Fully-Worked Solutions

CHAPTER **3** **Algebraic Formulae**

UPSKILL **3.1**

1 (a) $m - n = 9$

(b) $n(n + 1) = p$

(c) $a^2 + b^2 + c^2 = 140$

(d) $y = 5x + 6$

(e) $\sqrt[3]{x} = y - 40$

2 $P = 2(x + 1 + y - 2) = 2(x + y - 1)$

3 $A = \dfrac{1}{2}(h)(3k - 5 + 2k)$

$A = \dfrac{1}{2}(h)(5k - 5)$

$A = \dfrac{5h(k - 1)}{2}$

4 $y = 7(2x - 3) + 45$

$y = 14x - 21 + 45$

$y = 14x + 24$

5 $Q = 3\ 000 + 120t$

6 (a) No

(b) No

(c) Yes

(d) No

(e) No

7 (a) $p = m - 8$

(b) $c = a - 5b$

$5b = a - c$

$b = \dfrac{1}{5}(a - c)$

(c) $y = mx + 5$

$mx = y - 5$

$m = \dfrac{y - 5}{x}$

(d) $w = \dfrac{3v}{5u}$

$3v = 5uw$

$v = \dfrac{5uw}{3}$

(e) $e = \sqrt[3]{f - 2}$

$e^3 = f - 2$

$f = e^3 + 2$

(f) $t = r^2 + 1$

$r^2 = t - 1$

$r = \sqrt{t - 1}$

8 (a) $y = \dfrac{12}{x - 3}$

$x - 3 = \dfrac{12}{y}$

$x = \dfrac{12}{y} + 3$

(b) $a = k\sqrt{\dfrac{5}{b}}$

$\dfrac{a^2}{k^2} = \dfrac{5}{b}$

$b = \dfrac{5k^2}{a^2}$

(c) $e = \dfrac{f + 1}{f - 2}$

$ef - 2e = f + 1$

$f(e - 1) = 2e + 1$

$f = \dfrac{2e + 1}{e - 1}$

(d) $\dfrac{1}{x} = \dfrac{1}{2y} + \dfrac{1}{z}$

$\dfrac{1}{2y} = \dfrac{1}{x} - \dfrac{1}{z}$

$\dfrac{1}{2y} = \dfrac{z - x}{xz}$

$y = \dfrac{xz}{2(z - x)}$

9 (a) $5km = hm + 3k$

$m(5k - h) = 3k$

$m = \dfrac{3k}{5k - h}$

(b) $5km = hm + 3k$

$k(5m - 3) = hm$

$k = \dfrac{hm}{5m - 3}$

10 (a) $w = \dfrac{3^2 + 3}{6} = \dfrac{12}{6} = 2$

(b) $4 = \dfrac{x^2 + 3}{7}$

$x^2 = 25$

$x = 5$

11 (a) $k = \sqrt{\dfrac{18 - 2(3)}{3}} = \sqrt{4} = 2$

(b) $3 = \sqrt{\dfrac{7 - 2q}{3}}$

$9 = \dfrac{7 - 2q}{3}$

$27 = 7 - 2q$

$2q = -20$

$q = -10$

12 (a) $\dfrac{5b}{2} = c - 4$

$c = \dfrac{5b}{2} + 4$

(b) $c = \dfrac{5(2)}{2} + 4 = 9$

13 (a) $\dfrac{2}{w} = \dfrac{1}{y} - \dfrac{1}{x}$

$\dfrac{2}{w} = \dfrac{x - y}{xy}$

$w = \dfrac{2xy}{x - y}$

(b) $w = \dfrac{2(4)(3)}{4 - 3} = 24$

14 (a) $P = 6\ 000 + \dfrac{x}{100}(6\ 000)$

$= 6\ 000 + 60x$

(b) $6\ 900 = 6\ 000 + 60x$

$x = \dfrac{900}{60} = 15$

15 (a) $h = \dfrac{v^2}{2g}$

$v = \sqrt{2gh}$

(b) $v = \sqrt{2(10)(5)} = 10$ m/s

16 (a) $C = 20x + \dfrac{3y}{2} + \dfrac{5z}{3}$

(b) $C = 20(4) + \dfrac{3(20)}{2} + \dfrac{5(15)}{3}$

$= 80 + 30 + 25$

$= $ RM135

Summative Practice 3

Section A

1 $C = \dfrac{3}{2}m + \dfrac{5}{2}n$

$C = \dfrac{1}{2}(3m + 5n)$

Answer: **C**

2 $p - 5 = \dfrac{1}{2}(p + q)$

$2p - 10 = p + q$

$p = q + 10$

Answer: **D**

3 $y = x + (x - 2) + 2(x - 2)$

$y = x + x - 2 + 2x - 4$

$y = 4x - 6$

Answer: **A**

4 $3f = 2e - d + 4$

$f = \dfrac{1}{3}(-d + 2e + 4)$

Answer: **C**

5 $2w - 3 = \dfrac{15}{v}$

$w = \dfrac{1}{2}\left(\dfrac{15}{v} + 3\right)$

Answer: **B**

6 $\dfrac{h^2}{k^2} = \dfrac{5}{m - 1}$

$m - 1 = \dfrac{5k^2}{h^2}$

$m = \dfrac{5k^2}{h^2} + 1$

$m = \dfrac{5k^2 + h^2}{h^2}$

Answer: **B**

7 $xy - 3y = 2 - x$

$xy + x = 3y + 2$

$x(y + 1) = 3y + 2$

$x = \dfrac{3y + 2}{y + 1}$

Answer: **D**

8 $\dfrac{b}{c} = \dfrac{1}{b} - \dfrac{1}{a}$

$\dfrac{b}{c} = \dfrac{a - b}{ba}$

$\dfrac{1}{c} = \dfrac{a - b}{ab^2}$

$c = \dfrac{ab^2}{a - b}$

Answer: **A**

9 $x = \dfrac{5 + 3}{5 - 1}$

$x = \dfrac{8}{4} = 2$

Answer: **B**

10 $w = 2(16) - \dfrac{1}{2}(-4) - 3 = 31$

Answer: **D**

11 $4 = 2\sqrt{\dfrac{b + 7}{3}}$

$2 = \sqrt{\dfrac{b + 7}{3}}$

$4 = \dfrac{b + 7}{3}$

$b = 12 - 7 = 5$

Answer: **C**

12 $1 = n^2 - 6n + 10$

$n^2 - 6n + 9 = 0$

$(n - 3)^2 = 0$

$n = 3$

Answer: **D**

Section B

1 (a)

Square of the difference of two numbers is 100. → $(x + y)^2 = 100$

Sum of the squares of two numbers is 100. → $x^2 - y^2 = 100$

(b)

	Statements	TRUE or FALSE
(i)	Given the formula $x = \dfrac{1}{2}(y - x)$. x is the subject of the formula.	FALSE
(ii)	Given the formula $a = \sqrt{b^2 + c^2}$. a is the subject of the formula.	TRUE

2 (a) Given $x = \dfrac{3}{4}$, length $= 12 \times \dfrac{3}{4} = 9$ cm

(b) $y = \sqrt{(5x)^2 + (12x)^2}$

$y = \sqrt{169x^2}$

$y = 13x$

$A = \dfrac{1}{2}(5x)(12x)$

$A = 30x^2$

$P = 5x + 12x + y$

$P = 17x + y$

$P = 17x + 13x$

$P = 30x$

3 (a) $\dfrac{1}{a} = \dfrac{1}{b} + \dfrac{1}{c}$

$\dfrac{1}{a} = \dfrac{c + b}{bc}$

$a = \dfrac{bc}{b + c}$

(b) $\dfrac{1}{b} = \dfrac{1}{a} - \dfrac{1}{c}$

$\dfrac{1}{b} = \dfrac{c - a}{ac}$

$b = \dfrac{ac}{c - a}$

(c)

Values of a, b and c	$a = 2$, $b = 1$, $c = -2$	$a = \dfrac{1}{2}$, $b = 2$, $c = -1$	$a = \dfrac{4}{3}$, $b = 2$, $c = 4$
Answer	✓		✓

Section C

1 (a) $y = 8(x - 3) + 2$
$y = 8x - 24 + 2$
$y = 8x - 22$

(b) (i) $\dfrac{1}{2}k^3x = h + 4y$

$k^3 = \dfrac{2}{x}(h + 4y)$

$k = \sqrt[3]{\dfrac{2(h + 4y)}{x}}$

(ii) $k = \sqrt[3]{\dfrac{2(-4 + 4 \times 6)}{5}} = \sqrt[3]{8} = 2$

(c) (i) Using Pythagoras' theorem $x = \sqrt{25 - y^2}$

$L = \dfrac{1}{2}(3y + 2y)(x)$

$= \dfrac{1}{2}(5y)\left(\sqrt{25 - y^2}\right)$

(ii) When $x = 4$, $y = \sqrt{5^2 - 4^2} = 3$

$L = \dfrac{1}{2}(5 \times 3)(4)$ cm^2

$= 30$ cm^2

2 (a) $(w - 30) + (x - 30) = 75$
$w + x - 60 = 75$
$w = 135 - x$

(b) (i) $A = \dfrac{1}{2}(2y)(6x + 12x) = 18xy$

(ii) $A = 18 \times 5 \times 10 = 900$

(c) (i) $b = 80c + 1.2d$
$k = 100m + 1.5n$

(ii) $b = 256$, $d = 80$
$256 = 80c + 1.2 \times 80$

$c = \dfrac{256 - 96}{80} = 2$

Ali rented the Axia for 2 days.

(iii) $k = 525$, $m = 3$
$525 = 100 \times 3 + 1.5n$
$1.5n = 225$

$n = \dfrac{225}{1.5} = 150$

Latif travelled 150 km.