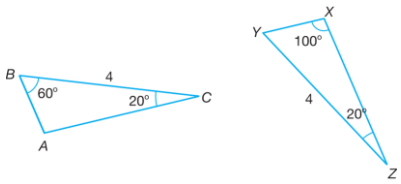
Form 5: Chapter 5
Congruency, Enlargement and Combined Transformations
Fully-worked Solutions

UPSKILL 5.1

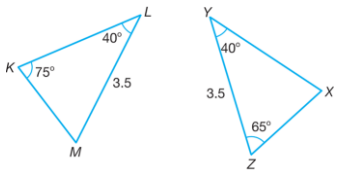
1 (a) Congruent
(b) Congruent
(c) Not congruent

2 (a)



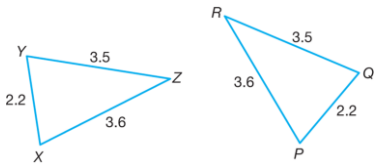
$\angle XYZ = 180^o - 100^o - 20^o = 60^o$
$\Delta BCA \cong \Delta YZX$ [Angle- Side-Angle]

(b)



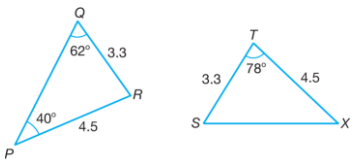
$\angle YXZ = 180^o - 40^o - 65^o = 75^o$
$\Delta KLM \cong \Delta XYZ$ [Angle-Angle-Side]

(c)



$\Delta XYZ \cong \Delta PQR$ [Side-Side-Side]

(d)



$\angle PRQ = 180^o - 40^o - 62^o = 78^o$
$\Delta PRQ \cong \Delta XTS$ [Side-Angle-Side]

3 ΔCAB and ΔBDC are congruent.
$x = \angle DBC = \angle ACB = 180^o - 101^o - 32^o = 47^o$

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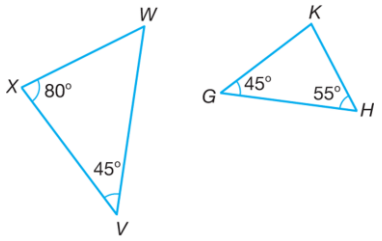
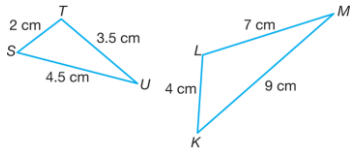
1 (a)



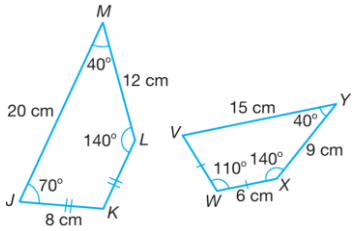
$\angle XWV = 180^o - 80^o - 45^o = 55^o$
$\angle GKH = 180^o - 45^o - 55^o = 80^o$
ΔXWV and KHG are similar because corresponding angles are equal.

(b)



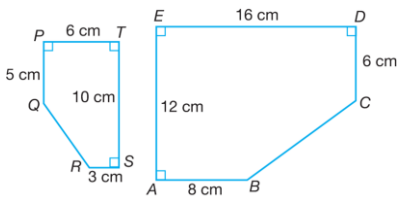
$\frac{US}{MK} = \frac{UT}{ML} = \frac{ST}{KL} = \frac{1}{2}$
ΔUST and ΔMKL are similar because the corresponding sides are proportional.

(c)



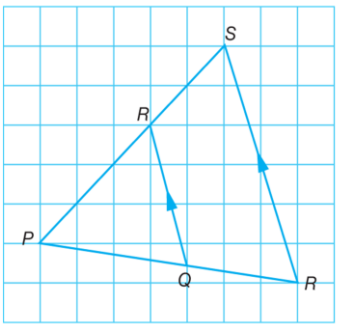
$\angle JKL = 360^o - 140^o - 40^o - 70^o = 110^o$
$\angle YVW = 360^o - 110^o - 140^o - 40^o = 70^o$
$\frac{JK}{VW} = \frac{LK}{XW} = \frac{ML}{YZ} = \frac{MJ}{YV} = \frac{4}{3}$
$LMJK$ and $XYVM$ are similar because the corresponding angles are equal and the corresponding sides are proportional.

(d)



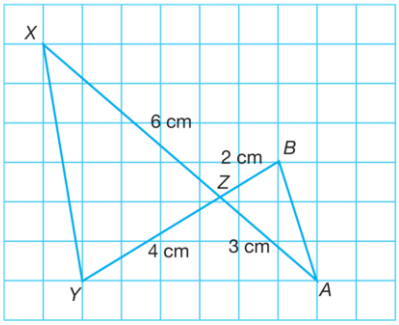
Not similar because $\frac{PQ}{AB} = \frac{ST}{DE} = \frac{5}{8}$ but $\frac{RS}{CD} = \frac{PT}{AE} = \frac{1}{2}$.

2 (a)



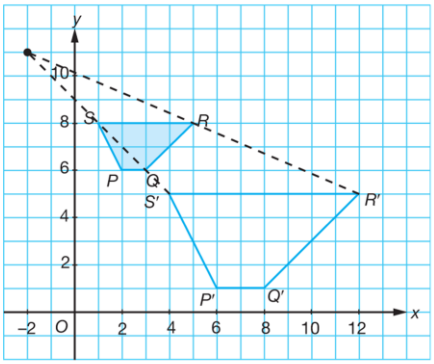
ΔPQR and ΔPRS are similar because the corresponding angles are equal.

(b)



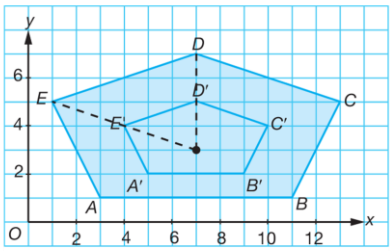
ΔZXY and ΔZAB are similar because they have two corresponding sides which are proportional and one equal angle.

3 (a)



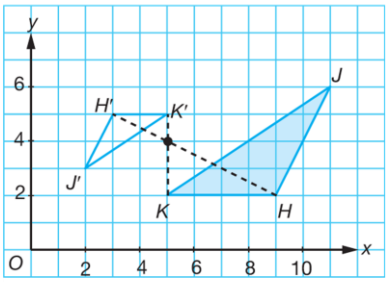
Centre of enlargement is (–2, 11).
Scale factor = 2

(b)



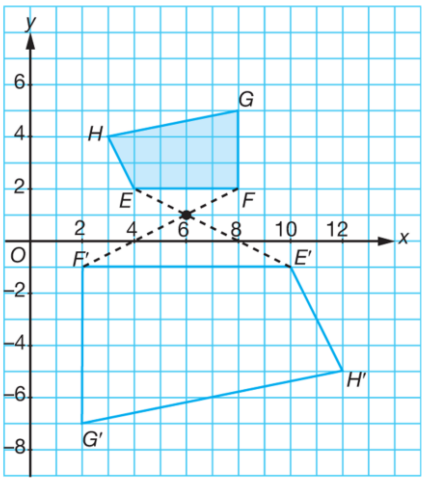
Centre of enlargement is (7, 3).
Scale factor = $\frac{1}{2}$

(c)



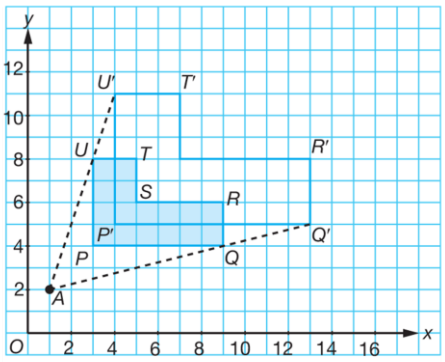
Centre of enlargement is (5, 4).
Scale factor = $-\frac{1}{2}$

(e)



Centre of enlargement is (6, 1).
Scale factor = –2

4



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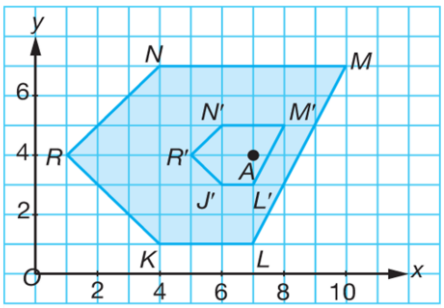

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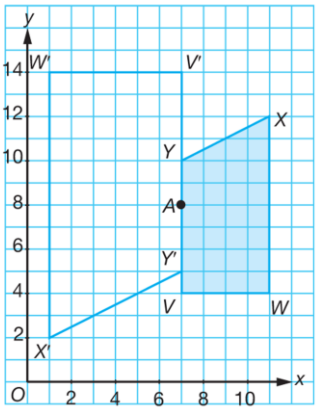

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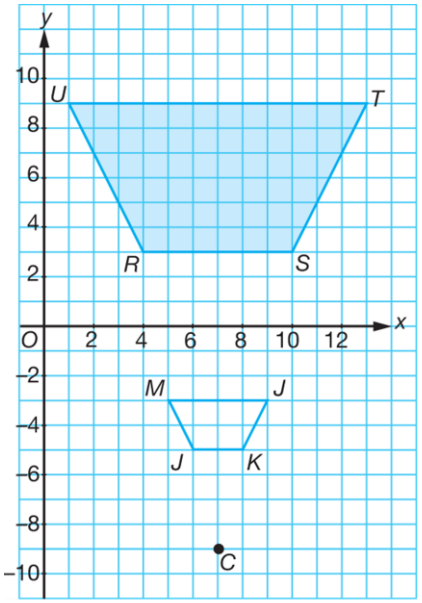

8 (a) Scale factor = 3

(b) Area of the shaded region

$= \frac{22}{7}(21)^2 - \frac{22}{7}(7)^2$

$= 1\,386 - 154$

$= 1\,232 \text{ cm}^2$

9 (a) Scale factor $= \frac{34}{6\,800} = \frac{1}{200}$

(b) Length of the plan $= \frac{1}{200} \times 10\,500$

$= 52.5 \text{ cm}$

(c) Area of the plan $= 34 \times 52.5 = 1\,785 \text{ cm}^2$

10 Area of $ACDE = \left(\frac{120}{30}\right)^2 \times$ Area of $ABGF$

Area of $ABGF$ + Area of the shaded region = $16 \times$ Area of $ABGF$

$15 \times$ Area of $ABGF = 225$

Area of $ABGF = 15 \text{ cm}^2$

11 Area of $HKMN = \left(\frac{30}{20}\right)^2 \times$ Area of $PQRS$

Area of $PQRS$ + Area of the shaded region

$= \frac{9}{4} \times$ Area of $PQRS$

$\frac{5}{4} \times$ Area of $PQRS$ = Area of the shaded region

$\frac{5}{4} \times$ Area of $PQRS = 100$

Area of $PQRS = 80 \text{ cm}^2$

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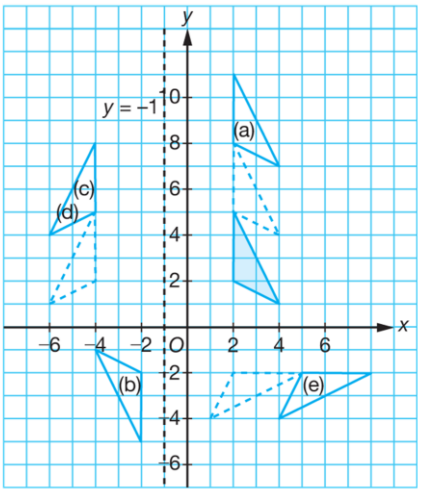
1



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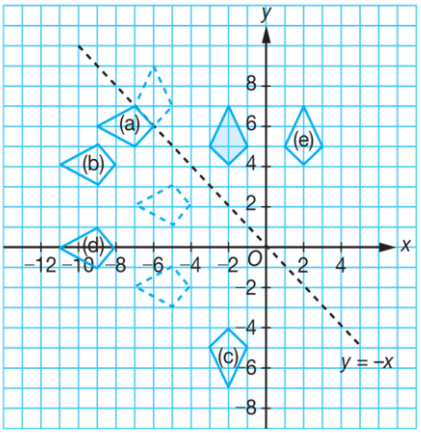

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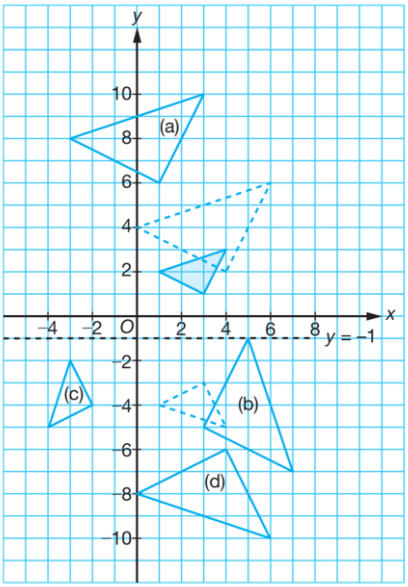


4 (a) (i) $A(2,-2) \xrightarrow{\mathbf{T}} (-4,3) \xrightarrow{\mathbf{P}} (2,3)$

(ii) $A(2,-2) \xrightarrow{\mathbf{P}} (-4,-2) \xrightarrow{\mathbf{T}} (-10,3)$

(b) Not equivalent

5 (a) (i) $B(-3,-4) \xrightarrow{\mathbf{R}} (4,-3) \xrightarrow{\mathbf{E}} (8,-6)$

(ii) $B(-3,-4) \xrightarrow{\mathbf{E}} (-6,-8) \xrightarrow{\mathbf{R}} (8,-6)$

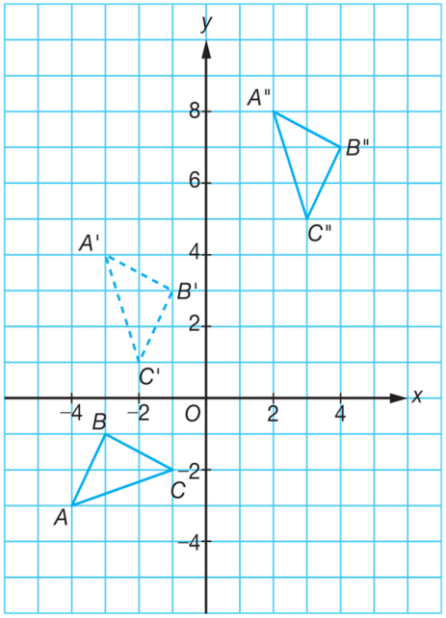
(b) Equivalent

6 (a) (i) $C(2,1) \xrightarrow{\mathbf{P}} (2,5) \xrightarrow{\mathbf{E}} (6,15)$

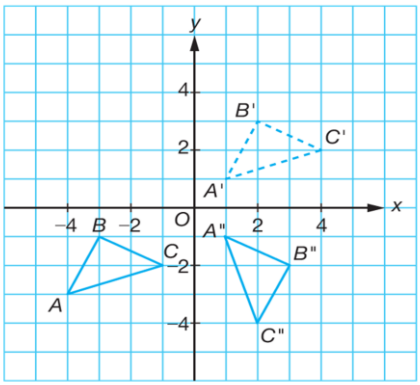
(ii) $C(2,1) \xrightarrow{\mathbf{E}} (6,3) \xrightarrow{\mathbf{P}} (6,3)$

(b) Not equivalent

7 (a) (i)

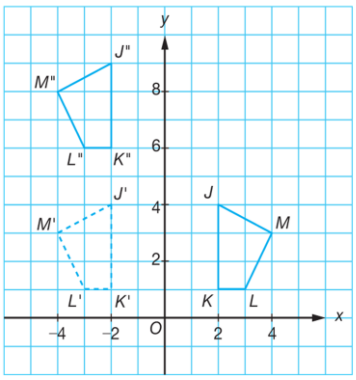


(ii)

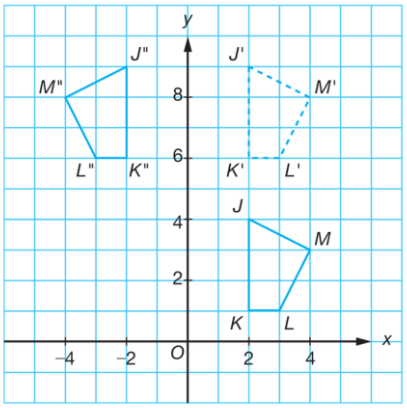


(b) Not equivalent

8 (a) (i)



(ii)



(b) Equivalent

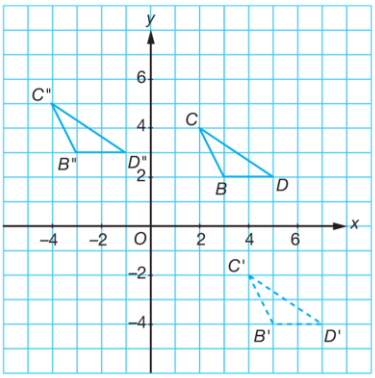
9 **V** is reflection in the straight line $y = 3$
U is an enlargement at the centre $A(11, 3)$ with a scale factor of 3

10 **W** is reflection in the straight line $x = 1$
V is a translation $\begin{pmatrix} 0 \\ -7 \end{pmatrix}$

11 **K** is a clockwise rotation of 90° about the centre $Q(2, -4)$
H is an enlargement at the centre $Q(2, -4)$ with a scale factor of 2

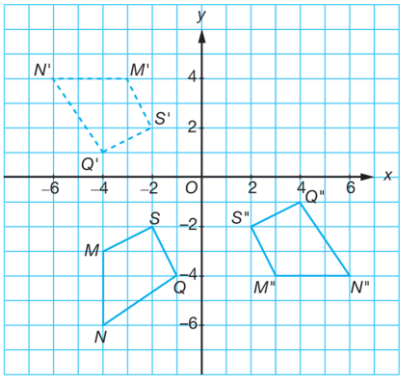
12 **S** is an anticlockwise rotation of 90° about the centre $F(2, 4)$
Q is a translation $\begin{pmatrix} -5 \\ 0 \end{pmatrix}$

13 (a)



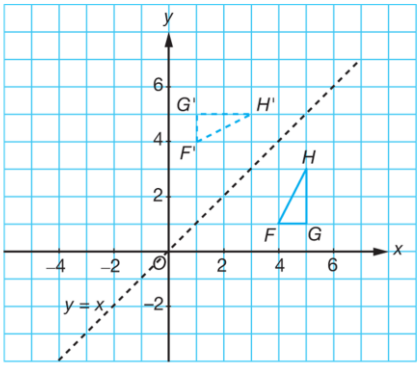
(b) Translation $\begin{pmatrix} -6 \\ 1 \end{pmatrix}$

14 (a)



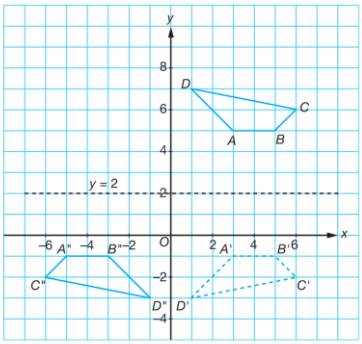
(b) Anticlockwise rotation of 90° about the origin

15 (a)



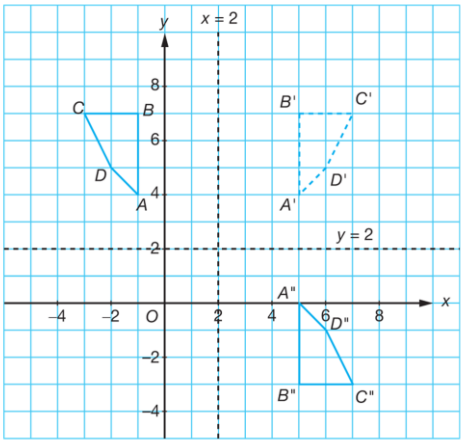
(b) Rotation of 360° about the origin

16 (a)



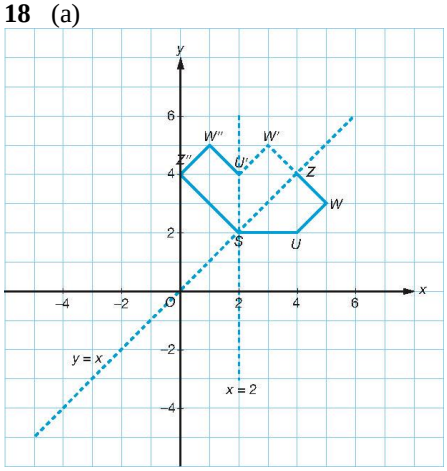
(b) Rotation of 180° about the centre (0, 2)

17 (a)



(b) Rotation of 180° about the centre (2, 2)

18 (a)



(b) Anticlockwise rotation of 90° about the centre $S(2, 2)$

19 (a) (i) $B(5, 3) \xrightarrow{\mathbf{P}} (3, 5)$

(ii) $B(5, 3) \xrightarrow{\mathbf{P}} (3, 5) \xrightarrow{\mathbf{T}} (5, 2)$

(iii) $B(5, 3) \xrightarrow{\mathbf{P}} (3, 5) \xrightarrow{\mathbf{R}} (3, 1)$

(b) **W** is an anticlockwise rotation of 90° about the centre $C(3, 3)$
V is an enlargement at the centre $M(4, 3)$ with a scale factor of 3

(c) Area of $\Delta MQN = 3^2 \times$ Area of ΔABC
$288 = 9 \times$ Area of ΔABC
Area of $\Delta ABC = 32$ cm^2

20 (a) (i) Reflection in the x-axis
(ii) Reflection in the y-axis
(iii) Rotation of 180° about the origin
(b) Enlargement at the centre (3, 0) with a scale factor of 3

(c) Area of $\Delta KLM = 3^2 \times$ Area of $\Delta A''B''C''$
$270 = 9 \times$ Area of $\Delta A''B''C''$
Area of $\Delta A''B''C'' = 30$ cm^2

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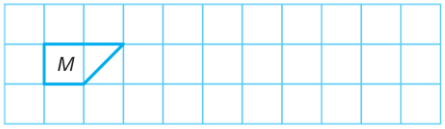
1



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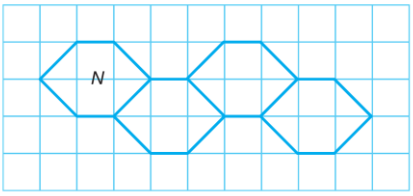


Summative Practice 5

Multiple-Choice Question

1 Area of $SKZWV = 4^2 \times$ Area of $SLMNU$

Area of $SLMNU$ + Area of the shaded region

$= 16 \times$ Area of $SLMNU$

$180 = 15 \times$ Area of $SLMNU$

Area of $SLMNU = 12\ \text{cm}^2$

Answer: B

2 Translation $\begin{pmatrix} a+h \\ b+k \end{pmatrix}$

Answer: C

3 Clockwise rotation of $\theta - \alpha$ about the origin

Answer: A

4 $(2, 2) \xrightarrow{\text{R}} (2, -2) \xrightarrow{\text{T}} (1, -4)$

Answer: B

5 $(2, 3) \xrightarrow{\text{T}} (3, 1) \xrightarrow{\text{E}} (5, 1)$

Answer: D

Structured Question

1 (a) $\angle PSQ = \angle RQS$ (Alternate angles, $PS \parallel QR$)

$\angle PQS = \angle RSQ$ (Alternate angles, $PQ \parallel SR$)

$SQ\ (\Delta PSQ) = SQ\ (\Delta RSQ)$ (Common side)

$\therefore \Delta PQS \cong \Delta RQS$ (ASA) [Angle-Side-Angle]

(b) $AC = EC$

$\angle ACB = \angle ECD$ (Vertically opposite angles)

$BC = DC$

$\therefore \Delta ABC \cong \Delta EDC$ (SAS) [Side-Angle-Side]

2 (a) ΔCED and ΔCHK are similar.

(b) $\dfrac{HK}{ED} = \dfrac{9}{5}$

$\dfrac{CK}{CD} = \dfrac{CK}{6}$

$\therefore \dfrac{CK}{6} = \dfrac{9}{5}$

$CK = 10\dfrac{4}{5}$ cm

3 (a) ΔRST and ΔRUV are similar.

(b) $\dfrac{UV}{ST} = \dfrac{x}{2}$

$\dfrac{RU}{RS} = \dfrac{7}{3}$

$\therefore \dfrac{x}{2} = \dfrac{7}{3}$

$x = 4\dfrac{2}{3}$ cm

$\dfrac{RT}{RV} = \dfrac{y}{y+6}$

$\dfrac{RS}{RU} = \dfrac{3}{7}$

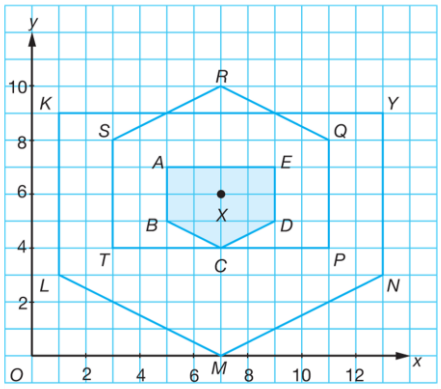
$\therefore \dfrac{y}{y+6} = \dfrac{3}{7}$

$7y = 3y + 18$

$4y = 18$

$y = 4\dfrac{1}{2}$ cm

4



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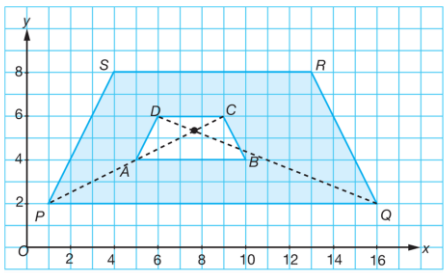


(a) Centre of enlargement is (7, 5).

(b) Scale factor = $\dfrac{SR}{DC} = \dfrac{9}{3} = 3$

(c) Area of $PQRS = 3^2 \times$ Area of $ABCD$
Area of $ABCD$ + Area of the shaded region $= 9 \times$ Area of $ABCD$
$8 \times$ Area of $ABCD$ = Area of the shaded region
$8 \times$ Area of $ABCD = 64$
Area of $ABCD = 8$ cm^2

6 (a) $T(10, 2) \xrightarrow{\mathbf{L}} (7, 4) \xrightarrow{\mathbf{L}} (4, 6)$

$R(7, 2) \xrightarrow{\mathbf{W}} (6, 5) \xrightarrow{\mathbf{L}} (3, 7)$

(b) (i) **U** is a reflection in the straight line $y = 8$.

V is an enlargement at the centre $P(4, 11)$ with a scale factor of 3.

(c) Area of $RQPST = 3^2 \times$ Area of $ABCDEF$
$= 9 \times 60$
$= 540$ cm^2
Area of the shaded region
$= 540 - 60 = 480$ cm^2

7 (a) (i) $B(2, 4) \xrightarrow{\mathbf{T}} (5, 2) \xrightarrow{\mathbf{R}} (2, -1)$

(ii) $B(2, 4) \xrightarrow{\mathbf{R}} (0, 2) \xrightarrow{\mathbf{T}} (3, 0)$

(b) (i) (a) **U** is an anticlockwise rotation of 90^o about the centre $C(6, 10)$.

(b) **V** is an enlargement at the centre $(10, 10)$ with a scale factor of 2.

(ii) Area of $DEGF = 2^2 \times$ Area of $DABG$
$= 4 \times 20$
$= 80$
Area of the shaded region
$= 80 - 20$
$= 60$ m^2

8 (a) (i) $(3, 4) \xrightarrow{\mathbf{T}} (1, 1)$

(ii) $(3, 4) \xrightarrow{\mathbf{R}} (4, -1)$

(iii) $(3, 4) \xrightarrow{\mathbf{R}} (4, -1) \xrightarrow{\mathbf{T}} (2, -4)$

(b) (i) (a) **V** is a reflection in the straight line $y = 1$

(b) W is an enlargement at the centre $(4, -2)$ with a scale factor of 3

(ii) Area of $PQRS = 3^2 \times 25 = 225$
Area of the shaded region
$= 225 - 25$
$= 200$ cm^2

9 (a) (i) $A(1, 2) \xrightarrow{\mathbf{P}} (2, 1) \xrightarrow{\mathbf{T}} (-3, 4)$

(ii) $A(1, 2) \xrightarrow{\mathbf{R}} (-3, 0) \xrightarrow{\mathbf{P}} (0, -3)$

(b) (i) (a) **W** is a reflection in the straight line $y = 3$

(b) **V** is an enlargement at the centre $(2, 1)$ with a scale factor of 3

(ii) Area of $DHFG = 3^2 \times$ Area of $MNKL$
Area of $DHFG = 9 \times 14 = 126$
Area of the shaded region
$= 126 - 14$
$= 112$ units^2

10 (i) $J(1, 2) \xrightarrow{\mathbf{R}} (5, 2) \xrightarrow{\mathbf{U}} (3, 4)$

(ii) $J(1, 2) \xrightarrow{\mathbf{T}} (3, 5) \xrightarrow{\mathbf{R}} (3, 5)$

(b) (i) **N** is a reflection in the straight line $y = 6$

(ii) **M** is an enlargement at the centre $F(7, 8)$ with a scale factor of 3

(c) Area of $EFGH = 3^2 \times$ Area of $ABCD$
Area of $EFGH = 9 \times 20 = 180$ m^2
Area of the shaded region
$= 180 - 20$
$= 160$ m^2

11 (a) (i) $A(5, 3) \xrightarrow{\mathbf{T}} (3, 7) \xrightarrow{\mathbf{T}} (1, 11)$

(ii) $A(5, 3) \xrightarrow{\mathbf{T}} (-3, 5) \xrightarrow{\mathbf{T}} (-5, 9)$

(b) (i) (a) **N** is a reflection in the straight line BC

(b) **M** is an enlargement at the centre $(4, 2)$ with a scale factor of 3

(ii) Area of shaded region
$=$ Area of $\Delta FDE -$ Area of ΔCGB
$= 270 - 30$
$= 240$ m^2

12 (a) (i) $K(5, 9) \xrightarrow{\mathbf{T}} (2, 5) \xrightarrow{\mathbf{T}} (-1, 1)$

(ii) $K(5, 9) \xrightarrow{\mathbf{P}} (5, 3) \xrightarrow{\mathbf{T}} (2, -1)$

(b) (i) (a) **N** is a clockwise rotation of 90^o about the centre $(4, 5)$.

(b) **M** is an enlargement at the centre $Q(5, 8)$ with a scale factor of 3

(ii) Area of $QRSTU = 3^2 \times$ Area of $KLMNP$
$180 = 9$ Area of $KLMNP$
Area of $KLMNP = 20$ m^2

13 (a) (i) $H''(4, 4) \xrightarrow{\mathbf{R^{-1}}} H'(4, -4) \xrightarrow{\mathbf{T^{-1}}} H(-2, 1)$

(ii) $H''(4, 4) \xrightarrow{\mathbf{E^{-1}}} H'(2, 2) \xrightarrow{\mathbf{P^{-1}}} H(2, -2)$

(b) (i) (a) $y = x$ (b) $x = 7$

(ii) Anticlockwise rotation of 90^o about the centre $(7, 7)$

(c) (i) **W** is an enlargement at the centre $(-2, 0)$ with a scale factor of 2

(ii) Area of hexagon $A = 56.5$ cm^2
Area of hexagon $P = 2^2 \times 56.5$
$= 226$ cm^2

SPM SPOT

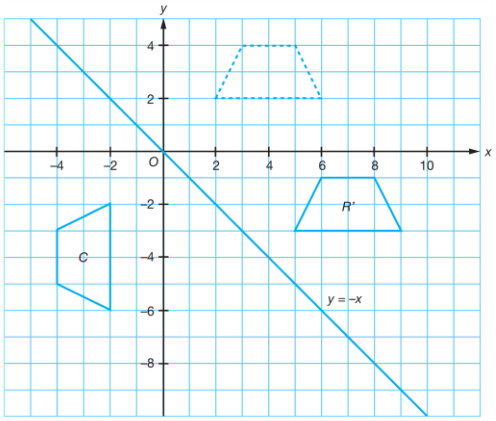
1 *Answer*: D

2 *Answer*: A

3 The inverse transformation of **M** is translation $\begin{pmatrix} -3 \\ 5 \end{pmatrix}$ and

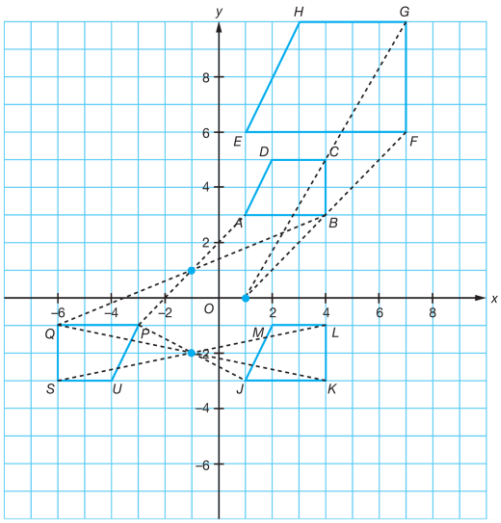
the inverse transformation of **N** is reflection in the straight line $y = -x$.

The object of *R'* is given by $\mathbf{N}^{-1}\mathbf{M}^{-1}$.



Answer: C

4



(a) (i) **V** is translation $\begin{pmatrix} 0 \\ -6 \end{pmatrix}$.

(ii) **W** is a 180° rotation about centre (–1, –2).

(iii) The single transformation is a 180° rotation about centre (–1, 1).

(b) (i) Scale factor = $\frac{HG}{DC} = \frac{4}{2} = 2$.

(ii) The coordinates of the centre of enlargement are (1, 0).

(iii) Area of *EFGH* = $2^2 \times$ Area of *ABCD*

$45 = 4 \times$ Area of *ABCD*

Area of *ABCD* = $\frac{45}{4} = 11.25$ cm^2

(c)