

# Fully-Worked Solutions

## CHAPTER 6 Angles and Tangents of Circles

### UPSKILL 6.1

1  $\angle SPR = \angle SQR$ ,  $\angle PQS = \angle PRS$ ,  $\angle PRQ = \angle PSQ$ ,  $\angle QSR = \angle QPR$

2  $\angle BAC$  and  $\angle BDC$  are angles subtended by arc  $BC$ .  $\therefore p = 38^\circ$   
 $\angle ABD$  and  $\angle ACD$  are angles subtended by arc  $AD$ .  $\therefore q = 66^\circ$

3 (a)  $\angle QSP = \angle QTP = 29^\circ$  (b)  $\angle QRP = \angle QTP = 29^\circ$   
 (c)  $\angle TPU = \angle TQU = 18^\circ$  (d)  $\angle RQS = \angle RPS = 21^\circ$

4 (a)  $x = 25^\circ$  (b)  $\frac{x}{17^\circ} = \frac{6}{2}$   
 $x = 3 \times 17^\circ = 51^\circ$

(c)  $\frac{15^\circ}{45^\circ} = \frac{7}{x}$   
 $x = 3 \times 7 \text{ cm} = 21 \text{ cm}$

5 (a)  $\angle CAD = \angle CBD = 45^\circ$

(b)  $\angle ADB = 3(\angle BDC)$   
 $= 3(20^\circ) = 60^\circ$

(c)  $\angle ACB = \angle ADB = 60^\circ$   
 (d)  $\angle BEC = 180^\circ - 45^\circ - 60^\circ = 75^\circ$

6 (a)  $p = \frac{1}{2} \times 136^\circ = 68^\circ$

(b)  $q = \frac{1}{2} \times 214^\circ = 107^\circ$

(c)  $r = \frac{1}{2} \times 80^\circ = 40^\circ$

(d)  $x = 360^\circ - 2 \times 58^\circ = 244^\circ$

$y = \frac{1}{2}x = \frac{1}{2}(244^\circ) = 122^\circ$

7  $x = 23^\circ$   $y = 2x = 2(23^\circ) = 46^\circ$

8 (a)  $\angle ABC = \frac{1}{2}$  reflex angle  $\angle AOC = \frac{1}{2}(360^\circ - 160^\circ) = 100^\circ$

(b)  $\angle ADE = \frac{1}{2} \times \angle AOE = \frac{1}{2} \times 20^\circ = 10^\circ$

(c)  $\angle CED = \angle FED = 30^\circ - 10^\circ = 20^\circ$

9 (a)  $\frac{y}{7.8} = \frac{28^\circ}{56^\circ}$   
 $y = \frac{28}{56} \times 7.8 = 3.9 \text{ cm}$

(b)  $\frac{y}{100^\circ} = \frac{12}{48}$   
 $y = \frac{12}{48} \times 100^\circ = 25^\circ$

10 (a)  $\angle ACB = 90^\circ$   
 $p + p + 90^\circ = 180^\circ$   
 $2p = 90^\circ$   
 $p = 45^\circ$

(b)  $\angle AOC = 2 \times 35^\circ = 70^\circ$   
 $q + q + 70^\circ = 180^\circ$   
 $2q = 110^\circ$   
 $q = 55^\circ$

(c)  $\angle BDC = \angle BAC = 63^\circ$  and  $\angle BCD = 90^\circ$   
 $r + 63^\circ + 90^\circ = 180^\circ$   
 $r = 180^\circ - 153^\circ = 27^\circ$

Alternative method:  $\angle ABC = 90^\circ$  and  $\angle ABO = 63^\circ$   
 $r = 90^\circ - 63^\circ = 27^\circ$

11  $\angle PQR = 90^\circ$   
 $x + 25^\circ = 90^\circ$   
 $x = 65^\circ$

$\angle PQS = \frac{1}{2}(\angle POS)$   
 $= \frac{1}{2}(80^\circ) = 40^\circ$   
 $y + 40^\circ = 90^\circ$   
 $y = 50^\circ$

12  $\angle COE = 2 \times 40^\circ = 80^\circ$   
 Conjugate angle of  $\angle COE = 360^\circ - 80^\circ = 280^\circ$

$x = \frac{1}{2}(280^\circ) = 140^\circ$

13  $\angle BAC = 90^\circ - 20^\circ = 70^\circ$

$\angle OAD = \frac{180^\circ - 56^\circ}{2} = 62^\circ$   
 $x = \angle BAC + \angle OAD = 70^\circ + 62^\circ = 132^\circ$

14  $x = 42^\circ$

$\angle FDE = \frac{180^\circ - 120^\circ}{2} = 30^\circ$

$\angle BDC = \angle FDE = 30^\circ$   
 $y = 30^\circ$

15 (a)  $\angle PQR = 90^\circ$   
 $x = 180^\circ - 90^\circ - 54^\circ = 36^\circ$

Reflex angle  $\angle TOR = 180^\circ + 35^\circ = 215^\circ$

$y = \frac{1}{2} \times 215^\circ = 107.5^\circ$

(b)  $\sin 54^\circ = \frac{PQ}{PR} = \frac{PQ}{20}$

$PQ = 20 \sin 54^\circ = 16.18 \text{ cm}$

### UPSKILL 6.2

1 (a) Quadrilateral  $BCDE$

(b) Quadrilateral  $FGHJ$

(c) Quadrilaterals  $PSTU$  and  $SPQR$

2 (a)  $x = 180^\circ - 55^\circ = 125^\circ$ ,  $y = 180^\circ - 68^\circ = 112^\circ$

- (b)  $x = 180^\circ - 45^\circ = 135^\circ$   
 $y + 2y = 180^\circ$   
 $3y = 180^\circ$   
 $y = 60^\circ$
- 3  $p = b$  and  $q = a$
- 4  $x = 138^\circ$   $y = 52^\circ$
- 5  $x = 2(30^\circ) = 60^\circ$   
 $30^\circ + y = 180^\circ$   
 $y = 150^\circ$

6 (a) The two cyclic quadrilaterals are  $PQRU$  and  $RSTU$ .

- (b) (i)  $\angle RST = \angle PUR$   
 $= 180^\circ - 68^\circ$   
 $= 112^\circ$
- (ii)  $\angle QPU = \angle URS$   
 $= 180^\circ - 95^\circ$   
 $= 85^\circ$

7 (a) Minor arc of  $DE = 2(\text{minor arc of } BC)$

- $50^\circ = 2x$   
 $x = 25^\circ$
- (b)  $\triangle BCE$ ,  $\angle CBE = 90^\circ$   
 $x + y = 90^\circ$   
 $25^\circ + y = 90^\circ$   
 $y = 90^\circ - 25^\circ$   
 $= 65^\circ$

- (c)  $\angle AEB = \angle ADB = 35^\circ$   
 $\angle AEC = 35^\circ + x$   
 $= 35^\circ + 25^\circ$   
 $= 60^\circ$

$ABCE$  is a cyclic quadrilateral.

$$\angle ABC + \angle AEC = 180^\circ$$

$$z + 90^\circ + 60^\circ = 180^\circ$$

$$z = 30^\circ$$

- 8 (a)  $x = \frac{1}{2} \times 130^\circ$   
 $= 65^\circ$

- (b)  $PQRS$  is a cyclic quadrilateral.  
 $\angle QRS + \angle QPS = 180^\circ$   
 $65^\circ + \angle QPS = 180^\circ$   
 $\angle QPS = 180^\circ - 65^\circ$   
 $= 115^\circ$

$$\text{Quadrilateral } PQOS, \quad y + y + 130^\circ + 115^\circ = 360^\circ$$

$$2y = 360^\circ - 245^\circ$$

$$2y = 115^\circ$$

$$y = 57.5^\circ$$

- (c)  $\angle OSR = 180^\circ - 93^\circ - 57.5^\circ = 29.5^\circ$   
 Conjugate angle of  $\angle QOS = 360^\circ - 130^\circ = 230^\circ$   
 Quadrilateral  $QRSO$ ,  $65^\circ + z + 230^\circ + 29.5^\circ = 360^\circ$   
 $z = 360^\circ - 324.5^\circ$   
 $= 35.5^\circ$

### UPSKILL 6.3

1 (a) **Tangent** **Point of tangency**

$KL$   $N$   
 $LM$   $R$

(b) **Tangent** **Point of tangency**

$PR$   $Q$   
 $RT$   $S$

2  $\angle URS = 90^\circ$ ,  $\angle STQ = 90^\circ$

3  $\angle OBC = 90^\circ$

$$\text{In } \triangle OBC, \quad x + 53^\circ + 90^\circ = 180^\circ$$

$$x + 143^\circ = 180^\circ$$

$$x = 180^\circ - 143^\circ$$

$$= 37^\circ$$

4 (a)  $\angle ORP$  and  $\angle OSP$  are right angles.

- (b) (i)  $\angle POR = \angle POS$   
 (ii)  $\angle SPO = \angle RPO$

(c)  $PS = PR$

5 (a)  $XZ = XY = 7.5$  cm

(b)  $\angle YXZ = 2 \times 25^\circ = 50^\circ$

$$\angle YOZ = 180^\circ - 50^\circ$$

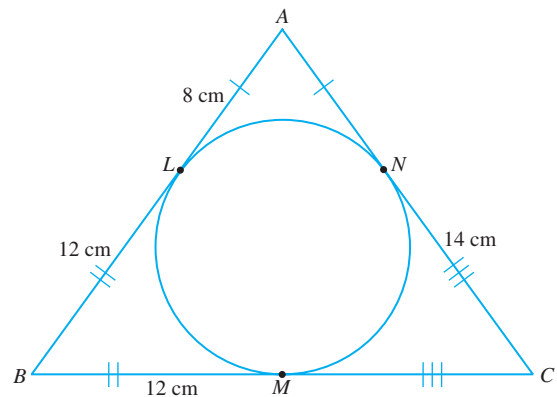
$$= 130^\circ$$

6 (a)  $x = 180^\circ - 40^\circ = 140^\circ$

$$(b) \quad y = \frac{180^\circ - 140^\circ}{2} = 20^\circ$$

$$(c) \quad z = \frac{180^\circ - 40^\circ}{2} = 70^\circ \text{ or } z = 90^\circ - 20^\circ = 70^\circ$$

7



(a)  $BL = BM = 12$  cm

(b)  $CM = CN = 14$  cm

(c)  $AN = AL = 8$  cm

8 (a)  $p = 180^\circ - 140^\circ$   
 $= 40^\circ$

(b)  $q = 180^\circ - 50^\circ$   
 $= 130^\circ$

(c)  $r = 180^\circ - 50^\circ - 40^\circ$   
 $= 90^\circ$

9 (a)  $p = 2\angle BFD$   
 $= 2(55^\circ)$   
 $= 110^\circ$

(b)  $q = 180^\circ - p$   
 $= 180^\circ - 110^\circ$   
 $= 70^\circ$

10 (a)  $x = 67^\circ$ ,  $y = 55^\circ$

(b)  $x = 20^\circ$ ,  $y = 180^\circ - 45^\circ - 85^\circ = 50^\circ$

(c)  $x = 20^\circ$ ,  $y = 180^\circ - 75^\circ - 35^\circ = 70^\circ$

11 (a)  $x = 43^\circ$

(b)  $y = 36^\circ$

(c)  $\angle EBG = 36^\circ$   
 $\angle HGB = 180^\circ - 100^\circ - 36^\circ = 44^\circ$

$$z = \angle EGB$$

$$= 43^\circ + 44^\circ$$

$$= 87^\circ$$

12 (a)  $p = 78^\circ$

(b)  $q = 40^\circ$

(c)  $r = \angle GDB$   
 $= 180^\circ - 92^\circ - 40^\circ$   
 $= 48^\circ$

13 (a)  $p = 70^\circ$

(b)  $\angle OBF = 20^\circ$

$$q = \angle DBF$$

$$= 22^\circ + 20^\circ$$

$$= 42^\circ$$

(c)  $r = 180^\circ - 42^\circ - 70^\circ$   
 $= 68^\circ$

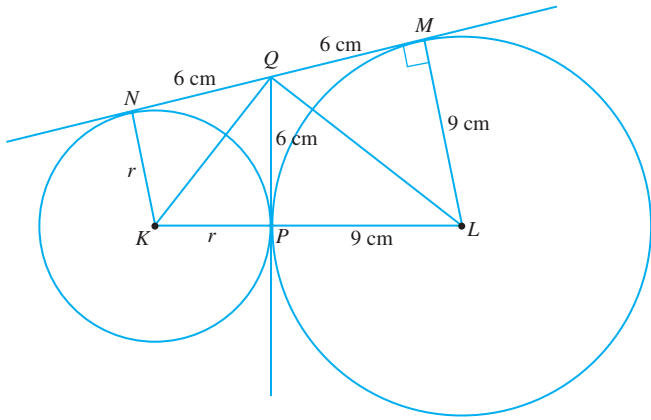
14 (a)  $p = 65^\circ$

(b)  $q = 75^\circ$

(c)  $\angle HJG = \angle HGJ$   
 $= \angle GLJ$   
 $= 180^\circ - 65^\circ - 75^\circ$   
 $= 40^\circ$

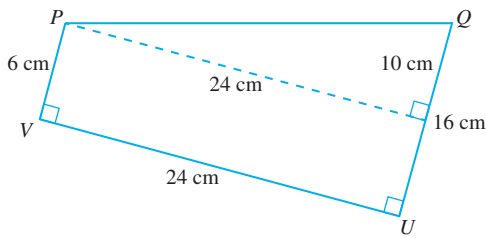
$$r = 180^\circ - 40^\circ - 40^\circ$$

$$= 100^\circ$$

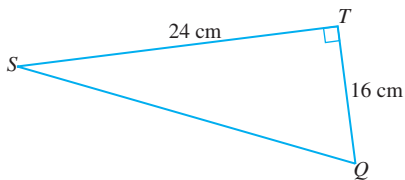


- (a)  $NM = 12$  cm
- (b)  $QL = \sqrt{6^2 + 9^2} = 10.82$  cm
- (c)  $(r + 9)^2 = 12^2 + (9 - r)^2$   
 $r^2 + 18r + 81 = 144 + 81 - 18r + r^2$   
 $18r + 18r = 144$   
 $36r = 144$   
 $r = \frac{144}{36} = 4$  cm

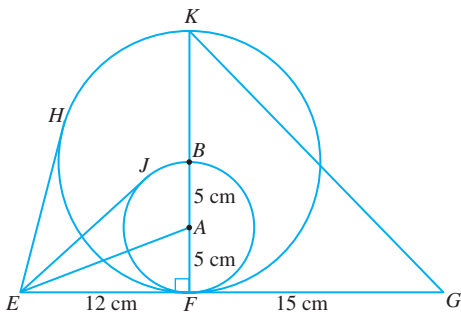
- 16 (a)  $UV = ST = 24$  cm  
 (b)



- (c)  $PQ = \sqrt{10^2 + 24^2} = 26$  cm

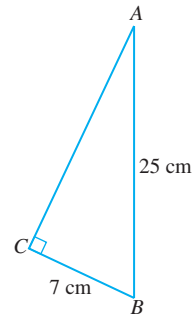


- 17  $QS = \sqrt{16^2 + 24^2} = 28.84$  cm



- (a)  $KF = 2BF = 2(10) = 20$  cm
- (b)  $HE = EF = 12$  cm
- (c)  $AE = \sqrt{12^2 + 5^2} = 13$  cm
- (d)  $KB = BF = 10$  cm  
 $KG = \sqrt{20^2 + 15^2} = 25$  cm

- 18 (a)  $AB = XY = 25$  cm



$$AC = \sqrt{25^2 - 7^2} = 24 \text{ cm}$$

(b)  $\cos \angle ABC = \frac{7}{25}$

$$\angle ABC = 73.74^\circ$$

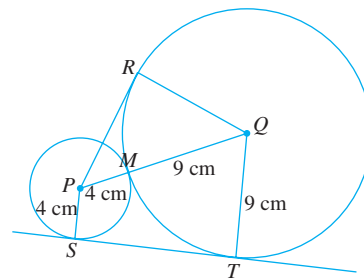
$$\text{Reflex angle } \angle CBY = 360^\circ - 73.74^\circ - 90^\circ = 196.26^\circ$$

$$\text{Major arc length } CY = \frac{196.26^\circ}{360^\circ} \times 2 \times \frac{22}{7} \times 7 = 23.99 \text{ cm}$$

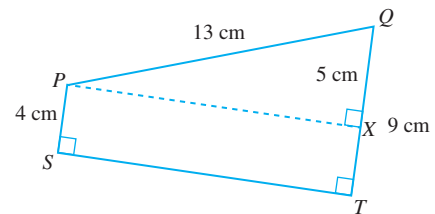
$$\begin{aligned} \text{Length of string from A to X} &= AC + \text{major arc length } CY + XY \\ &= 24 + 23.99 + 25 \\ &= 72.99 \text{ cm} \end{aligned}$$

**UPSKILL 6.4**

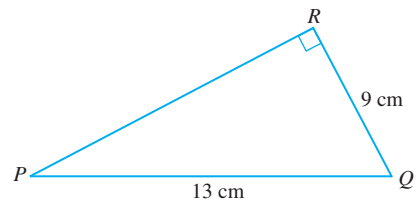
1



(a)



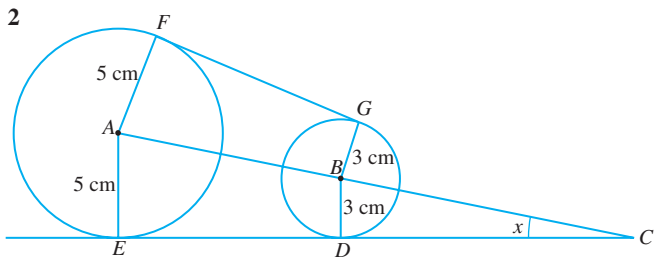
$$ST = PX = \sqrt{13^2 - 5^2} = 12 \text{ cm}$$



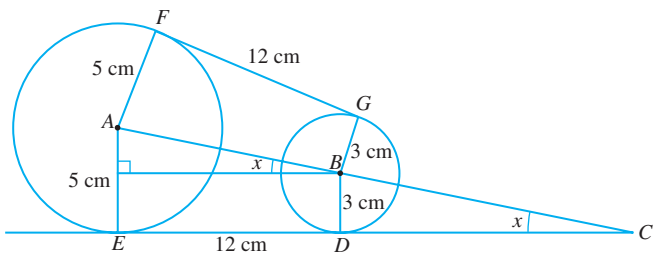
$$PR = \sqrt{13^2 - 9^2} = 9.381 \text{ cm}$$

$$\begin{aligned} \text{Perimeter of pentagon } PSTQR &= 4 + 12 + 9 + 9 + 9.381 \\ &= 43.381 \text{ cm} \end{aligned}$$

- (b) Area of pentagon  $PSTQR$   
 = area of trapezium  $PSTQ$  + area of right-angled triangle  $PRQ$   
 $= \left[ \frac{1}{2} \times (4 + 9) \times 12 \right] + \left[ \frac{1}{2} \times 9 \times 9.381 \right]$   
 $= 120.21 \text{ cm}^2$



- (a)  $ED = FG = 12$  cm  
 (b)

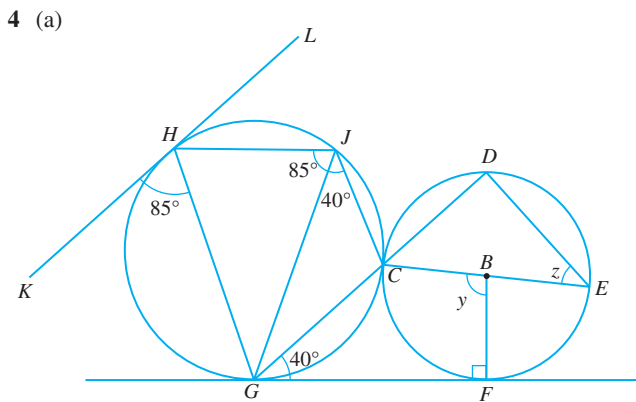


$$\tan x = \frac{2}{12} = \frac{1}{6}$$

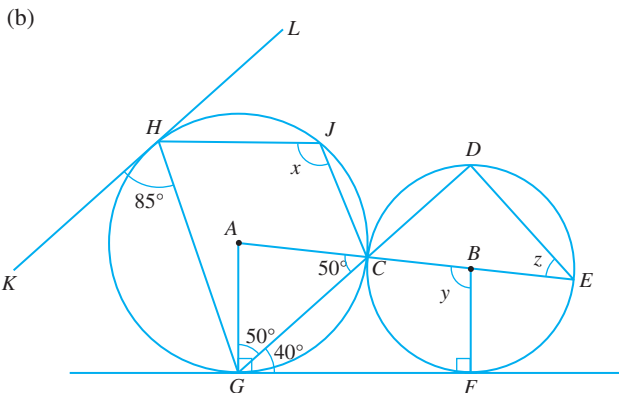
(c)  $\tan x = \frac{3}{CD} = \frac{1}{6}$

$$\therefore CD = 3 \times 6 = 18 \text{ cm}$$

- 3 (a)  $x = 40^\circ$   
 (b)  $\angle UPS = \angle PRS = 60^\circ$   
 $\angle MPN = 180^\circ - 60^\circ - 40^\circ = 80^\circ$   
 $y = 180^\circ - 80^\circ - 40^\circ = 60^\circ$   
 (c)  $\angle RSP = \angle RPT$   
 $= 115^\circ - y$   
 $= 115^\circ - 60^\circ = 55^\circ$   
 $z = 180^\circ - 55^\circ - 80^\circ = 45^\circ$



$$x = 85^\circ + 40^\circ = 125^\circ$$



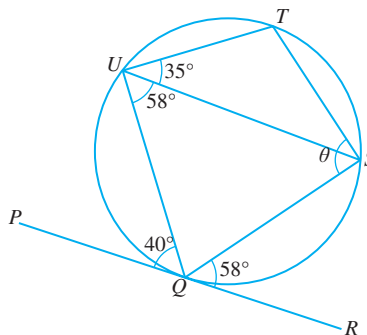
$$y = 360^\circ - 40^\circ - 90^\circ - 130^\circ = 100^\circ$$

(c)  $\angle DCE = \angle ACG = 50^\circ$   
 $\angle CDE = 90^\circ$   
 $\therefore z = 180^\circ - 90^\circ - 50^\circ = 40^\circ$

## Summative Practice 6

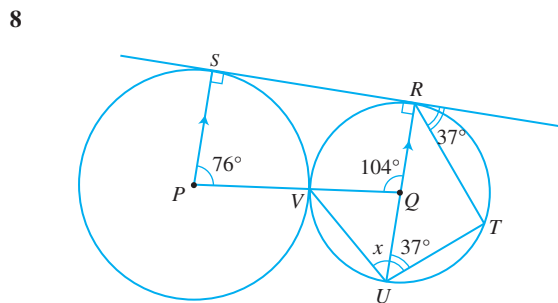
### Section A

- 1 Answer: B  
 2 Reflex angle at O =  $360^\circ - 105^\circ = 255^\circ$   
 $p = \frac{1}{2} \times 255^\circ = 127.5^\circ$   
 Answer: D  
 3 Answer: B  
 4  $x = 180^\circ - 72^\circ = 108^\circ$   
 $3y + y = 180^\circ$   
 $y = 45^\circ$   
 $\therefore x + y = 108^\circ + 45^\circ = 153^\circ$   
 Answer: C  
 5  $x = 99^\circ$ ;  $y = 180^\circ - 65^\circ = 115^\circ$   
 Answer: A  
 6 Answer: D  
 7



$QRSTU$  is a cyclic quadrilateral.  $\theta + 58^\circ + 35^\circ = 180^\circ$   
 $\theta = 180^\circ - 93^\circ = 87^\circ$

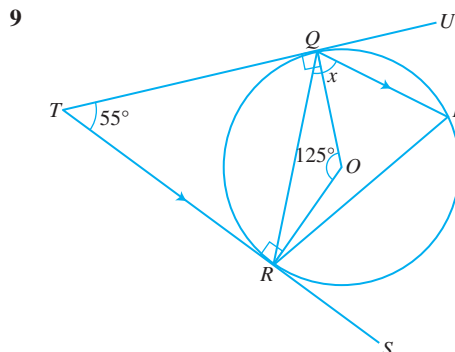
Answer: B



$$\angle QVU = \angle QUV = \frac{104^\circ}{2} = 52^\circ$$

$$x = 37^\circ + 52^\circ = 89^\circ$$

Answer: C



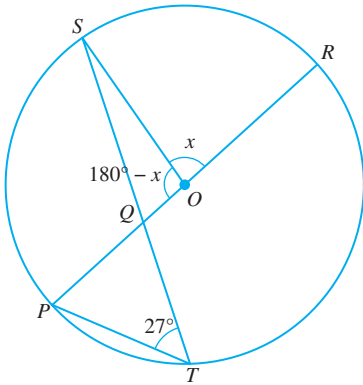
$$\angle TQR = \angle TRQ = \frac{180^\circ - 55^\circ}{2} = 62.5^\circ$$

$PQ \parallel TR$

$$x = \angle TRQ = 62.5^\circ$$

Answer: **B**

10



$$180^\circ - x = 2(27^\circ)$$

$$180^\circ - x = 54^\circ$$

$$x = 180^\circ - 54^\circ = 126^\circ$$

Answer: **C**

11  $PR = \sqrt{6^2 + 9^2}$   
 $= 10.82 \text{ cm}$

$$\angle PRK = \angle QRL$$

$$\sin \angle PRK = \sin \angle QRL$$

$$\frac{PK}{PR} = \frac{QL}{QR}$$

$$\frac{6}{10.82} = \frac{2}{QR}$$

$$6QR = 2 \times 10.82$$

$$QR = 3.61 \text{ cm}$$

$$\therefore PQ = 10.82 + 3.61 = 14.43 \text{ cm}$$

Answer: **C**

12  $3x - 1^\circ = 78^\circ + x + 9^\circ$

$$2x = 88^\circ$$

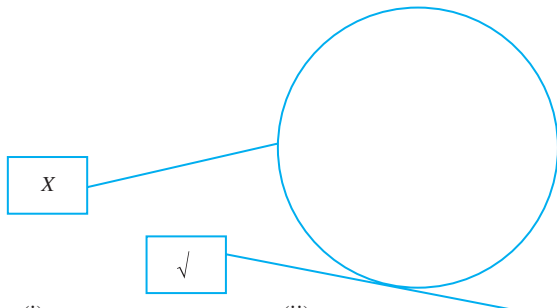
$$x = 44^\circ$$

Answer: **B**

### Section B

1  $p = q$       $p = 2q$       $p = \frac{1}{2}q$       $p = 90^\circ$

2 (a)



(b) (i)  $p = s$

(ii)  $w = t$

3 (a) 1

(b) 2

(c) 3

(d) 0

### Section C

1 (a)  $p + 48^\circ = 81^\circ$

$$p = 33^\circ$$

$$q = p + 60^\circ$$

$$= 33^\circ + 60^\circ$$

$$= 93^\circ$$

(b) (i)  $BC = BD = 4 \text{ cm}$

(ii)  $\tan \theta = \frac{4}{6}$

$$\theta = \tan^{-1} \frac{4}{6} = 33.69^\circ$$

$$x = 2\theta$$

$$= 2 \times 33.69$$

$$= 67.38^\circ$$

(iii)  $\cos x = \frac{6}{OA}$

$$\cos 67.38^\circ = \frac{6}{OA}$$

$$OA = \frac{6}{\cos 67.38^\circ} = 15.6 \text{ cm}$$

$$AD = 15.6 - 6 = 9.6 \text{ cm}$$

2 (a)  $x = 180^\circ - 90^\circ - \frac{82^\circ}{2} = 49^\circ$

(b) (i)  $\angle PAM = \angle PCM = \angle QDM = \angle QBM = 90^\circ$

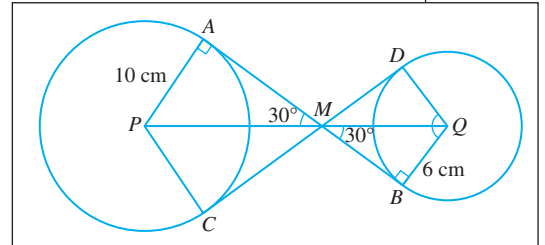
$$\angle DMB = 60^\circ$$

$$\therefore x = 180^\circ - \angle DMB$$

$$= 180^\circ - 60^\circ$$

$$= 120^\circ$$

(ii)  $\tan 30^\circ = \frac{10}{AM}$ ;  $\tan 30^\circ = \frac{6}{BM}$



$$AM = \frac{10}{\tan 30^\circ}; \quad BM = \frac{6}{\tan 30^\circ}$$

$$= 17.32 \text{ cm} \quad = 10.39 \text{ cm}$$

$$\therefore AB = AM + MB$$

$$= 17.32 + 10.39$$

$$= 27.71 \text{ cm}$$

(iii) Area of shaded region = Area of kite  $BQDM$  - area of sector  $BQD$

$$\text{Area of kite } BQDM = 2(\text{area of } \triangle MDQ)$$

$$= 2\left(\frac{1}{2} \times 6 \times 10.39\right)$$

$$= 62.34 \text{ cm}^2$$

$$\text{Area of sector } BQD = \frac{120^\circ}{360^\circ} \times \frac{22}{7} \times 6^2$$

$$= 37.71 \text{ cm}^2$$

$$\text{Area of shaded region} = 62.34 - 37.71$$

$$= 24.63 \text{ cm}^2$$

3 (a) (i)  $x = \frac{1}{2}(100^\circ) = 50^\circ$     (ii)  $y = \frac{1}{2}(260^\circ) = 130^\circ$

(b) (i)  $\angle MNQ = 75^\circ$

$$\angle PNQ = 180^\circ - 75^\circ = 105^\circ$$

$$\angle PQN = 180^\circ - 105^\circ - 40^\circ$$

$$= 35^\circ$$

$$\therefore x = 35^\circ$$

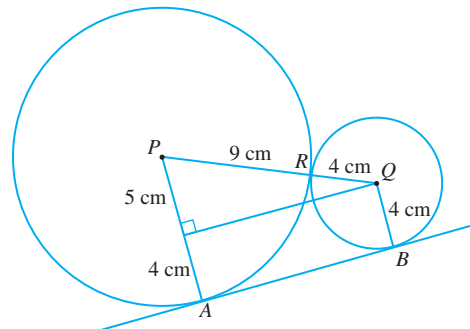
(ii)  $\angle NLQ = \angle PQN = 35^\circ$

$$\angle QLM = \angle MQR = 75^\circ$$

$$\therefore y = 35^\circ + 75^\circ$$

$$= 110^\circ$$

(c)



$$AB = 13^2 - 5^2 = 12 \text{ cm}$$

$$\text{Area of trapezium } ABQP$$

$$= \frac{1}{2} \times (4 + 9) \times 12$$

$$= 78 \text{ cm}^2$$