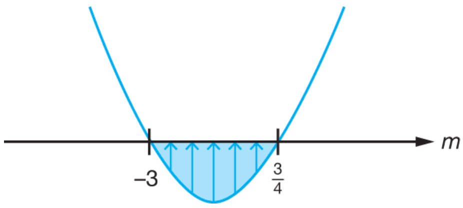
$$b^2 - 4ac < 0$$

$$(-4m)^2 - 4(1 - m)(9) < 0$$

$$16m^2 + 36m - 36 < 0$$

$$4m^2 + 9m - 9 < 0$$

$$(4m - 3)(m + 3) < 0$$



The range of values of m is
$-3 < m < \frac{3}{4}$

(b) $f(x) = 4x^2 - (m + 2)x + m - 1$

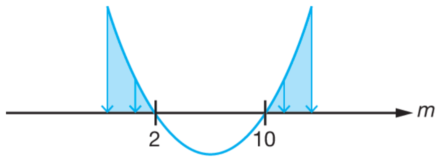
$$b^2 - 4ac > 0$$

$$[-(m + 2)]^2 - 4(4)(m - 1) > 0$$

$$m^2 + 4m + 4 - 16m + 16 > 0$$

$$m^2 - 12m + 20 > 0$$

$$(m - 2)(m - 10) > 0$$



The range of values of m is
$m < 2$ or $m > 10$

20 $3x^2 - 3x + 4 + q\left(2x^2 - x - 1\right) = 0$

$$3x^2 - 3x + 4 + 2qx^2 - qx - q = 0$$

$$(2q + 3)x^2 + (-3 - q)x + 4 - q = 0$$

$$b^2 - 4ac < 0$$

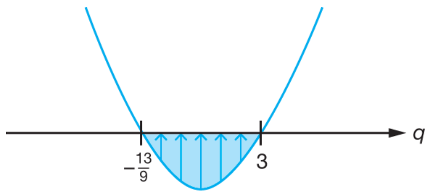
$$(-3 - q)^2 - 4(2q + 3)(4 - q) < 0$$

$$9 + 6q + q^2 - 4\left(-2q^2 + 5q + 12\right) < 0$$

$$9 + 6q + q^2 + 8q^2 - 20q - 48 < 0$$

$$9q^2 - 14q - 39 < 0$$

$$(q - 3)(9q + 13) < 0$$



The range of values of q is
$-\frac{13}{9} < q < 3$

21 (a) $f(x)=-x^2+4kx-5k^2-1$

$f(x)=-\left(x^2-4kx+5k^2+1\right)$

$f(x)=-\left[x^2-4kx+(-2k)^2-(-2k)^2+5k^2+1\right]$

$f(x)=-\left[x^2-4kx+(4k^2)-(4k^2)+5k^2+1\right]$

$f(x)=-\left[(x-2k)^2+k^2+1\right]$

$f(x)=-(x-2k)^2-k^2-1$

The maximum value of $f(x)$ is $-k^2-1$ when $x-2k=0 \Rightarrow x=2k$.

But it is given that the maximum value of $f(x)$ is $-r^2-2k$.

By comparison,

$-k^2-1=-r^2-2k$

$r^2=k^2-2k+1$

$r^2=(k-1)^2$

$r=k-1$ [Shown]

(b) The axis of symmetry is $x=2k$.

But it is given that the axis of symmetry is $x=r^2-1$.

By comparison,

$r^2-1=2k \quad \ldots (1)$

Substitute $r=k-1$ into (1):

$(k-1)^2-1=2k$

$k^2-2k+1-1-2k=0$

$k^2-4k=0$

$k(k-4)=0$

Given that $k\neq 0$, thus $k=4$

Therefore $r=k-1=4-1=3$

22 (a) $f(x)=a(x-p)^2+q$

Since $f(x)$ has a maximum value, therefore $a<0$.

(b) $f(x)=a(x-2)^2+3$

But it is given that $f(x)=a(x-p)^2+q$.

By comparison, $p=2$ and $q=3$.

(c) $f(x)=-2(x-2)^2+3$

$=-2\left(x^2-4x+4\right)+3$

$=-2x^2+8x-5$

(d) (i) $f(x)=2x^2-8x+5$ ← The sign of each term is changed.

(ii) $f(x)=-2x^2-8x-5$ ← The sign of the coefficient of x is changed

23 (a) $h(x)=-x^2+8x-8$

$=-\left(x^2-6x+8\right)$

$=-\left[x^2-6x+\left(\frac{-6}{2}\right)^2-\left(\frac{-6}{2}\right)^2+8\right]$

$=-\left(x^2-6x+9-9+8\right)$

$=-\left[(x-3)^2-1\right]$

$=-(x-3)^2+1$

Thus, $p=1$

(b) The maximum point is (3, 1).

y-intercept $=-8$

At the x-axis, $y=0$

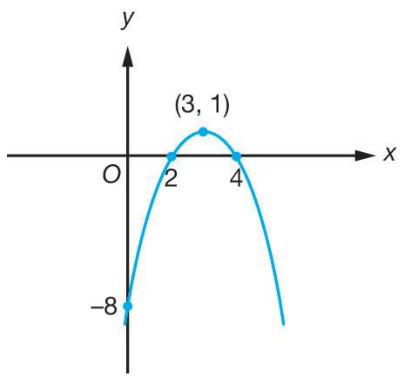
$-x^2+8x-8=0$

$x^2-8x+8=0$

$(x-2)(x-4)=0$

$x=2$ or 4

Thus, the curve will intersect the x-axis at (2, 0) and (4, 0).



24 (a) $f(x)=2x-3-4x^2$

$=-4x^2+2x-3$

$=-4\left(x^2-\frac{1}{2}x+\frac{3}{4}\right)$

$=-4\left[x^2-\frac{1}{2}x+\left(-\frac{1}{(2)(2)}\right)^2-\left(-\frac{1}{(2)(2)}\right)^2+\frac{3}{4}\right]$

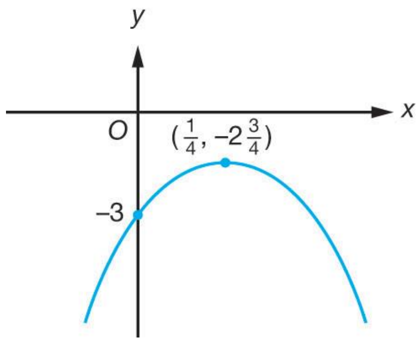
$=-4\left(x^2-\frac{1}{2}x+\frac{1}{16}-\frac{1}{16}+\frac{3}{4}\right)$

$=-4\left[\left(x-\frac{1}{4}\right)^2+\frac{11}{16}\right]$

$=-4\left(x-\frac{1}{4}\right)^2-\frac{11}{4}$

Maximum value $=-\frac{11}{4}$ when $x=\frac{1}{4}$

(b) y-intercept = –3



25 (a) $f(x) = 4 - 3x - x^2$

$= -x^2 - 3x + 4$

$= -(x^2 + 3x - 4)$

$= -\left[x^2 + 3x + \left(\frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2 - 4\right]$

$= -\left(x^2 + 3x + \frac{9}{4} - \frac{9}{4} - 4\right)$

$= -\left[\left(x + \frac{3}{2}\right)^2 - \frac{25}{4}\right]$

$= -\left(x + \frac{3}{2}\right)^2 + \frac{25}{4}$

Hence, the maximum point is

$\left(-\frac{3}{2}, \frac{25}{4}\right)$, i.e. $\left(-1\frac{1}{2}, 6\frac{1}{4}\right)$.

(b) y-intercept = 4

At the x-axis, $y = 0$

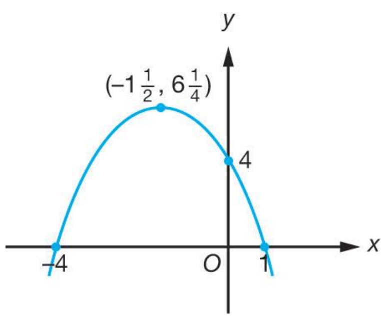
$-x^2 - 3x + 4 = 0$

$x^2 + 3x - 4 = 0$

$(x + 4)(x - 1) = 0$

$x = -4$ or $x = 1$

Thus, the curve will intersect the x-axis at the points (–4, 0) and (1, 0).



(c) $k = -1\frac{1}{2}$

26 (a) $f(x) = x^2 - x - 6$

$= (x + 2)(x - 3)$

y-intercept = –6

x-intercept = –2 and 3

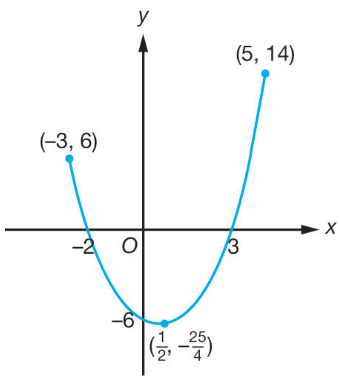
$f(x) = x^2 - x - 6$

$= x^2 - x + \left(-\frac{1}{2}\right)^2 - \left(-\frac{1}{2}\right)^2 - 6$

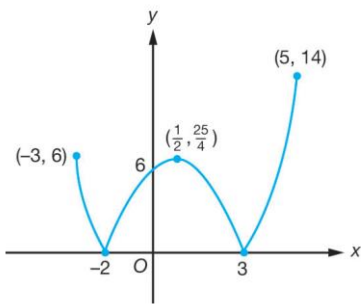
$= x^2 - x + \frac{1}{4} - \frac{1}{4} - 6$

$= \left(x - \frac{1}{2}\right)^2 - \frac{25}{4}$

x	–3	5
$f(x)$	6	14



(b)



27 (a) $h(x) = -x^2 - 4kx + 5k$

$= -(x^2 + 4kx - 5k)$

$= -\left[x^2 + 4kx + \left(\frac{4k}{2}\right)^2 - \left(\frac{4k}{2}\right)^2 - 5k\right]$

$= -(x^2 + 4kx + 4k^2 - 4k^2 - 5k)$

$= -\left[(x - 2k)^2 - 4k^2 - 5k\right]$

Maximum value = 6

$-4k^2 - 5k = 6$

$4k^2 + 5k + 6 = 0$

$(4k-3)(k+2)=0$

$k=\frac{3}{4}$ or -2

(b) $h(x) = -x^2 - 4(-2)x - 10$

$= -x^2 + 8x - 10$

$= -(x^2 - 8x + 10)$

$= -\left[x^2 - 8x + \left(\frac{-8}{2}\right) - \left(\frac{-8}{2}\right) + 10\right]$

$= (x^2 - 8x + 16 - 16 + 10)$

$= (x-4)^2 - 6$

Maximum point is $(4, -6)$.

y-intercept = –10

At the x-axis, y = 0.

$-x^2 + 8x - 10 = 0$

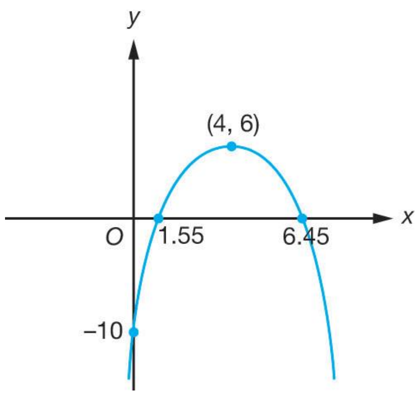
$x^2 - 8x + 10 = 0$

$x = \frac{8 \pm \sqrt{(-8)^2 - 4(1)(10)}}{2(1)}$

$x = \frac{8 \pm \sqrt{24}}{2}$

$x = 6.45$ or 1.55

Hence, the curve will intersect the x-axis at (1.55, 0) and (6.45, 0).



28 (a) $f(x) = x^2 + hx + 5$

$= \left(x + hx + \left(\frac{h}{2}\right)^2 - \left(\frac{h}{2}\right)^2 + 5\right)$

$= \left(x + \frac{h}{2}\right)^2 - \frac{h^2}{4} + 5$

Given $h(x) = (x+k)^2 + \frac{11}{4}$.

By comparison,

$-\frac{h^2}{4} + 5 = \frac{11}{4}$

$-\frac{h^2}{4} = -\frac{9}{3}$

$h = -3$ [Given $h < 0$]

$k = \frac{h}{2}$

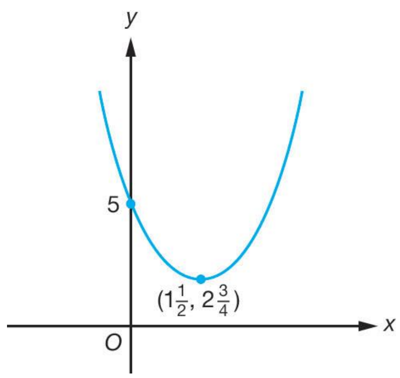
$k = -\frac{3}{2}$

(b) When $h = -3$, $k = -\frac{3}{2}$,

$f(x) = \left(x - \frac{3}{2}\right)^2 + \frac{11}{4}$

Minimum point $\left(\frac{3}{2}, \frac{11}{4}\right)$ i.e. $\left(1\frac{1}{2}, 2\frac{3}{4}\right)$

y-intercept = 5



29 (a) The midpoint between (1, 0) and (5, 0) is $\left(\frac{1+5}{2}, 0\right)$, i.e. $(3, 0)$.

The maximum value is 8.

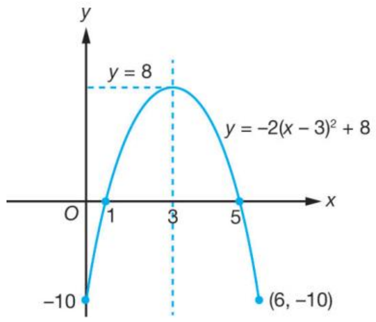
Therefore, (3, 8) is the maximum point.

Hence, $f(x) = -2(x-3) + 8$

But it is given that $f(x) = -2(x-h) - 2k$.

By comparison, $h = 3$ and $-2k = 8 \Rightarrow k = -4$.

(b) y-intercept is 8 and the x-intercepts are 1 and 5.



(c) $f(x) = -2(x-3)^2 + 8$

If the graph is reflected in the x-axis, the sign of each term is changed.

$f(x) = 2(x-3)^2 - 8$

(d) $f(x) = a(x-h) + k$

If the graph is reflected in the x-axis, the sign of h is changed.

$f(x) = -2(x+3)^2 + 8$

30 $y = px + 4 \ \ldots (1)$

$y = x^2 - 4x + 5 \ \ldots (2)$

Substitute (2) into (1) :

$x^2 - 4x + 5 = px + 4$

$x^2 - 4x - px + 1 = 0$

$a = 1, b = -4 - p, c = 1$

$b^2 - 4ac = 0$

$(-p-4)^2 - 4 = 0$

$p^2 + 8x + 16 - 4 = 0$

$p^2 + 8x + 12 = 0$

$(p+2)(p+6) = 0$

$p = -2$ or -6

31 $y = h - 2x \ \ldots (1)$

$y^2 + xy + 8 = 0 \ \ldots (2)$

Substitute (1) into (2) :

$(h-2x)^2 + x(h-2x) + 8 = 0$

$h^2 - 4hx + 4x^2 + hx - 2x^2 + 8 = 0$

$2x^2 - 3hx + h^2 + 8 = 0$

$a = 2, b = -3h, ch^2 + 8$

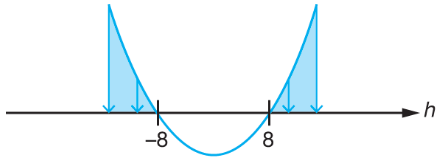
$b^2 - 4ac > 0$

$(-3h)^2 - 4(2)(h^2 + 8) > 0$

$9h^2 - 8h^2 - 64 > 0$

$h^2 - 64 > 0$

$(h+8)(h-8) > 0$



The range of values of h is $h < -8$ or $h > 8$.

32 $y = x + k \ \ldots (1)$

$y^2 + x^2 = 2 \ \ldots (2)$

Substitute (1) into (2) :

$(x+k)^2 + x^2 = 2$

$x^2 + 2kx + k^2 + x^2 - 2 = 0$

$2x^2 + 2kx + k^2 - 2 = 0$

$a = 2, b = 2k, c = k^2 - 2$

$b^2 - 4ac < 0$

$(2k)^2 - 4(2)(k^2 - 2) < 0$

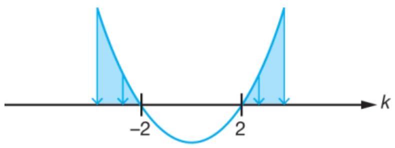
$4k^2 - 8k^2 + 16 < 0$

$-4k^2 + 16 < 0$

$4k^2 - 16 > 0$

$k^2 - 4 > 0$

$(k+2)(k-2) > 0$



The range of values of k is $k < -2$ or $k > 2$.

33 $f(x) = x^2 + (k-2)x + 16 - 2k$

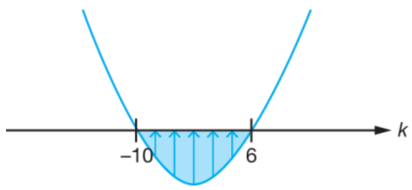
$b^2 - 4ac < 0$

$(k-2)^2 - 4(1)(16 - 2k < 0$

$k^2 - 4k + 4 - 64 + 8k < 0$

$k^2 + 4k - 60 < 0$

$(k-6)(k+10) < 0$



The range of values of k is $-10 < k < 6$.

But it is given that $m < k < n$.

By comparison, $m = -10$ and $n = 6$.