Fully-Worked Solutions

CHAPTER 13 Simple Probability

UPSKILL 13.1

1 (a) $\frac{19}{100} = 0.19$

(b) Not possible to obtain a number '7'. Hence, the probability of such event is 0.

(c) The event of obtaining a number more than 0 will definitely happen. Hence, the probability of such event is 1.

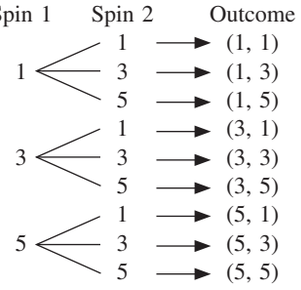
2 (a) $\frac{72}{150} = 0.48$ (b) 0.5

UPSKILL 13.2

1 (a) Red, Blue and Yellow

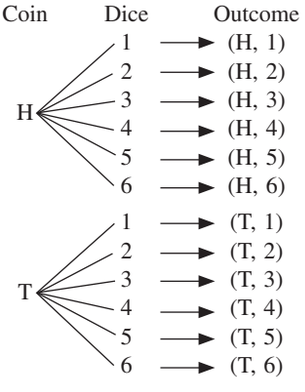
(b) Let R represents red ball, B represents blue balls and Y represents yellow balls
$S = \{R, B_1, B_2, Y_1, Y_2, Y_3\}$

2 (a)



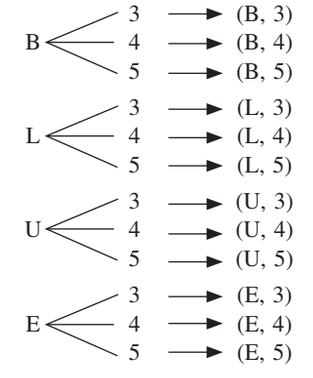
$S = \{(1, 1), (1, 3), (1, 5), (3, 1), (3, 3), (3, 5), (5, 1), (5, 3), (5, 5)\}$

(b)



$S = \{(H, 1), (H, 2), (H, 3), (H, 4), (H, 5), (H, 6), (T, 1), (T, 2), (T, 3), (T, 4), (T, 5), (T, 6)\}$

(c)



$S = \{(B, 3), (B, 4), (B, 5), (L, 3), (L, 4), (L, 5), (U, 3), (U, 4), (U, 5), (E, 3), (E, 4), (E, 5)\}$

3 (a)

Dice 1 / Dice 2	1	2	3	4	5	6
1	(1, 1)	(2, 1)	(3, 1)	(4, 1)	(5, 1)	(6, 1)
2	(1, 2)	(2, 2)	(3, 2)	(4, 2)	(5, 2)	(6, 2)
3	(1, 3)	(2, 3)	(3, 3)	(4, 3)	(5, 3)	(6, 3)
4	(1, 4)	(2, 4)	(3, 4)	(4, 4)	(5, 4)	(6, 4)
5	(1, 5)	(2, 5)	(3, 5)	(4, 5)	(5, 5)	(6, 5)
6	(1, 6)	(2, 6)	(3, 6)	(4, 6)	(5, 6)	(6, 6)

Sample space, S
$= \{(1, 1), (2, 1), (3, 1), (4, 1), (5, 1), (6, 1), (1, 2), (2, 2), (3, 2), (4, 2), (5, 2), (6, 2), (1, 3), (2, 3), (3, 3), (4, 3), (5, 3), (6, 3), (1, 4), (2, 4), (3, 4), (4, 4), (5, 4), (6, 4), (1, 5), (2, 5), (3, 5), (4, 5), (5, 5), (6, 5), (1, 6), (2, 6), (3, 6), (4, 6), (5, 6), (6, 6)\}$

(b) (i) $\{(2, 2), (2, 3), (2, 5), (3, 2), (3, 3), (3, 5), (5, 2), (5, 3), (5, 5)\}$

(ii) $\{(4, 6), (5, 5), (6, 4)\}$

(iii) $\{(1, 5), (2, 6), (5 ,1), (6, 2)\}$

4 (a) $\frac{30}{150} = 0.2$

(b) $\frac{30 + 35 + 20 + 25 + 25}{150} = 0.9$

5 (a) $\frac{15}{50} = 0.3$

(b) $\frac{5 + 8 + 15}{50} = 0.56$

(c) $\frac{6 + 5 + 4}{50} = 0.3$

6 (a) $\frac{65}{250} = 0.26$

(b) $\frac{30}{250} = 0.12$

(c) $\frac{30 + 65 + 45 + 30}{250} = 0.68$

UPSKILL 13.3

1 (a) Event A' is not getting a left-handed boy when choosing a student at random from a group of students.

(b) Event B' is not getting a multiple of 5 when a number is chosen randomly from the set of two-digit numbers.

(c) Event C' is not winning the U-15 football tournament at the inter-district level.

2 (a) Sample space, $S = \{S, A, M, P, L, E\}$

$D = \{A, E\}$

$\therefore D' = \{S, M, P, L\}$

(b) Sample space, S

$= \{(1, 1), (2, 1), (3, 1), (4, 1), (5, 1), (6, 1), (1, 2), (2, 2), (3, 2), (4, 2), (5, 2), (6, 2), (1, 3), (2, 3), (3, 3), (4, 3), (5, 3), (6, 3), (1, 4), (2, 4), (3, 4), (4, 4), (5, 4), (6, 4), (1, 5), (2, 5), (3, 5), (4, 5), (5, 5), (6, 5), (1, 6), (2, 6), (3, 6), (4, 6), (5, 6), (6, 6)\}$

$E = \{(6, 2), (5, 3), (6, 3), (4, 4), (5, 4), (6, 4), (3, 5), (4, 5), (5, 5), (6, 5), (2, 6), (3, 6), (4, 6), (5, 6), (6, 6)\}$

$\therefore E' = \{(1, 1), (2, 1), (3, 1), (4, 1), (5, 1), (6, 1), (1, 2), (2, 2), (3, 2), (4, 2), (5, 2), (1, 3), (2, 3), (3, 3), (4, 3), (1, 4), (2, 4), (3, 4), (1, 5), (2, 5), (1, 6)\}$

(c) Sample space, S
$= \{(H, H, H, H), (H, H, H, T), (H, H, T, H), (H, H, T, T), (H, T, H, H), (H, T, H, T), (H, T, T, H), (H, T, T, T), (T, H, H, H), (T, H, H, T), (T, H, T, H), (T, H, T, T), (T, T, H, H), (T, T, H, T), (T, T, T, H), (T, T, T, T)\}$

$F = \{(H, H, H, H), (H, H, H, T), (H, H, T, H), (H, T, H, H), (T, H, H, H)\}$

$F' = \{(H, H, T, T), (H, T, H, T), (H, T, T, H), (H, T, T, T), (T, H, H, T), (T, H, T, H), (T, H, T, T), (T, T, H, H), (T, T, H, T), (T, T, T, H), (T, T, T, T)\}$

3 W = event of getting a white ball

$$P(W) = 1 - \frac{13}{20} = \frac{7}{20}$$

4 P(no rain on sports day)

$$= 1 - \frac{2}{9} = \frac{7}{9}$$

5 $n(S) = 3 + 6 + 9 = 18$

$$P(\text{not red}) = 1 - \frac{3}{18} = \frac{15}{18} = \frac{5}{6}$$

6 Sample space, $S = \{(H, H, H), (H, H, T), (H, T, H), (H, T, T), (T, H, H), (T, H, T), (T, T, H), (T, T, T)\}$

A = event of getting all tails

A' = event of getting at least one head

$$P(A) = \frac{1}{8}$$

$$P(A') = 1 - P(A) = 1 - \frac{1}{8} = \frac{7}{8}$$

7 Sample space, $S = \{(3, 4), (3, 6), (3, 8), (5, 4), (5, 6), (5, 8), (7, 4), (7, 6), (7, 8)\}$

$n(S) = 9$

(a) D = difference between the numbers is more than 2
$= \{(3, 6), (3, 8), (5, 8), (7, 4)\}$

$n(D) = 4$

$$P(D) = \frac{n(D)}{n(S)} = \frac{4}{9}$$

(b) B = sum of the two numbers is a prime number
$= \{(3, 4), (3, 8), (5, 6), (5, 8), (7, 4), (7, 6)\}$

$n(B) = 6$

$$P(B') = 1 - P(B) = 1 - \frac{6}{9} = \frac{1}{3}$$

8 $S = \{(1, 1), (2, 1), (3, 1), (4, 1), (5, 1), (6, 1), (1, 2), (2, 2), (3, 2), (4, 2), (5, 2), (6, 2), (1, 3), (2, 3), (3, 3), (4, 3), (5, 3), (6, 3), (1, 4), (2, 4), (3, 4), (4, 4), (5, 4), (6, 4), (1, 5), (2, 5), (3, 5), (4, 5), (5, 5), (6, 5), (1, 6), (2, 6), (3, 6), (4, 6), (5, 6), (6, 6)\}$

$n(S) = 36$

(a) $A = \{(5, 1), (6, 2), (1, 5), (2, 6)\}$

$n(A) = 4$

$$P(A') = 1 - P(A) = 1 - \frac{4}{36} = \frac{32}{36} = \frac{8}{9}$$

(b) $B = \{(6, 5), (5, 6), (6, 6)\}$

$$P(B') = 1 - P(B) = 1 - \frac{3}{36} = \frac{33}{36} = \frac{11}{12}$$

(c) $C = \{(6, 2), (4, 3), (3, 4), (2, 6)\}$

$$P(C') = 1 - P(C) = 1 - \frac{4}{36} = \frac{32}{36} = \frac{8}{9}$$

UPSKILL 13.4

1 (a) Probability of selecting an orange-flavoured sweet $= \frac{3}{8}$

$$\frac{15}{15 + x} = \frac{3}{8}$$
$$120 = 45 + 3x$$
$$3x = 75$$
$$x = 25$$

(b) $15 + 25 = 40$

2 (a) R = event of choosing a red ball
$n(R) = 6$, $n(S) = 6 + 12 = 18$

$$P(R) = \frac{n(R)}{n(S)} = \frac{6}{18} = \frac{1}{3}$$

(b) $P(R) = \frac{1}{2}$, $n(R) = 6 + x$, $n(S) = 18 + x$

$$\frac{6 + x}{18 + x} = \frac{1}{2}$$
$$12 + 2x = 18 + x$$
$$x = 6$$

3 (a) (i) $\frac{24}{80} = \frac{3}{10}$

(ii) $\frac{10 + 12}{80} = \frac{11}{40}$

(b) $n(S) = 80 + y$

$$\frac{6 + 24 + y}{80 + y} = \frac{4}{9}$$
$$270 + 9y = 320 + 4y$$
$$5y = 50$$
$$y = 10$$

Summative Practice 13

Section A

1 H = obtaining at least one head
= {(H, H), (H, T), (T, H)}
$n(H) = 22 + 26 + 24 = 72$
$n(S) = 100$

$$P(H) = \frac{n(H)}{n(S)} = \frac{72}{100} = \frac{18}{25}$$

Answer: **A**

2 $n(4) = 15$, $n(S) = 80$

$$P(4) = \frac{15}{80} = \frac{3}{16}$$

Answer: **C**

3 *Answer*: **C**

4 Sample space, S = {S_1, S_2, S_3, U_1, U_2, C_1, C_2, E, F, L}
$n(S) = 10$
C = obtaining a 'C'
$n(C) = 2$

$$P(C) = \frac{n(C)}{n(S)} = \frac{2}{10} = \frac{1}{5}$$

Answer: **D**

5 Sample space, S = {(H, 1), (H, 2), (H, 3), (H, 4), (H, 5), (H, 6), (T, 1), (T, 2), (T, 3), (T, 4), (T, 5), (T, 6)}
$n(S) = 12$
A = obtaining a tail and a multiple of 3
A = {(T, 3), (T, 6)}

$$P(A) = \frac{n(A)}{n(S)} = \frac{2}{12} = \frac{1}{6}$$

Answer: **B**

6 Sample space, S = {10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30}
$n(S) = 21$
A = obtaining a card with number such that the difference between the two digits of the number is 3
A = {14, 25, 30}

$$P(A) = \frac{n(A)}{n(S)} = \frac{3}{21} = \frac{1}{7}$$

Answer: **A**

7 $n(S) = 5 + 7 + 8 = 20$
R = choosing a red chip, $n(R) = 5$

$$P(A) = \frac{n(R)}{n(S)} = \frac{5}{20} = \frac{1}{4}$$

B = choosing a red chip, $n(B) = 7$

$$P(B) = \frac{n(B)}{n(S)} = \frac{7}{20}$$

Y = choosing a red chip, $n(Y) = 8$

$$P(Y) = \frac{n(Y)}{n(S)} = \frac{8}{20} = \frac{2}{5}$$

Answer: **D**

8 W = winning a badminton match
$P(W) = 0.85$
$P(W') = 1 - 0.85 = 0.15$

Answer: **B**

9 $n(S) = 36$
A = {(1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)}

$$P(A) = \frac{n(A)}{n(S)} = 1 - \frac{6}{36} = \frac{30}{36} = \frac{5}{6}$$

Answer: **D**

10 Sample space, S = two-digit numbers
= {10, 11, 12, 13, …, 99}
$n(S) = 90$
$120 = 1 \times 120$
$= 2 \times 60$
$= 3 \times 40$
$= 4 \times 30$
$= 5 \times 24$
$= 6 \times 20$
$= 8 \times 15$
$= 10 \times 12$
F = selecting two-digit numbers which is a factor of 120 and multiple of 5
= {10, 15, 20, 30, 40, 60}
$n(F) = 6$

$$P(F') = 1 - \frac{n(F)}{n(S)} = 1 - \frac{6}{90} = \frac{1}{15} = \frac{14}{15}$$

Answer: **B**

Section B

1 (a) {1, 2, 3, 4, 5, 6}
Equally likely

(b) S = {(H, H, H), (H, H, T), (H, T, H), (T, H, H), (H, T, T), (T, H, T), (T, T, H), (T, T, T)}
Equally likely

(c) S = {E_1, E_2, E_3, L_1, L_2, X, C, N, T}
Not equally likely

(d) S = {R_1, R_2, B_1, B_2, B_3, Y_1, Y_2, Y_3, Y_4}
Not equally likely

2 Sample space, S = {(1, 1), (1, 3), (1, 5), (1, 7), (3, 1), (3, 3), (3, 5), (3, 7), (5, 1), (5, 3), (5, 5), (5, 7), (7, 1), (7, 3), (7, 5), (7, 7)}

P(Both spins show odd numbers) = 1

P(Sum of the two numbers shown is an odd number) = 0

S = {(1, 7), (3, 5), (5, 3), (7, 1)}

P(Sum of the two numbers shown is 8) $= \frac{4}{16} = \frac{1}{4}$

S = {(1, 5), (1, 7), (3, 7), (5, 1), (7, 1), (7, 3)}

P(Difference between the two numbers shown is at least 4)

$= \frac{6}{16} = \frac{3}{8}$

3 $n(S) = 6 + 10 + 20 + 13 + 11 = 60$

(a) $P(\text{chess}) = \frac{6}{60} = \frac{1}{10}$

(b) P(bowling or squash)

$= \frac{13 + 11}{60}$

$= \frac{2}{5}$

(c) P(not table tennis)

$= 1 - P(\text{table tennis})$

$= 1 - \frac{20}{60} = \frac{2}{3}$

(d) P(not badminton and table tennis)

$= 1 - P$(chess and bowling and squash)

$= 1 - \frac{6 + 13 + 11}{60} = \frac{1}{2}$

4 (a) (i) A = {a month with 31 days from the calendar}

= {January, March, May, July, August, October, December}

A' = {February, April, June, September, November}

TRUE

(ii) Q = {prime numbers from 1 to 9}

= {2, 3, 5, 7}

Q' = {1, 4, 6, 8, 9}

$n(Q') = 5$

FALSE

(b) P(Team A does not win the football competition)

$= 1 - \frac{1}{4}$

$= \frac{3}{4}$

P(All the three teams do not win the football competition)

$= 1 - \left(\frac{1}{4} + \frac{1}{8} + \frac{5}{12}\right)$

$= 1 - \frac{6 + 3 + 10}{24}$

$= \frac{5}{24}$

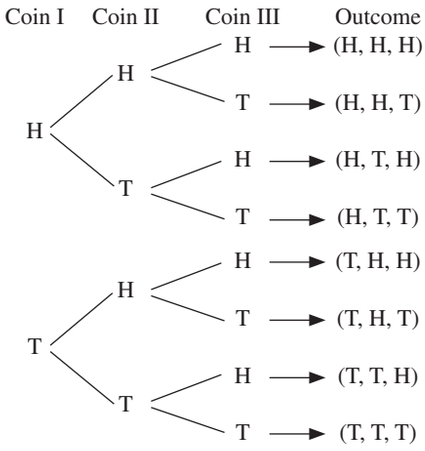
5 (a) A' is the event of obtaining all heads

A' = {H, H}

B' is the event of obtaining all tails

A' = {T, T}

(b)



$n(s) = 8$

A = Event that the three coins show the same outcome

= {(H, H, H), (T, T, T)}

$P(A) = \frac{n(A)}{n(S)}$

$= \frac{2}{8}$

$= \frac{1}{4}$

B = Event of obtaining exactly two head

= {(H, H, T), (H, T, H), (T, H, H)}

$P(B) = \frac{n(B)}{n(S)}$

$= \frac{3}{8}$

Section C

1 (a) (i) Sample space, $S = \{S_1, S_2, E, A, K\}$

(ii) $n(S) = 5$

(b) (i) Y = yellow marble

$P(Y) = \frac{1}{5}$

$P(Y') = 1 - \frac{1}{5} = \frac{4}{5}$

(ii) $\frac{7 + 8 + 5 + 4}{7 + 8 + 5 + 4 + x} = \frac{4}{5}$

$24 \times 5 = 4(24 + x)$

$120 = 96 + 4x$

$4x = 24$

$x = 6$

Alternative method

$\frac{x}{7 + 8 + 5 + 4 + x} = \frac{1}{5}$

$\frac{x}{24 + x} = \frac{1}{5}$

$5x = 24 + x$

$4x = 24$

$x = 6$

(c) (i) $P(\text{female worker}) = 1 - \frac{3}{10} = \frac{7}{10}$

(ii) $P(\text{female worker}) = \frac{y}{12 + y}$

$\frac{y}{12 + y} = \frac{7}{10}$

$10y = 84 + 7y$

$3y = 84$

$y = 28$

2 (a) (i) V' = {B, S, T, R}

(ii) V' = {Monday, Tuesday, Wednesday, Thursday, Friday}

(b) (i) $P(\text{student with brown eyes}) = \frac{3 + 6}{15 + 15} = \frac{9}{30}$

$P(\text{student does not have brown eyes}) = 1 - \frac{9}{30}$

$= \frac{21}{30}$

$= \frac{7}{10}$

(ii) $P(\text{student is a boy who has brown eyes}) = \frac{3}{30} = \frac{1}{10}$

(iii) $P(\text{student is not a girl who has brown eyes})$

$= 1 - \frac{6}{30} = \frac{4}{5}$

(c) $n(S) = 6 + 12 + 15 + 8 + 4 = 45$

(i) $P(<12) = \frac{6 + 12 + 15 + 8}{10} = \frac{41}{45}$

(ii) $\frac{6 + n}{45 + n} = \frac{1}{4}$

$24 + 4n = 45 + n$

$3n = 21$

$n = 7$

3 (a) (i) Probability that the champion is not A

$= P(A')$

$= 1 - \frac{7}{12}$

$= \frac{5}{12}$

(ii) Probability that the champion is B

$= 1 - \frac{3}{4}$

$= \frac{1}{4}$

Probability that the champion is C

$= 1 - \frac{7}{12} - \frac{1}{4}$

$= \frac{12 - 7 - 3}{12}$

$= \frac{1}{6}$

(b) (i) $S = \{(O, S), (O, I), (O, X), (N, S), (N, I), (N, X), (E, S), (E, I), (E, X)\}$

(ii) Let A be the event that the two cards drawn are vowels.

$A = (O, I), (E, I)$

$P(A) = \frac{2}{9}$

(iii) Let B be the event that exactly one consonant is drawn.

$B = (O, S), (O, X), (N, I), (E, S), (E, X)$

$P(B) = \frac{5}{9}$

(c) (i) Probability that the student chosen is not a prefect

$= 1 - \frac{3}{8}$

$= \frac{5}{8}$

(ii) $\frac{3}{8} = \frac{9}{24}$

Therefore, the total number of students in the class is 24

(iii) $\frac{9 + x}{24 + x} = \frac{4}{9}$

$81 + 9x = 96 + 4x$

$5x = 15$

$x = 3$