**Form 4 Chapter 1**
**Functions**
**Fully-Worked Solutions**

## UPSKILL 1.1a

**1** (a) Each image is obtained by changing the sign of each object. Hence, $f(x) = -x$.

(b) (i) Domain = {6, 7, 8}
(ii) Codomain = {–6, –7, –8, –9}
(iii) Range = {–6, –7, –8}
(iv) The object of –6 is 6.
(v) The image of 7 is –7.

**2** (a) Since the vertical line intersects the graph only once, then it is a function.
(b) Since the vertical line intersects the graph twice, then it is not a function.

**3** $2x - 1 \neq 0$

$x \neq \frac{1}{2}$

But it is given that $h \neq \frac{1}{2}$.

Hence by comparison, $h = \frac{1}{2}$.

## UPSKILL 1.1b

**1** (a) Domain = $-4 \leq x \leq 4$
Range = $1 \leq f(x) \leq 5$

(b) Domain = $-1 \leq x \leq 2$
Range = $0 \leq f(x) \leq 9$

## UPSKILL 1.1c

**1** (a) $f(x) = \frac{18}{2x - 9}$

(i) $f(0) = \frac{18}{2(0) - 9} = -2$

(ii) $f(3) = \frac{18}{2(3) - 9} = \frac{18}{-3} = -6$

(b) (i) $f(x) = 2$

$\frac{18}{2x - 9} = 2$

$18 = 4x - 18$

$4x = 36$

$x = 9$

(ii) $f(x) = 6$

$\frac{18}{2x - 9} = 6$

$18 = 12x - 54$

$12x = 72$

$x = 6$

**2** (a) $f(3) = -5$

$\frac{a}{3 - b} = -5$

$a = -15 + 5b \ \dots (1)$

$f(-5) = -1$

$\frac{a}{-5 - b} = -1$

$a = 5 + b \ \dots (2)$

Substituting (2) into (1):

$5 + b = -15 + 5b$

$4b = 20$

$b = 5$

From (2): $a = 5 + 5 = 10$

(b) $f(x) = \frac{10}{x - 5}$

$x - 5 \neq 0$

$x \neq 5$

Hence, the value of $x$ such that f is undefined is 5.

**3** (a) $f(x) = \frac{px + q}{x - 2}$

$f(3) = 4$

$\frac{3p + q}{3 - 2} = 4$

$3p + q = 4$

$q = 4 - 3p \ \dots (1)$

$f(1) = 2$

$\frac{p + q}{1 - 2} = 2$

$p + q = -2 \ \dots (2)$

Substituting (1) into (2):

$p + 4 - 3p = -2$

$-2p = -6$

$p = 3$

From (1):

$q = 4 - 3(3) = -5$

(b) $f(x)=\frac{3x-5}{x-2}$

The value of $x$ such that $f$ is undefined is 2.

(c) $f(x)=\frac{4}{3}x$

$\frac{3x-5}{x-2}=\frac{4x}{3}$

$9x-15=4x^2-8x$

$4x^2-17x+15=0$

$(x-3)(4x-5)=0$

$x=3$ or $x=\frac{5}{4}$

**4** (a) $g(x)=ax+\frac{b}{x}$

$g(2)=7$

$2a+\frac{b}{2}=7$

$4a+b=14$

$b=14-4a \ \ldots (1)$

$g(-1)=-5$

$-a+\frac{b}{-1}=-5$

$-a-b=-5 \ \ldots (2)$

Substituting (1) into (2) :

$-a-(14-4a)=-5$

$-a-14+4a=-5$

$3a=9$

$a=3$

From (1) : $b=14-4a$

$b=14-4(3)=2$

(b) $g(x)=3x+\frac{2}{x}$

$g$ is undefined when $x$ = 0.

(c) $g(x)=7$

$3x+\frac{2}{x}=7$

$3x^2+2=7x$

$3x^2-7x+2=0$

$(3x-1)(x-2)=0$

$x=\frac{1}{3}$ or $x=2$

$x=2$ is not accepted.

$\therefore x=\frac{1}{3}$

**5** (a) $g(x)=a+bx$

$g(1)=-3$

$a+b=-3$

$a=-3-b \ \ldots (1)$

$g(-2)=3$

$a-2b=3 \ \ldots (2)$

Substituting (1) into (2) :

$-3-b-2b=3$

$-3-3b=3$

$-3b=6$

$b=-2$

From (1): $a=-3-(-2)=-1$

(b) $g(x)=-1-2x$

$g(n^2+1)=5n-6$

$-1-2(n^2+1)=5n-6$

$-2n^2-2-1-5n+6=0$

$-2n^2-5n+3=0$

$2n^2+5n-3=0$

$(2n-1)(n+3)=0$

$n=\frac{1}{2}$ or $-3$

**6** $f(x)=x$

$\frac{5x-4}{x+1}=x$

$5x-4=x^2+x$

$x^2-4x+4=0$

$(x-2)^2=0$

$x=2$

**7** $f(x)=x$

$\frac{12}{x-4}=x$

$12=x^2-4x$

$x^2-4x-12=0$

$(x-6)(x+2)=0$

$x=6$ or $-2$

**8** (a) $f(x)=px+qx^2$

$f(-1)=-5$

$-p+q=-5$

$q=p-5 \ \ldots (1)$

$f(-2) = -16$
$-2p + 4q = -16$
$p - 2q = 8 \quad \ldots (2)$

Substituting (1) into (2) :
$p - 2(p - 5) = 8$
$p - 2p + 10 = 8$
$-p = -2$
$p = 2$

From (1) : $q = 2 - 5 = -3$

(b) $f(x) = 2x - 3x^2$

$f(x) = x$
$2x - 3x^2 = x$
$3x^2 - x = 0$
$x(3x - 1) = 0$
$x = 0$ or $\frac{1}{3}$

**UPSKILL 1.1d**

**1** $h(x) = |x^2 - 4x - 3|$

(a) $h(-3) = |(-3)^2 - 4(-3) - 3|$
$= |18|$
$= 18$

(b) $h(0) = |0^2 - 4(0) - 3| = |-3| = 3$

(c) $h(2) = h(x) = |2^2 - 4(2) - 3| = |-7| = 7$

**2** (a) (i) $f(2)$
$= |2 - 5(2)|$
$= |-8|$
$= 8$

(ii) $f(-2)$
$= |2 - 5x|$
$= |2 - 5(-2)|$
$= 12$

(b) $f(x) = 7$
$|2 - 5x| = 7$
$2 - 5x = \pm 7$

$2 - 5x = 7$
$5x = -5$
$x = -1$

$2 - 5x = -7$
$-5x = -9$
$x = \frac{9}{5}$

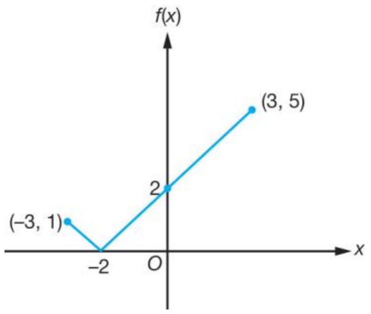
**3** $g(x) = 7$
$|2x + 1| = 7$
$2x + 1 = \pm 7$

$2x + 1 = 7$
$2x = 6$
$x = 3$

$2x + 1 = -7$
$2x = -8$
$x = -4$

**4** (a) $f(x) = |x + 2|$

| $x$ | –3 | –2 | –1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $f(x)$ | 1 | 0 | 1 | 2 | 3 | 4 | 5 |



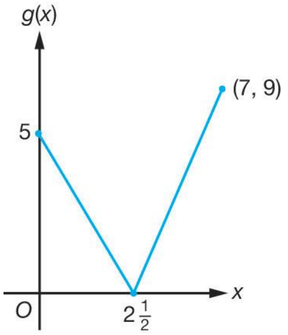

The range of $f(x)$ is $0 \leq f(x) \leq 5$.

(b) $g(x) = |2x - 5|$

| $x$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| $g(x)$ | 5 | 3 | 1 | 1 | 3 | 5 | 7 | 9 |

When $|2x - 5| = 0$
$2x - 5 = 0$
$x = \frac{5}{2} = 2\frac{1}{2}$

The graph touches the $x$-axis at $\left(2\frac{1}{2}, 0\right)$.




The range of $g(x)$ is $0 \le g(x) \le 9$.

(c) $h(x) = |3 - 2x|$

| $x$ | –3 | –2 | –1 | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|---|---|---|
| $h(x)$ | 9 | 7 | 5 | 3 | 1 | 1 | 3 | 5 |

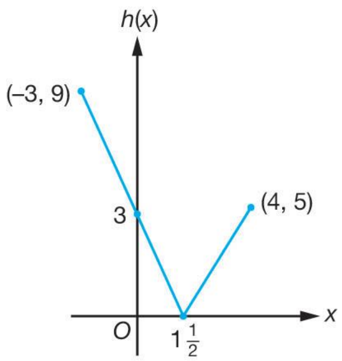
On the $x$-axis, $y = 0$,

$|3 - 2x| = 0$

$3 - 2x = 0$

$x = 1\frac{1}{2}$

The graph touches the $x$-axis at $\left(1\frac{1}{2}, 1\right)$.




The range of $h(x)$ is $0 \le h(x) \le 9$.

**UPSKILL 1.2a**

**1** $f(x) = |4 - 5x|$

$g(x) = \sqrt{x - 2}$

(a) $fg(6) = f\left(\sqrt{6 - 2}\right)$

$= f(2)$

$= |4 - 5(2)|$

$= |-6| = 6$

(b) $gf(2) = g(|4 - 5(2)|)$

$= g(|-6|)$

$= g(6)$

$= \sqrt{6 - 2}$

$= \sqrt{4}$

$= 2$

(c) $f^2(0) = ff(0)$

$= f(|4 - 5(0)|)$

$= f(4)$

$= |4 - 5(4)|$

$= |-16|$

$= 16$

(d) $g^2(51) = gg(51)$

$= g\left(\sqrt{51 - 2}\right)$

$= g\left(\sqrt{49}\right)$

$= g(7)$

$= \sqrt{7 - 2}$

$= \sqrt{5}$

$= 2.236$

**2** (a) $fg(x) = f[g(x)]$

$= f(3x + 1)$

$= (3x + 1)^2 - 1$

$= 9x^2 + 6x + 1 - 1$

$= 9x^2 + 6x$

$gf(x) = g(x^2 - 1)$

$= 3(x^2 - 1) + 1$

$= 3x^2 - 3 + 1$

$= 3x^2 - 2$

(b) $fg(x) = f(1 - 3x)$

$= (1 - 3x + 1)^2$

$= (2 - 3x)^2$

$= 4 - 12x + 9x^2$

$gf(x) = g\left[(x + 1)^2\right]$

$= 1 - 3(x + 1)^2$

$= 1 - 3(x^2 + 2x + 1)$

$= -2 - 3x^2 - 6x$

$= -3x^2 - 6x - 2$

(c) $fg(x) = f\left(\frac{1}{x^2+2}\right)$

$= 2 - \left(\frac{1}{x^2+2}\right)$

$= \frac{2(x^2+2)-1}{x^2+2}$

$= \frac{2x^2+3}{x^2+2}$

$gf(x) = g(2-x)$

$= \frac{1}{(2-x)^2+2}$

$= \frac{1}{4-4x+x^2+2}$

$= \frac{1}{x^2-4x+6}$

**3** (a) $f^2(x) = ff(x)$

$= f(4x-3)$

$= 4(4x-3)-3$

$= 16x-12-3$

$= 16x-15$

(b) $g^2(x) = gg(x)$

$= g(x+1)$

$= (x+1)+1$

$= x+2$

$f^2(x) = g^2(x)$

$16x-15 = x+2$

$15x = 17$

$x = \frac{17}{15}$

**4** (a) $f^2(x) = ff(x)$

$= f\left(\frac{x-1}{x+1}\right)$

$= \frac{\left(\frac{x-1}{x+1}\right)-1}{\left(\frac{x-1}{x+1}\right)+1}$

$= \frac{\frac{x-1-(x+1)}{x+1}}{\frac{x-1+(x+1)}{x+1}}$

$= \frac{-2}{2x}$

$= -\frac{1}{x},\ x \neq 0$

(b) $f^4(x) = f^2f^2(x)$

$= f^2\left(-\frac{1}{x}\right)$

$= -\frac{1}{\left(-\frac{1}{x}\right)}$

$= x$

(c) $f^{12}(x) = f^4f^4f^4(x)$

$= f^4f^4(x)$

$= f^4(x)$

$= x$

(d) $f^{13}(x) = f\,f^{12}(x)$

$= f(x)$

$= \frac{x-1}{x+1},\quad x \neq -1$

**5** (a) $f(8) = 4$

(b) $g(4) = 16$

(c) $gf(8) = g(4) = 16$

**6** The function that maps *x* straight away to *z* is *nm*.

$nm(x) = n(3x+2)$

$= (3x+2)^2 - 10$

$= 9x^2+12x+4-10$

$= 9x^2+12x-6$

**7** (a) $fg(x) = f(x-2)$

$= [(x-2)+1]^2$

$= (x-1)^2$

$= x^2-2x+1$

$gf(x) = g[(x+1)^2]$

$= (x+1)^2-2$

$= x^2+2x-1$

(b) (i)
$$fg(x) = 4$$
$$x^2 - 2x + 1 = 4$$
$$x^2 - 2x - 3 = 0$$
$$(x+1)(x-3) = 0$$
$$x = -1 \text{ or } 3$$

(ii)
$$gf(x) = 7$$
$$g(x+1)^2 = 7$$
$$(x+1)^2 - 2 = 7$$
$$x^2 + 2x + 1 - 9 = 0$$
$$x^2 + 2x - 8 = 0$$
$$(x-2)(x+4) = 0$$
$$x = 2 \text{ or } -4$$

(c)
$$fg(x) = gf(x)$$
$$x^2 - 2x + 1 = x^2 + 2x - 1$$
$$-4x = -2$$
$$x = \frac{1}{2}$$

**UPSKILL 1.2b**

**1** (a)
$$fg(x) = 3x - 2$$
$$f[g(x)] = 3x - 2$$
$$g(x) + 2 = 3x - 2$$
$$g(x) = 3x - 4$$

(b)
$$fg(x) = \frac{2x+5}{x-2}$$
$$f[g(x)] = \frac{2x+5}{x-2}$$
$$3[g(x)] + 2 = \frac{2x+5}{x-2}$$
$$3g(x) = \frac{2x+5}{x-2} - 2$$
$$3g(x) = \frac{2x+5-2(x-2)}{x-2}$$
$$3g(x) = \frac{9}{x-2}$$
$$g(x) = \frac{3}{x-2}, \; x \neq 2$$

(c)
$$fg(x) = x^2 + 4x + 3$$
$$f[g(x)] = x^2 + 4x + 3$$
$$[g(x)]^2 - 1 = x^2 + 4x + 3$$
$$[g(x)]^2 = x^2 + 4x + 3 + 1$$
$$[g(x)]^2 = x^2 + 4x + 4$$
$$[g(x)]^2 = (x+2)^2$$
$$g(x) = x + 2$$

**2** (a)
$$gf(x) = \frac{3}{x-2}$$
$$g[f(x)] = \frac{3}{x-2}$$
$$g(x+1) = \frac{3}{x-2}$$
Let $x + 1 = u$
$$x = u - 1$$
$$g(u) = \frac{3}{(u-1)-2}$$
$$g(u) = \frac{3}{u-3}$$
$$g(x) = \frac{3}{x-3}, \; x \neq 3$$

(b)
$$gf(x) = \frac{5}{10x-1}$$
$$g[f(x)] = \frac{5}{10x-1}$$
$$g\left(\frac{1}{x}\right) = \frac{5}{10x-1}$$
Let $\frac{1}{x} = u$
$$x = \frac{1}{u}$$
$$g(u) = \frac{5}{10\left(\frac{1}{u}\right) - 1}$$
$$g(u) = \frac{5u}{10-u}$$
$$g(x) = \frac{5x}{10-x}, \; x \neq 10$$

(c) $gf(x) = 9x^2 + 9x + 2$

$g[f(x)] = 9x^2 + 9x + 2$

$g[3x+2] = 9x^2 + 9x + 2$

Let $3x + 2 = u$

$x = \frac{u-2}{3}$

$g(u) = 9\left(\frac{u-2}{3}\right)^2 + 9\left(\frac{u-2}{3}\right) + 2$

$g(u) = 9\left(\frac{u^2 - 4u + 4}{9}\right) + 3(u-2) + 2$

$g(u) = u^2 - 4u + 4 + 3u - 6 + 2$

$g(u) = u^2 - u$

$g(x) = x^2 - x$

**3** (a) $fg(x) = 4x - 12$

$f[g(x)] = 4x - 12$

$2g(x) = 4x - 12$

$g(x) = 2x - 6$

$hf(x) = \frac{2x+1}{2}$

$h[f(x)] = \frac{2x+1}{2}$

$h(2x) = \frac{2x+1}{2}$

Let $2x = u$

$x = \frac{u}{2}$

$h(u) = \frac{2\left(\frac{u}{2}\right) + 1}{2}$

$h(u) = \frac{u+1}{2}$

$h(x) = \frac{x+1}{2}$

(b) $gf(x) = \frac{2x-1}{3}$

$g[f(x)] = \frac{2x-1}{3}$

$g(2x-2) = \frac{2x-1}{3}$

Let $2x - 2 = u$

$x = \frac{u+2}{2}$

$g(u) = \frac{2\left(\frac{u+2}{2}\right) - 1}{3}$

$g(u) = \frac{u+2-1}{3}$

$g(u) = \frac{u+1}{3}$

$g(x) = \frac{x+1}{3}$

$fh(x) = 2x^2$

$f[h(x)] = 2x^2$

$2h(x) - 2 = 2x^2$

$h(x) - 1 = x^2$

$h(x) = x^2 + 1$

(c) $fg(x) = x^2 + 6x + 7$

$f[g(x)] = x^2 + 6x + 7$

$[g(x)]^2 - 2 = x^2 + 6x + 7$

$[g(x)]^2 = x^2 + 6x + 9$

$[g(x)]^2 = (x+3)^2$

$g(x) = x + 3$

$hf(x) = 2x^2 - 7$

$h[f(x)] = 2x^2 - 7$

$h(x^2 - 2) = 2x^2 - 7$

Let $x^2 - 2 = u$

$x^2 = u + 2$

$h(u) = 2(u+2) - 7$

$h(u) = 2u - 3$

$h(x) = 2x - 3$

## UPSKILL 1.2c

**1** $gf(x) = g[f(x)]$

$= g(1-x)$

$= p(1-x)^2 + h$

$= p(1-2x+x^2) + h$

$= p - 2px + px^2 + h$

$= px^2 - 2px + p + h$

But it is given that $gf(x) = 3x^2 - 6x + 5$.

By comparison,

$p = 3$ and $p + h = 5$

$3 + h = 5$

$h = 2$

**2** (a) $f(x) = hx + k$

$f^2(x) = ff(x)$

$= f(hk + k)$

$= h(hk + k) + k$

$= h^2k + hk + k$

But it is given that $f^2(x) = 81x - 16$.

By comparison,

$h^2 = 81$

$h = \pm 9$

$hk + k = -16$

When $h = 9$,

$9k + k = -16$

$10k = -16$

$k = -\frac{8}{5}$

When $h = -9$,

$-9k + k = -16$

$-8k = -16$

$k = 2$

(b) When $h = -9$ and $k = 2$,

$f(x) = -9x + 2$

$f(x^2) = 3x$

$-9x^2 + 2 = 3x$

$9x^2 + 3x - 2 = 0$

$(3x-1)(3x+2) = 0$

$x = \frac{1}{3}$ or $-\frac{2}{3}$

## UPSKILL 1.3

**1** Let $f^{-1}(4) = y$

$f(y) = 4$

(a) $3 - 2y = 4$

$y = \frac{1}{-2}$

$f^{-1}(4) = -\frac{1}{2}$

(b) $6 - \frac{5}{y} = 4$

$\frac{5}{y} = 2$

$y = \frac{5}{2}$

$f^{-1}(4) = \frac{5}{2}$

(c) $\frac{3y+2}{2y+3} = 4$

$3y + 2 = 8y + 12$

$5y = -10$

$y = -2$

$f^{-1}(4) = -2$

**2** (a) The horizontal line intersects the curve at only one point. Hence, $f(x) = \frac{2x-1}{x+2}$, $x \neq -2$ has inverse function.

(b) The horizontal line intersects the curve at more than one point. Hence, $f(x) = x^2 - 5x + 6$ $x \neq -2$ does not have inverse function.

**3** (a) Let $f^{-1}(x) = y$

$f(y) = x$

$5 - 4y = x$

$4y = 5 - x$

$y = \frac{5-x}{4}$

$f^{-1}(x) = \frac{5-x}{4}$

(b) Let $g^{-1}(x) = y$

$$g(y) = x$$
$$\frac{3y-4}{2} = x$$
$$3y - 4 = 2x$$
$$y = \frac{2x+4}{3}$$
$$g^{-1}(x) = \frac{2x+4}{3}$$

(c) Let $h^{-1}(x) = y$

$$h(y) = x$$
$$9 - \frac{3}{y} = x$$
$$\frac{3}{y} = 9 - x$$
$$y = \frac{3}{9-x}$$
$$h^{-1}(x) = \frac{3}{9-x}, \ x \neq 9$$

(d) Let $m^{-1}(x) = y$

$$m(y) = x$$
$$\frac{2y+2}{5y-3} = x$$
$$2y + 2 = 5xy - 3x$$
$$2y - 5xy = -3x - 2$$
$$y(2-5x) = -3x - 2$$
$$y = \frac{-3x-2}{2-5x}$$
$$m^{-1}(x) = \frac{3x+2}{5x-2}, \ x \neq \frac{2}{5}$$

(e) Let $n^{-1}(x) = y$

$$n(y) = x$$
$$\sqrt{2-y} = x$$
$$2 - y = x^2$$
$$y = 2 - x^2$$
$$n^{-1}(x) = 2 - x^2$$

$$f(x) = 5 - 4x$$
$$f^{-1}(x) = \frac{5-x}{4}$$
$$ff^{-1}(x) = f\left(\frac{5-x}{4}\right)$$
$$= 5 - 4\left(\frac{5-x}{4}\right)$$
$$= 5 - (5 - x)$$
$= x$ [Shown]

$$f^{-1}f(x) = f^{-1}(5-4x)$$
$$= \frac{5-(5-4x)}{4}$$
$$= \frac{4x}{4}$$
$= x$ [Shown]

**4** (a) Let $g^{-1}(x) = y$

$$g(y) = x$$
$$\frac{y-1}{y-2} = x$$
$$y - 1 = xy - 2x$$
$$y - xy = 1 - 2x$$
$$y(1-x) = 1 - 2x$$
$$y = \frac{1-2x}{1-x}$$
$$g^{-1}(x) = \frac{1-2x}{1-x}, \ x \neq 1$$

(b) $gg^{-1}(x) = g\left(\frac{1-2x}{1-x}\right)$

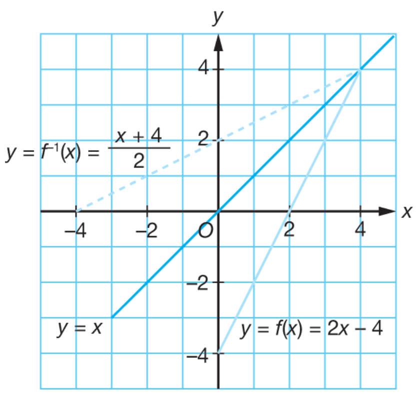
$$= \frac{\left(\frac{1-2x}{1-x}\right)-1}{\left(\frac{1-2x}{1-x}\right)-2}$$
$$= \frac{\frac{1-2x-(1-x)}{1-x}}{\frac{1-2x-2(1-x)}{1-x}}$$
$$= \frac{1-2x-1+x}{1-2x-2+2x}$$
$$= \frac{-x}{-1}$$
$= x$ [Shown]

$$g^{-1}g(x) = g^{-1}\left(\frac{x-1}{x-2}\right)$$
$$= \frac{1-2\left(\frac{x-1}{x-2}\right)}{1-\left(\frac{x-1}{x-2}\right)}$$
$$= \frac{\frac{x-2-2x+2}{x-2}}{\frac{x-2-x+1}{x-2}}$$
$$= \frac{-x}{-1}$$
$= x$ [Shown]

**5** (a) Let $f^{-1}(x) = y$

$$f(y) = x$$
$$2y - 4 = x$$
$$y = \frac{x+4}{2}$$
$$f^{-1}(x) = \frac{x+4}{2}$$

(b)




The graph of $f^{-1}$ is the reflection of the graph of $f$ in the straight line $y = x$.

(c) (i) The domain of $f(x)$ is $0 \le x \le 4$.
The range of $f(x)$ is $-4 \le f(x) \le 4$.

(ii) The domain of $f^{-1}(x)$ is $-4 \le x \le 4$.

The range of $f^{-1}(x)$ is

$0 \le f^{-1}(x) \le 4$.

*Conclusion*

- The domain of $f^{-1}(x)$ is the range of $f(x)$.
- The range of $f^{-1}(x)$ is the domain of $f(x)$.

**6** (a) $f(x) = \frac{3x-1}{x-2}$, $x \ne 2$

It is given that $x \ne h$.
By comparison, $h = 2$.

(b) $f^2(x) = ff(x)$ $\quad f(x) = \frac{3x-1}{x-2}$

$$= f\left(\frac{3x-1}{x-2}\right)$$
$$= \frac{3\left(\frac{3x-1}{x-2}\right)-1}{\left(\frac{3x-1}{x-2}\right)-2}$$
$$= \frac{\frac{3(3x-1)-(x-2)}{x-2}}{\frac{3x-1-2(x-2)}{x-2}}$$
$$= \frac{9x-3-x+2}{3x-1-2x+4}$$
$$= \frac{8x-1}{x+3}, \ x \ne -3$$

(c) Let $f^{-1}(x) = y$

$$f(y) = x$$
$$\frac{3y-1}{y-2} = x$$
$$3y - 1 = xy - 2x$$
$$3y - xy = 1 - 2x$$
$$y(3-x) = 1 - 2x$$
$$y = \frac{1-2x}{3-x}$$
$$f^{-1}(x) = \frac{2x-1}{x-3}, \ x \ne 3$$

**7** $\quad f(x) = \frac{4}{x}$

$g(x) = 2x + 3$

(a) Let $f^{-1}(x) = y$

$$f(y) = x$$
$$\frac{4}{y} = x$$
$$y = \frac{4}{x}$$
$$f^{-1}(x) = \frac{4}{x}, \ x \ne 0$$

(b) Let $g^{-1}(x) = y$

$$g(y) = x$$
$$2y + 3 = x$$
$$y = \frac{x-3}{2}$$
$$g^{-1}(x) = \frac{x-3}{2}$$

(c) $f^{-1}g^{-1} = f^{-1}\left(\frac{x-3}{2}\right)$

$$= \frac{4}{\frac{x-3}{2}}$$
$$= \frac{8}{x-3}, \ x \neq 3$$

(d) $g^{-1}f^{-1}(x) = g^{-1}\left(\frac{4}{x}\right)$

$$= \frac{\frac{4}{x} - 3}{2}$$
$$= \frac{4-3x}{2x}, \ x \neq 0$$

**8** $f(x) = 1 - 2x$

$$g(x) = \frac{x+2}{x-2}$$

(a) Let $f^{-1}(x) = y$

$$f(y) = x$$
$$1 - 2y = x$$
$$y = \frac{1-x}{2}$$
$$f^{-1}(x) = \frac{1-x}{2}$$

(b) Let $g^{-1}(x) = y$

$$g(y) = x$$
$$\frac{y+2}{y-2} = x$$
$$y + 2 = xy - 2x$$
$$xy - y = 2x + 2$$
$$y(x-1) = 2x + 2$$
$$y = \frac{2x+2}{x-1}$$
$$g^{-1}(x) = \frac{2x+2}{x-1}, \ x \neq 1$$

(c) $g^{-1}f^{-1} = g^{-1}\left(\frac{1-x}{2}\right)$

$$= \frac{2\left(\frac{1-x}{2}\right) + 2}{\left(\frac{1-x}{2}\right) - 1}$$
$$= \frac{3-x}{\frac{1-x-2}{2}}$$
$$= \frac{6-2x}{-x-1}$$
$$= \frac{2x-6}{x+1}, \ x \neq -1$$

(d) $fg(x) = f\left(\frac{x+1}{x-2}\right)$

$$= 1 - 2\left(\frac{x+1}{x-2}\right)$$
$$= \frac{x-2-2(x+1)}{x-2}$$
$$= \frac{x-2-2x-4}{x-2}$$
$$= \frac{-x-6}{x-2}$$
$$fg(x) = \frac{x+6}{2-x}, \ x \neq 2$$

(e) $y = \frac{x+6}{2-x}$

$$y(2-x) = x + 6$$
$$2y - xy = x + 6$$
$$x + xy = -2y - 6$$
$$x(1+y) = 2y - 6$$
$$x = \frac{2y-6}{y+1}$$
$$(fg)^{-1}(x) = \frac{2x-6}{x+1}, \ x \neq -1$$

Yes, $(fg)^{-1}(x) = g^{-1}f^{-1}(x)$

**9** (a) $f^2(x) = ff(x)$

$$= f(2x-1)$$
$$= 2(2x-1) - 1$$
$$= 4x - 3$$

Let $f^{-1}(x)=y$

$$f(y)=x$$
$$2y-1=x$$
$$y=\frac{x+1}{2}$$
$$f^{-1}(x)=\frac{x+1}{2}$$

(b) $\left(f^{-1}\right)^2(x)=f^{-1}f^{-1}(x)$

$$=f^{-1}\left(\frac{x+1}{2}\right)$$
$$=\frac{\left(\frac{x+1}{2}\right)+1}{2}$$
$$=\frac{x+1+2}{4}$$
$$=\frac{x+3}{4}$$

$$f^2(x)=ff(x)$$
$$=f(2x-1)$$
$$=2(2x-1)-1$$
$$=4x-3$$

Let $y=4x-3$

$$x=\frac{y+3}{4}$$
$$\left(f^2\right)^{-1}(x)=\frac{x+3}{4}$$

Hence, $\left(f^{-1}\right)^2(x)=\left(f^2\right)^{-1}(x)$

[Shown]

**10** Let $f^{-1}(x)=y$

$$f(y)=x$$
$$\frac{y+p}{y-5}=x$$
$$y+p=xy-5x$$
$$xy-y=p+5x$$
$$y(x-1)=p+5x$$
$$y=\frac{5x+p}{x-1}$$
$$f^{-1}(x)=\frac{5x+p}{x-1},\ x\neq 1$$

But it is given that $f^{-1}(x)=\frac{qx+6}{x-1}$, $x\neq 1$.

Hence, by comparison, $q=5$ and $p=6$.

**11** (a) Let $f^{-1}(x)=y$

$$f(y)=x$$
$$4y+h=x$$
$$y=\frac{x-h}{4}$$
$$f^{-1}(x)=\frac{x-h}{4}$$

But it is given that $f^{-1}(x)=\frac{x+5}{k}$.

Hence, by comparison,
$h=-5$ and $k=4$.

(b) $f^{-1}f(b)=b^2-2$

$$b=b^2-2$$
$$b^2-b-2=0$$
$$(b-2)(b+1)=0$$
$$b=2 \text{ or } -1$$

**12** (a) Let $f^{-1}(x)=y$

$$f(y)=x$$
$$\frac{3y-1}{y}=x$$
$$3y-1=xy$$
$$3y-xy=1$$
$$y(3-x)=1$$
$$y=\frac{1}{3-x}$$
$$f^{-1}(x)=\frac{1}{3-x},\ x\neq 3$$

But it is given that $f^{-1}(x)=\frac{1}{m-x}$.

Hence, by comparison, $m=3$.

(b) $f^{-1}f\left(k^2+2\right)=(k+2)^2+2$

$$k^2+2=k^2+4k+4+2$$
$$4k+4=0$$
$$k=-1$$

**13** (a) Let $y=\frac{3x+8}{4}$

$$4y=3x+8$$
$$x=\frac{4y-8}{3}$$
$$f(x)=\frac{4x-8}{3}$$

(b) Let $y = \frac{2x+1}{x+4}$

$y(x+4) = 2x+1$

$xy + 4y = 2x+1$

$xy - 2x = 1-4y$

$x(y-2) = 1-4y$

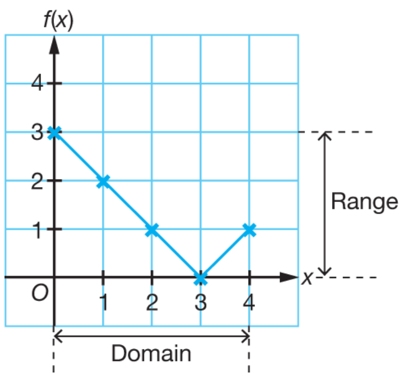
$x = \frac{1-4y}{y-2}$

$g(x) = \frac{1-4x}{x-2},\ x \neq 2$

## Summative Practice 1

**1** (a)

| $x$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $f(x)$ | 3 | 2 | 1 | 0 | 1 |




(b) The corresponding range of $f(x)$ is $0 \leq f(x) \leq 3$.

**2** $fg : x \to x^2 + 1$

$f[g(x)] = x^2 + 1$

$3g(x) - 7 = x^2 + 1$

$3g(x) = x^2 + 8$

$g(x) = \frac{x^2+8}{3}$

**3** $gf : x \to x^2 + 1$

$g[f(x)] = x^2 + 1$

$g(x-2) = x^2 + 1$

Let $x - 2 = u$

$x = u + 2$

$g(u) = (u+2)^2 + 1$

$g(u) = u^2 + 4u + 4 + 1$

$g(u) = u^2 + 4u + 5$

$g(x) = x^2 + 4x + 5$

**4** $g(x) = px + q$

$g^2 : x \to 49x - 32$

$g^2(x) = 49x - 32$

$g[g(x)] = 49x - 32$

$g(px+q) = 49x - 32$

$p(px+q) + q = 49x - 32$

$p^2x + pq + q = 49x - 32$

By comparison,

$p^2 = 49$ and $pq + q = -32$

$p = 7$ (Given $p > 0$)

When $p = 7$, $7q + q = -32$

$8q = -32$

$q = -4$

**5** (a) Let $f^{-1}(x) = y$

$f(y) = x$

$2y - 5 = x$

$y = \frac{x+5}{2}$

$f^{-1}(x) = \frac{x+5}{2}$

$f^{-1}g(x)$

$= f^{-1}[g(x)]$

$= f^{-1}\left(\frac{3x}{x+3}\right)$

$= \frac{\left(\frac{3x}{x+3}\right)+5}{2}$

$= \frac{\frac{3x+5(x+3)}{x+3}}{2}$

$= \frac{8x+15}{2(x+3)}$

$= \frac{8x+15}{2x+6},\ x \neq -3$

(b) $g[f(x)]$

$= g(2x-5)$

$= \frac{3(2x-5)}{(2x-5)+3}$

$= \frac{6x-15}{2x-2}$

$$f^2(4) = ff(4) = f(2\times4-5) = f(3) = 2\times3-5 = 1$$

$$gf(-k) = f^2(4)$$

$$\frac{6(-k)-15}{2(-k)-2} = 1$$

$$-6k-15 = -2k-2$$

$$-4k = 13$$

$$k = -\frac{13}{4}$$

**6** (a) Let $f^{-1}(x) = y$

$$f(y) = x$$

$$p - qy = x$$

$$y = \frac{p-x}{q}$$

$$f^{-1}(x) = \frac{p-x}{q}$$

(b) $f(2) = 7$

$$p - 2q = -7$$

$$p = 2q - 7 \ \dots (1)$$

$$f^{-1}(8) = -1$$

$$\frac{p-8}{q} = -1$$

$$p - 8 = -q$$

$$p = 8 - q \ \dots (2)$$

Substituting (1) into (2) :

$$2q - 7 = 8 - q$$

$$3q = 15$$

$$q = 5$$

From (2): $p = 8 - 5 = 3$

**7** (a) Let $f^{-1}(x) = y$

$$f(y) = x$$

$$2y - 3 = x$$

$$y = \frac{x+3}{2}$$

$$f^{-1}(x) = \frac{x+3}{2}$$

$$f^{-1}g(x) = f^{-1}\left(\frac{x}{2}+2\right) = \frac{\frac{x}{2}+2+3}{2} = \frac{x+10}{4}$$

(b) $hg(x) = 2x + 4$

$$h[g(x)] = 2x + 4$$

$$h\left(\frac{x}{2}+2\right) = 2x + 4$$

Let $\frac{x}{2} + 2 = u$

$$\frac{x}{2} = u - 2$$

$$x = 2u - 4$$

$$h(u) = 2(u-4) + 4$$

$$h(u) = 2u - 8 + 4$$

$$h(u) = 2u - 4$$

$$h(x) = 2x - 4$$

**8** (a) $f(y) = ay + b$

$$g(y) = \frac{5}{3y-b}$$

| | |
|---|---|
| $f(3) = -2$ | $g(3) = 1$ |
| $3a + b = -2$ | $\frac{5}{3(3)-b} = 1$ |
| $3a + 4 = -2$ | $9 - b = 5$ |
| $a = -2$ | $b = 4$ |

(b) The function that maps $x$ onto $y$ is $f^{-1}(x)$.

It is found that $f(y) = -2y + 4$ .

Let $w = -2y + 4$,

$$y = \frac{4-w}{2}$$

$$f^{-1}(x) = \frac{4-x}{2}$$

(c) Hence, the function that maps $x$ onto $z$ is $gf^{-1}(x)$.

$$gf^{-1}(x)$$
$$= g\left(\frac{4-x}{2}\right)$$
$$= \frac{5}{3\left(\frac{4-x}{2}\right)-4}$$
$$= \frac{10}{3(4-x)-8}$$
$$= \frac{10}{12-3x-8}$$
$$= \frac{10}{4-3x},\ x \neq \frac{4}{3}$$

**9** (a) $gf(x) = g(h-x^2)$
$$= k(h-x^2)+2$$
$$= hk-kx^2+2$$
$$= hk+2-kx^2$$

But it is given that $gf(x) = 14-3x^2$.
By comparison, $k = 3$
$3h + 2 = 14$
$h = 4$

(b) Let $g^{-1}(-13) = y$
$$g(y) = -13$$
$$3y + 2 = -13$$
$$y = -5$$
$$g^{-1}(-13) = -5$$

$$f(t) = -5$$
$$4-t^2 = -5$$
$$t^2 = 9$$
$$t = \pm 3$$

**10** (a) $f(x) = \frac{hx}{x-3},\ x \neq 3$

Let $f^{-1}(x) = y$.
$$f(y) = x$$
$$\frac{hy}{y-3} = x$$
$$hy = x(y-3)$$
$$hy = xy-3x$$
$$xy-hy = 3x$$
$$y(x-h) = 3x$$
$$y = \frac{3x}{x-h}$$
$$f^{-1}(x) = \frac{3x}{x-h}$$

But it is given that $f^{-1}(x) = \frac{kx}{x-2}$.
By comparison,
$k = 3$ and $h = 2$

(b) When $h = 2$, $f^{-1}(x) = \frac{3x}{x-2}$
$$gf^{-1}(x) = -5x$$
$$g\left(\frac{3x}{x-2}\right) = -5x$$
$$\frac{1}{\left(\frac{3x}{x-2}\right)} = -5x$$
$$\frac{x-2}{3x} = -5x$$
$$x-2 = -15x^2$$
$$15x^2+x-2 = 0$$
$$(3x-1)(5x+2) = 0$$
$$x = \frac{1}{3} \text{ or } x = -\frac{2}{5}$$

**11** (a) $fg(x) = f[g(x)]$
$$= f\left(\frac{2+x}{4-3x}\right)$$
$$= \frac{2\left(\frac{2+x}{4-3x}\right)-1}{\left(\frac{2+x}{4-3x}\right)-3}$$
$$= \frac{\frac{2(2+x)-(4-3x)}{4-3x}}{\frac{2+x-3(4-3x)}{4-3x}}$$
$$= \frac{4+2x-4+3x}{2+x-12+9x}$$
$$= \frac{5x}{10x-10}$$
$$= \frac{x}{2x-2},\ x \neq 1$$

(b) Let $y = \frac{x}{2x-2}$

$$2xy - 2y = x$$
$$2xy - x = 2y$$
$$x(2y-1) = 2y$$
$$x = \frac{2y}{2y-1}$$
$$(fg)^{-1} = \frac{2x}{2x-1}, \quad x \neq \frac{1}{2}$$

**12** (a) $fg : x \to x^2 + 1$

$$f[g(x)] = x^2 + 1$$
$$2g(x) + 2 = x^2 + 1$$

Substitute the $x$ in $f(x) = 2x + 2$ with $g(x)$.

$$2g(x) = x^2 - 1$$
$$g(x) = \frac{x^2 - 1}{2}$$

Hence, $g(3) = \frac{3^2 - 1}{2} = 4$

(b) $f : x \to 2 - x$

$$f(x) = 2 - x$$

Let $f^{-1}(x) = y$

$$f(y) = x$$
$$2 - y = x$$
$$y = 2 - x$$
$$\therefore f^{-1}(x) = 2 - x$$

$$gf^{-1} : x \to 3x^2 - 12x + 13$$
$$g\left[f^{-1}(x)\right] = 3x^2 - 12x + 13$$
$$g(2-x) = 3x^2 - 12x + 13$$
$$a + b(2-x)^2 = 3x^2 - 12x + 13$$
$$a + b\left(4 - 4x + x^2\right) = 3x^2 - 12x + 13$$
$$a + 4b - 4bx + bx^2 = 3x^2 - 12x + 13$$

By comparison,
$b = 3$

$$a + 4b = 13$$
$$a + 4(3) = 13$$
$$a = 1$$

**13** (a) $fg : x \to x + 2$

$$f\left[g(x)\right] = x + 2$$
$$a[g(x)] + b = x + 2$$
$$a(2x-1) + b = x + 2$$
$$2ax - a + b = x + 2$$

By comparison,
$2a = 1$

$$a = \frac{1}{2}$$

$$-a + b = 2$$
$$-\frac{1}{2} + b = 2$$
$$b = \frac{5}{2}$$

(b) Let $h^{-1}(x) = y$

$$h(y) = x$$
$$\frac{1}{y-3} = x$$
$$1 = xy - 3x$$
$$xy = 1 + 3x$$
$$y = \frac{1+3x}{x}$$
$$\therefore h^{-1}(x) = \frac{1+3x}{x}$$

$$h^{-1}g(x) = 1$$
$$h^{-1}\left[g(x)\right] = 1$$
$$\frac{1+3g(x)}{g(x)} = 1$$
$$1 + 3g(x) = g(x)$$
$$2g(x) = -1$$
$$2(2x-1) = -1$$
$$4x - 2 = -1$$
$$4x = 1$$
$$x = \frac{1}{4}$$

**14** (a) $fg(x) = 5x - 3$

$$f\left[g(x)\right] = 5x - 3$$
$$g(x) + 14 = 5x - 3$$
$$g(x) = 5x - 17$$

(b) Let $h^{-1}(x) = y$

$$h(y) = x$$
$$\frac{y-1}{y+3} = x$$
$$y - 1 = x(y+3)$$
$$y - 1 = xy + 3x$$

$$y - xy = 3x + 1$$
$$y(1 - x) = 3x + 1$$
$$y = \frac{3x+1}{1-x}$$
$$h^{-1}(x) = \frac{3x+1}{1-x}$$

$$g(x) = h^{-1}(x)$$
$$5x - 17 = \frac{3x+1}{1-x}$$
$$(5x-17)(1-x) = 3x+1$$
$$5x - 5x^2 - 17 + 17x = 3x + 1$$
$$5x^2 - 19x + 18 = 0$$
$$(5x-9)(x-2) = 0$$
$$x = \frac{9}{5} \text{ or } 2$$

**15** (a) $g(x) = \frac{2}{x-4},\ x \neq k$

Denominator $\neq 0$

$x - 4 \neq 0$

$x \neq 4$

By comparison,

$k = 4$

(b) $f(x) = \frac{5}{2}x - h$

Let

$$f^{-1}(x) = y$$
$$f(y) = x$$
$$\frac{5}{2}y - h = x$$
$$\frac{5}{2}y = x + h$$
$$y = \frac{2(x+h)}{5}$$
$$f^{-1}(x) = \frac{2(x+h)}{5} = \frac{2x+2h}{5}$$

But it is given that

$f^{-1}(x) = \frac{mx+6}{5}$.

By comparison

$m = 2$ and $2h = 6 \Rightarrow h = 3$

(c) When $h = 3$,

$$f(x) = \frac{5}{2}x - 3$$

$$gf(p+1) = p+2$$
$$g[f(p+1)] = p+2$$
$$g\left[\frac{5}{2}(p+1) - 3\right] = p+2$$

Substitute the $x$ in $f(x) = \frac{5}{2}x - 3$ with $(p+1)$.

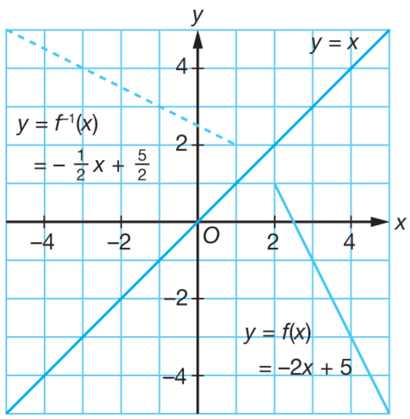
$$g\left(\frac{5}{2}p + \frac{5}{2} - 3\right) = p+2$$
$$g\left(\frac{5}{2}p - \frac{1}{2}\right) = p+2$$
$$\frac{2}{\left(\frac{5}{2}p - \frac{1}{2}\right) - 4} = p+2$$
$$\frac{2}{\left(\frac{5}{2}p - \frac{9}{2}\right)} = p+2$$
$$\frac{4}{(5p-9)} = p+2$$
$$4 = (p+2)(5p-9)$$
$$4 = 5p^2 + p - 18$$
$$5p^2 + p - 22 = 0$$
$$(5p+11)(x-2) = 0$$
$$p = -\frac{11}{5} \text{ or } p = 2$$

**16** (a) $f f^{-1}(p^2) = f(p-5)$

$$p^2 = -2(p-5)+5$$
$$p^2 = -2p+10+5$$
$$p^2 + 2p - 15 = 0$$
$$(p-3)(p+5) = 0$$
$$p = 3 \text{ or} -5$$

(b) Let

$$f^{-1}(x) = y$$
$$f(y) = x$$
$$-2y+5 = x$$
$$2y = 5-x$$
$$y = \frac{5-x}{2}$$
$$f^{-1}(x) = -\frac{1}{2}x + \frac{5}{2}$$




The graph of $f^{-1}$ is the reflection of the graph of $f$ in the straight line $y = x$.

(c) The range of $f(x)$ is $-5 \le x \le 1$.

The domain of $f^{-1}(x)$ is $-5 \le f^{-1}(x) \le 1$.

The range of $f^{-1}(x)$ is $2 \le f^{-1}(x) \le 5$.

The domain of $f^{-1}(x)$ is the range of $f(x)$.

The range of $f^{-1}(x)$ is the domain of $f(x)$.