Fully-Worked Solutions

CHAPTER 12 Measures of Central Tendencies

UPSKILL 12.1

1 (a) 3 6 6 6 7 8 9 11

Mode = 6

Median = $\frac{6 + 7}{2} = 6.5$

Mean = $\frac{56}{8} = 7$

(b) 2 9 11 11 12 14 16 18

Mode = 11

Median = $\frac{11 + 12}{2} = 11.5$

Mean = $\frac{93}{8} = 11.625$

(c) 9.8 11.2 13.0 13.9 14.1 15.5

Mode = None

Median = $\frac{13 + 13.9}{2} = 13.45$

Mean = $\frac{77.5}{6} = 12.92$

(d) 21.2 22.5 23.6 23.6 24.8 30.2

Mode = 23.6

Median = $\frac{23.6 + 23.6}{2} = 23.6$

Mean = $\frac{145.9}{6} = 24.32$

2 (a) Mode = 2 people

Total number of cars

= 10 + 15 + 12 + 8

= 45

$\frac{n + 1}{2} = \frac{46}{2} = 23$

∴ Median = 23rd value = 2 people

Mean = $\frac{(1 \times 10) + (2 \times 15) + (3 \times 12) + (4 \times 8)}{45}$

= $\frac{108}{45} = 2.4$

(b) Mode = 2 children

Total number of families

= 3 + 7 + 13 + 12 + 11

= 46

$\frac{n}{2} = \frac{46}{2} = 23$

$\frac{n}{2} + 1 = \frac{46}{2} + 1 = 24$

∴ Median is the average of the values at the 23rd and 24th positions.

Median = $\frac{2 + 3}{2} = 2.5$

Mean = $\frac{(0 \times 3) + (1 \times 7) + (2 \times 13) + (3 \times 12) + (4 \times 11)}{46}$

= $\frac{113}{46} = 2.46$

3 (a) 52 53 53 54 55 55 59 59 59 62

Mode = 59 cm

(b) 52 53 53 54 55 55 59 59 62

Mode = 53 cm, 55 cm, 59 cm

4 (a) 6 9 9 12 12 12 15 15 18

Mode = 12

Median = 12

Mean = $\frac{108}{9} = 12$

(b) (i) Every data is subtracted by 4

Mode = 12 − 4 = 8

Median = 12 − 4 = 8

Mean = 12 − 4 = 8

(ii) Every data is divided by 3

Mode = 12 ÷ 3 = 4

Median = 12 ÷ 3 = 4

Mean = 12 ÷ 3 = 4

5 Mean = $\frac{(20 \times 170) + (175 + 178 + 174 + 171 + 177)}{25}$

= $\frac{3\,400 + 875}{25}$

= 171 cm

6 Mean = $\frac{(25 \times 55) - 79}{24}$

= $\frac{1\,375 - 79}{24}$

= 54 kg

7 (6 × 6.5) − (6.2 + 6.6 + 6.3 + 5.9 + 6.5)

= 39 − 31.5

= 7.5 kg

8 Mean = $\frac{6 + 9 + p + 12 + 7 + 8 + q}{7}$

$8 = \frac{p + q + 42}{7}$

$p + q = 56 - 42$

= 14

9

Marks	Tally	Frequency
50 – 54	////	4
55 – 59	𝍸 /	6
60 – 64	𝍸 ///	8
65 – 69	𝍸 𝍸	10
70 – 74	𝍸	5
74 – 79	𝍸 //	7

10 (a)

Height (cm)	Midpoint	Frequency, f
135 – 139	137	6
140 – 144	142	8
145 – 149	147	11
150 – 154	152	19
155 – 159	157	6

Modal class = (150 – 154)

Mean = $\frac{(137 \times 6) + (142 \times 8) + (147 \times 11) + (152 \times 19) + (157 \times 6)}{50}$

= $\frac{7\,405}{50} = 148.1$

(b)

Score	Midpoint	Frequency, f
0 – 9	4.5	5
10 – 19	14.5	10
20 – 29	24.5	25
30 – 39	34.5	35
40 – 49	44.5	25

Modal class = (30 – 39)

$$\text{Mean} = \frac{(4.5 \times 5) + (14.5 \times 10) + (24.5 \times 25) + (34.5 \times 35) + (44.5 \times 25)}{100}$$

$$= \frac{3\ 100}{100}$$

$$= 31$$

11 6.5 6.8 6.9 7.0 7.1 7.2 7.7 8.0 8.0 17.1

Mode = 8

$$\text{Median} = 5.5^{\text{th}} \text{ value} = \frac{7.1 + 7.2}{2} = 7.15$$

$$\text{Mean} = \frac{6.5 + 6.8 + 6.9 + 7.0 + 7.1 + 7.2 + 7.7 + 8.0 + 8.0 + 17.1}{10}$$

$$= \frac{82.3}{10}$$

$$= 8.23$$

Median is the best measure of central tendency.
Mode is not suitable because it lies at one of the ends of the ordered set of data.
Mean is not suitable because there is an extreme value of 17.1.

12 Mode is the best measure of central tendency. Shoe sizes are categorical data. Mode can provide valuable information on the sizes of shoes with the highest demand in the market.

13 Mode is the best measure of central tendency since mode can provide valuable information on the best-selling colour of cars in the market.

14 (a) Mode = 4

Total frequency = 1 + 2 + 3 + 5 + 2 + 4 + 2
= 19

$$\frac{n+1}{2} = \frac{19+1}{2} = 10$$

$\therefore$ Median = 10^{th} value = 4

$$\text{Mean} = \frac{(1 \times 1) + (2 \times 2) + (3 \times 3) + (4 \times 5) + (5 \times 2) + (6 \times 4) + (7 \times 2)}{19}$$

$$= \frac{82}{19}$$

$$= 4.32$$

(b) Mode = 60 kg

Total frequency = 2 + 5 + 7 + 6 + 4 + 3
= 27

$$\frac{n+1}{2} = \frac{27+1}{2} = 14$$

Median = 14^{th} value = 60

$$\text{Mean} = \frac{(50 \times 2) + (55 \times 5) + (60 \times 7) + (65 \times 6) + (70 \times 4) + (75 \times 3)}{27}$$

$$= \frac{1\ 690}{27}$$

$$= 62.59$$

15 Team A: Range = 3 – 0 = 3; Mean = $\frac{17}{12}$ = 1.42 goals per game

Team B: Range = 5 – 0 = 5; Mean = $\frac{26}{12}$ = 2.17 goals per game

$$\frac{2.17}{1.42} = 1.53$$

Team B has scored around 1.5 times more goals per game compared to team A.

Summative Practice 12

Section A

1 5 6 6 8 9 10 11 11

$$\text{Median} = \frac{8+9}{2} = 8.5$$

Answer: **C**

2 $$\text{Mean} = \frac{12 + 16 + 19 + 13 + 9 + 13 + 11 + 13}{8}$$

$$= \frac{106}{8}$$

$$= 13.25$$

Answer: **B**

3

Score	0	1	2	3	4
Number of participants	5	8	9	8	7

Score 2 has the highest frequency of 9.
Thus, mode is 2.

Answer: **A**

4 (8 × 9.25) – (7 × 10.36) = 74 – 72.52 = 1.48 kg

Answer: **A**

5 6 6 6 7 8 10

Mode = 6

$$\text{Median} = \frac{6+7}{2} = 6.5$$

6.5 – 6 = 0.5

Answer: **C**

6 3, x, 9, 1, x, 8, 7, 8

$$\frac{2x + 36}{8} = 5.5$$

$$2x = 44 - 36$$

$$x = 4$$

1 3 4 4 7 8 8 9

$$\text{Median} = \frac{4+7}{2} = 5.5$$

Answer: **D**

7

Mass (kg)	Midpoint	Frequency, f
40 – 44	42	5
45 – 49	47	4
50 – 54	52	6
55 – 59	57	3
60 – 64	62	2

$$\text{Mean} = \frac{(42 \times 5) + (47 \times 4) + (52 \times 6) + (57 \times 3) + (62 \times 2)}{20}$$

$$= \frac{1\ 005}{20}$$

$$= 50.25$$

Answer: **B**

8 *Answer*: **C**

9 7 9 9 9 10 10 11 11 12 28

Median is the best representative of the data because there is an extreme value.

Answer: **D**

10

Stem	Leaf
2	3 5 8 8 9
3	1 7 9
4	1 3

Mode = 28

Total frequency = 10

$\frac{n}{2} = \frac{10}{2} = 5$

$\frac{n}{2} + 1 = \frac{10}{2} + 1 = 6$

$\therefore$ Median is the average of the values at the 5^{th} and 6^{th} positions.

Median $= \frac{29 + 31}{2} = 30$

Mode + Median = 30 + 28 = 58

Answer: **A**

Section B

1 (a) The total points scored by the 8 players is $46 + p + q$

(b) $\frac{(p + q) + 46}{8} = 8$

$p + q = 64 - 46$

$= 18$

(c) $p + q = 18$

$p = q = 9$

$\therefore$ Mode = 9

(d) $p \neq q$, $\therefore$ Mode = 5

2 (a) 145 150 155 157 158 159 161

(b) (i) Mean $= \frac{145 + 150 + 155 + 157 + 158 + 159 + 161}{7}$

$= \frac{1\ 085}{7}$

$= 155$ cm

(ii) Median = 157 cm

(iii) 145 150 152 155 157 158 159 161

Median $= \frac{155 + 157}{2}$

$= 156$ cm

3 (a) Midpoint $= \frac{4 + 6}{2} = 5$

$4 + 6 + y + 2y + 3 = 25$

$13 + 3y = 25$

$3y = 12$

$y = 4$

(b) Frequency: 4 6 4 8 3

Mode lies in the class interval (10 – 12)

Total frequency = 25

$\frac{n + 1}{2} = \frac{25 + 1}{2} = 13$

$\therefore$ Median is the value at the 13^{th} position in the class interval (7 – 9)

4

	Statement	TRUE or FALSE
(a)	The class interval size is 1.	FALSE
(b)	There are data with extreme values.	FALSE
(c)	The modal class is RM(8 – 9).	TRUE
(d)	The mean is the most appropriate measure of central tendency.	TRUE

Section C

1 (a) Total wages of 20 workers = RM3 100 × 20 = RM62 000

Total wages of the female workers

= RM2 500 × 8 = RM20 000

Mean wage of the male workers $= \frac{62\ 000 - 20\ 000}{12}$

$=$ RM3 500

(b) 3 5 5 7 9 10

(i) Mode = 5

(ii) Median $= \frac{5 + 7}{2} = 6$

(iii) Mean $= \frac{3 + 5 + 5 + 7 + 9 + 10}{6}$

$= \frac{39}{6} = 6.5$

(c) (i) 14, 18, 10, 6, 10, 20

Every data given is multiplied by 2

Mean = 6.5 × 2 = 13

(ii) 20, 22, 18, 16, 18, 23

Every data given is added to 13

Mean = 6.5 + 13 = 19.5

2 (a) (i) Total height of the 3 plants = 40 × 3 = 120 cm

$\therefore$ The height of plant $Q = \frac{7}{20} \times 120 = 42$ cm

(ii) Total height of the 4 plants = 45 × 4 = 180 cm

$\therefore$ The height of plant S = 180 − 120 = 60 cm

(b) (i)

Class interval	Tally	Frequency	Midpoint
10 – 14	//	2	12
15 – 19	////	4	17
20 – 24	𝍸 𝍸 //	12	22
25 – 29	𝍸 𝍸	10	27
30 – 34	𝍸 ///	8	32
35 – 39	////	4	37

(ii) (a) Modal class is (20 – 24)

(b) Mean $= \frac{(12 \times 2) + (17 \times 4) + (22 \times 12) + (27 \times 10) + (32 \times 8) + (37 \times 4)}{40}$

$= \frac{1\ 030}{40}$

$= 25.75$