Fully-Worked Solutions

CHAPTER 6 Three-Dimensional Geometrical Shapes

UPSKILL 6.1

1 (a) Two polygonal surfaces and vertical rectangular surfaces.

(b) One polygonal flat base and slanting triangular surfaces that meet at an apex.

(c) One curved surface with all the points on the surface are equidistant from the centre.

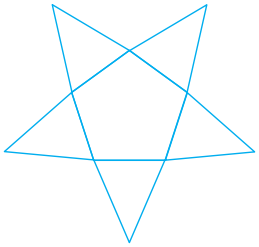
2 (a) Triangular based pyramid

(b) Cylinder

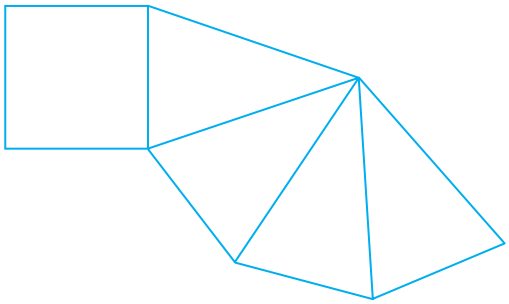
(c) Prism

UPSKILL 6.2

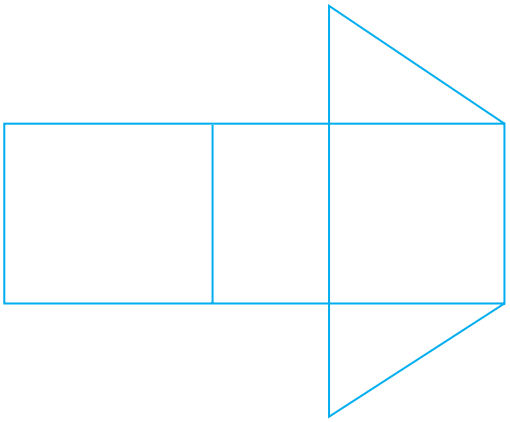
1 (a)



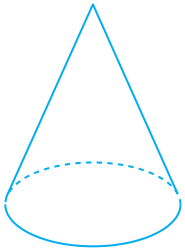
(b)



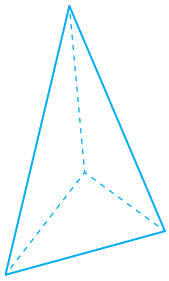
(c)



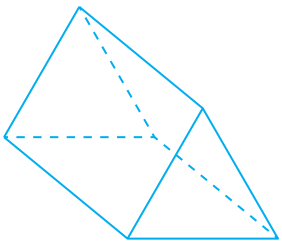
2 (a) Cone



(b) Pyramid



(c) Prism



UPSKILL 6.3

1 (a) Surface area of the cube
$= 6 \times (8 \times 8)$
$= 6 \times 64$
$= 384 \text{ cm}^2$

(b) Surface area of the cuboid $= 2[5 \times 15 + 5 \times 10 + 15 \times 10]$
$= 550 \text{ cm}^2$

(c) Surface area of the prism
$= 4 \times 12 + 5 \times 12 + 2\left[\frac{1}{2} \times (5 + 8) \times 4\right] + 8 \times 12 + 5 \times 12$
$= 316 \text{ cm}^2$

2 (a) Surface area of the cone
$= \pi r^2 + \pi rs$
$= \left(\frac{22}{7} \times 7^2\right) + \left(\frac{22}{7} \times 7 \times 20\right)$
$= 594 \text{ cm}^2$

(b) Surface area of the cylinder
$= 2\pi r^2 + 2\pi rh$
$= \left(2 \times \frac{22}{7} \times 7^2\right) + \left(2 \times \frac{22}{7} \times 7 \times 12\right)$
$= 308 + 528$
$= 836 \text{ cm}^2$

(c) Surface area of the pyramid
$= 10 \times 10 + 4\left[\frac{1}{2} \times 10 \times 13\right]$
$= 360 \text{ cm}^2$

3 (a) Surface area of sphere $= 4 \times \frac{22}{7} \times 1.05^2$
$= 13.86 \text{ cm}^2$

(b) Surface area of sphere $= 4 \times \frac{22}{7} \times \left(\frac{4}{2}\right)^2$
$= 50.29 \text{ cm}^2$

(c) Surface area of hemisphere $= 3 \times \frac{22}{7} \times \left(\frac{7}{2}\right)^2$
$= 115.5 \text{ cm}^2$

(d) Surface area of hemisphere $= 3 \times \frac{22}{7} \times \left(\frac{2.1}{2}\right)^2$
$= 10.40 \text{ cm}^2$

4 Surface area of the solid
= Surface area of hemisphere + Surface area of cube − Area of circle
$= 3 \times 3.142 \times 10^2 + 6(20 \times 20) - 3.142 \times 10^2$
$= 3\,028.4 \text{ cm}^2$

5 Total surface area
$= \frac{1}{2} \times$ Area of curved surface of cylinder + $2 \times$ Area of semicircle + Area of rectangular top
$= \left(\frac{1}{2} \times 2\pi rh\right) + \left(2 \times \frac{1}{2}\pi r^2\right) + 2rh$
$= \left(\frac{22}{7} \times 3.5 \times 84\right) + \left(\frac{22}{7} \times 3.5^2\right) + (2 \times 3.5 \times 84)$
$= 924 + 38.5 + 588$
$= 1\,550.5 \text{ cm}^2$

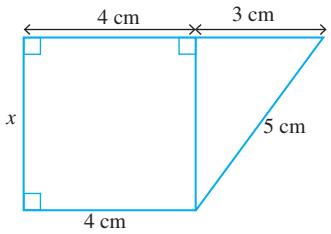
6 Surface area of the solid
= Surface area of sphere − $2 \times$ Area of circles + Area of curved surface of cylinder
$3\,036 = 4 \times \frac{22}{7} \times 10.5^2 - 2 \times \frac{22}{7} \times 10.5^2 + 2 \times \frac{22}{7} \times 10.5 \times t$
$3\,036 = 693 + 66t$
$2\,343 = 66t$
$t = 35.5 \text{ cm}$

7 $2\left(\frac{1}{2} \times 25 \times 10\right) + 2\left(\frac{1}{2} \times 15 \times x\right) + 25x = 790$
$250 + 15x + 25x = 790$
$40x = 540$
$x = 13.5 \text{ cm}$

UPSKILL 6.4

1 (a) Volume of prism
= Base area × Height
$= 16.5 \times 12$
$= 198 \text{ cm}^3$

(b) Uniform cross section of the prism is a trapezium.



$x = \sqrt{5^2 - 3^2}$
$= 4 \text{ cm}$

Volume of prism
= Base area × Height
$= \left[\frac{1}{2} \times (7 + 4) \times 4\right] \times 8$
$= 176 \text{ cm}^3$

2 (a) Volume of cylinder = Base area × Height
$= 13 \times 5$
$= 65 \text{ cm}^3$

(b) Volume of cylinder $= \pi r^2 h$
$= \frac{22}{7} \times 3.5^2 \times 10$
$= 385 \text{ cm}^3$

(c) Volume of cylinder $= \pi r^2 h$
$= \frac{22}{7} \times \left(\frac{14}{2}\right)^2 \times 21$
$= 3\,234 \text{ cm}^3$

3 (a) Volume of pyramid $= \frac{1}{3} \times$ Base area × Height
$= \frac{1}{3} \times 30 \times 16$
$= 160 \text{ cm}^3$

(b) Volume of pyramid $= \frac{1}{3} \times$ Base area × Height
$= \frac{1}{3} \times 6 \times 6 \times 9$
$= 108 \text{ cm}^3$

(c) Volume of pyramid $= \frac{1}{3} \times$ Base area × Height
$= \frac{1}{3} \times \left(\frac{1}{2} \times 6 \times 8\right) \times 7$
$= 56 \text{ cm}^3$

4 (a) Volume of cone $= \frac{1}{3} \times$ Base area × Height
$= \frac{1}{3} \times 45 \times 21$
$= 315 \text{ cm}^3$

(b) $V = \frac{1}{3}\pi r^2 h$
$= \frac{1}{3} \times \frac{22}{7} \times 14^2 \times 18$
$= 3\,696 \text{ mm}^3$

(c) $V = \frac{1}{3}\pi r^2 h$
$= \frac{1}{3} \times \frac{22}{7} \times \left(\frac{2.1}{2}\right)^2 \times 1.5$
$= 1.7325 \text{ m}^3$

5 (a) Volume of sphere $= \frac{4}{3}\pi r^3$
$= \frac{4}{3} \times \frac{22}{7} \times 6^3$
$= 905\frac{1}{7} \text{ cm}^3$

(b) Volume of sphere $= \frac{4}{3}\pi r^3$
$= \frac{4}{3} \times \frac{22}{7} \times \left(\frac{8}{2}\right)^3$
$= 268\frac{4}{21} \text{ cm}^3$

(c) Volume of the hemisphere $= \frac{1}{2} \times \frac{4}{3}\pi r^3$
$= \frac{2}{3} \times \frac{22}{7} \times (1.2)^3$
$= 3.621 \text{ m}^3$

6 (a) Volume of cylinder $= \pi r^2 h$
$= \frac{22}{7} \times \left(\frac{21}{2}\right)^2 \times 25$
$= 8\,662.5 \text{ cm}^3$

Volume of hemisphere $= \frac{1}{2} \times \frac{4}{3}\pi r^3$
$= \frac{2}{3} \times \frac{22}{7} \times \left(\frac{21}{2}\right)^3$
$= 2\,425.5 \text{ cm}^3$

$\therefore$ Volume of the composite solid $= 8\,662.5 + 2\,425.5$
$= 11\,088 \text{ cm}^3$

(b) Volume of pyramid $= \frac{1}{3} \times$ Base area × Height
$= \frac{1}{3} \times (4 \times 6) \times 5$
$= 40 \text{ cm}^3$

Volume of cuboid $= 4 \times 6 \times 7$
$= 168 \text{ cm}^3$

$\therefore$ Volume of the composite solid = 40 + 168
= 208 cm^3

(c) Volume of prism = Base area × Height
$= \left(\frac{1}{2} \times 3 \times 4\right) \times 7$
= 42 cm^3

Volume of a half cylinder = $\frac{1}{2}\pi r^2 h$
$= \frac{1}{2} \times \frac{22}{7} \times \left(\frac{4}{2}\right)^2 \times 7$
= 44 cm^3

$\therefore$ Volume of the composite solid = 42 + 44
= 86 cm^3

7 Volume of prism = Base area × Height
$= \left[\frac{1}{2} \times (10 + 14) \times 8\right] \times 21$
= 2 016 cm^3

Volume of cylinder = $\pi r^2 h$
$= \frac{22}{7} \times \left(\frac{7}{2}\right)^2 \times 21$
= 808.5 cm^3

$\therefore$ Volume of the remaining solid = 2 016 − 808.5
= 1 207.5 cm^3

Summative Practice 6

Section A

1 *Answer*: **C**

2 *Answer*: **B**

3 *Answer*: **C**

4 *Answer*: **C**

5 Total surface area of pyramid = $16 \times 16 + 4\left(\frac{1}{2} \times 16 \times 6\right)$
= 448 cm^2

Answer: **B**

6 Total surface area = $3\pi r^2$
$= 3 \times \frac{22}{7} \times 3.5^2$
= 115.5 cm^2

Answer: **C**

7 Total surface area
$= 8(\sqrt{1^2 + 20^2}) + 8(1) + 8(2) + 2\left[\frac{1}{2} \times (1 + 2) \times 20\right]$
= 244.2 m^2
Cost = 244.2 × RM25
= RM6 105
Answer: **A**

8 Volume of pyramid = $\frac{1}{3}$ × Base area × Height
$1\,000 = \frac{1}{3} \times A \times 20$
$\frac{20}{3}A = 1\,000$
$A = 1\,000 \times \frac{3}{20}$
= 150 mm^2

Answer: **D**

9 Volume of pyramid = $\frac{1}{3}$ × Base area × Height
$96 = \frac{1}{3} \times (4 \times 4) \times h$
$\frac{16}{3}h = 96$
$h = 96 \times \frac{3}{16}$
= 18 cm

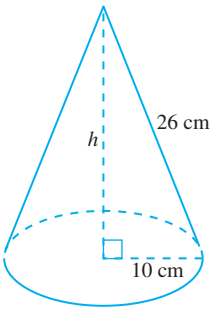
Answer: **D**

10 $V = \frac{1}{3}\pi r^2 h$
$198 = \frac{1}{3} \times \frac{22}{7} \times r^2 \times 21$
$22r^2 = 198$
$r^2 = 9$
$r = \sqrt{9}$
$\therefore$ r = 3 cm
Answer: **B**

11 $V = \frac{4}{3}\pi r^2 h$
$\frac{4}{3} \times \frac{22}{7} \times r^3 = 600$
$\frac{88}{21}r^3 = 600$
$r^3 = 600 \times \frac{21}{88}$
= 143.18
$r = \sqrt[3]{143.18}$
= 5.23 mm

Answer: **C**

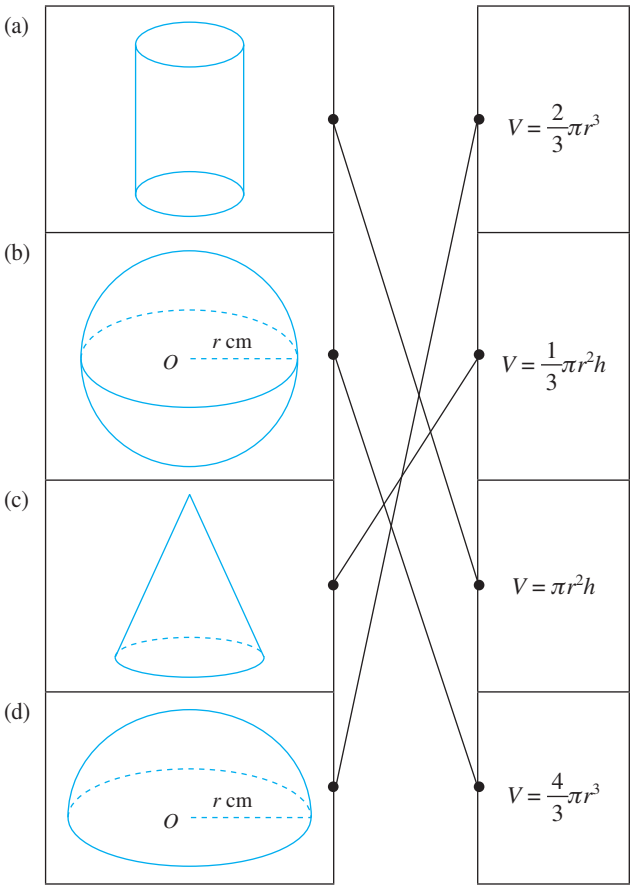
12 $h = \sqrt{26^2 - 10^2}$ = 24 cm
$V = \frac{1}{3}\pi r^2 h$
$= \frac{1}{3} \times \frac{22}{7} \times 10^2 \times 24$
= 2 514 cm^3
Answer: **A**



Section B

1 (a) FALSE
(b) TRUE
(c) TRUE
(d) FALSE

2



3 (a) Prism (b) Pyramid (c) Cube (d) Cuboid

4 Volume of the solid

$= \frac{1}{3}\pi r^2\left[2r + (3)h\right]$

$= \frac{1}{3} \times \frac{22}{7} \times (10.5)^2\left[2(10.5) + (126)\right]$

$= 16\ 978.5\ \text{cm}^3$

Section C

1 (a) (i) Prism has two polygonal flat surfaces which are congruent and parallel.

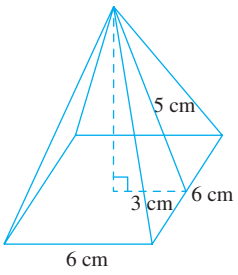
(ii) Prism has other surfaces which are vertical rectangular planes.

(b) Volume of the pyramid

$= \frac{1}{3} \times \text{Base area} \times \text{Height}$

$= \frac{1}{3} \times 6 \times 6 \times 4$

$= 48\ \text{cm}^3$



$h = \sqrt{5^2 - 3^2} = 4\ \text{cm}$

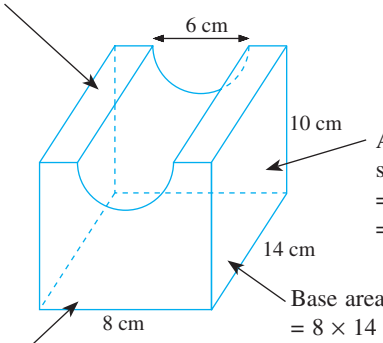
(c) Area of circle $= \pi r^2$

Area of curved surface of cylinder $= 2\pi rh$

Area of top surface

$= 2 \times 14 + \frac{1}{2} \times \left(2 \times \frac{22}{7} \times 3\right) \times 14$

$= 160$



Area of vertical surface
$= 14 \times 10$
$= 140$

Base area
$= 8 \times 14$
$= 112$

Area of vertical front surface

$= 8 \times 10 - \frac{1}{2} \times \frac{22}{7} \times 3^2$

$= 65.86$

Total surface area of the solid

$= 2 \times 65.86 + 2 \times 140 + 112 + 160$

$= 683.72\ \text{cm}^2$

2 (a) (i) Prism

(ii) Cylinder

(b) Volume of prism $= \frac{1}{2} \times 10 \times 14 \times 11$

$= 770\ \text{cm}^3$

Volume of cylinder $= \frac{22}{7} \times 7^2 \times h$

$= 154h$

$\therefore 154h = 770$

$h = \frac{770}{154}$

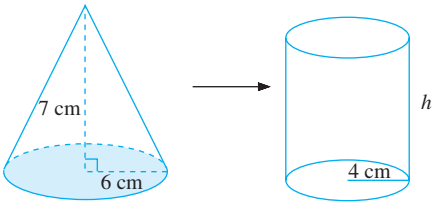
$= 5\ \text{cm}$

(c) Volume of cone $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi(6^2)(7)$

Volume of cylinder $= \pi r^2 h$

$= \pi(4^2)h$



$\pi(4^2)h = \frac{1}{3}\pi(6^2)(7)$

$\therefore 16h = 84$

$h = 5.25\ \text{cm}$