

Form 5: Chapter 1
Variation
Fully-worked Solutions

UPSKILL 1.1

1

x	9	27	45	63
y	8	24	40	56
$\frac{y}{x}$	$\frac{8}{9}$	$\frac{8}{9}$	$\frac{8}{9}$	$\frac{8}{9}$

Since the value of $\frac{y}{x} = \frac{8}{9}$ (a constant), thus y varies directly as x .

2

x	12	20	50
y	10	64	400
$\frac{y}{x}$	$\frac{5}{6}$	$\frac{16}{5}$	2

Since $\frac{y}{x}$ is not a constant, thus y does not vary directly as x .

3 (a) $m \propto \sqrt{n}$

$$m = k\sqrt{n} \text{ [} k \text{ is a constant.]}$$

$$m = 1 \text{ when } n = 16$$

$$1 = k\sqrt{16}$$

$$k = \frac{1}{4}$$

$$m = \frac{1}{4}\sqrt{n}$$

(b) When $n = 36$,

$$m = \frac{1}{4}\sqrt{36} = \frac{3}{2}$$

4 $y \propto x^{\frac{1}{3}}$

$$y = kx^{\frac{1}{3}}$$

Given that $x = 8$ and $y = 10$,

$$10 = k(8)^{\frac{1}{3}} \text{ [} k \text{ is a constant.]}$$

$$10 = 2k$$

$$k = 5$$

$$y = 5x^{\frac{1}{3}}$$

Given that $x = p$ and $y = 15$,

$$15 = 5p^{\frac{1}{3}}$$

$$p^{\frac{1}{3}} = 3$$

$$p = 3^3 = 27$$

5 (a) $p \propto x^3 y^2 z^{\frac{1}{3}}$

$$p = kx^3 y^2 z^{\frac{1}{3}} \text{ [} k \text{ is a constant.]}$$

When $x = 3$, $y = \frac{1}{9}$ and $z = 64$, thus

$$p = 72.$$

$$72 = k(3)^3 \left(\frac{1}{9}\right)^2 (64)^{\frac{1}{3}}$$

$$72 = k(27) \left(\frac{1}{3}\right)(4)$$

$$k = \frac{72(3)}{27(4)} = 2$$

$$\therefore p = 2x^3 y^2 z^{\frac{1}{3}}$$

(b) $p = 2x^3 y^2 z^{\frac{1}{3}}$

When $x = 2$, $y = \frac{1}{81}$, $z = 125$,

$$p = 2(2)^3 \left(\frac{1}{81}\right)^2 (125)^{\frac{1}{3}}$$

$$p = 8\frac{8}{9}$$

6 (a) $M \propto vn$

$$M = kvn \text{ [} k \text{ is a constant.]}$$

$M = 60$ when $v = 1000$ and $n = 3$.

$$60 = k(1000)(3)$$

$$k = \frac{60}{3000}$$

$$k = \frac{1}{50}$$

$$M = \frac{1}{50}vn$$

(b) When $v = 1500$ and $n = 4$,

$$M = \frac{1}{50}(1500)(4) = 120$$

7 (a) $V \propto j^3$

$$V = kj^3 \text{ [} k \text{ is a constant.]}$$

$$\text{When } j = 7, V = \frac{2\ 156}{3}$$

$$\frac{2\ 156}{3} = k(7)^3$$

$$k = \frac{2156}{3} \times \frac{1}{343}$$

$$k = \frac{44}{21}$$

$$V = \frac{44}{21} j^3$$

(b) When $j = 3$,

$$V = \frac{44}{21} (3)^3$$

$$V = 56\frac{4}{7} \text{ cm}^3$$

8 $V \propto T$

$$V = kT \text{ [} k \text{ is a constant.]}$$

$$V = 13 \text{ when } T = 650$$

$$13 = k(650)$$

$$k = \frac{13}{650}$$

$$V = \frac{13}{650} T$$

When $T = 400$,

$$V = \frac{13}{650} (400) = 8$$

UPSKILL 1.2

1

x	2	4	5	8
y	10	5	4	2.5
xy	20	20	20	20

Since the value of $xy = 20$ (a constant), thus y varies inversely as x .

2 $m \propto \frac{1}{y^3}$

$$m = k \left(\frac{1}{y^3} \right) \text{ [} k \text{ is a constant.]}$$

$$m = 1 \text{ when } y = 8$$

$$1 = k \left(\frac{1}{8^3} \right)$$

$$1 = k \left(\frac{1}{2} \right)$$

$$k = 2$$

$$m = 2 \left(\frac{1}{y^3} \right)$$

When $m = 10$,

$$10 = 2 \left(\frac{1}{y^3} \right)$$

$$y^3 = \frac{1}{5}$$

$$y = \frac{1}{125}$$

3 $y \propto \frac{1}{x^3}$

$$y = \frac{k}{x^3} \text{ [} k \text{ is a constant.]}$$

$$y = 5 \text{ when } x = 2$$

$$5 = \frac{k}{2^3}$$

$$k = 40$$

$$y = \frac{40}{x^3}$$

When $y = \frac{5}{8}$,

$$\frac{5}{8} = \frac{40}{x^3}$$

$$x^3 = 40 \times \frac{8}{5}$$

$$x^3 = 64$$

$$x = 4$$

$$4 \quad h \propto \frac{1}{r^2}$$

$$h = k \frac{1}{r^2} \text{ [} k \text{ is a constant.]}$$

$$h = 7 \text{ when } r = 3$$

$$7 = k \left(\frac{1}{3^2} \right)$$

$$k = 63$$

$$h = \frac{63}{r^2}$$

$$\text{When } r = 5,$$

$$h = \frac{63}{5^2} = 2.52 \text{ cm}$$

$$5 \quad I \propto \frac{1}{R}$$

$$I = \frac{k}{R} \text{ [} k \text{ is a constant.]}$$

$$\text{Given that } I = 0.4 \text{ and } R = 30,$$

$$0.4 = \frac{k}{30}$$

$$k = 12$$

$$I = \frac{12}{R}$$

$$\text{When } R = 48,$$

$$I = \frac{12}{48} = 0.25$$

UPSKILL 1.3

$$1 \quad w \propto \frac{x^2}{\sqrt{y}}$$

$$w = \frac{kx^2}{\sqrt{y}} \text{ [} k \text{ is a constant.]}$$

$$\text{Given that } w = 4, x = 2 \text{ and } y = 9,$$

$$4 = \frac{k(2)^2}{\sqrt{9}}$$

$$k = 3$$

$$w = \frac{3x^2}{\sqrt{y}}$$

$$\text{When } w = 8, x = p \text{ and } y = 36,$$

$$8 = \frac{3p^2}{\sqrt{36}}$$

$$8 = \frac{3p^2}{6}$$

$$p^2 = 16$$

$$p = \pm 4$$

$$2 \quad p \propto \frac{q^2}{r}$$

$$p = \frac{kq^2}{r} \text{ [} k \text{ is a constant.]}$$

$$\text{Given that } p = 25, q = 5, r = 3,$$

$$25 = \frac{k(5)^2}{3}$$

$$k = 3$$

$$p = \frac{3q^2}{r}$$

$$\text{When } p = 12, q = m \text{ and } r = 9,$$

$$12 = \frac{3m^2}{9}$$

$$m^2 = 36$$

$$m = \pm 6$$

$$3 \text{ (a) } K \propto mv^2$$

$$K = kmv^2 \text{ [} k \text{ is a constant.]}$$

$$\text{If } m = 3 \text{ and } v = 12, \text{ thus } K = 216.$$

$$216 = k(3)(12)^2$$

$$k = \frac{1}{2}$$

$$K = \frac{1}{2}mv^2$$

$$\text{When } K = 250 \text{ and } v = 10,$$

$$250 = \frac{1}{2}m(10)^2$$

$$250 = 50m$$

$$m = 5 \text{ kg}$$

$$\text{(b) When } m = 6 \text{ and } K = 300,$$

$$300 = \frac{1}{2}(6)v^2$$

$$v^2 = 100$$

$$v = 10 \text{ m s}^{-1}$$

$$4 \quad F \propto \frac{q_1q_2}{r^2}$$

$$F = \frac{kq_1q_2}{r^2} \text{ [} k \text{ is a constant.]}$$

$$125 = \frac{k(1.0 \times 10^{-5})(2.0 \times 10^{-5})}{0.12^2}$$

$$125 = \frac{k(1.0 \times 10^{-5})(2.0 \times 10^{-5})}{0.12^2}$$

$$1.8 = k(2.0 \times 10^{-10})$$

$$k = \frac{1.8}{(2.0 \times 10^{-10})}$$

$$k = 9 \times 10^9$$

Summative Practice 1

Multiple-Choice Questions

1

x	3	5	7
y	12	20	60
$\frac{y}{x}$	4	4	$8\frac{4}{7}$

Since $\frac{y}{x}$ is not a constant, thus y does not vary directly as x .

Answer: B

2 $m \propto n$

$$m = kn \text{ [} k \text{ is a constant.]}$$

When $n = 3$, $m = 9$

$$9 = k(3)$$

$$k = 3$$

$$m = 3n$$

Answer: A

3

x	2	6	8
y	24	8	6
xy	48	48	48

Since the value of xy is a constant, thus y varies inversely as x .

Answer: C

4 $h \propto \frac{1}{t^3}$

$$h = \frac{k}{t^3} \text{ [} k \text{ is a constant.]}$$

When $t = 2$, $h = 4$

$$4 = \frac{k}{2^3}$$

$$k = 32$$

$$h = \frac{32}{t^3}$$

Answer: A

5 $y \propto \frac{x^2}{p}$

$$y = \frac{kx^2}{p} \text{ [} k \text{ is a constant.]}$$

When $x = 2$, $y = 4$ and $p = 3$,

$$4 = \frac{k(2)^2}{3}$$

$$k = 3$$

$$y = \frac{3x^2}{p}$$

When $y = 25$ and $p = 3$,

$$25 = \frac{3x^2}{3}$$

$$x^2 = 25$$

$$x = \pm 5$$

Answer: B

6 $m \propto \frac{n}{\sqrt{h}}$

$$m = \frac{kn}{\sqrt{h}} \text{ [} k \text{ is a constant.]}$$

When $n = 8$, $\underline{h} = 16$ and $m = 2$,

$$2 = \frac{k(8)}{\sqrt{16}}$$

$$2 = \frac{k(8)}{4}$$

$$k = 1$$

$$m = \frac{n}{\sqrt{h}}$$

Answer: A

Structured Questions

1 $p \propto q$

$$p = kq \text{ [} k \text{ is a constant.]}$$

$$q = 5 \text{ when } p = 20$$

$$20 = k(5)$$

$$k = 4$$

$$p = 4q$$

$$\text{If } q = 8,$$

$$p = 4(8) = 32$$

2 $y \propto x^2$

$$y = kx^2 \text{ [} k \text{ is a constant.]}$$

$$\text{When } x = 4, y = 48,$$

$$48 = k(4)^2$$

$$k = 3$$

$$y = 3x^2$$

$$\text{When } x = 6 \text{ and } y = p,$$

$$p = 3(6)^2 = 108$$

3 $p \propto \sqrt{q}$

$$p = k\sqrt{q} \text{ [} k \text{ is a constant.]}$$

$$p = 10 \text{ when } q = 25$$

$$10 = k\sqrt{25}$$

$$k = 2$$

$$p = 2\sqrt{q}$$

$$\text{If } q = 100,$$

$$p = 2\sqrt{100} = 20$$

4 $y \propto \frac{x^{\frac{1}{2}} p^{\frac{1}{3}}}{q^3}$

$$y = k \frac{x^{\frac{1}{2}} p^{\frac{1}{3}}}{q^3} \text{ [} k \text{ is a constant.]}$$

$$\text{When } x = 16, p = 125, q = 2, \text{ thus } y = 5,$$

$$5 = k \left[\frac{16^{\frac{1}{2}} 125^{\frac{1}{3}}}{2^3} \right]$$

$$5 = k \left[\frac{4(5)}{8} \right]$$

$$k = 2$$

$$y = \frac{2x^{\frac{1}{2}} p^{\frac{1}{3}}}{q^3}$$

$$\text{When } x = 4, p = 8 \text{ and } q = 4,$$

$$y = \frac{2(4)^{\frac{1}{2}} (8)^{\frac{1}{3}}}{4^3} = \frac{1}{8}$$

5 (a) $T \propto j$

$$T = kj \text{ [} k \text{ is a constant.]}$$

$$T = 10 \text{ when } j = 5.$$

$$10 = 5k$$

$$k = 2$$

$$T = 2j$$

(b) When $j = 3.5,$

$$T = 2(3.5) = 7 \text{ g cm}^{-3}$$

6 (a) $v \propto \sqrt{T}$

$$v = k\sqrt{T} \text{ [} k \text{ is a constant.]}$$

$$T = 100 \text{ when } v = 5$$

$$5 = k\sqrt{100}$$

$$5 = k(10)$$

$$k = 0.5$$

$$v = 0.5\sqrt{T}$$

(b) (i) When $T = 64,$

$$v = 0.5\sqrt{64} = 4 \text{ m s}^{-1}$$

(ii) When $v = 8,$

$$8 = 0.5\sqrt{T}$$

$$\sqrt{T} = 16$$

$$T = 256 \text{ J}$$

7 $N \propto dv^2$

$$N = kd^2v^2 \text{ [} k \text{ is a constant.]}$$

$$N = 7\frac{1}{2} \text{ when } d = 30 \text{ and } v = 18$$

$$\frac{15}{2} = k(30)(18)^2$$

$$k = \frac{1}{1296}$$

$$N = \frac{1}{1296} dv^2$$

$$N = \frac{1}{1296} (200)(36)^2$$

$$N = 200 \text{ barrels}$$

8 $R \propto Av$

$$R = kAv \text{ [} k \text{ is a constant.]}$$

When $A = 25$ and $v = 40$, thus $R = 225$.

$$225 = k(25)(40)$$

$$k = \frac{9}{40}$$

$$R = \frac{9}{40} Av$$

When $v = 350$ and $R = 236.25$,

$$236.25 = \frac{9}{40} A(350)$$

$$A = \frac{236.25}{350} \times \frac{40}{9}$$

$$A = 3 \text{ m}^2$$

9 $q \propto \frac{1}{p^2}$

$$q = \frac{k}{p^2} \text{ [} k \text{ is a constant.]}$$

Given that $p = 1$ and $q = 12$,

$$12 = \frac{k}{1^2}$$

$$k = 12$$

$$q = \frac{12}{p^2}$$

When $p = 2$ and $q = x$,

$$x = \frac{12}{2^2} = \frac{12}{4} = 3$$

10 $y \propto \frac{1}{x^3}$

$$y = \frac{k}{x^3} \text{ [} k \text{ is a constant.]}$$

$y = 5$ when $x = 2$

$$5 = \frac{k}{2^3}$$

$$k = 40$$

$$y = \frac{40}{x^3}$$

When $y = \frac{5}{8}$,

$$\frac{5}{8} = \frac{40}{x^3}$$

$$x^3 = 40 \times \frac{8}{5}$$

$$x^3 = 64$$

$$x = 4$$

11 (a) $b \propto \frac{1}{j^3}$

$$b = \frac{k}{j^3} \text{ [} k \text{ is a constant.]}$$

When $j = 2$, $b = 500$,

$$500 = \frac{k}{2^3}$$

$$k = 4\,000$$

$$b = \frac{4\,000}{j^3}$$

(b) (i) When $j = 5$,

$$b = \frac{4\,000}{5^3}$$

$$b = 32$$

(ii) When $b = 4\,000$

$$4\,000 = \frac{4\,000}{j^3}$$

$$j^3 = 1$$

$$j = 1 \text{ cm}$$

12 (a) $T \propto \frac{1}{d^2}$

$$T = \frac{k}{d^2} \text{ [} k \text{ is a constant.]}$$

$T = 3.84$, when $d = 2$

$$3.84 = \frac{k}{2^2}$$

$$k = 15.36$$

$$T = \frac{15.36}{d^2}$$

(b) When $T = 6$,

$$6 = \frac{15.36}{d^2}$$

$$d^2 = \frac{15.36}{6}$$

$$d^2 = 2.56$$

$$d = 1.6 \text{ cm}$$

13 $w \propto \frac{x^2}{\sqrt{y}}$

$$w = \frac{kx^2}{\sqrt{y}} \text{ [} k \text{ is a constant.]}$$

Given that $w = 4$, $x = 2$ and $y = 9$,

$$4 = \frac{k(2^2)}{\sqrt{9}}$$

$$k = 3$$

$$w = \frac{3x^2}{\sqrt{y}}$$

When $w = 8$, $x = 4$ and $y = p$,

$$8 = \frac{3(4)^2}{\sqrt{p}}$$

$$\sqrt{p} = 6$$

$$p = 36$$

14 $p \propto \frac{1}{q\sqrt{r}}$

$$p = \frac{k}{q\sqrt{r}}$$

When $p = \frac{2}{3}$, $q = 2$ and $r = 9$,

$$\frac{2}{3} = \frac{k}{2\sqrt{9}} \quad [k \text{ is a constant.}]$$

$$\frac{2}{3} = \frac{k}{6}$$

$$k = 4$$

$$p = \frac{4}{q\sqrt{r}}$$

When $p = \frac{1}{9}$, $q = 4$ and $r = n$,

$$\frac{1}{9} = \frac{4}{4\sqrt{n}}$$

$$\sqrt{n} = 9$$

$$n = 81$$

15 $p \propto \frac{\sqrt{r}}{q^2}$

$$p = \frac{k\sqrt{r}}{q^2} \quad [k \text{ is a constant.}]$$

When $p = 3$, $q = 2$ and $r = 9$,

$$3 = \frac{k\sqrt{9}}{2^2}$$

$$3 = \frac{k(3)}{3^2}$$

$$k = 9$$

$$p = \frac{9\sqrt{r}}{q^2}$$

When $p = \frac{4}{9}$, $q = k$ and $r = 16$,

$$\frac{4}{9} = \frac{9\sqrt{16}}{k^2}$$

$$k^2 = 36 \times \frac{9}{4}$$

$$k^2 = 81$$

$$k = \pm 9$$

16 $V \propto AR$

$$V = kAR$$

When $V = 8$, $A = 4$ and $R = 2$,

$$8 = k(4)(2)$$

$$k = 1$$

$$V = AR$$

(a) When $A = 5$ and $R = 3$,

$$V = 5(3) = 15 \text{ volts}$$

(b) When $V = 16$ and $R = 4$,

$$16 = A(4)$$

$$A = 4 \text{ amperes}$$

(c) When $A = 6$ and $V = 3$,

$$3 = 6R$$

$$R = 0.5 \text{ ohm}$$

17 (a) $F \propto \frac{Mm}{r^2}$

$$F = G \frac{Mm}{r^2}$$

(b) $F = (6.670 \times 10^{-11}) \left[\frac{5.972 \times 10^{24} \times 7.348 \times 10^{22}}{(3.844 \times 10^8)^2} \right]$

$$F = 1.981 \times 10^{20} \text{ N}$$

1 $t \propto \frac{1}{N}$

$$t = \frac{k}{N} \text{ [} k \text{ is a constant.]}$$

When $N = 2$ and $t = 1.5$,

$$1.5 = \frac{k}{2}$$

$$k = 3$$

Thus, $t = \frac{3}{N}$

When $N = 6$,

$$t = \frac{3}{6}$$

$$t = 0.5 \text{ hours}$$

Hence, the time taken by 6 workers to pack and sort the same number of boxes of shoes is 30 minutes.

Answer: C

2 Let N represent the number of reference books sold.

$$N \propto \frac{B}{H}$$

$$N = k \left(\frac{B}{H} \right) \text{ [} k \text{ is a constant.]}$$

When $N = 4\,800$, $B = 5\,000$, $H = 40$,

$$N = k \left(\frac{B}{H} \right)$$

$$4\,800 = k \left(\frac{5\,000}{40} \right)$$

$$k = 4\,800 \times \frac{40}{5\,000}$$

$$k = \frac{192}{5}$$

$$\therefore N = \frac{192}{5} \left(\frac{B}{H} \right)$$

When $B = 7\,500$ dan $H = 40$,

$$N = \frac{192}{5} \left(\frac{7\,500}{40} \right) = 7\,200 \text{ books}$$