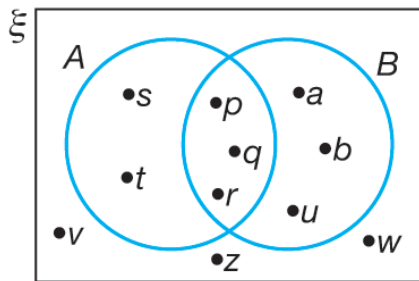


Form 4: Chapter 4
Operations on Sets
Fully-worked Solutions

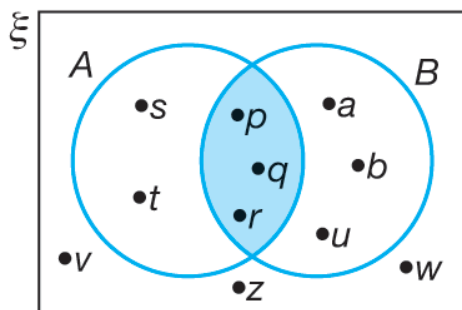
UPSKILL 4.1

- 1 (a) $\{b, h, k\}$
 (b) $\{5, 6\}$
 (c) $P = \{2, 3, 5, 7, \dots\}$
 $Q = \{1, 2, 3, 4, 5, 6\}$
 $P \cap Q = \{2, 3, 5\}$
 (d) $\{ \}$

- 2 (a) $\{p, q, r\}$
 (b) (i)



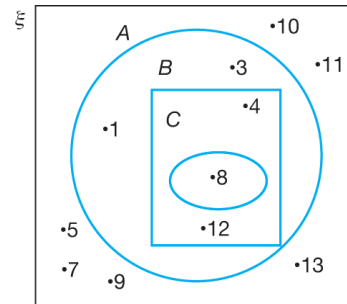
(ii)



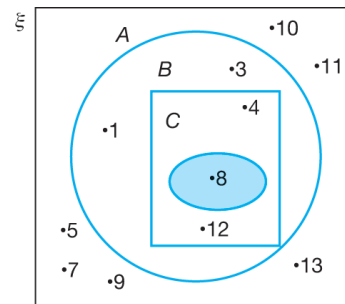
- 3 (a) $\{2, 4, 5\}$
 (b) $\{5, 7, 9\}$

- 4 $A = \{1, 2, 3, 4, 6, 8, 12\}$
 $B = \{4, 8, 12\}$
 $C = \{8\}$

(a)

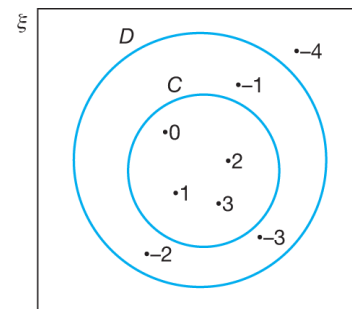


(b)

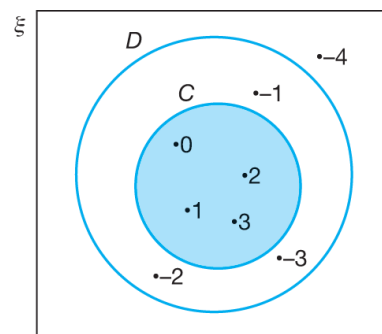


5 (a) $\{0, 1, 2, 3\}$

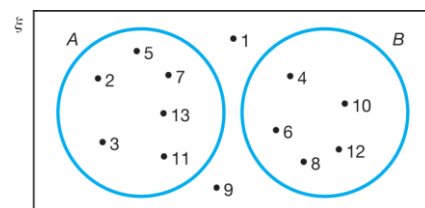
(b) (i)



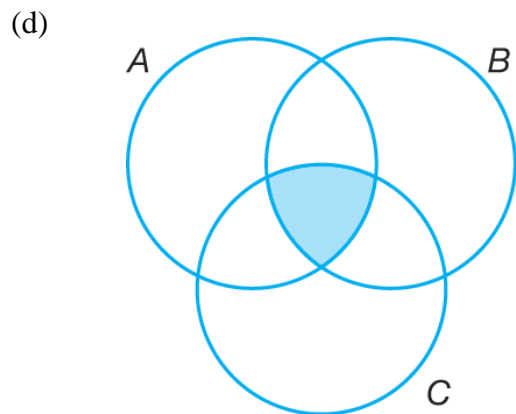
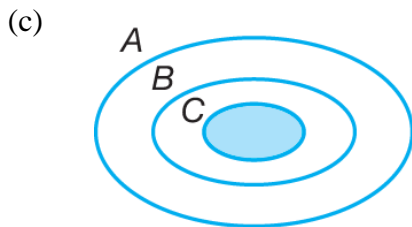
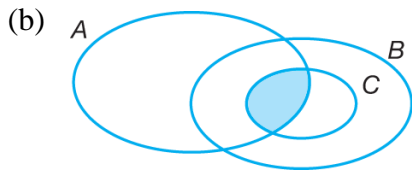
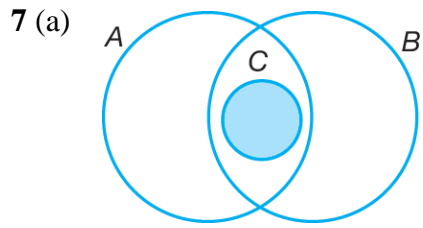
(ii)



- 6 (a) $A = \{2, 3, 5, 7, 11, 13\}$
 $B = \{4, 6, 8, 10, 12\}$

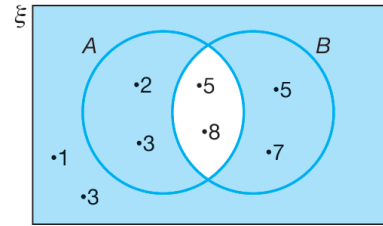


(b) $A \cap B = \emptyset$

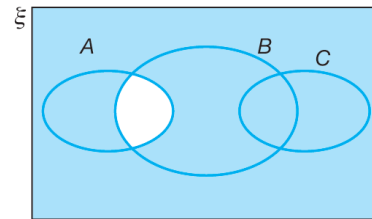


- 8 (a) (i) $A \cap B = \{6, 8\}$
(ii) $(A \cap B)' = \{1, 2, 3, 4, 5, 7\}$

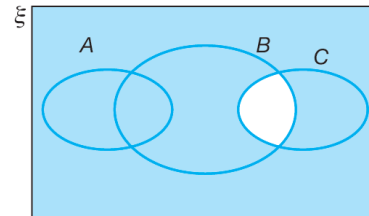
(b)



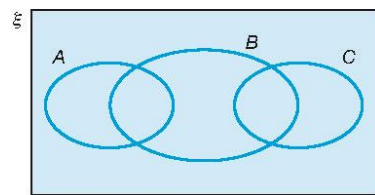
9 (a)



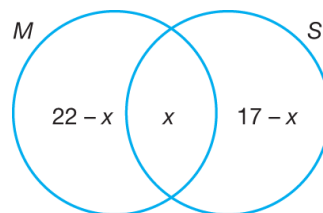
(b)



(c)



10

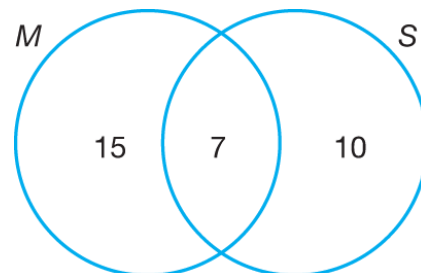


$$n(\xi) = 32$$

$$22 - x + x + 17 - x = 32$$

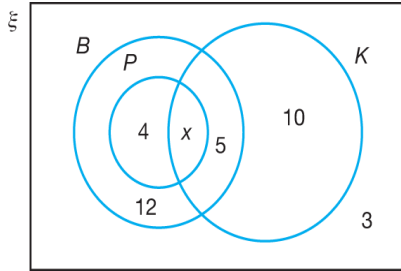
$$7 - x = 0$$

$$x = 7$$



- (a) The number of students who are interested in both Mathematics and Science is 7.
- (b) The number of students who are interested in Science but not Mathematics is 10.

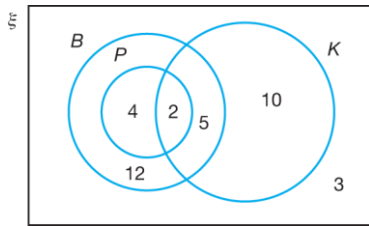
11



$$n(\xi) = 36$$

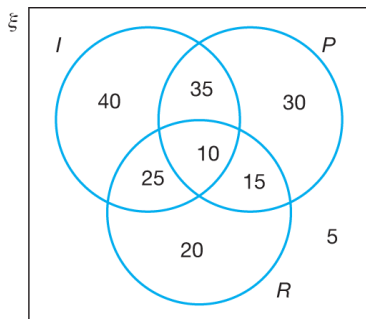
$$12 + 4 + x + 5 + 10 + 3 = 36$$

$$x = 2$$



- (a) $n(B \cap P \cap K) = 2$ students
- (b) $n(P' \cap K) = 5 + 10 = 15$ students
- (c) $n(B \cap P') = 5 + 12 = 17$ students
- (d) The number of student who like to play one type of game only = $12 + 10 = 22$

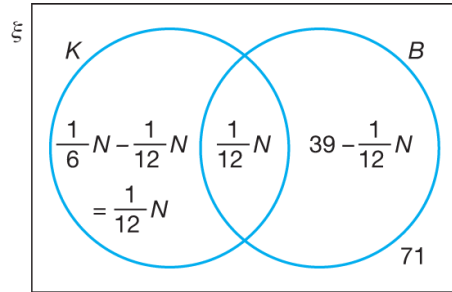
12



- (a) $n(\text{internet and smart phone but not reading}) = 35$ students
- (b) $n(\text{two activities only}) = 35 + 25 + 15 = 75$ students

- (c) $n(\text{one activity only}) = 40 + 30 + 20 = 90$ students
- (d) $n(\text{not interested in any of the activities}) = 5$ students

13



$$(a) \frac{1}{12}N + \frac{1}{12}N + \left(39 - \frac{1}{12}N + 71\right) = N$$

$$\frac{11}{12}N = 110$$

$$N = 120$$

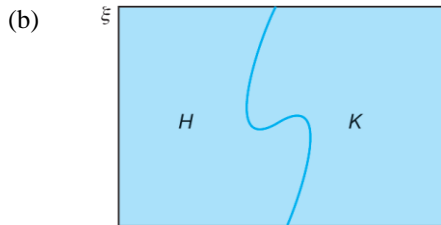
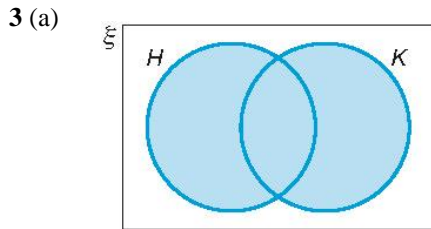
Number of undergraduates = 120

- (b) $\frac{1}{12}N = \frac{1}{12} \times 120 = 10$ undergraduates
- (c) $120 - 10 = 110$ undergraduates

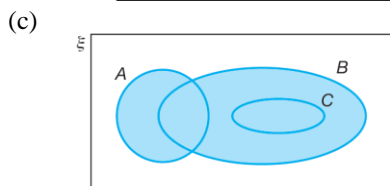
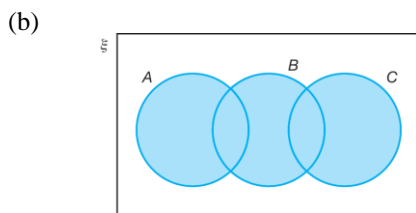
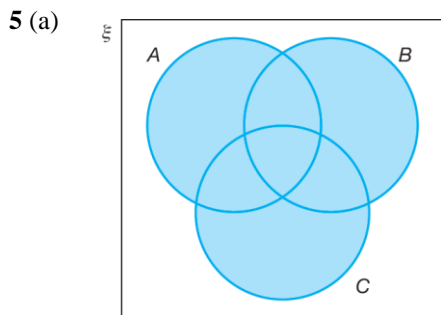
UPSKILL 4.2

- 1 (a) {5, 7, 15, 20}
 (b) {a, b, c, d, x, y, z}
 (c) {p, q, r, s}

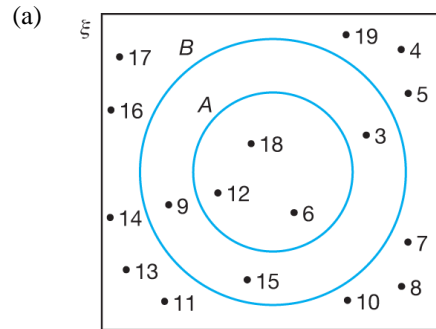
- 2 $P = \{3, 6, 9, 12\}$
 $Q = \{1, 2, 3, 4, 6, 12\}$
 $P \cup Q = \{1, 2, 3, 4, 6, 9, 12\}$



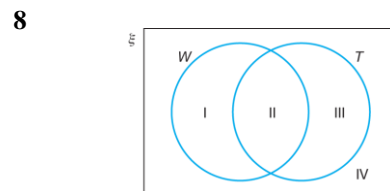
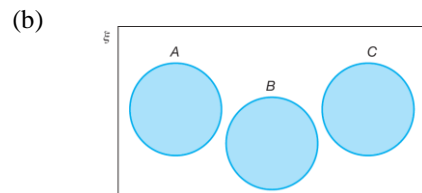
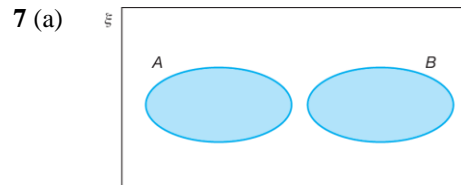
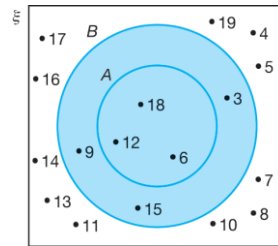
- 4 (a) {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12}
 (b) {a, b, c, d, e, f, h, j, i, l, o}



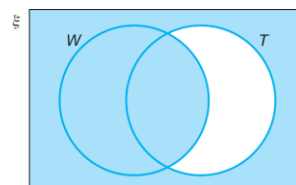
- 6 $A = \{6, 12, 18\}$
 $B = \{3, 6, 9, 12, 15, 18\}$



- (b) (i) $A \cup B = \{3, 6, 9, 12, 15, 18\}$

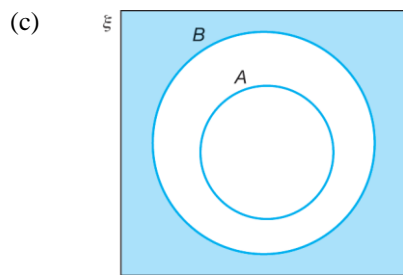
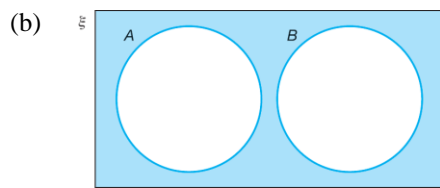
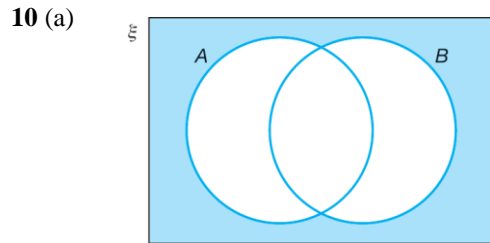


- $W = \{I, II\}$
 $T' = \{I, IV\}$
 $W \cap T' = \{I, II, IV\}$

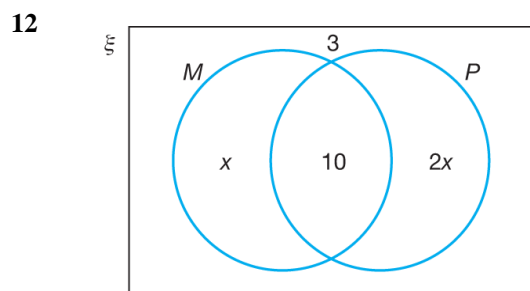


9 (a) $A \cup B = \{1, 3, 5, 6, 7, 9\}$
 $(A \cup B)' = \{2, 4, 8, 10\}$

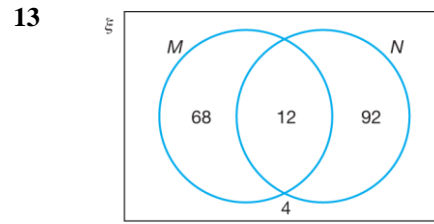
(b) $P \cup Q = \{a, b, c, d, e, f\}$
 $(P \cup Q)' = \{g, h\}$



11 $n(K \cup M)' = n(L \cup M)'$
 $4 + 5 = x + 5$
 $x = 4$



$x + 10 + 2x + 3 = 40$
 $3x = 27$
 $x = 9$
 $n(\text{girl guide}) = 10 + 2x$
 $= 10 + 2(9)$
 $= 28$



$n(\xi) = 68 + 12 + 92 + 4 = 176$ students

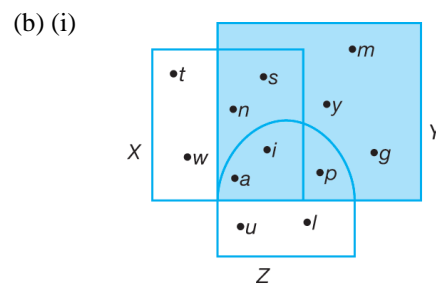
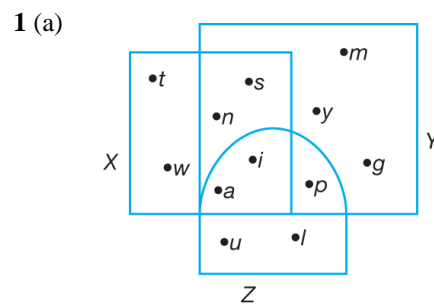
14 (a) $2k + k + 3 = 18$
 $3k = 15$
 $k = 5$

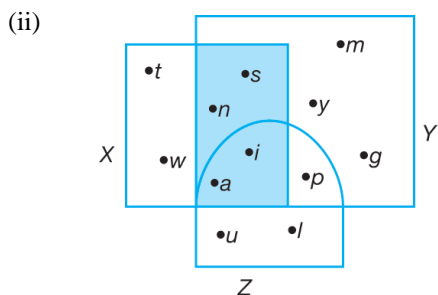
(b) $n(\xi) = 4k + 3 = 4(5) + 3 = 23$
participants

15 (a) $n(E \cap K) = 8$
 $2 + 2h = 8$
 $h = 3$

(b) Number of customers who like to drink two types of drinks only
 $= 5 + 1 + 2(3)$
 $= 12$

UPSKILL 4.3





- 2 (a) (i) $A = \{1, 2, 4, 5, 10\}$
(ii) $B = \{4, 8\}$
(iii) $C = \{2, 4, 6, 8, 10\}$

(b) (i) $B \cap C = \{4, 8\}$
 $A \cup (B \cap C) = \{1, 2, 4, 5, 8, 10\}$

(ii) $A \cup B = \{1, 2, 4, 5, 8, 10\}$
 $C \cap (A \cup B) = \{2, 4, 8, 10\}$

(iii) $A \cup C = \{1, 2, 4, 5, 6, 8, 10\}$
 $B \cap (A \cup C) = \{4, 8\}$

- (c) (i) $n[A \cup (B \cap C)] = 6$
(ii) $n[C \cap (A \cup B)] = 4$
(iii) $n[B \cap (A \cup C)] = 2$

- 3 (a) (i) $(P \cap Q) \cup R$
(ii) $(P \cap R) \cup (Q \cap R)$ or $(P \cup Q) \cap R$

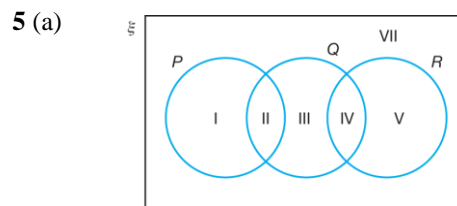
- 4 $A = \{1, 3, 5, 7, 9\}$
 $B = \{2, 4, 6, 8, 10\}$
 $C = \{2, 3, 4, 5, 6\}$

(a) $A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$
 $A \cup B \cap C = \{2, 3, 4, 5, 6\}$
 $(A \cup B \cap C)' = \{1, 7, 8, 9, 10\}$

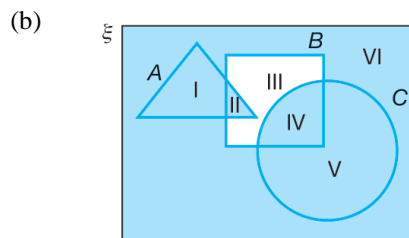
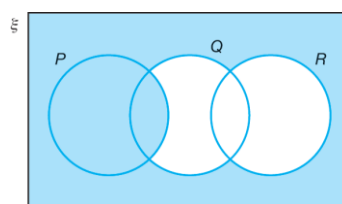
(b) $B' = \{1, 3, 5, 7, 9\}$
 $B' \cap C = \{3, 5\}$
 $A \cup (B' \cap C) = \{1, 3, 5, 7, 9\}$

(c) $A' = \{2, 4, 6, 8, 10\}$
 $B \cup C = \{2, 3, 4, 5, 6, 8, 10\}$
 $A' \cap (B \cup C) = \{2, 4, 6, 8, 10\}$

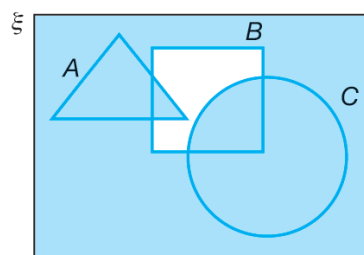
(d) $A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$
 $C' = \{1, 7, 8, 9, 10\}$
 $A \cup B \cap C' = \{1, 7, 8, 9, 10\}$



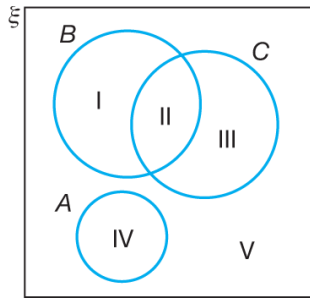
$P = \{I, II\}$
 $Q \cup R = \{II, III, IV, V\}$
 $(Q \cup R)' = \{I, VII\}$
 $P \cup (Q \cup R)' = \{I, II, VII\}$



$A = \{I, II\}$
 $B' = \{I, V, VI\}$
 $C = \{IV, V\}$
 $A \cup B' \cup C = \{I, II, IV, V, VI\}$



6 (a)



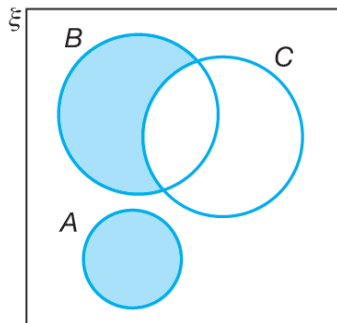
$$C' = \{I, IV, V\}$$

$$B = \{I, II\}$$

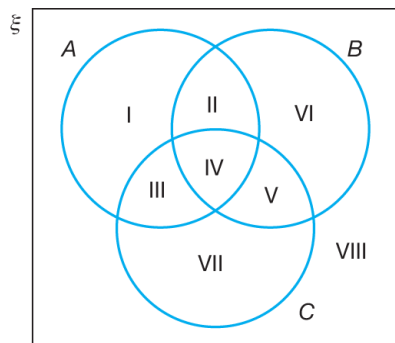
$$C' \cap B = \{I\}$$

$$A = \{IV\}$$

$$(C' \cap B) \cup A = \{I, IV\}$$



(b)



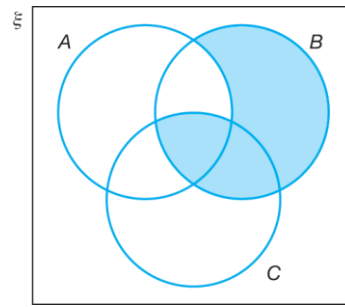
$$A' = \{V, VI, VII, VIII\}$$

$$C = \{III, IV, V, VII\}$$

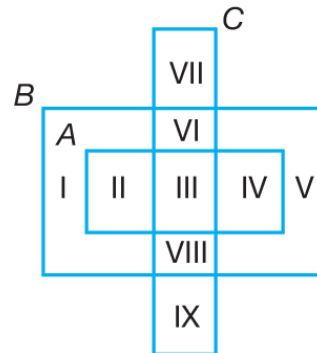
$$A' \cup C = \{III, IV, V, VI, VII, VIII\}$$

$$B = \{II, IV, V, VI\}$$

$$(A' \cup C) \cap B = \{IV, V, VI\}$$



7

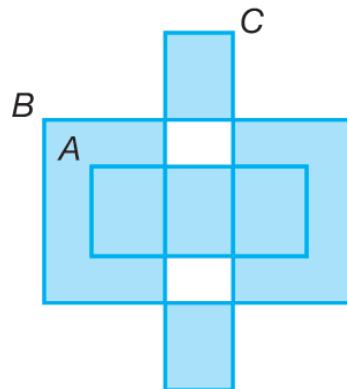


$$B \cap C = \{III, VI, VIII\}$$

$$(B \cap C)' = \{I, II, IV, V, VII, IX\}$$

$$A = \{II, III, IV\}$$

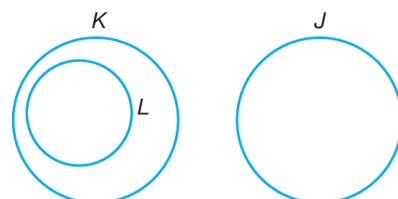
$$(B \cap C)' \cup A = \{I, II, III, IV, V, VII, IX\}$$



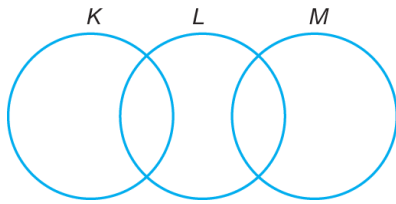
8 $(Q' \cap P) \cap R$

9 $(A \cap B) \cup (B' \cap C)$

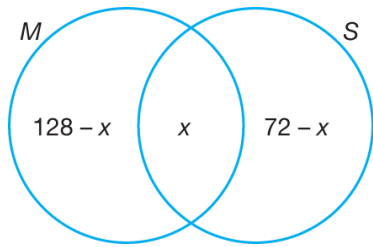
10



11



12



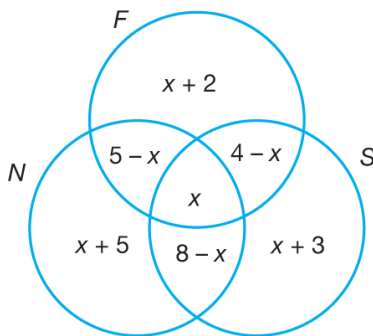
$$n(\xi) = 160$$

$$128 + 72 - x = 160$$

$$x = 40$$

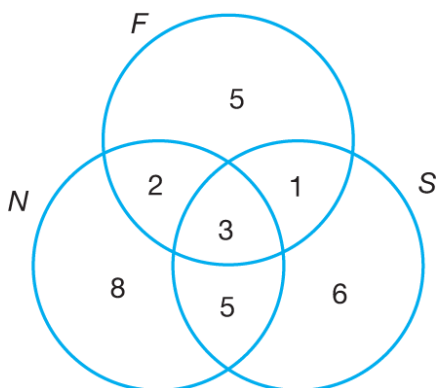
Hence, the number of students who achieve grade A in both Mathematics and Science is 40.

13 (a)



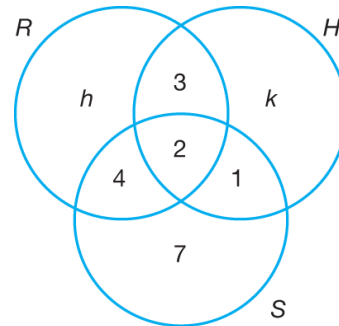
$$n(\xi) = x + 27 = 30$$

$$x = 3$$



- (b) (i) The number of students who like to read all three types of books is 3.
 (ii) The number of students who like to read non-fiction or sports but not fiction story books is $8 + 5 + 6 = 19$.
 (iii) The number of students who like to read fiction story books and sports books only = 1.

14 (a)



(b) $n(\xi) = 40$
 $h + k + 17 = 40$
 $h + k = 23 \dots (1)$

$$n(R) = n(H)$$

$$h + 9 = k + 6$$

$$h - k = -3 \dots (2)$$

$$(1) - (2) : 2k = 26$$

$$k = 13$$

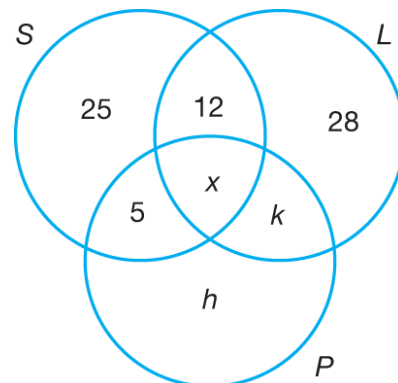
From (1) :

$$h + 13 = 23$$

$$h = 10$$

- (c) The number of students who have two characteristics only = $3 + 4 + 1 = 8$

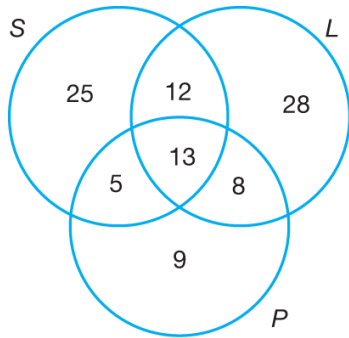
15



(a) $n(S) = 55$
 $x + 42 = 55$

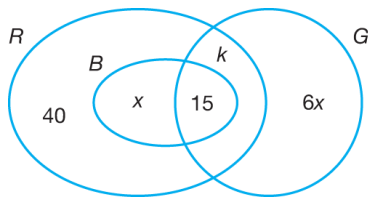
$$\begin{aligned}
 x &= 13 \\
 n(L) &= 61 \\
 40 + 13 + k &= 91 \\
 k &= 8
 \end{aligned}$$

$$\begin{aligned}
 n(\xi) &= 100 \\
 h + 91 &= 100 \\
 h &= 9
 \end{aligned}$$



Number of movie viewers who like to watch scaring or comedy and investigation genres = $5 + 13 + 8 = 26$

16



$$(a) n(B) = \frac{1}{6} \times n(R)$$

$$x + 15 = \frac{1}{6}(x + k + 55)$$

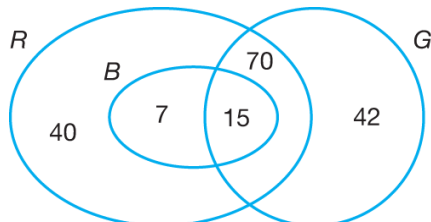
$$x + 15 = \frac{1}{6}(x + 70 + 55)$$

$$6x + 90 = x + 125$$

$$5x = 35$$

$$x = 7$$

(b)



Number of members who join not more than two clubs
 $= 40 + 7 + 70 + 42$
 $= 159$

Summative Practice 4

Multiple-Choice Questions

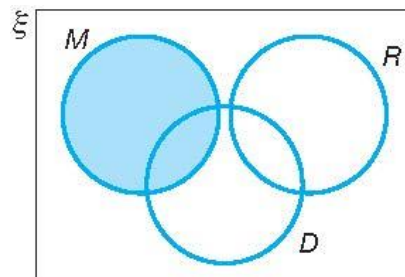
1 The set which represents the shaded region is $P' \cup Q$.

Answer: B

2 The set which represents the shaded region is $(P \cup R)' \cap Q$.

Answer: C

3 The Venn diagram which represents the students who like pineapples but not star fruits is



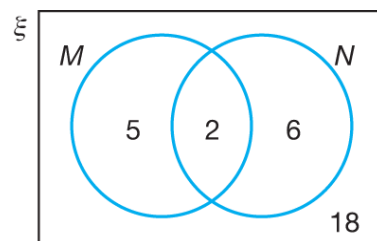
4 $M = \{x : x \text{ is a multiple of } 5\}$
 $M = \{10, 15, 20, 25, 30, 35, 40\}$

$N = \{x : x \text{ is a multiple of } 4\}$
 $N = \{12, 16, 20, 24, 28, 32, 36, 40\}$

$M \cap N = \{20, 40\}$

$(M \cup N)' = \{11, 13, 14, 17, 18, 19, 21, 22, 23, 26, 27, 29, 31, 33, 34, 37, 38, 39\}$

The required Venn diagram is



Answer: D

- 5 $P = \{2, 4, 6, 8, 10\}$
 $Q = \{2, 3, 5, 7\}$
 $R = \{1, 2, 3, 4, 5\}$
 $P' = \{1, 3, 5, 7, 9\}$
 $P' \cup R = \{1, 2, 3, 4, 5, 7, 9\}$
 $Q' = \{1, 4, 6, 8, 9, 10\}$
 $Q' \cap (P' \cup R) = \{1, 4, 9\}$

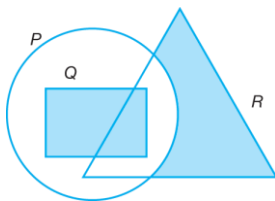
Answer: A

- 6 The Venn diagram which represents the shaded region is $(P \cup R)'$.

Answer: C

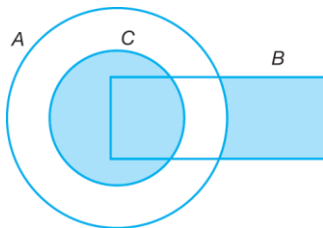
Structured Questions

1



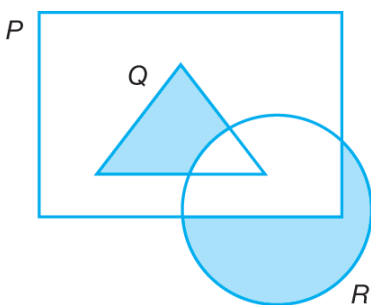
$$(P' \cap R) \cup Q$$

2



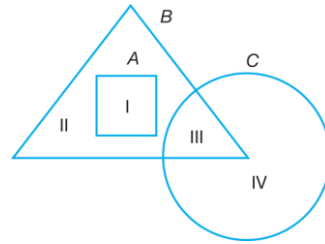
$$(A' \cap B) \cup C$$

3



$$(P' \cap R) \cup Q$$

4



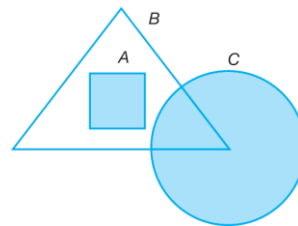
$$A = \{I\}$$

$$B' = \{IV\}$$

$$A \cup B' = \{I, IV\}$$

$$C = \{III, IV\}$$

$$(A \cup B)' \cup C = \{I, III, IV\}$$

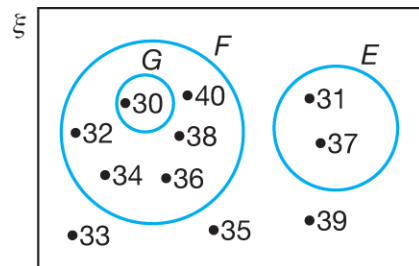


5 (a) $A \cup (B \cap C)$

(b) $(A \cup B) \cap C'$

- 6 (a) $\xi = \{30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40\}$
 $E = \{31, 37\}$
 $F = \{30, 32, 34, 36, 38, 40\}$
 $G = \{30\}$

(b)



(c) $F \cap G = \{30\}$

$$E \cup (F \cap G) = \{30, 31, 37\}$$

(d) $n[E \cup (F \cap G)] = 3$

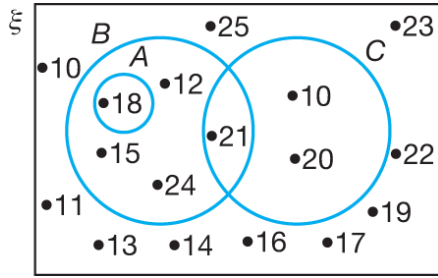
- 7 (a) $\xi = \{10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25\}$

$$A = \{18\}$$

$$B = \{12, 15, 18, 21, 24\}$$

$$C = \{10, 20, 21\}$$

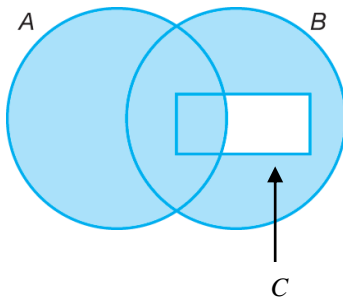
(b)



(c) $(A \cap B) \cup (B \cap C)$
 $= \{18\} \cup \{21\}$
 $= \{18, 21\}$

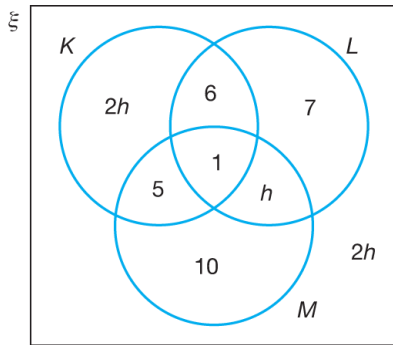
(d) $n(B \cap C)' = 15 + 1 - 1 = 15$

8



$A \cup (B \cap C)'$

9



(a) $n(K) = n(L \cup M)'$

$2h + 12 = 4h$

$2h = 12$

$h = 6$

(b) $n[(K \cap L) \cup (L \cap M) \cup (K \cap M)]'$

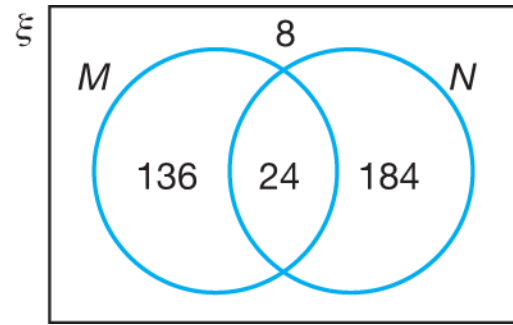
$= 2h + 7 + 10 + 2h$

$= 4h + 17$

$= 4(6) + 17$

$= 41$

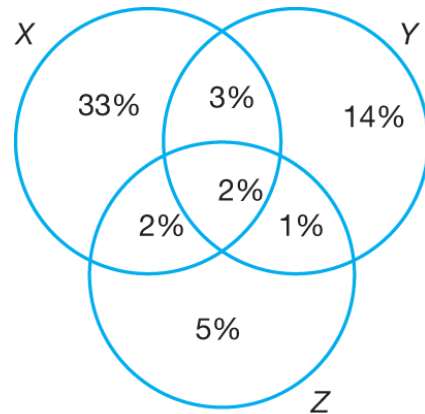
10



(a) Number of students
 $= 136 + 24 + 184 + 8$
 $= 352$

(b) The number of students who like to read one type of magazines only
 $= 136 + 184$
 $= 320$

11 (a)



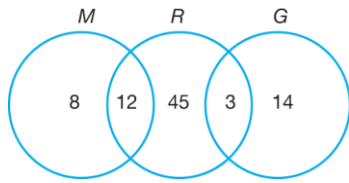
(b) The percentage of customers who buy one brand of facial cleanser only
 $= 33\% + 14\% + 5\%$
 $= 52\%$

The number of customers who buy one brand of facial cleanser only

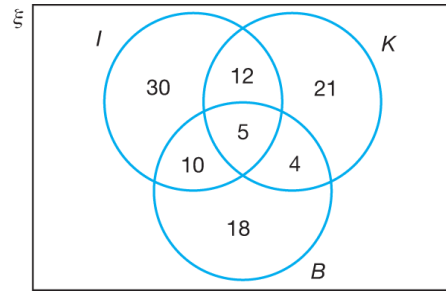
$= \frac{52}{100} \times 10\,000$

$= 5\,200$

12 (a)



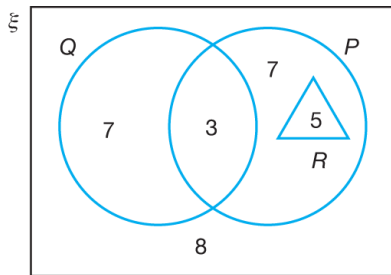
- (b) The number of students who like two types of hobbies only
 $= 12 + 3$
 $= 15$



(a) The number of housewives who like to cook

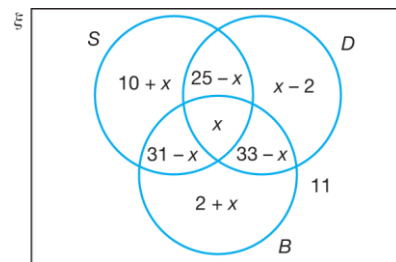
- (i) all three types of dishes = 5
 (ii) one type of dish only
 $= 30 + 21 + 18$
 $= 69$
 (iii) fried chicken or chicken curry but not broccoli
 $= 30 + 12 + 21$
 $= 63$

13 (a)



- (a) $h = 7, k = 5, m = 8$
 (b) (i) The number of customers who do not buy brand R soaps but buy brand P soaps
 $= 7 + 3$
 $= 10$
 (ii) The number of customers who do not buy brand Q soaps but buy brand P soaps
 $= 7 + 5$
 $= 12$
 (c) The number of customers buy one brand of soaps only
 $= 7 + 7$
 $= 14$
 (d) $P' \cap R = \phi$
 $\therefore n(P' \cap R) = 0$

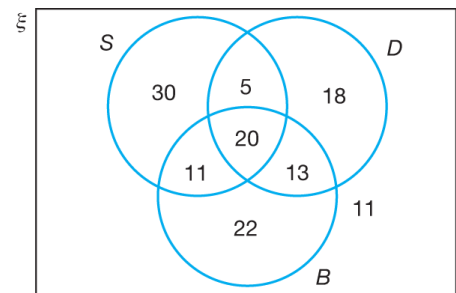
15



$$10 + x + 25 - x + x + 31 - x + x - 2 + 33 - x + 2 + x + 11 = 130$$

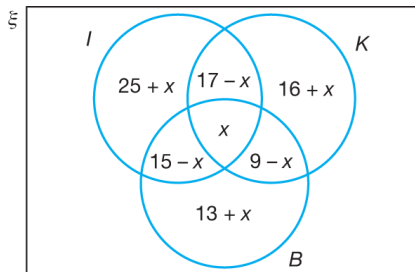
$$x + 110 = 130$$

$$x = 20$$



- (b) The number of customers who like
 (i) all the three types of programmes
 $= 20$
 (ii) only one type of programme
 $= 30 + 18 + 22$
 $= 70$
 (iii) at least two types of programmes
 $= 5 + 11 + 13 + 20$
 $= 49$

14



$$25 + x + 15 - x + x + 17 - x + 16 + x + 9 - x + 13 + x = 100$$

$$95 + x = 100$$

$$x = 5$$

SPM SPOT

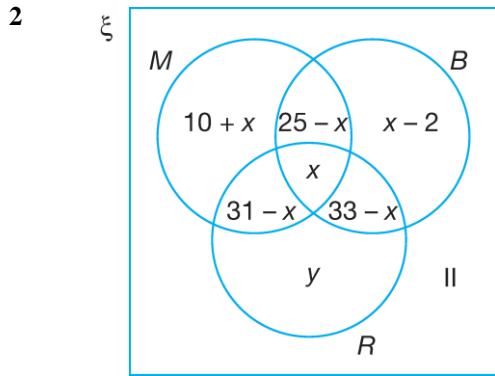
1 $n(\xi) = 100$

$95 + x = 5$

$x = 5$

Hence, the number of customers who like all three types of curry dishes is 5.

Answer: A



$n(M) = n(R)$

$66 = 64 + y - x$

$y - x = 2 \dots (1)$

$n(\xi) = 130$

$56 + 41 + y + 11 = 130$

$y = 22$

Substitute $y = 22$ into (1):

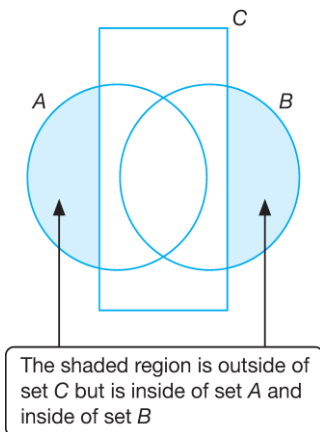
$22 - x = 2$

$x = 20$

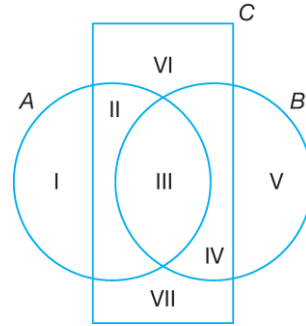
Hence, the number of customers who like to read all the three types of books is 20.

Answer: A

3 (a)



(b)



$A' = \{IV, V, VI, VII\}$

$B' = \{I, II, VI, VII\}$

$A' \cup B' = \{I, II, IV, V, VI, VII, VIII\}$

$C = \{II, III, IV, VI, VII\}$

$A' \cup B' \cap C = \{II, IV, VI, VII\}$

