Fully-Worked Solutions

## CHAPTER 11 Isometric Transformations

UPSKILL 11.1

1 (a) Yes

(b) Yes

(c) Yes

(d) No

2 (a) Congruent

(b) Congruent

(c) Not congruent

UPSKILL 11.2

1 (a) Yes, because all the points of the object are moved through the same distance in the same direction to their corresponding points on the image.

(b) No, because the shapes and sizes are not the same.

2 (a) Object A is moved 2 units to the left and 4 units downwards.

(b) Translation $\begin{pmatrix} -2 \\ -4 \end{pmatrix}$

3
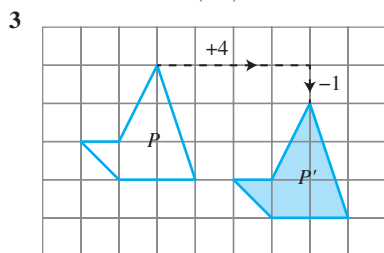
4 (a) $\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 6 \\ -4 \end{pmatrix} + \begin{pmatrix} -5 \\ 2 \end{pmatrix}$

$= \begin{pmatrix} 6 - 5 \\ -4 + 2 \end{pmatrix}$

$= \begin{pmatrix} 1 \\ -2 \end{pmatrix}$

Coordinates of image, $P' = (1, -2)$

(b) $\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} -4 \\ 9 \end{pmatrix} + \begin{pmatrix} 7 \\ -4 \end{pmatrix}$

$= \begin{pmatrix} -4 + 7 \\ 9 - 4 \end{pmatrix}$

$= \begin{pmatrix} 3 \\ 5 \end{pmatrix}$

Coordinates of image, $Q' = (3, 5)$

5 $\begin{pmatrix} 5 \\ 1 \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} -2 \\ 8 \end{pmatrix}$

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ 1 \end{pmatrix} - \begin{pmatrix} -2 \\ 8 \end{pmatrix}$

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 + 2 \\ 1 - 8 \end{pmatrix}$

$= \begin{pmatrix} 7 \\ -7 \end{pmatrix}$

Coordinates of $A = (7, -7)$

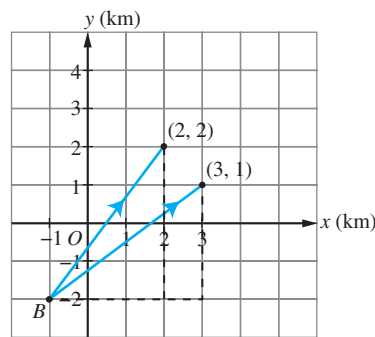**6** Let the translation be $\begin{pmatrix} x \\ y \end{pmatrix}$

For P,

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 + 2 \\ -2 - 1 \end{pmatrix} = \begin{pmatrix} 5 \\ -3 \end{pmatrix}$

Let the coordinates of Q be (a, b)

$\begin{pmatrix} 6 \\ -3 \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix} - \begin{pmatrix} 5 \\ -3 \end{pmatrix}$

$\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 6 - 5 \\ -3 + 3 \end{pmatrix}$

$= \begin{pmatrix} 1 \\ 0 \end{pmatrix}$
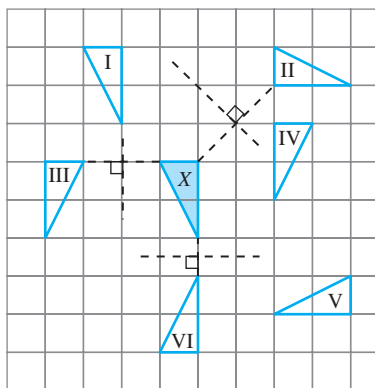
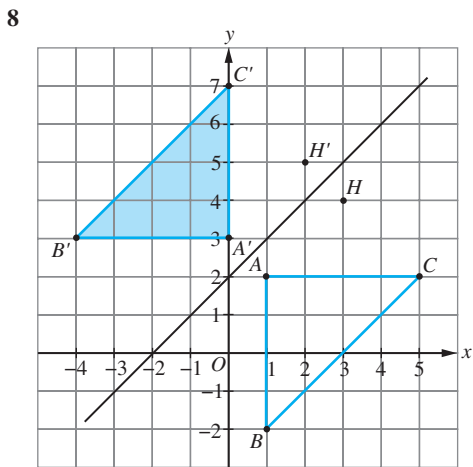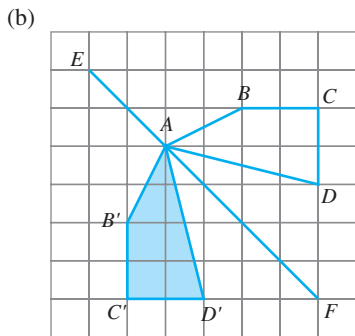$\therefore Q(1, 0)$

7

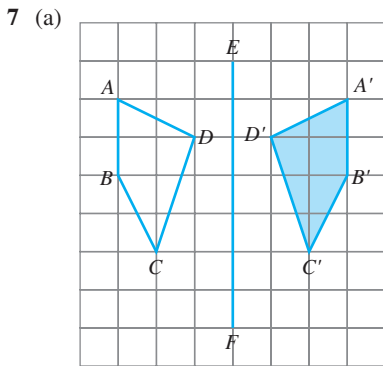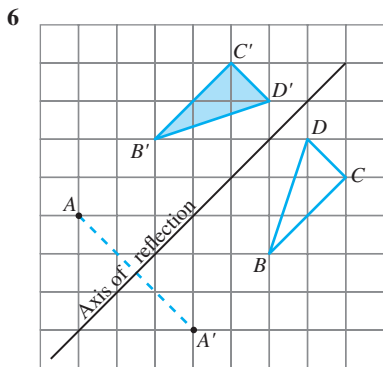$(2, 2)$ and $(3, 1)$

UPSKILL 11.3

1 (a) Yes

(b) No

2 (a) Object P is mapped to image Q under a reflection in the y-axis.

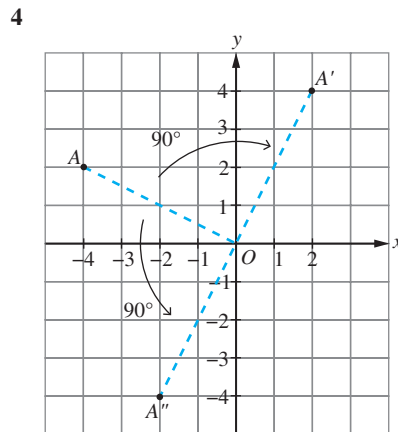(b) Object P is mapped to image Q under a reflection in the line CF.
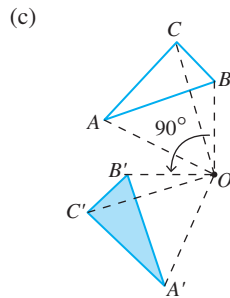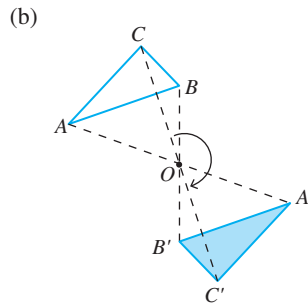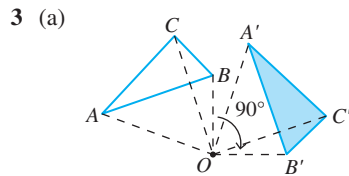
3

II, III and VI

4 (a) $K'(5, 3)$, $L'(-2, -7)$, $M'(0, -4)$ and $N'(6, 0)$

(b) $K'(-5, -3)$, $L'(2, 7)$, $M'(0, 4)$ and $N'(-6, 0)$

5 (a) $(2, -2)$ or any point on the line PQ

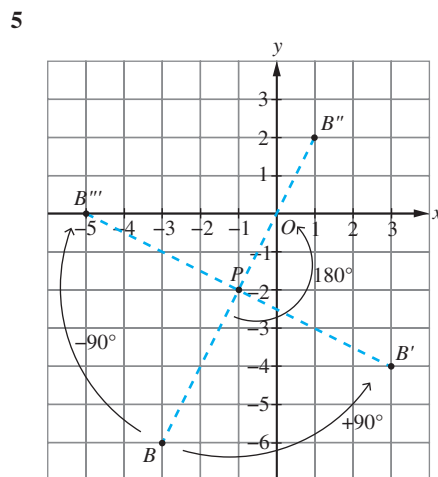(b) $A'(-1, 3)$, $B'(-2, -3)$

(c) $(-k, -h)$

6

7 (a)

(b)

8

$A'(0, 3)$, $B'(-4, 3)$, $C'(0, 7)$

UPSKILL 11.4

1 (a) Yes

(b) No

(c) Yes

2 (a) Anticlockwise rotation of 90° about the point (3, 1).

(b) Clockwise rotation of 90° about the origin (0, 0).
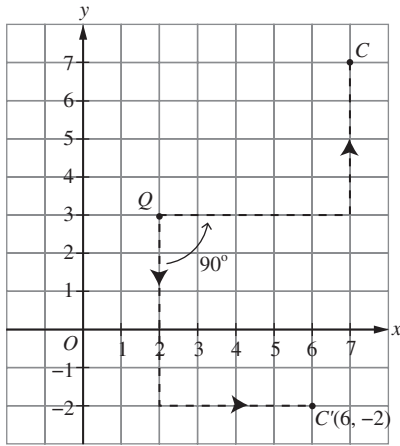
(c) Rotation of 180° about the point (1, −1).
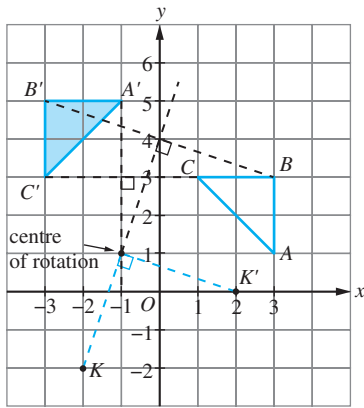
3 (a)

(b)

(c)

4

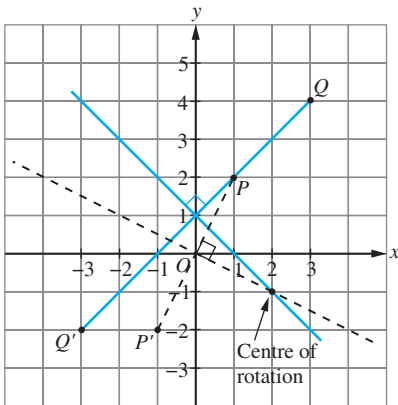(a) (2, 4)

(b) (−2, −4)

5

(a) (3, −4)

(b) (1, 2)

(c) (−5, 0)

6

$C \to C'$ is a clockwise rotation of 90°,
implies that $C' \to C$ is an anticlockwise rotation of 90°

∴ $C(7, 7)$

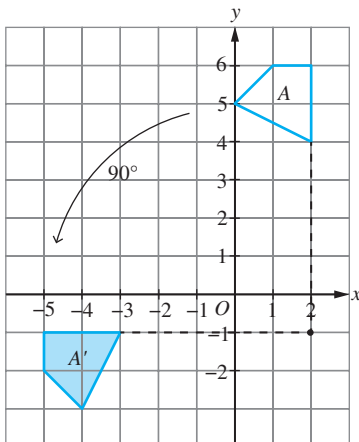**7** (a) Anticlockwise rotation of 90° about the point $(-1, 1)$.
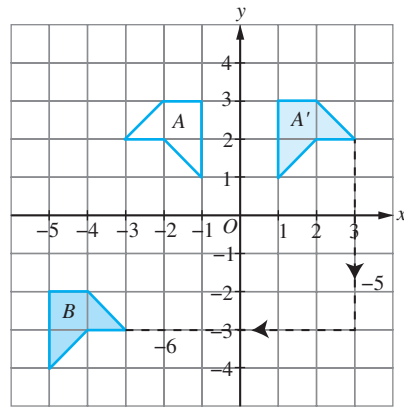
(b) $K'(2, 0)$

8 (a)

Anticlockwise rotation of 90° about the centre $(2, -1)$
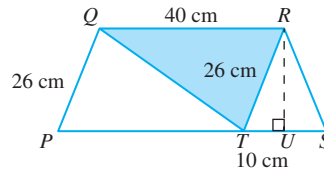
(b)

1 (a) I, IV

(b) II, IV

2

(a) Congruent

(b) Reflection in the y-axis followed by translation $\begin{pmatrix} -6 \\ -5 \end{pmatrix}$

3 (a) $T \to Q$; $P \to R$; $Q \to T$

The transformation is a rotation of 180° about the midpoint of QT.

(b)

In $\triangle TRS$, RU is the perpendicular bisector of TS. Hence, $TU = 10$ cm

In $\triangle RTU$, using Pythagoras' theorem,

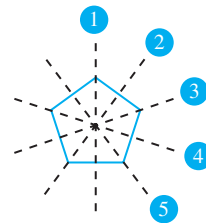$RU^2 = 26^2 - 10^2$

$\quad\; = 576$

$RU = \sqrt{576}$

$\quad\; = 24$ cm

Area of $\triangle QRT = \dfrac{1}{2} \times QR \times RU$

$\qquad\qquad\quad = \dfrac{1}{2} \times 40 \times 24$

$\qquad\qquad\quad = 480$ cm^2
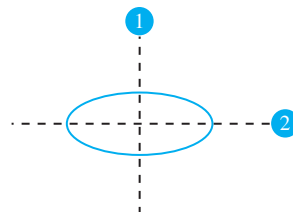
1 (a) No   (b) Yes   (c) No   (d) Yes

2 (a)

Order = 5

(b)

Order = 2

3 (a) 4

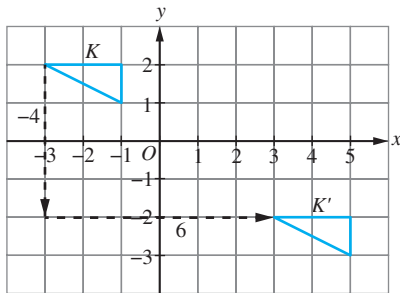   (b) 1

   (c) 1

   (d) Infinite

4 (a) 4

   (b) 1

   (c) 2

   (e) 2

Summative Practice 11

Section A

1 *Answer*: **B**

2

Translation $\begin{pmatrix} 6 \\ -4 \end{pmatrix}$

Answer: **A**

3 $\begin{pmatrix} 3 \\ 8 \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} -5 \\ 4 \end{pmatrix}$

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 + 5 \\ 8 - 4 \end{pmatrix}$
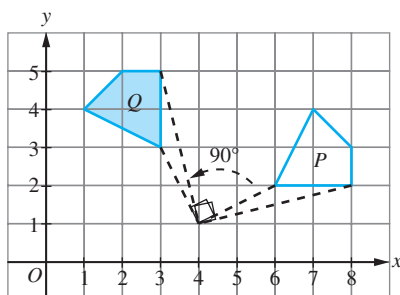
$\phantom{\begin{pmatrix} x \\ y \end{pmatrix}} = \begin{pmatrix} 8 \\ 4 \end{pmatrix}$

Answer: **D**

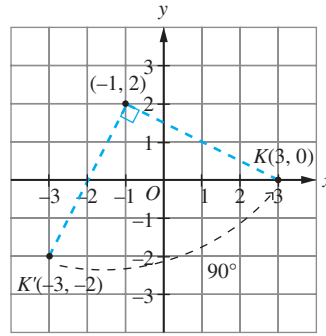**4** Reflection in the *y*-axis: $K(-4, 7) \to K'(4, 7)$

Answer: **D**

5 *Answer*: **C**

6

Answer: **B**

7 $K \to K'$: Anticlockwise rotation of 270°

$K' \to K$: Anticlockwise rotation of 90°

Answer: **A**

8 *Answer*: **C**

9 *Answer*: **D**

10 *Answer*: B

Section B

1 (a) FALSE

   (b) FALSE

   (c) TRUE

   (d) TRUE

2 (a) ✗

   (b) ✓

   (c) ✓

   (d) ✗

3 (a) *R*

   (b) *S*

   (c) *Q*

   (d) *P*

4

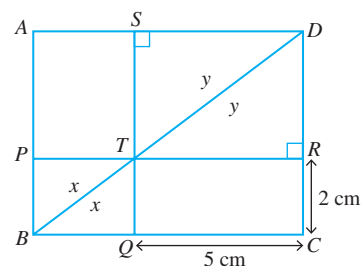| Shape	Number of axis of symmetry		Order of rotational symmetry	
(a)	0	✓	0	
	1		1	✓
(b)	2	✓	2	✓
	4		4	

Section C

1 (a) 5 units to the left and 9 units upwards

   (b) (i) Δ*BQT*
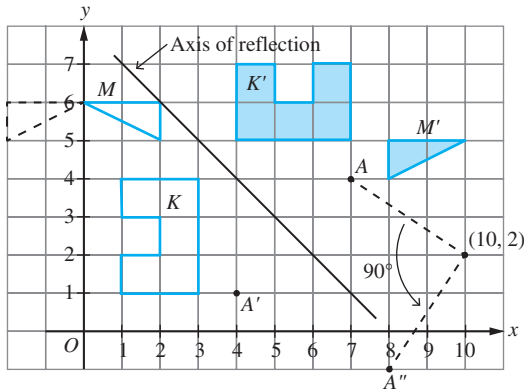
      (ii) Rotation of 180° about the midpoint of *TD*

      (iii)

      Area of rectangle $TQCR = 2 \times 5 = 10$ cm^2

Let area of ΔPBT = area of ΔQBT = x

Let area of ΔSTD = area of ΔRTD = y

ΔABD and ΔBCD are congruent.

Area of ΔABD = Area of ΔCBD

Area of $APTS$ + x + y = Area of $TQCR$ + x + y

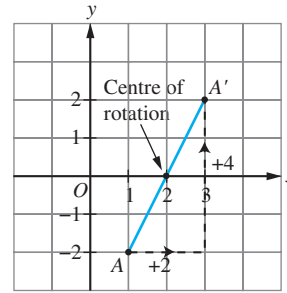Area of $APTS$ = Area of $TQCR$
$= 10$ cm^2

(c)

2 (a) (i) $\begin{pmatrix} 3 \\ -4 \end{pmatrix} = \begin{pmatrix} 5 \\ 7 \end{pmatrix} + \begin{pmatrix} x \\ y \end{pmatrix}$

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 - 5 \\ -4 - 7 \end{pmatrix}$

$= \begin{pmatrix} -2 \\ -11 \end{pmatrix}$

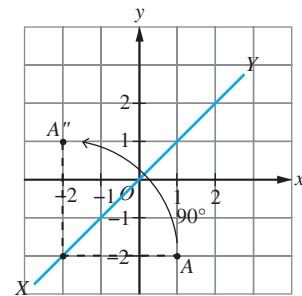$\therefore$ Translation $\begin{pmatrix} -2 \\ -11 \end{pmatrix}$

(ii) $\begin{pmatrix} -8 \\ -1 \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} -2 \\ -11 \end{pmatrix}$

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -8 + 2 \\ -1 + 11 \end{pmatrix}$

$= \begin{pmatrix} -6 \\ 10 \end{pmatrix}$

$\therefore Q(-6, 10)$

(b) (i)

Rotation of 180° about the point (2, 0)

(ii)

Coordinates of centre of rotation = (−2, −2)

(c) (i) Clockwise rotation of 90° about the point E.

(ii) $DE = EF = 6.3$ cm

$DJ = DE - EJ$
$= 6.3 - 4.5$ cm
$= 1.8$ cm

Perimeter = $AB + BC + CD + DJ + JH + HG + GF + EF + AE$
$= 4.5 + 2.7 + 4.5 + (6.3 - 4.5) + 4.5 + 2.7 + 4.5 + 6.3 + 4.5$
$= 36$ cm