UPSKILL 9.1

1 (a) (i) The event of obtaining a multiple of 3
   or a factor of 12, i.e. $A \cup B$
   (ii) The event of obtaining a multiple of 3
   and a factor of 12, i.e. $A \cap B$
   (b) (i) $A \cup B = \{1, 2, 3, 4, 6, 9, 12\}$
   (ii) $A \cap B = \{3, 6, 12\}$

2 (a) (i) The event such that the first digit is
   an even number or the last digit is
   an odd number, i.e. $P \cup Q$
   (ii) The event such that the first digit is
   an even number and the last digit is
   an odd number, i.e. $P \cap Q$
   (b) (i) $P \cup Q = \{618, 681, 816, 861\}$
   (ii) $P \cap Q = \{681, 861\}$

UPSKILL 9.2

1 (a) Dependent
   (b) Dependent
   (c) Independent

2

Box A \ Box B	1	2	3
1	(1, 1)	(1, 2)	(1, 3)
2	(2, 1)	(2, 2)	(2, 3)
3	(3, 1)	(3, 2)	(3, 3)
4	(4, 1)	(4, 2)	(4, 3)

(a) $\dfrac{7}{12}$ (b) $\dfrac{1}{6}$

3 $S = \{(R, 2), (R, 3), (R, 7), (R, 8), (U, 2),$
   $(U, 3), (U, 7), (U, 8)\}$
   (a) $\{(R, 3), (R, 7), \{(U, 2),$
   $(U, 3), (U, 7), (U, 8)\}$
   $\dfrac{3}{4}$
   (b) $\{(R, 2), (R, 3), (R, 7)\}$
   $\dfrac{3}{8}$

4 (a) P(all three of them strike the target)
$$= \frac{2}{5} \times \frac{3}{4} \times \frac{2}{3} = \frac{1}{5}$$
(b) P(only one of them strikes the target)
$$= \left(\frac{2}{5} \times \frac{1}{4} \times \frac{1}{3}\right) + \left(\frac{3}{5} \times \frac{3}{4} \times \frac{1}{3}\right) + \left(\frac{3}{5} \times \frac{1}{4} \times \frac{2}{3}\right)$$
$$= \frac{1}{30} + \frac{3}{20} + \frac{1}{10}$$
$$= \frac{17}{60}$$

(c) P(at least one of them strike the target)
$$= 1 - \frac{3}{5} \times \frac{1}{4} \times \frac{1}{3}$$
$$= \frac{19}{20}$$

5 (a) P(win all the games)
$$= \frac{3}{5} \times \frac{1}{4} \times \frac{5}{6} = \frac{1}{8}$$
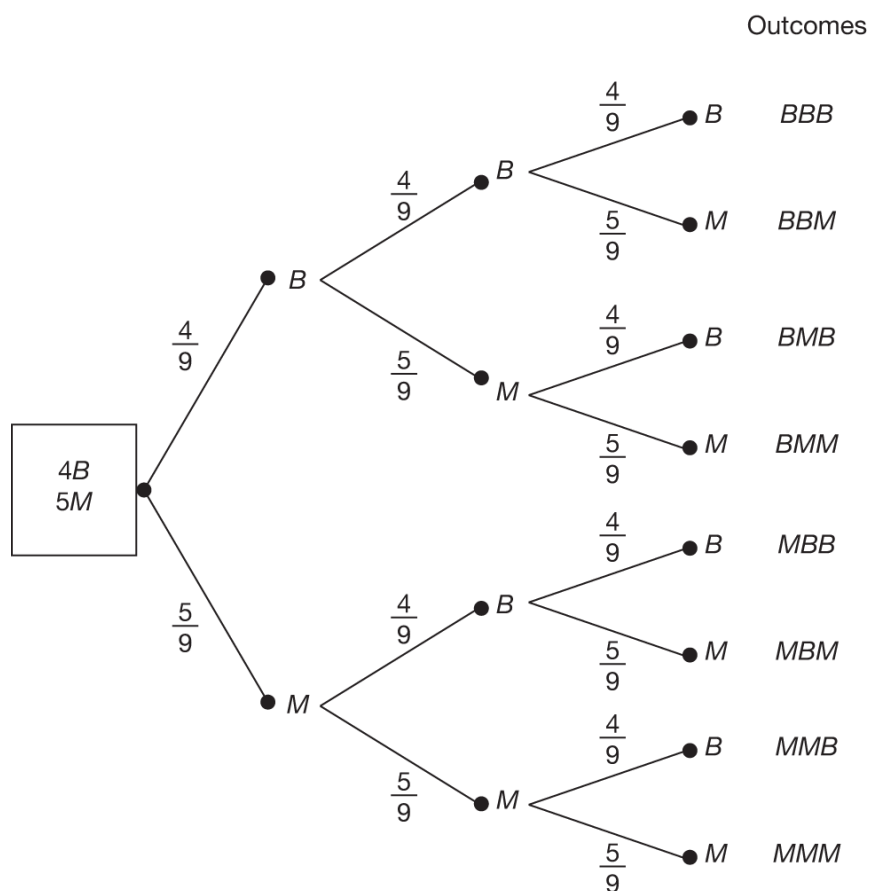(b) P(win at least two of the games)
$$= \left(\frac{3}{5} \times \frac{1}{4} \times \frac{5}{6}\right) + \left(\frac{3}{5} \times \frac{1}{4} \times \frac{1}{6}\right)$$
$$+ \left(\frac{2}{5} \times \frac{1}{4} \times \frac{5}{6}\right) + \left(\frac{3}{5} \times \frac{3}{4} \times \frac{5}{6}\right)$$
$$= \frac{1}{8} + \frac{1}{40} + \frac{1}{12} + \frac{3}{8}$$
$$= \frac{73}{120}$$

6 B – Blue $\qquad M$ – Red

Outcomes

(a) $P(MM) = \dfrac{5}{9} \times \dfrac{5}{9} = \dfrac{25}{81}$

(b) $P(MMM) = \dfrac{5}{9} \times \dfrac{5}{9} \times \dfrac{5}{9} = \dfrac{125}{729}$

(c) $P(BBB) + P(BBM) + P(BMB) + P(MBB)$

$= \left(\dfrac{4}{9} \times \dfrac{4}{9} \times \dfrac{4}{9} \right) + \left(\dfrac{4}{9} \times \dfrac{4}{9} \times \dfrac{5}{9} \right) \times 3$

$= \dfrac{304}{729}$

(d) $P(BBM) + P(BMB) + P(MBB)$

$= \left(\dfrac{5}{9} \times \dfrac{4}{9} \times \dfrac{4}{9} \right) \times 3 = \dfrac{80}{243}$

7 (a) P(both cards have the same colour)

$= P(BB) + P(MM) + P(HH)$

$= \left(\dfrac{4}{14} \times \dfrac{3}{13} \right) + \left(\dfrac{3}{14} \times \dfrac{2}{13} \right) + \left(\dfrac{7}{14} \times \dfrac{6}{13} \right)$

$= \dfrac{30}{91}$

2

(b) P(the cards are of different colours)

$$= 1 - \frac{30}{91} = \frac{61}{91}$$

8 (a) $P(BH) + P(HB) = \left(\frac{4}{15} \times \frac{5}{14}\right) \times 2 = \frac{4}{21}$

(b) $P(HH) + P(BB) = \left(\frac{5}{15} \times \frac{4}{14}\right) + \left(\frac{6}{15} \times \frac{5}{14}\right) = \frac{5}{21}$

9 P(3 of them obtain the pens with the same colour)

$= P(BBB) + P(MMM)$

$$= \left(\frac{7}{10} \times \frac{6}{9} \times \frac{5}{8}\right) + \left(\frac{3}{10} \times \frac{2}{9} \times \frac{1}{8}\right)$$

$$= \frac{7}{24} + \frac{1}{120}$$

$$= \frac{3}{10}$$

10 (a) $P(MM) = \frac{1}{6} \times \frac{4}{6} = \frac{1}{9}$

(b) $P(MK) + P(KM) = \left(\frac{1}{6} \times \frac{1}{6}\right) + \left(\frac{3}{6} \times \frac{4}{6}\right) = \frac{13}{36}$
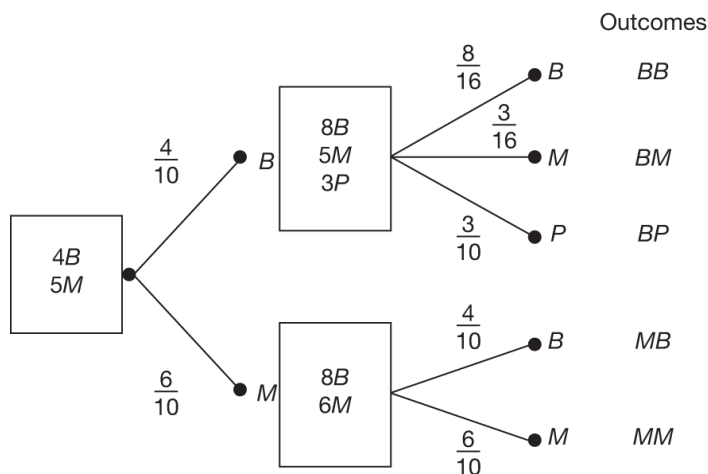
(c) P(different colours) = 1 – P(same colour)

$$= 1 - \left(\frac{1}{6} \times \frac{4}{6}\right) - \left(\frac{2}{6} \times \frac{1}{6}\right) - \left(\frac{3}{6} \times \frac{1}{6}\right)$$

$$= \frac{3}{4}$$

11 (a) $P(HH) = \frac{2}{9} \times \frac{8}{14} = \frac{8}{63}$

(b) $P(KB) + P(BK) = \frac{3}{9} \times \frac{4}{14} + \frac{4}{9} \times \frac{2}{14} = \frac{10}{63}$

12 B – Blue
   M – Red
   P – White

(a) $P(MB) + P(BM) = \dfrac{6}{10} \times \dfrac{4}{10} + \dfrac{4}{10} \times \dfrac{5}{16} = \dfrac{73}{200}$

(b) $P(BP) = \dfrac{4}{10} \times \dfrac{3}{16} = \dfrac{3}{40}$

UPSKILL 9.3

1 (a) Mutually exclusive events
(b) Not mutually exclusive events

2 (a) (i) $P(A) = \dfrac{4}{10} = \dfrac{2}{5}$

(ii) $P(B) = \dfrac{3}{10}$

(b) Yes. A and B are mutually exclusive events.

(c) $P(A \cup B) = \dfrac{2}{5} + \dfrac{3}{10} = \dfrac{7}{10}$

3 (a) Yes, A and B are mutually exclusive events.

(b) $P(A \cup B) = \dfrac{4}{6} + \dfrac{2}{6} = 1$

4

B ⁄ A	1	2	3	4	5
1	√			×	
2		√	×		
3		×	√		
4	×			√	
5					√

Mark √ for the event both cards have the same number.
Mark × for the event the sum of the numbers on the two cards is 5.

(a) P(both cards have the same number) $= \dfrac{5}{25} = \dfrac{1}{5}$

(b) P(the sum of the numbers on the two cards is 5) $= \dfrac{4}{25}$

(c) $P(A \cup B) = \dfrac{1}{5} + \dfrac{4}{25} = \dfrac{9}{25}$

5 $(X \cup Y) = P(X) + P(Y) - P(X \cap Y)$

$\qquad = \dfrac{3}{10} + \dfrac{2}{5} - \left(\dfrac{3}{10} \times \dfrac{2}{5} \right)$

$\qquad = \dfrac{29}{50}$

6 (a) Yes, both events are mutually exclusive.

(b) (i) $P(T \cup B) = \dfrac{2}{9} + \dfrac{4}{9} = \dfrac{2}{3}$

(ii) $P(T' \cap B') = P(T \cup B)' = 1 - \dfrac{2}{3} = \dfrac{1}{3}$

4

7

Machine \ Condition	Good	Defective	Total
P	16	20	36
Q	9	15	24
Total	25	35	60

(a) P(in good condition or produced by machine *P*)

$$= \frac{25}{60} + \frac{36}{60} - \frac{16}{60}$$

$$= \frac{3}{4}$$

(b) P(defective or produced by machine *Q*)

$$= \frac{35}{60} + \frac{24}{60} - \frac{15}{60}$$

$$= \frac{11}{15}$$

UPSKILL 9.4

1

A \ B	1	2	3	4
1	√		√	
2		√		√×
3	√		√×	×
4		√×	×	√×

Mark √ for the event that the sum of the numbers is even.
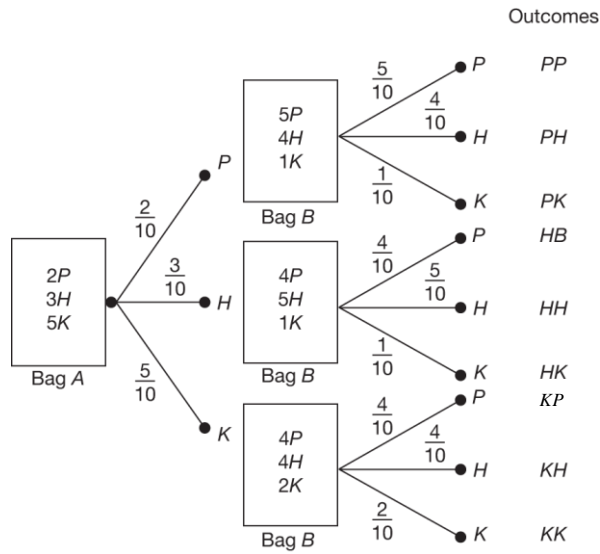Mark × for the event the sum of the numbers is greater than 5.

(a) P(the sum of the two numbers is even and greater than 5)

$$= \frac{4}{16} = \frac{1}{4}$$

(b) P(the sum of the two numbers is even or greater than 5)

$$= \frac{10}{16} = \frac{5}{8}$$

Success Plus Mathematics SPM

2 *P* – White
B – Black
K – Grey

Outcomes

Bag A: 2P 3H 5K

$\frac{2}{10}$ → P → Bag B: 5P 4H 1K
- $\frac{5}{10}$ → P — PP
- $\frac{4}{10}$ → H — PH
- $\frac{1}{10}$ → K — PK

$\frac{3}{10}$ → H → Bag B: 4P 5H 1K
- $\frac{4}{10}$ → P — HB
- $\frac{5}{10}$ → H — HH
- $\frac{1}{10}$ → K — HK

$\frac{5}{10}$ → K → Bag B: 4P 4H 2K
- $\frac{4}{10}$ → P — KP
- $\frac{4}{10}$ → H — KH
- $\frac{2}{10}$ → K — KK

$$\text{P}(PP) + \text{P}(HH) + \text{P}(KK)$$
$$= \left(\frac{2}{10} \times \frac{5}{10}\right) + \left(\frac{3}{10} \times \frac{5}{10}\right) + \left(\frac{5}{10} \times \frac{2}{10}\right)$$
$$= \frac{7}{20}$$

3 (a) $\text{P}\left(A\,\overline{R}\,\overline{Z}\right) + \text{P}\left(\overline{A}\,R\,\overline{Z}\right) + \text{P}\left(\overline{A}\,\overline{R}\,Z\right)$
$$= \left(\frac{9}{10} \times \frac{1}{5} \times \frac{3}{10}\right) + \left(\frac{1}{10} \times \frac{4}{5} \times \frac{3}{10}\right)$$
$$+ \left(\frac{1}{10} \times \frac{1}{5} \times \frac{7}{10}\right)$$
$$= \frac{23}{150}$$

(b) $\text{P}\left(A\,R\,\overline{Z}\right) + \text{P}\left(A\,\overline{R}\,Z\right) + \text{P}\left(\overline{A}\,R\,Z\right)$
$$= \left(\frac{9}{10} \times \frac{4}{5} \times \frac{3}{10}\right) + \left(\frac{9}{10} \times \frac{1}{5} \times \frac{7}{10}\right)$$
$$+ \frac{1}{10} \times \frac{4}{5} \times \frac{7}{10}$$
$$= \frac{199}{500}$$

(c) P(at least one of them strike the target)
$$= 1 - \frac{1}{10} \times \frac{1}{5} \times \frac{3}{10}$$
$$= \frac{497}{500}$$

6

Success Plus Mathematics SPM

4 P(*MH*) + P(*HM*)

$$= \left(\frac{6}{10} \times \frac{4}{10} \right) + \frac{4}{10} \times \frac{6}{9}$$

$$= \frac{38}{75}$$

5 (a) P(both are SPM books)

$$= \frac{5}{8} \times \frac{4}{7}$$

$$= \frac{5}{14}$$

(b) P(both are of the same subject)

$$= \left(\frac{6}{18} \times \frac{5}{17} \right) + \left(\frac{7}{18} \times \frac{6}{17} \right) + \left(\frac{5}{18} \times \frac{4}{17} \right)$$

$$= \frac{46}{153}$$

6 (a) P(both are males)

$$= \frac{5}{12} \times \frac{4}{11}$$

$$= \frac{5}{33}$$

(b) P(both are from the same country)

$$= \left(\frac{7}{16} \times \frac{6}{15} \right) + \left(\frac{4}{16} \times \frac{3}{15} \right) + \left(\frac{5}{16} \times \frac{4}{15} \right)$$

$$= \frac{37}{150}$$

Summative Practice 9

Multiple-Choice Questions

1

A\B	1	3	5
1			
2	×	×	√×
3			
4	×	√×	×
5			

Mark √ for the event that the sum of the numbers is 7.
Mark × for the event that the product of the numbers is greater than 5.

P(the sum of the numbers is 7 or the product of the numbers is greater than 5)

$$= \frac{6}{15} = \frac{2}{5}$$

Answer: C

2 P(multiple of 3 or perfect square)

$$= \frac{3}{9} + \frac{2}{9} - \frac{1}{9} = \frac{4}{9}$$

Answer: B

3 P(two pass)

$$= \left(\frac{1}{3} \times \frac{2}{5} \times \frac{1}{4} \right) + \left(\frac{2}{3} \times \frac{2}{5} \times \frac{3}{4} \right) + \left(\frac{1}{3} \times \frac{3}{4} \times \frac{3}{5} \right)$$

$$= \frac{1}{30} + \frac{1}{5} + \frac{3}{20}$$

$$= \frac{23}{60}$$

Answer: D

4 P(same colour)

$$= \left(\frac{5}{11} \times \frac{6}{10} \right) + \left(\frac{6}{10} \times \frac{5}{11} \right)$$

$$= \frac{6}{11}$$

Answer: C

5 P(*RG*) + P(*GR*)

$$= \left(\frac{4}{8} \times \frac{4}{8} \right) + \left(\frac{4}{8} \times \frac{4}{7} \right)$$

$$= \frac{15}{18}$$

Answer: B

Structured Questions

1 (a) P(*FFF*)

$$= \frac{8}{9} \times \frac{23}{25} \times \frac{19}{20}$$

$$= \frac{874}{1\,125}$$

(b) P(only one of them pass)

$$= \left(\frac{1}{9} \times \frac{23}{25} \times \frac{19}{20} \right) + \left(\frac{8}{9} \times \frac{2}{25} \times \frac{19}{20} \right) +$$

$$\left(\frac{1}{20} \times \frac{8}{9} \times \frac{23}{25} \right)$$

$$= \frac{437}{4\,500} + \frac{76}{1\,125} + \frac{46}{1\,125}$$

$$= \frac{37}{180}$$

Success Plus Mathematics SPM

(c) P(at least one of them pass)

$= 1 -$ P(all fail)

$= 1 - \dfrac{8}{9} \times \dfrac{23}{25} \times \dfrac{19}{20}$

$= 1 - \dfrac{874}{1\ 125}$

$= \dfrac{251}{1\ 125}$

2 (a) P(Physics, Chemistry)

$= \dfrac{3}{14} \times \dfrac{5}{13}$

$= \dfrac{15}{182}$

(b) P(the books are of the same category)

$= \left(\dfrac{3}{14} \times \dfrac{2}{13} \right) + \left(\dfrac{5}{14} \times \dfrac{4}{13} \right) + \left(\dfrac{6}{14} \times \dfrac{5}{13} \right)$

$= \dfrac{4}{13}$

3 (a) Yes, T and S are mutually exclusive events.

(b) (i) P(play tennis or squash)

$= \dfrac{4}{7} + \dfrac{2}{7}$

$= \dfrac{6}{7}$

(ii) P(did not play tennis nor squash)

$= 1 - \dfrac{6}{7}$

$= \dfrac{1}{7}$

4

A \ B	1	2	3
1	√		×
2		√×	
3	×		√

Mark √ for the event the numbers of the cards are the same.
Mark × for the event the sum of the numbers on the cards is 4.

(a) P(the numbers of the cards are the same)

$= \dfrac{3}{9} = \dfrac{1}{3}$

(b) P(the sum of the numbers on the cards is 4)

$= \dfrac{3}{9} = \dfrac{1}{3}$

(c) P(the numbers of the cards are the same or the sum of the numbers on the cards is 4)

$$= \frac{3}{9} + \frac{3}{9} - \frac{1}{9} = \frac{5}{9}$$

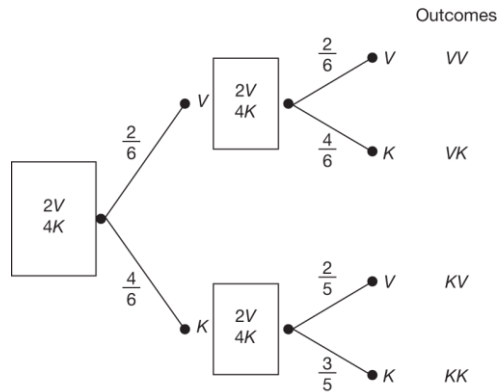**5** (a) P(both are STPM books)

$$= \frac{7}{13} \times \frac{6}{12} = \frac{7}{26}$$

(b) P(both books are of the same subject)

$$= \left(\frac{6}{18} \times \frac{5}{17} \right) + \left(\frac{7}{18} \times \frac{6}{17} \right) + \left(\frac{5}{18} \times \frac{4}{17} \right)$$

$$= \frac{46}{153}$$

6

(a) $P(KK) = \frac{4}{6} \times \frac{3}{5} = \frac{2}{5}$

(b) $P(VK) + P(KV) = \frac{2}{6} \times \frac{4}{6} + \frac{4}{6} \times \frac{2}{5} = \frac{22}{45}$

7 (a) $P(AA) = \frac{3}{8} \times \frac{2}{9} = \frac{1}{12}$

(b) $P(A\bar{A}) + P(\bar{A}A) = \left(\frac{3}{8} \times \frac{7}{9} \right) + \left| \frac{5}{8} \times \frac{1}{9} \right| = \frac{13}{36}$

8 (a) $P(20, 20) + P(20, 20) = \frac{2}{10} \times \frac{3}{12} = \frac{1}{20}$

(b) $P(20, 50) + P(50, 20) = \frac{2}{10} \times \frac{4}{12} + \frac{5}{10} \times \frac{3}{12} = \frac{23}{120}$

9 (a) $P(PP) = \frac{6}{11} \times \frac{5}{10} = \frac{3}{11}$

(b) $P(LL) + P(PP) = \frac{5}{9} \times \frac{4}{8} + \frac{4}{9} \times \frac{3}{8} = \frac{4}{9}$

10 $S = \{(2, A), (2, B), (4, A), (4, B), (7, A), (7, B), (9, A), (9, B)\}$

(a) $P\{(2, A), P(4, A)\}$,

P(even number and letter A) $= \frac{2}{8} = \frac{1}{4}$

10

(b) P{(2, *A*), (2, *B*), (7, *A*), (7, *B*), (4, *B*), (9, *B*)}

P(prime number or letter *B*) = $\frac{6}{8} = \frac{3}{4}$

11 (a) P(white, blue ball) = $\frac{120}{360} \times \frac{2}{6} = \frac{1}{9}$

(b) P(purple, red card) + P(white, red ball)

$= \frac{240}{360} \times \frac{6}{14} + \frac{120}{360} \times \frac{4}{6}$

$= \frac{32}{63}$

12 P(same colour)

$= \left(\frac{4}{15} \times \frac{3}{14} \times \frac{2}{13}\right) + \left(\frac{5}{15} \times \frac{4}{14} \times \frac{3}{13}\right) + \left(\frac{6}{15} \times \frac{5}{14} \times \frac{4}{13}\right)$

$= \frac{4}{455} + \frac{2}{91} + \frac{4}{91}$

$= \frac{34}{455}$

13 (a) P(strike the target 1 time)

$= \left(\frac{4}{5} \times \frac{1}{5} \times \frac{1}{5}\right) \times 3$

$= \frac{12}{125}$

(b) P(strike the target 2 times)

$= \left(\frac{4}{5} \times \frac{4}{5} \times \frac{1}{5}\right) \times 3$

$= \frac{48}{125}$

(c) P(straight the target at least 1 time)
   = 1 – P(did not strike the target)

$= 1 - \left(\frac{1}{5} \times \frac{1}{5} \times \frac{1}{5}\right)$

$= \frac{124}{125}$

(d) P(strike the target at least 2 times)

$= \frac{48}{125} + \left(\frac{4}{5} \times \frac{4}{5} \times \frac{4}{5}\right)$

$= \frac{48}{125} + \frac{64}{125}$

$= \frac{112}{125}$

Success Plus Mathematics SPM

14 P(2 same lettters)

$$= \left(\frac{2}{9} \times \frac{1}{8}\right) + \left(\frac{2}{9} \times \frac{1}{8}\right) + \left(\frac{2}{9} \times \frac{1}{8}\right)$$

$$= \frac{1}{12}$$

SPM SPOT

1 P(Nordin eats either an orange or an apple)
$$= 0.31 + 0.45 - (0.31 \times 0.45)$$
$$= 0.6205$$
Answer: C

2 P(Nordin and Chee Meng draw vouchers of the same value)
$$= P(RM5, RM5) + P(RM10, RM10) + P(RM20, RM20)$$
$$= \left(\frac{4}{10} \times \frac{3}{10}\right) + \left(\frac{3}{10} \times \frac{4}{10}\right) + \left(\frac{3}{10} \times \frac{3}{10}\right)$$
$$= \frac{12}{100} + \frac{12}{100} + \frac{9}{100}$$
$$= \frac{33}{100}$$
Answer: D

3 (a) $P(RM20, RM20) = \dfrac{4}{20} \times \dfrac{3}{19} = \dfrac{3}{95}$

(b) $P(RM100, RM100) = \dfrac{2}{20} \times \dfrac{1}{19} = \dfrac{1}{190}$

(c) P(RM20, RM50) + P(RM50, RM20)

$$= \left(\frac{4}{12} \times \frac{6}{11}\right) + \left(\frac{6}{12} \times \frac{4}{11}\right)$$

$$= \frac{4}{11}$$

(d) P(RM50, RM100) + P(RM100, RM50)

$$= \left(\frac{6}{12} \times \frac{2}{11}\right) + \left(\frac{2}{12} \times \frac{6}{11}\right)$$

$$= \frac{2}{11}$$

Success Plus Mathematics SPM