Fully-Worked Solutions

CHAPTER 1 **Indices**

UPSKILL 1.1

1 (a) 9^5 (b) $(-10)^7$ (c) x^{17} (d) $(-y)^{12}$

2

	a^n	Repeated multiplications	Base	Index
(a)	5^4	$5 \times 5 \times 5 \times 5$	5	4
(b)	7^8	$7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7$	7	8
(c)	k^6	$k \times k \times k \times k \times k \times k$	k	6
(d)	w^{20}	$w \times w \times w \times ... \times w$ (20 factors)	w	20
(e)	m^n	$m \times m \times m \times ... \times m$ (n factors)	m	n

3 (a) $3 \times 3 \times 3 \times 3$; 81
 (b) $(-2) \times (-2) \times (-2) \times (-2) \times (-2) \times (-2) \times (-2) \times (-2)$; 256
 (c) $(-5) \times (-5) \times (-5) \times (-5) \times (-5)$; $-3\,125$
 (d) $\left(\frac{1}{2}\right) \times \left(\frac{1}{2}\right) \times \left(\frac{1}{2}\right) \times \left(\frac{1}{2}\right) \times \left(\frac{1}{2}\right) \times \left(\frac{1}{2}\right) \times \left(\frac{1}{2}\right) \times \left(\frac{1}{2}\right) \times \left(\frac{1}{2}\right) \times \left(\frac{1}{2}\right)$; $\frac{1}{1\,024}$

4 (a) $1\,296 = 6 \times 6 \times 6 \times 6 = 6^4$
 (b) $2\,187 = 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 = 3^7$
 (c) $1\,024 = 4 \times 4 \times 4 \times 4 \times 4 = 4^5$
 (d) $-2\,048 = (-2) \times (-2) \times (-2) \times (-2) \times (-2) \times (-2) \times$
  $(-2) \times (-2) \times (-2) \times (-2) \times (-2)$
  $= (-2)^{11}$

5 (a) $4\,096 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$
  $= 2^{12}$
 (b) $6\,561 = 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 = 3^8$
 (c) $1\,331 = 11 \times 11 \times 11 = 11^3$
 (d) $2\,401 = 7 \times 7 \times 7 \times 7 = 7^4$

6 (a) $512 = 8 \times 8 \times 8 = 8^3$, $\therefore x = 3$
 (b) $1\,296 = 6 \times 6 \times 6 \times 6 = 6^4$, $\therefore x = 4$
 (c) $3\,125 = 5 \times 5 \times 5 \times 5 \times 5 = 5^5$, $\therefore x = 5$
 (d) $1\,024 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^{10}$, $\therefore x = 10$

7 (a) $12^3 = 12 \times 12 \times 12 = 1\,728$
 (b) $(0.9)^4 = 0.9 \times 0.9 \times 0.9 \times 0.9 = 0.6561$
 (c) $(1.5)^3 = 1.5 \times 1.5 \times 1.5 = 3.375$
 (d) $\left(\frac{4}{3}\right)^5 = \frac{4}{3} \times \frac{4}{3} \times \frac{4}{3} \times \frac{4}{3} \times \frac{4}{3} = \frac{1\,024}{243}$

8 (a)
```
2 | 288
2 | 144
2 | 72
2 | 36
2 | 18
3 | 9
3 | 3
  | 1
```
 $288 = 2^5 \times 3^2$

(b)
```
5 | 1 125
5 | 225
5 | 45
3 | 9
3 | 3
  | 1
```
 $1\,125 = 3^2 \times 5^3$

(c)
```
3 | 3 969
3 | 1 323
3 | 441
3 | 147
7 | 49
7 | 7
  | 1
```
 $3\,969 = 3^4 \times 7^2$

(d)
```
2 | 10 800
2 | 5 400
2 | 2 700
2 | 1 350
3 | 675
3 | 225
3 | 75
5 | 25
5 | 5
  | 1
```
 $10\,800 = 2^4 \times 3^3 \times 5^2$

UPSKILL 1.2A

1 (a) $6^5 \times 6^9 = 6^{5+9} = 6^{14}$
 (b) $8^4 \times 8^{11} \times 8^{13} = 8^{4+11+13} = 8^{28}$
 (c) $-3m^2 \times 2m \times 4m^9 = (-3 \times 2 \times 4) \times m^{2+1+9} = -24m^{12}$
 (d) $2k^{10} \times k^7 \times k^6 \times 4k = (2 \times 4)k^{10+7+6+1} = 8k^{24}$

2 (a) $h^2 \times k^5 \times h^4 \times k^3 = h^{2+4}k^{5+3}$
  $= h^6 k^8$
 (b) $2^3 \times 3^4 \times 5 \times 2^2 \times 5^6 \times 3^3 = 2^{3+2} \times 3^{4+3} \times 5^{1+6}$
  $= 2^5 \times 3^7 \times 5^7$
 (c) $4mn^7 \times 12m^2n^6 \times \frac{1}{2}m^3n^8 = \left(4 \times 12 \times \frac{1}{2}\right)m^{1+2+3}n^{7+6+8}$
  $= 24m^6n^{21}$
 (d) $\frac{1}{3}p^4q^2 \times \frac{1}{4}p^{11}q^3 \times 78p^{15}q = \left(\frac{1}{3} \times \frac{1}{4} \times 78\right)p^{4+11+15}q^{2+3+1}$
  $= \frac{13}{2}p^{30}q^6$

3 (a) $2^{12} \div 2^9 = 2^{12-9}$
  $= 2^3$
 (b) $7^{15} \div 7^8 \div 7^2 = 7^{15-8-2}$
  $= 7^5$
 (c) $8x^5 \div 2x^3 \div x = \frac{8}{2}x^{5-3-1}$
  $= 4x$
 (d) $-45y^{14} \div 9y^7 \div y^5 = -\frac{45}{9}y^{14-7-5}$
  $= -5y^2$

4 (a) $24w^{10}y^3 \div 8w^2y = \left(\frac{24}{8}\right)w^{10-2}y^{3-1}$
  $= 3w^8y^2$
 (b) $\frac{18m^{12}n^6}{15m^4n^3} = \left(\frac{18}{15}\right)m^{12-4}n^{6-3}$
  $= \frac{6}{5}m^8n^3$
 (c) $\frac{20n^{16}p^8}{5n^4p^2} \div 2n^3p^3 = \left(\frac{20}{5 \times 2}\right)n^{16-4-3}p^{8-2-3}$
  $= 2n^9p^3$
 (d) $36x^{12}y^{10} \div 15x^3y \div 4xy^3 = \left(\frac{36}{15 \times 4}\right)x^{12-3-1}y^{10-1-3}$
  $= \frac{3}{5}x^8y^6$

5 (a) $(3^2)^6 = 3^{12}$
 (b) $[(-h)^3]^2 = h^6$
 (c) $[(-k)^3]^5 = -k^{15}$

6 (a) $(2^m)^5 = 2^{m \times 5} = 2^{5m}$
 (b) $(5^{4a+8})^{\frac{1}{2}} = 5^{\frac{4a+8}{2}} = 5^{2a+4}$
 (c) $(6^{2n})^{3n} = 6^{2n \times 3n} = 6^{(2 \times 3)(n \times n)} = 6^{6n^2}$
 (d) $(7^{x+2})^{x-2} = 7^{(x+2)(x-2)}$
  $= 7^{x^2-4}$

7 (a) $(2^4 \times 3^5)^3 = 2^{4(3)} \times 3^{5(3)}$
  $= 2^{12} \times 3^{15}$
 (b) $(3^8 \times 5^4)^{\frac{1}{4}} = 3^{\frac{8}{4}} \times 5^{\frac{4}{4}}$
  $= 3^2 \times 5^1$
 (c) $\left(a^{\frac{1}{4}} \div b^{\frac{5}{6}}\right)^{12} = a^{\frac{12}{4}} \div b^{\frac{5 \times 12}{6}}$
  $= a^3 \div b^{10}$
  $= \frac{a^3}{b^{10}}$

(d) $\left(\dfrac{e^6}{f^8}\right)^{\frac{5}{2}} = \dfrac{e^{6 \times \frac{5}{2}}}{f^{8 \times \frac{5}{2}}}$

$\quad = \dfrac{e^{15}}{f^{20}}$

(e) $\left(\dfrac{2g^6}{3h^8}\right)^4 = \dfrac{2^4 g^{6(4)}}{3^4 h^{8(4)}}$

$\quad = \dfrac{16g^{24}}{81h^{32}}$

8 (a) $\dfrac{(2x^4y^3)^2}{6x^5y} = \dfrac{2^2 x^{4(2)} y^{3(2)}}{6x^5 y^1}$

$\quad = \dfrac{4x^8 y^6}{6x^5 y^1}$

$\quad = \left(\dfrac{4}{6}\right) x^{8-5} y^{6-1}$

$\quad = \dfrac{2}{3} x^3 y^5$

(b) $\dfrac{(5mn^2)^3 \times m^2 n}{100mn^3} = \dfrac{5^3 m^{1(3)} n^{2(3)} \times m^2 n}{100mn^3}$

$\quad = \dfrac{125 m^3 n^6 \times m^2 n}{100 m^1 n^3}$

$\quad = \left(\dfrac{125}{100}\right) m^{3+2-1} n^{6+1-3}$

$\quad = \dfrac{5}{4} m^4 n^4$

(c) $\dfrac{(3a^2b^5)^3 \times (2a^3b)^4}{(6ab^2)^2 \times 12a^3b^4} = \dfrac{3^3 a^{2(3)} b^{5(3)} \times 2^4 a^{3(4)} b^{1(4)}}{6^2 a^{1(2)} b^{2(2)} \times 12a^3 b^4}$

$\quad = \dfrac{27a^6 b^{15} \times 16 a^{12} b^4}{36 a^2 b^4 \times 12 a^3 b^4}$

$\quad = \left(\dfrac{27 \times 16}{36 \times 12}\right) a^{6+12-2-3} b^{15+4-4-4}$

$\quad = a^{13} b^{11}$

UPSKILL 1.2B

1 (a) $g^{-3} = \dfrac{1}{g^3}$

(b) $\dfrac{1}{2^{-8}} = 2^8$

(c) $5m^{-6} = \dfrac{5}{m^6}$

(d) $\left(\dfrac{3a}{5b}\right)^{-5} = \left(\dfrac{5b}{3a}\right)^5$

2 (a) $2^{-8} = \dfrac{1}{2^8}$

(b) $4^{-4} = \dfrac{1}{4^4} = \dfrac{1}{(2^2)^4} = \dfrac{1}{2^{2 \times 4}} = \dfrac{1}{2^8}$

(c) $8^{-3} = \dfrac{1}{8^3} = \dfrac{1}{(2^3)^3} = \dfrac{1}{2^{3 \times 3}} = \dfrac{1}{2^9}$

(d) $32^{-2} = \dfrac{1}{32^2} = \dfrac{1}{(2^5)^2} = \dfrac{1}{2^{5 \times 2}} = \dfrac{1}{2^{10}}$

3 (a) $\dfrac{1}{3^{11}} = 3^{-11}$

(b) $\dfrac{1}{(9)^5} = \dfrac{1}{(3^2)^5} = \dfrac{1}{3^{2 \times 5}} = \dfrac{1}{3^{10}} = 3^{-10}$

(c) $\dfrac{1}{(27)^3} = \dfrac{1}{(3^3)^3} = \dfrac{1}{3^{3 \times 3}} = \dfrac{1}{3^9} = 3^{-9}$

(d) $\dfrac{1}{(81)^2} = \dfrac{1}{(3^4)^2} = \dfrac{1}{3^{4 \times 2}} = \dfrac{1}{3^8} = 3^{-8}$

4 (a) $3^{\frac{1}{7}} = \sqrt[7]{3}$

(b) $9^{\frac{1}{10}} = (3^2)^{\frac{1}{10}} = 3^{2 \times \frac{1}{10}} = 3^{\frac{1}{5}} = \sqrt[5]{3}$

(c) $27^{\frac{1}{6}} = (3^3)^{\frac{1}{6}} = 3^{3 \times \frac{1}{6}} = 3^{\frac{1}{2}} = \sqrt{3}$

(d) $81^{\frac{1}{12}} = (3^4)^{\frac{1}{12}} = 3^{4 \times \frac{1}{12}} = 3^{\frac{1}{3}} = \sqrt[3]{3}$

5 (a) $8^{\frac{4}{3}} = (2^3)^{\frac{4}{3}} = (2^{12})^{\frac{1}{3}}; \therefore m = 12, n = 3$

(b) $25^{\frac{5}{4}} = \left(5^2\right)^{\frac{5}{4}} = \left(5^{\frac{1}{4}}\right)^{10}; m = 10, n = 4$

(c) $243^{\frac{3}{5}} = \left(3^5\right)^{\frac{3}{5}} = \left(3^{15}\right)^{\frac{1}{5}}; m = 15, n = 5$

(d) $343^{\frac{5}{3}} = \left(7^3\right)^{\frac{5}{3}} = \left(7^{\frac{1}{3}}\right)^{15}; m = 15, n = 3$

6 (a) $27^{\frac{2}{3}} = (3^3)^{\frac{2}{3}} = \sqrt[3]{3^6}; \therefore b = 6, c = 3$

(b) $1\,000^{\frac{5}{4}} = (10^3)^{\frac{5}{4}} = (\sqrt[4]{10})^{15}; \therefore b = 15, c = 4$

(c) $625^{\frac{4}{5}} = (5^4)^{\frac{4}{5}} = \sqrt[5]{5^{16}}; \therefore b = 5, c = 16$

(d) $1\,296^{\frac{3}{7}} = (6^4)^{\frac{3}{7}} = (\sqrt[7]{6})^{12}; \therefore b = 12, c = 7$

7 (a) $(m^4 n^{\frac{2}{3}} k^{-6})^{\frac{3}{2}} = m^6 n k^{-9} = \dfrac{m^6 n}{k^9}$

(b) $\left(\dfrac{p^6}{q^8 r^2}\right)^{-\frac{3}{2}} = \left(\dfrac{q^8 r^2}{p^6}\right)^{\frac{3}{2}} = \dfrac{q^{12} r^3}{p^9}$

(c) $\dfrac{a^5 b^4 \times ab^{-2} c^3}{(a^2 bc^4)^2} = a^{5+1-4} b^{4-2-2} c^{3-8}$

$\quad = a^2 c^{-5} = \dfrac{a^2}{c^5}$

(d) $\dfrac{(\sqrt{x^6} y^{-9})^{\frac{2}{3}} \times (x^{\frac{5}{2}} y^2 z)^{-4}}{(x^2 y z^{-2})^3} = \dfrac{x^2 y^{-6} \times x^{-10} y^{-8} z^{-4}}{x^6 y^3 z^{-6}}$

$\quad = x^{2-10-6} y^{-6-8-3} z^{-4+6}$

$\quad = x^{-14} y^{-17} z^2$

$\quad = \dfrac{z^2}{x^{14} y^{17}}$

8 (a) $\dfrac{49^{\frac{3}{2}} \times 64^{-\frac{1}{2}}}{\sqrt[3]{343} \times \sqrt[5]{32}} = \dfrac{(7^2)^{\frac{3}{2}} \times (2^6)^{-\frac{1}{2}}}{(7^3)^{\frac{1}{3}} \times (2^5)^{\frac{1}{5}}}$

$\quad = \dfrac{7^3 \times 2^{-3}}{7^1 \times 2^1}$

$\quad = 7^{3-1} \times 2^{-3-1}$

$\quad = 7^2 \times 2^{-4}$

$\quad = 49 \times \dfrac{1}{2^4}$

$\quad = \dfrac{49}{16}$

(b) $\dfrac{(3^6 \times 5^{-4})^{\frac{1}{2}}}{\sqrt{9^3} \times \sqrt[6]{25^3}} = \dfrac{3^3 \times 5^{-2}}{\sqrt{(3^2)^3} \times \sqrt[6]{(5^2)^3}}$

$\quad = \dfrac{3^3 \times 5^{-2}}{(3^6)^{\frac{1}{2}} \times (5^6)^{\frac{1}{6}}}$

$\quad = \dfrac{3^3 \times 5^{-2}}{3^3 \times 5^1}$

$\quad = 3^{3-3} \times 5^{-2-1}$

$\quad = 3^0 \times \dfrac{1}{5^3}$

$\quad = \dfrac{1}{125}$

UPSKILL 1.2C

1 (a) $125^{\frac{1}{3}} \times 25^{\frac{1}{2}} = (5^3)^{\frac{1}{3}} \times (5^2)^{\frac{1}{2}}$

$\qquad\qquad\quad = 5 \times 5$

$\qquad\qquad\quad = 25$

(b) $96^{-1} \times 3^3 = (32 \times 3)^{-1} \times 3^3$

$\qquad\qquad = \dfrac{9}{32}$

(c) $(0.04)^{\frac{1}{2}} \times 16^{\frac{3}{4}} = (0.2^2)^{\frac{1}{2}} \times (2^4)^{\frac{3}{4}}$
$= 0.2 \times 8$
$= 1.6$

(d) $\left(\dfrac{9}{16}\right)^5 \times \left(\dfrac{64}{9}\right)^4 = \left(\dfrac{3^2}{4^2}\right)^5 \times \left(\dfrac{4^3}{3^2}\right)^4$
$= 3^2 \times 4^2$
$= 144$

(e) $1\,000^{\frac{5}{3}} \times 100^{-\frac{3}{2}} \div 20^2 = 10^5 \times 10^{-3} \div (2 \times 10)^2$
$= \dfrac{10^0}{2^2}$
$= \dfrac{1}{4}$

(f) $243^{\frac{2}{5}} \times (6^{-2})^2 \times 8^{\frac{4}{3}} = 3^2 \times (2 \times 3)^{-4} \times 2^4$
$= 3^{2-4} \times 2^{-4+4}$
$= \dfrac{1}{9}$

2 (a) $5^{2x} \times 5^{x+3} = 5^6 \div 25^{2x-9}$
$5^{2x} \times 5^{x+3} = 5^6 \div 5^{2(2x-9)}$
$2x + x + 3 = 6 - 2(2x - 9)$
$3x + 3 = 6 - 4x + 18$
$3x + 4x = 18 + 6 - 3$
$7x = 21$
$x = 3$

(b) $\dfrac{1}{9} \times 27^{x+2} = 81^x$

$\dfrac{1}{3^2} \times 3^{3(x+2)} = 3^{4x}$

$3^{-2} \times 3^{3x+6} = 3^{4x}$
$-2 + 3x + 6 = 4x$
$x = 4$

3 (a) $8^m \times 4^n = 2^3$
$(2^3)^m \times (2^2)^n = 2^3$
$3m + 2n = 3$ ...(1)
$343^m \times 7(7^{n+1}) = 49$
$(7^3)^m \times 7^{1+n+1} = 7^2$
$3m + 1 + n + 1 = 2$
$3m + n = 0$
$n = -3m$
Substitute $n = -3m$ into equation (1):
$3m + 2(-3m) = 3$
$3m - 6m = 3$
$-3m = 3$
$m = -1$
From (1): $3(-1) + 2n = 3$
$2n = 3 + 3$
$n = 3$
$\therefore m = -1, n = 3$

(b) $2^{5x} \times \dfrac{1}{2^{y-2}} = 8$

$2^{5x-(y-2)} = 2^3$
$5x - y + 2 = 3$
$5x - y = 1$ ...(1)
$9(3^{4x-y}) \times 3^{5x-y} = 27$
$3^2(3^{4x-y}) \times 3^{5x-y} = 3^3$
$2 + 4x - y + 5x - y = 3$
$9x - 2y = 1$ ...(2)
(1) × 2: $10x - 2y = 2$ ...(3)
(3) − (2): $10x - 2y - (9x - 2y) = 2 - 1$
$10x - 2y - 9x + 2y = 1$
$x = 1$
Substitute $x = 1$ into (1):
$5(1) - y = 1$
$y = 4$
$\therefore x = 1; y = 4$

Summative Practice 1

Section A

1
2	4 096
2	2 048
2	1 024
2	512
2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

$4\,096 = 2^{12}$
$x = 12$
Answer: **A**

2 $1\,296 = 36^2$
$= (4 \times 9)^2$
$= (2^2 \times 3^2)^2$
$= 2^4 \times 3^4$
Answer: **D**

3 $2^x \times 3^y \times 64 \times 27 = 2^x \times 3^y \times 2^6 \times 3^3$
$= 2^{x+6} \times 3^{y+3}$
Answer: **B**

4 $m^{2x} \times m^{x-6} = m^{24}$
$2x + x - 6 = 24$
$3x = 30$
$x = 10$
Answer: **C**

5 $\left(\dfrac{a^2}{b^3}\right)^3 \times \dfrac{b}{c^3} \times \left(\dfrac{a}{c}\right)^{-2} = \dfrac{a^6}{b^9} \times \dfrac{b}{c^3} \times \dfrac{a^{-2}}{c^{-2}}$
$= a^{6-2}b^{1-9}c^{-3+2}$
$= \dfrac{a^4}{b^8 c}$
Answer: **C**

6 $(a^{12}b^5 \div a^3 b^2 c^{-6})^{\frac{2}{3}} = (a^{12-3}b^{5-2}c^{-6})^{\frac{2}{3}}$
$= a^6 b^2 c^{-4}$
Answer: **A**

7 $(64p^2)^{-\frac{1}{3}} \times (8p)^{\frac{5}{3}} = \dfrac{1}{4} \times p^{-\frac{2}{3}} \times 32 \times p^{\frac{5}{3}}$
$= 8p$
Answer: **C**

8 $w^2 = 32^{\frac{8}{5}} \times 125^{\frac{2}{3}}$
$= 2^8 \times 5^2$
$w = \sqrt{2^8 \times 5^2}$
$= 2^4 \times 5$
$= 80$
Answer: **A**

9 $\sqrt{(mn^3)^3} \div \sqrt{(mn)^{-1}} = (m^3 n^9 \div m^{-1} n^{-1})^{\frac{1}{2}}$
$= (m^{3+1} n^{9+1})^{\frac{1}{2}}$
$= m^2 n^5$
Answer: **D**

10 $729^{\frac{1}{6}} \div 9^{\frac{3}{2}} \times (243)^{-\frac{1}{5}} = (3^6)^{\frac{1}{6}} \div (3^2)^{\frac{3}{2}} \times (3^5)^{-\frac{1}{5}}$
$= 3^{1-3-1}$
$= \dfrac{1}{27}$
Answer: **B**

Section B

1 $\sqrt[3]{-8} = -2$; $(-2)^6 = 64$; $(-64)^{\frac{1}{3}} = -4$; $32^{\frac{2}{5}} = 4$

2

Index Notation					
(a) $(p^{-2})^3$		p^{-5}		✓	p^{-6}
(b) $p^3 \times p^2$	✓	p^5			p^6
(c) $p^{\frac{3}{2}}$		$p^3 \div p^2$		✓	$\sqrt{p^3}$
(d) $\dfrac{p^{-3}}{p^2}$		p^{-1}		✓	p^{-5}

3 $\left(\sqrt{1\frac{9}{16}} - \sqrt[3]{4\frac{17}{27}}\right)^2 = \left(\sqrt{\frac{25}{16}} - \sqrt[3]{\frac{125}{27}}\right)^2$

$\qquad = \left(\frac{5}{4} - \frac{5}{3}\right)^2$

$\qquad = \left(\frac{-5}{12}\right)^2$

$\qquad = \frac{25}{144}$

$a = 5 \qquad b = 5 \qquad c = -5 \qquad d = \frac{25}{144}$

4 (a) $3^{-2} \times 81^{\frac{1}{4}} = 3^{-2} \times (3^4)^{\frac{1}{4}}$

$\qquad = 3^{-2+1}$

$\qquad = 3^{-1}$

$\qquad = \frac{1}{3}$

$8^{-\frac{2}{3}} \div 16^{\frac{1}{4}} = (2^3)^{-\frac{2}{3}} \div (2^4)^{\frac{1}{4}}$

$\qquad = 2^{-2-1}$

$\qquad = 2^{-3}$

$\qquad = \frac{1}{8}$

(b) $\frac{a^{-2}b^5}{c^3} \times \left(\frac{c^2}{ab^{-1}}\right)^2 = a^{-2}b^5c^{-3} \times a^{-2}b^2c^4$

$\qquad = a^{-2-2}b^{5+2}c^{-3+4}$

$\qquad = a^{-4}b^7c$

$\qquad = \frac{b^7c}{a^4}$

$\frac{a^2b^5}{c^{-2}} \div \frac{ab^{-3}}{c} = a^2b^5c^2 \div ab^{-3}c^{-1}$

$\qquad = a^{2-1}b^{5+3}c^{2+1}$

$\qquad = ab^8c^3$

Section C

1 (a) $\frac{18}{6}n^{3+2} = 3n^5$

(b) $\left(\frac{2}{\sqrt{49}} \times \sqrt[3]{343^2}\right)^{-2} = \left(\frac{2}{7} \times 343^{\frac{2}{3}}\right)^{-2}$

$\qquad = \left(\frac{2}{7} \times 7^2\right)^{-2}$

$\qquad = (14)^{-2}$

$\qquad = \frac{1}{196}$

(c) $\sqrt[3]{a^2} \times \sqrt{a^y} \div \sqrt[4]{a^3} = 1$

$a^{\frac{2}{3}} \times a^{\frac{y}{2}} \div a^{\frac{3}{4}} = a^0$

$\frac{2}{3} + \frac{y}{2} - \frac{3}{4} = 0$

$\frac{y}{2} = \frac{3}{4} - \frac{2}{3}$

$\qquad = \frac{9-8}{12}$

$\qquad = \frac{1}{12}$

$y = \frac{1}{6}$

(d) $\sqrt[3]{m} \times \sqrt[3]{m^2n^6} \div \sqrt{mn^{-2}} = m^{\frac{1}{3}} \times m^{\frac{2}{3}}n^{\frac{6}{3}} \div m^{\frac{1}{2}}n^{-\frac{2}{2}}$

$\qquad = m^{\frac{1}{3}+\frac{2}{3}-\frac{1}{2}}n^{2-(-1)}$

$\qquad = m^{\frac{1}{2}}n^3$

(e) $(x^2y)^2 \times \left(xy^{\frac{1}{2}}\right)^3 \div (x^2y^3)^{\frac{3}{2}} = x^4y^2 \times x^3y^{\frac{3}{2}} \div x^3y^{\frac{9}{2}}$

$\qquad = x^{4+3-3}y^{2+\frac{3}{2}-\frac{9}{2}}$

$\qquad = x^4y^{-1}$

$\qquad = \frac{x^4}{y}$

2 (a) $(p^r)^4 = p^{12}$

$p^{4r} = p^{12}$

$4r = 12$

$r = \frac{12}{4}$

$\quad = 3$

(b) $\sqrt{\frac{1}{25}} + \sqrt[4]{\frac{1}{16}} = \frac{1}{5} + \frac{1}{2}$

$\qquad = \frac{2+5}{10}$

$\qquad = \frac{7}{10}$

(c) $\sqrt[6]{p} \times \sqrt[3]{p^4q^3} \times \sqrt{pq^4}$

$= p^{\frac{1}{6}} \times p^{\frac{4}{3}}q^{\frac{3}{3}} \times p^{\frac{1}{2}}q^{\frac{4}{2}}$

$= p^{\frac{1}{6}+\frac{4}{3}+\frac{1}{2}}q^{1+2}$

$= p^{\frac{2+16+6}{12}}q^3$

$= p^2q^3$

(d) Height $= \dfrac{a^{\frac{3}{2}}b^5c^{\frac{5}{2}}}{(ab^4c^3)^{\frac{1}{2}} \times \left(a^2b^{\frac{1}{2}}c^{\frac{3}{2}}\right)^2}$

$= a^{\frac{3}{2}}b^5c^{\frac{5}{2}} \div a^{\frac{1}{2}}b^2c^{\frac{3}{2}} \div a^4bc^3$

$= a^{\frac{3}{2}-\frac{1}{2}-4}b^{5-2-1}c^{\frac{5}{2}-\frac{3}{2}-3}$

$= a^{-3}b^2c^{-2}$

(e) $\qquad 16^x \times 2^{2y} = 4$

$(2^4)^x \times 2^{2y} = 2^2$

$4x + 2y = 2$

$2x + y = 1 \qquad \text{...(1)}$

$27^x \times 9(3^{y-1}) = 81$

$(3^3)^x \times 3^2(3^{y-1}) = 3^4$

$3^{3x} \times 3^{2+y-1} = 3^4$

$3x + 2 + y - 1 = 4$

$3x + y = 3 \qquad \text{...(2)}$

(2) − (1): $3x - 2x = 3 - 1$

$\qquad\qquad\qquad x = 2$

Substitute $x = 2$ into equation (1):

$2(2) + y = 1$

$\qquad y = -3$

$\therefore x = 2, y = -3$