Fully-Worked Solutions

CHAPTER 7 Coordinates

UPSKILL 7.1

1 Scale on both axes is 1 unit
Distance of $AB = 4 \times 1 = 4$ units
Distance of $CD = 4 \times 1 = 4$ units
Distance of $EF = 6 \times 1 = 6$ units
Distance of $GH = 5 \times 1 = 5$ units

2 (a) Distance of $AB = y_2 - y_1$
$= 6 - 2$
$= 4$ units

(b) Distance of $CD = y_2 - y_1$
$= 9 - 5$
$= 4$ units

(c) Distance of $EF = y_2 - y_1$
$= -3 - (-6)$
$= 3$ units

(d) Distance of $PQ = x_2 - x_1$
$= 5 - 0$
$= 5$ units

(e) Distance of $RS = x_2 - x_1$
$= 8 - 2$
$= 6$ units

(f) Distance of $TU = x_2 - x_1$
$= -1 - (-5)$
$= 4$ units

3 (a) Distance of $AB = \sqrt{3^2 + 4^2}$
$= \sqrt{25}$
$= 5$ units

(b) Distance of $CD = \sqrt{3^2 + 5^2}$
$= \sqrt{34}$
$= 5.83$ units

(c) Distance of $FG = \sqrt{4^2 + 2^2}$
$= \sqrt{20}$
$= 4.47$ units

4 (a) Distance of $PQ = \sqrt{8^2 + 15^2}$
$= \sqrt{289}$
$= 17$ units

(b) Distance of $RS = \sqrt{8^2 + 10^2}$
$= \sqrt{164}$
$= 12.81$ units

(c) Distance of $TU = \sqrt{6^2 + 20^2}$
$= \sqrt{436}$
$= 20.88$ units

5 (a) Distance of $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$= \sqrt{(3 - 2)^2 + (8 - 4)^2}$
$= \sqrt{1^2 + 4^2}$
$= \sqrt{17}$
$= 4.12$ units

(b) Distance of $CD = \sqrt{[0 - (-3)]^2 + (9 - 0)^2}$
$= \sqrt{3^2 + 9^2}$
$= \sqrt{90}$
$= 9.49$ units

(c) Distance of $EF = \sqrt{(-1 - 7)^2 + (4 - (-2))^2}$
$= \sqrt{(-8)^2 + 6^2}$
$= \sqrt{100}$
$= 10$ units

(d) Distance of $GH = \sqrt{[3 - (-6)]^2 + (10 - (-2))^2}$
$= \sqrt{9^2 + 12^2}$
$= \sqrt{225}$
$= 15$ units

(e) Distance of $IJ = \sqrt{(-5 - 3)^2 + [-2 - (-8)]^2}$
$= \sqrt{(-8)^2 + 6^2}$
$= \sqrt{100}$
$= 10$ units

(f) Distance of $KL = \sqrt{[-9 - (-2)]^2 + [-6 - (-1)]^2}$
$= \sqrt{(-7)^2 + (-5)^2}$
$= \sqrt{74}$
$= 8.6$ units

6 Distance of $UV = \sqrt{[6 - (-4)]^2 + [(-2) - 4]^2}$
$= \sqrt{10^2 + 6^2}$
$= \sqrt{136}$
$= 11.66$ units

7 Distance of $PQ = 6 - (-2)$
$= 8$ units

Distance of $PR = 5 - (-1)$
$= 6$ units

Distance of $QR = \sqrt{[6 - (-2)]^2 + [5 - (-1)]^2}$
$= \sqrt{8^2 + 6^2}$
$= \sqrt{100}$
$= 10$ units

$\therefore$ Perimeter of $\Delta PQR = PQ + PR + QR$
$= 8 + 6 + 10$
$= 24$ units

8 $5 - p = 7$ or $p - 5 = 7$
$-p = 7 - 5$ $p = 7 + 5$
$p = -2$ $= 12$

9 $\dfrac{1}{2} \times [k - (-1)] \times (8 - 2) = 30$
$3(k + 1) = 30$
$3k + 3 = 30$
$3k = 27$
$k = 9$

UPSKILL 7.2

1 (a) Midpoint of $AB = (5, 2)$

(b) Midpoint of $CD = (2, -3)$

(c) Midpoint of $EF = (-7, 5)$

(d) Midpoint of $GH = (2, -1)$

(e) Midpoint of $IJ = (1, 6)$

(f) Midpoint of $KL = (2, 0)$

2 (a) Midpoint of $PQ = (3, -5)$

(b) Midpoint of $QR = (7, -3)$

3 (a) Midpoint of $AB = (4, 2)$

(b) Midpoint of $CD = (2, -1)$

(c) Midpoint of $EF = (-4.5, -4)$

(d) Midpoint of $GH = (-3, 1)$

4 (a) Midpoint of $MN = \left(\dfrac{1 + (-1)}{2}, \dfrac{9 + 5}{2}\right)$
$= (0, 7)$

(b) Midpoint of $PQ = \left(\dfrac{2 + 3}{2}, \dfrac{-1 + (-3)}{2}\right)$
$= (2.5, -2)$

(c) Midpoint of $RS = \left(\dfrac{6 + 4}{2}, \dfrac{9 + (-3)}{2}\right)$
$= (5, 3)$

(d) Midpoint of $TU = \left(\dfrac{-5 + (-4)}{2}, \dfrac{-10 + 8}{2}\right)$
$= (-4.5, -1)$

(e) Midpoint of $VW = \left(\dfrac{7 + 6}{2}, \dfrac{11 + 1}{2}\right)$
$= (6.5, 6)$

(f) Midpoint of $XY = \left(\dfrac{0 + (-7)}{2}, \dfrac{-8 + (-6)}{2}\right)$
$= (-3.5, -7)$

5 Let the coordinates of point P be (x, y)
$(2, 9) = \left(\dfrac{x + (-3)}{2}, \dfrac{y + 12}{2}\right)$
$\dfrac{x - 3}{2} = 2 \qquad \dfrac{y + 12}{2} = 9$
$x - 3 = 4 \qquad y + 12 = 18$
$x = 7 \qquad y = 6$
$\therefore$ Coordinates of P is $(7, 6)$

6 Let the coordinates of point R be (x, y)
Q is the midpoint of PR
$(0, 7) = \left(\dfrac{x + (-4)}{2}, \dfrac{y + 11}{2}\right)$
$\dfrac{x - 4}{2} = 0 \qquad \dfrac{y + 11}{2} = 7$
$x - 4 = 0 \qquad y + 11 = 14$
$x = 4 \qquad y = 3$
$\therefore$ Coordinates of R is $(4, 3)$

7 L is the midpoint of KM
$(5, q) = \left(\dfrac{1 + p}{2}, \dfrac{3 + 10}{2}\right)$
$\dfrac{1 + p}{2} = 5 \qquad \dfrac{3 + 10}{2} = q$
$1 + p = 10 \qquad q = \dfrac{13}{2}$
$p = 9 \qquad q = 6.5$

UPSKILL 7.3

1 Let N be the midpoint of LM.
Coordinates of $N = \left(\dfrac{6 + (-10)}{2}, \dfrac{-3 + (-3)}{2}\right)$
$= (-2, -3)$
Since $KL = KM$, coordinates of $K = (-2, 3)$
Midpoint of $KL = \left(\dfrac{-2 - 10}{2}, \dfrac{3 - 3}{2}\right) = (-6, 0)$
Midpoint of $KM = \left(\dfrac{-2 + 6}{2}, \dfrac{3 - 3}{2}\right) = (2, 0)$

Area of the shaded region $= \dfrac{1}{2} \times 16 \times 6 - \dfrac{1}{2} \times 8 \times 3$
$= 36$ units^2

2 Let $P(x, y)$
$\dfrac{x + 6}{2} = 2 \qquad\qquad \dfrac{y + 2}{2} = 5$
$x = -2 \qquad\qquad y = 8$
Hence, $P(-2, 8)$
Let M be midpoint of PR. Thus, $M(-2, 2)$
Distance of $PM = 8 - 2 = 6$ units
Distance of $PR = 2 \times 6 = 12$ units
Distance of $PQ = \sqrt{(-2 - 6)^2 + (8 - 2)^2}$
$= 10$ units
Perimeter of $PQR = 10 + 10 + 12$
$= 32$ units
Joseph needs 32 m of wire thus, the length of the wire is insufficient.

3 Coordinates of $M = \left(\dfrac{6 + (-12)}{2}, 10\right) = (-3, 10)$
Area $= 108$ units^2
$\dfrac{1}{2} \times 18 \times (10 - h) = 108$
$10 - h = 12$
$10 - 12 = h$
$\therefore h = -2$

4 (a) Distance of $KL = 2KM$
$= 2 \times 5$
$= 10$ units

(b) $K(-5, -4)$

(c) $L(5, -4)$
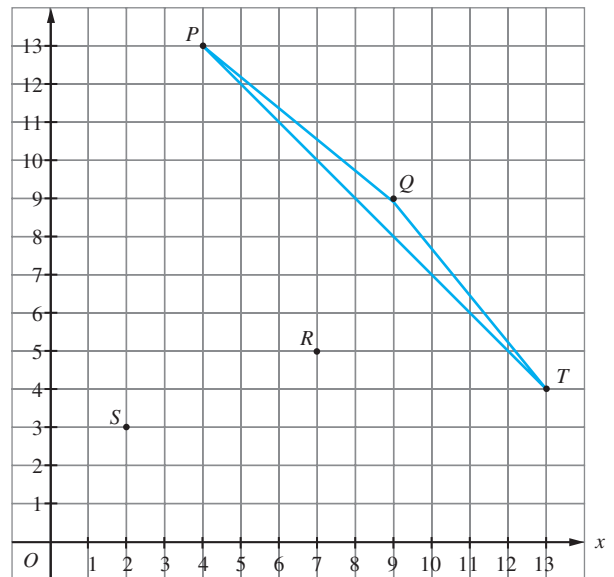
Summative Practice 7

Section A

1 Distance of L from origin $= \sqrt{(-6 - 0)^2 + (8 - 0)^2}$
$= 10$ units
Answer: **B**

2 Distance of $DK = \sqrt{4^2 + 3^2}$
$= 5$ units
Answer: **D**

3

Answer: **D**

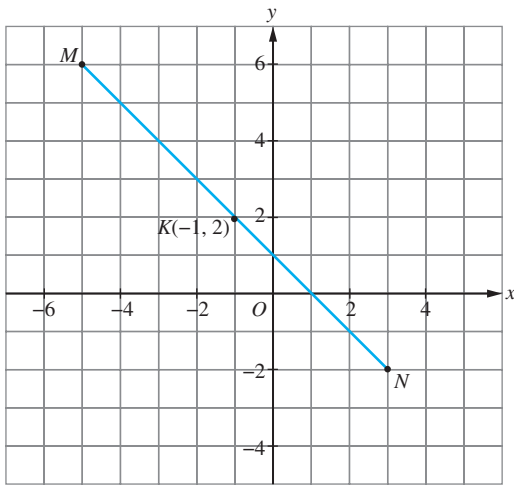4 $(-7, 0)$: Distance $= \sqrt{(-7 - 0)^2 + (0 - 0)^2} = 7$ units
$(-5, -12)$: Distance $= \sqrt{(-5 - 0)^2 + (-12 - 0)^2} = 13$ units
$(-8, 6)$: Distance $= \sqrt{(-8 - 0)^2 + (6 - 0)^2} = 10$ units
$(0, 9)$: Distance $= \sqrt{(0 - 0)^2 + (9 - 0)^2} = 9$ units
Answer: **B**

5

Answer: **C**

6 $RT = 2RN$
$RN = NT$
N is the midpoint of RT
$N = \left(\dfrac{6 + (-4)}{2}, \dfrac{7 + (-1)}{2}\right)$
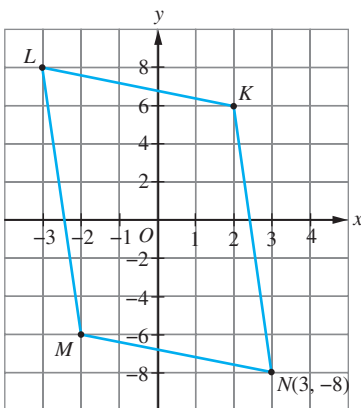$= (1, 3)$
Answer: **D**

7 $S(12, 0)$
R is the midpoint of QS
Coordinates of $R = \left(\dfrac{6 + 12}{2}, \dfrac{4 + 0}{2}\right)$
$= (9, 2)$
Answer: **C**

8

Answer: **B**

9 M is the midpoint of the line PQ
$(3, h) = \left(\dfrac{k + 8}{2}, \dfrac{-4 + 6}{2}\right)$

$\dfrac{k + 8}{2} = 3$ $\qquad$ $h = \dfrac{-4 + 6}{2}$
$k + 8 = 6$ $\qquad\qquad$ $h = 1$
$\quad k = 6 - 8$
$\quad k = -2$
Answer: **A**

10 $W = \left(\dfrac{-4 + 10}{2}, \dfrac{-3 + (-13)}{2}\right)$
$= (3, -8)$

Distance of $WU = \sqrt{(15 - 3)^2 + [-3 - (-8)]^2} = 13$ units

Answer: **D**

Section B

1 (a) ✗ $\qquad$ (b) ✓ $\qquad$ (c) ✗ $\qquad$ (d) ✓

2 (a) Distance of $PQ = \sqrt{8^2 + 6^2}$
$= 10$ units

(b) Midpoint of $PR = \left(\dfrac{0 + 12}{2}, \dfrac{3 - 9}{2}\right)$
$= (6, -3)$

Section C

1 (a) Midpoint $= \left(\dfrac{5 - 7}{2}, \dfrac{4 + 8}{2}\right)$
$= (-1, 6)$

(b) $3 - h = 5$ $\qquad$ or $\qquad$ $h - 3 = 5$
$\quad\; h = 3 - 5$ $\qquad\qquad\quad$ $h = 5 + 3$
$\quad\; h = -2$ $\qquad\qquad\qquad\;$ $h = 8$

(c) (i) Coordinates of $M = (3, 4)$

(ii) $(3, 4) = \left(9, \dfrac{p - 2}{2}\right)$

$\dfrac{p - 2}{2} = 4$
$p - 2 = 8$
$\quad\; p = 10$

2 (a) Distance $= \sqrt{(4 - 6)^2 + [1 + 5]^2}$
$= \sqrt{40}$
$= 6.324$ units

(b) Distance of $KL = 3 - (-1) = 4$ units
Distance of $LM = 5 - 2 = 3$ units
Distance of $KM = \sqrt{[3 - (-1)]^2 + [5 - 2]^2} = 5$ units
∴ Perimeter of the triangle $KLM = 4 + 3 + 5$
$= 12$ units

(c) (i) $\qquad\qquad$ Area $= 24$
$\dfrac{1}{2} \times (8 - 2) \times (q - 1) = 24$
$3(q - 1) = 24$
$q - 1 = 8$
$q = 9$

(ii) Midpoint of $PR = \left(\dfrac{4 + 2}{2}, \dfrac{9 + 1}{2}\right) = (3, 5)$