

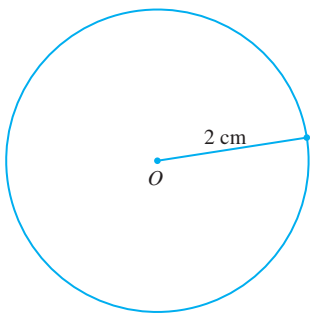
Fully-Worked Solutions

CHAPTER 5 Circles

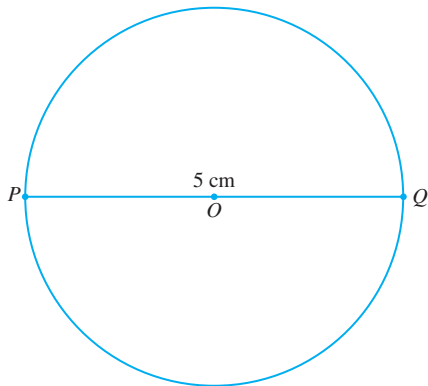
UPSKILL 5.1

- (a) Centre
(b) Radius
(c) Chord
(d) Diameter
(e) Minor arc
(f) Circumference
- (a) Minor sector
(b) Major sector
(c) Minor segment
(d) Major segment

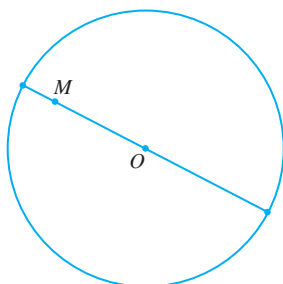
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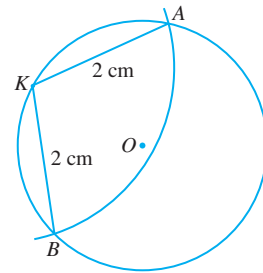
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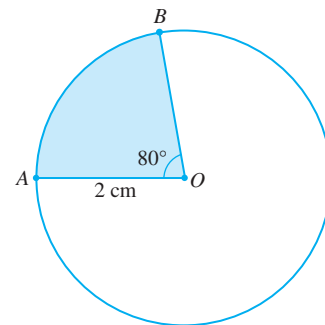
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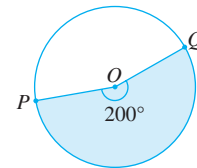
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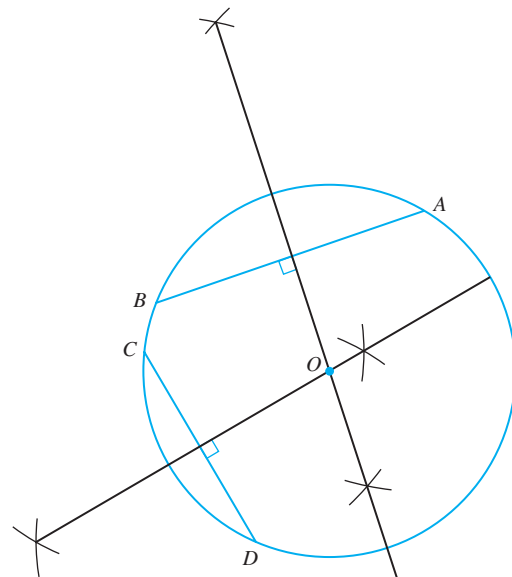
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UPSKILL 5.2

- (a) $CD = BD = 6\text{ cm}$
(b) $PR = \frac{1}{2}PQ$
 $= \frac{1}{2} \times 16\text{ cm}$
 $= 8\text{ cm}$
- (a) $CD = AB = 6\text{ cm}$
(b) $r = \sqrt{4^2 + 3^2} = 5\text{ cm}$

3



s

$$4 \text{ (a) } AD = \frac{1}{2}AB$$

$$= \frac{1}{2} \times 8$$

$$= 4 \text{ cm}$$

$$(b) OD = \sqrt{5^2 - 4^2}$$

$$= 3 \text{ cm}$$

$$\therefore CD = 5 - 3 = 2 \text{ cm}$$

$$5 \text{ } AB = 6 \text{ cm}$$

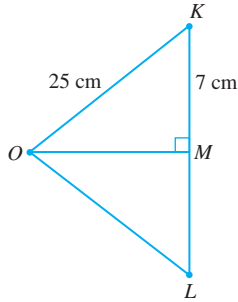
$$OB = \sqrt{10^2 - 6^2} = 8 \text{ cm}$$

$$DE = 8 \text{ cm}$$

$$OE = \sqrt{10^2 - 8^2} = 6 \text{ cm}$$

$$BE = 8 + 6 = 14 \text{ cm}$$

6 (a)



$$KM = \frac{1}{2} \times KL = \frac{1}{2} \times 14 = 7 \text{ cm}$$

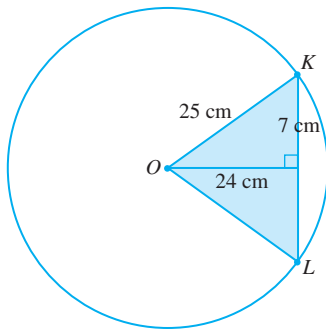
Using Pythagoras' theorem,

$$OM = \sqrt{OK^2 - KM^2}$$

$$= \sqrt{25^2 - 7^2}$$

$$= 24 \text{ cm}$$

(b)



$$\text{Area of } \triangle OKL = \frac{1}{2} \times 14 \times 24 = 168 \text{ cm}^2$$

UPSKILL 5.3

$$1 \text{ (a) } \text{Diameter} = 84 \text{ cm}$$

$$\text{Circumference} = \pi d$$

$$= \frac{22}{7} \times 84$$

$$= 264 \text{ cm}$$

$$(b) \text{ Radius} = 6.3 \text{ cm}$$

$$\text{Circumference} = 2\pi r$$

$$= 2(3.142)(6.3)$$

$$= 39.59 \text{ cm}$$

$$2 \text{ Circumference} = 60 \text{ cm}$$

$$2\pi r = 60$$

$$2(3.142)r = 60$$

$$6.284r = 60$$

$$r = \frac{60}{6.284}$$

$$= 9.55 \text{ cm}$$

$$\text{Diameter} = 2(9.55)$$

$$= 19.10 \text{ cm}$$

$$3 \text{ (a) } \text{Area of circle} = \pi r^2$$

$$= \frac{22}{7} \left(\frac{21}{2} \right)^2$$

$$= 346.5 \text{ m}^2$$

$$(b) \text{ Area of circle} = \pi r^2$$

$$= \frac{22}{7} (14)^2$$

$$= 616 \text{ cm}^2$$

$$4 \text{ Area of circle} = 616 \text{ cm}^2$$

$$\frac{22}{7} \times r^2 = 616$$

$$r^2 = 616 \times \frac{7}{22}$$

$$= 196$$

$$r = \sqrt{196} = 14 \text{ cm}$$

$$\text{Diameter} = 2r$$

$$= 14(2)$$

$$= 28 \text{ cm}$$

$$5 \quad 2\pi r = 88$$

$$2 \times \frac{22}{7} \times r = 88$$

$$r = 88 \times \frac{7}{44}$$

$$= 14 \text{ cm}$$

$$\therefore \text{Area of circle} = \pi r^2$$

$$= \frac{22}{7} (14)^2$$

$$= 616 \text{ cm}^2$$

$$6 \quad \pi r^2 = 125$$

$$3.142r^2 = 125$$

$$r^2 = 39.78$$

$$r = \sqrt{39.78}$$

$$= 6.307 \text{ cm}$$

$$\therefore \text{Circumference of circle} = 2\pi r$$

$$= 2 \times 3.142 \times 6.307$$

$$= 39.63 \text{ cm}$$

$$7 \quad \frac{\text{Length of arc}}{2\pi r} = \frac{120^\circ}{360^\circ}$$

$$\text{Length of arc} = \frac{1}{3} \times 2 \times \frac{22}{7} \times 35$$

$$= 73.33 \text{ cm}$$

$$8 \text{ (a) } \frac{\text{Length of major arc } AB}{2\pi r} = \frac{240^\circ}{360^\circ}$$

$$\text{Length of arc} = \frac{2}{3} \times 2 \times \frac{22}{7} \times 42$$

$$= 176 \text{ cm}$$

$$(b) \frac{\text{Length of major arc } AB}{2\pi r} = \frac{290^\circ}{360^\circ}$$

$$\text{Length of arc} = \frac{29}{36} \times 2 \times \frac{22}{7} \times 0.9$$

$$= 4.557 \text{ cm}$$

$$9 \text{ (a) } \frac{\text{Length of arc}}{2\pi r} = \frac{\theta}{360^\circ}$$

$$\frac{11}{2 \times \frac{22}{7} \times 7} = \frac{\theta}{360^\circ}$$

$$\theta = \frac{1}{4} \times 360^\circ$$

$$= 90^\circ$$

$$(b) \frac{\text{Length of arc}}{2\pi r} = \frac{\theta}{360^\circ}$$

$$\frac{3.3}{2 \times \frac{22}{7} \times 2.8} = \frac{\theta}{360^\circ}$$

$$\theta = \frac{3}{16} \times 360^\circ$$

$$= 67.5^\circ$$

$$10 \frac{\text{Length of arc}}{2\pi r} = \frac{\theta}{360^\circ}$$

$$\frac{82.5}{2 \times \frac{22}{7} \times r} = \frac{75^\circ}{360^\circ}$$

$$\frac{105}{8r} = \frac{5}{24}$$

$$r = \frac{105}{8} \times \frac{24}{5}$$

$$= 63 \text{ cm}$$

$$11 (a) \frac{\text{Area of sector}}{3.142 \times 15^2} = \frac{30^\circ}{360^\circ}$$

$$\text{Area of sector} = \frac{1}{12} \times 3.142 \times 15^2$$

$$= 58.91 \text{ cm}^2$$

$$(b) \frac{\text{Area of sector}}{3.142 \times 8^2} = \frac{125^\circ}{360^\circ}$$

$$\text{Area of sector} = \frac{125}{360} \times 3.142 \times 8^2$$

$$= 69.82 \text{ cm}^2$$

$$(c) \frac{\text{Area of sector}}{3.142 \times 21^2} = \frac{265^\circ}{360^\circ}$$

$$\text{Area of sector} = \frac{265}{360} \times 3.142 \times 21^2$$

$$= 1\,019.97 \text{ cm}^2$$

$$12 \frac{\text{Area of sector}}{\frac{22}{7} \times r^2} = \frac{\theta}{360^\circ}$$

$$\frac{1.4}{\frac{22}{7} \times 8.4^2} = \frac{\theta}{360^\circ}$$

$$\theta = \frac{1.4}{\frac{22}{7} \times 8.4^2} \times 360^\circ$$

$$= 2.273^\circ$$

$$13 \frac{\text{Area of sector}}{\frac{22}{7} \times r^2} = \frac{\theta}{360^\circ}$$

$$\frac{4.62}{\frac{22}{7} \times 2.1^2} = \frac{\theta}{360^\circ}$$

$$\theta = \frac{4.62}{\frac{22}{7} \times 2.1^2} \times 360^\circ$$

$$= 120^\circ$$

$$14 \text{ Length of arc } BDC = \frac{1}{2} \times 2 \times \frac{22}{7} \times 10.5$$

$$= \frac{1}{2} \times 2 \times \frac{22}{7} \times 10.5$$

$$= 33 \text{ cm}$$

$$\text{Perimeter of the whole diagram} = 33 + 21 + 21$$

$$= 75 \text{ cm}$$

$$15 \text{ Area of the shaded region} = \text{Area of rectangle } PQRS - \text{Area of circle}$$

$$= (7 \times 20) - \left(\frac{22}{7} \times 3.5^2\right)$$

$$= 140 - 38.5$$

$$= 101.5 \text{ cm}^2$$

Summative Practice 5

Section A

1 The longest chord in a circle is diameter.

Answer: D

2 A chord divides a circle into two segments.

Answer: C

$$3 AO = \frac{1}{2}AD$$

$$= \frac{1}{2} \times 26$$

$$= 13 \text{ cm}$$

$$PB = \frac{1}{2}PQ$$

$$= \frac{1}{2} \times 10$$

$$= 5 \text{ cm}$$

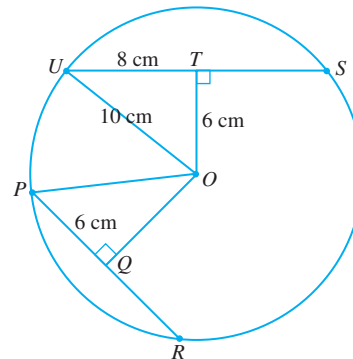
$$BO = \sqrt{13^2 - 5^2} = 12 \text{ cm}$$

$$BO = OC = 12 \text{ cm}$$

$$\therefore BC = 12 + 12 = 24 \text{ cm}$$

Answer: D

4

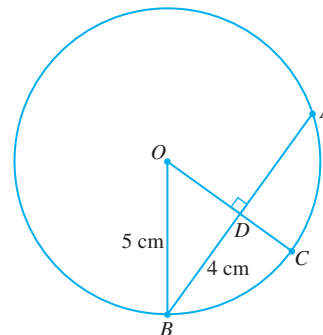


$$OP = OU = 10 \text{ cm}$$

$$OQ = \sqrt{10^2 - 6^2} = 8 \text{ cm}$$

Answer: B

5



$$OD = \sqrt{5^2 - 4^2} = 3 \text{ cm}$$

$$CD = OC - OD$$

$$= 5 - 3$$

$$= 2 \text{ cm}$$

Answer: A

6 Perimeter of circle = $2\pi r$
 $2 \times \frac{22}{7} \times r = 176$
 $r = 28 \text{ cm}$

Area of circle = πr^2
 $= \frac{22}{7} \times 28^2$
 $= 2464 \text{ cm}^2$

Answer: D

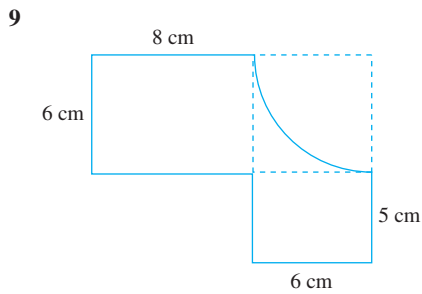
7 Area of the whole diagram = $\frac{1}{2} \times 6 \times 6 + \frac{270}{360} \times \pi \times 6^2$
 $= 18 + 27\pi$

Answer: A

8 $\pi r^2 = 154$
 $\frac{22}{7} r^2 = 154$
 $r^2 = 49$
 $r = \sqrt{49}$
 $= 7 \text{ cm}$

\therefore Circumference of circle = $2\pi r$
 $= 2 \times \frac{22}{7} \times 7$
 $= 44 \text{ cm}$

Answer: C



Area of the diagram = $6 \times 5 + 6(8 + 6) - \frac{1}{4}\pi(6)^2$
 $= 30 + 84 - \frac{1}{4} \times \frac{22}{7} \times 36$
 $= 85.7 \text{ cm}^2$

Answer: C

10 $\frac{22}{7} \times r = 33$
 $r = 10.5$

$PS = 2r$
 $= 2 \times 10.5$
 $= 21 \text{ cm}$

Area of the remaining card = Area of rectangle – Area of circle
 $= 40 \times 21 - \frac{22}{7} \times 10.5^2$
 $= 493.5 \text{ cm}^2$

Answer: A

Section B

- 1 (a) Diameter
 (b) Chord
 (c) Minor sector
 (d) Minor segment

- 2 (a) ✗ (b) ✓ (c) ✓ (d) ✗

Section C

- 1 (a) (i) Chord
 (ii) Diameter

(b) $\pi r^2 = 38.5$
 $\frac{22}{7} r^2 = 38.5$
 $r^2 = 12.25$
 $r = 3.5 \text{ cm}$

Circumference of circle = $2\pi r$
 $= 2 \times \frac{22}{7} \times 3.5$
 $= 22 \text{ cm}$

(c) Area of the shaded region
 $= \frac{80}{360} \times \frac{22}{7} \times 10.5^2 - \frac{80}{360} \times \frac{22}{7} \times 7^2$
 $= 77 - \frac{308}{9}$
 $= \frac{385}{9} \text{ cm}^2$

- 2 (a) (i) Equal chords produce arc of the same length
 (ii) Equal chords are equidistant from the centre of the circle.

(b) (i) $OB = \sqrt{8^2 + 15^2}$
 $= 17 \text{ cm}$

$EF = 17 - 15$
 $= 2 \text{ cm}$

(ii) $DE = \sqrt{17^2 - 15^2}$
 $= 8 \text{ cm}$

(iii) $CD = 2DE$
 $= 2 \times 8$
 $= 16 \text{ cm}$

(c) Area of the shaded region
 $= 3.5 \times 3.5 - 4 \left[\frac{1}{4} \times \frac{22}{7} \times \left(\frac{3.5}{2} \right)^2 \right]$
 $= 12.25 - 9.625$
 $= 2.625 \text{ cm}^2$

3 (a) $2 \left(\frac{22}{7} \right) r = 132$
 $r = 21$

$d = 2r = 42 \text{ cm}$

(b) Area of the remaining plank
 $= \text{Area of rectangle } ABCD - \text{Area of semicircle} - \text{Area of quadrant}$
 $= 7 \times 10 - \frac{1}{2} \times \frac{22}{7} \times 3.5^2 - \frac{1}{4} \times \frac{22}{7} \times 7^2$
 $= 70 - \frac{77}{4} - \frac{77}{2}$
 $= 12.25 \text{ cm}^2$

(c) $CD = 2 \times OF = 2 \times 5 = 10 \text{ cm}$
 $AC = DB = \frac{1}{2}(34 - 10) = 12 \text{ cm}$

Perimeter of salinon
 $= \frac{1}{2} \times \left[2 \left(\frac{22}{7} \times 12 \right) + \frac{22}{7} \times 10 + \frac{22}{7} \times 34 \right]$
 $= \frac{1}{2} \times \left(\frac{528}{7} + \frac{220}{7} + \frac{748}{7} \right)$
 $= \frac{748}{7} \text{ cm}$