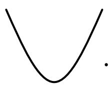
Form 4: Chapter 1
Quadratic Functions and Equations in One Variable
Fully-worked Solutions

UPSKILL 1.1

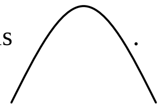
1 (a) No because the highest power of $p + 8$ is 1 and not 2.
(b) Yes because the highest power of $q^2 - 9$ is 2.
(c) Yes because the highest power of $r^2 + 10r$ is 2.
(d) Yes because the highest power of $s^2 + 4s + 5$ is 2.
(e) No because the highest power of $4t^3 + 3t^2 - 6t + 8$ is 3 and not 2.
(f) No because the highest power of $0u^2 + 18u + 3 = 18u + 3$ is 1 and not 2.
(g) No because $12 - 5vw + 2v^2$ has two variables and not one variable
(h) No because $z^2 + 4z - \dfrac{5}{z}$ is not in the form $az^2 + bz + c$.

2 (a) Yes because the highest power of $f(d) = (3 - d)(2 + 5d) = 6 + 13d - 5d^2$ is 2.
(b) No because the highest power of $g(e) = \dfrac{4}{e^2} - 5 = 4e^{-2} - 5$ is –2.
(c) Yes because the highest power of $h(j) = (2 - 3j)^2 = 4 - 12j + 9j^2$ is 2.
(d) No because the highest power of $m(k) = k - \dfrac{3}{k}$ is 1.
(e) No because the highest power of $n(p) = p(3p+1)^2 = 3p^3 + 6p^2 + p$ is 3.
(f) No because the highest power of $p(u) = \dfrac{u^2 + 5}{u^2} = 1 + 5u^{-2}$ is –2.

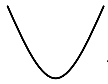
3 (a) Since the coefficient of x^2 is positive,

then the shape of the graph is .

(b) Since the coefficient of x^2 is negative,

then the shape of the graph is .

(c) Since the coefficient of x^2 is positive,

then the shape of the graph is .

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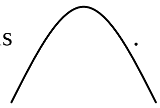
then the shape of the graph is .

4 (a) The equation of the axis of symmetry is $x = \dfrac{-2 + 8}{2} = 3$
(b) When $x = 2$,
$y = -(3)^2 + 6(3) + 16 = 25.$
The coordinates of the maximum point are (3, 25).
(c) When the curve is reflected in the x-axis, its function is $f(x) = x^2 - 6x - 16$.
(d) When the curve is reflected in the y-axis, its function is $f(x) = -x^2 - 6x + 16$.

5 (a) The equation of the axis of symmetry is $x = \dfrac{-6 + (-2)}{2} = -4$
(b) When $x = -4$,
$y = (-4)^2 + 8(-4) + 12 = -4$.
The coordinates of the maximum point are $(-4, -4)$.
(c) When the curve is reflected in the x-axis, its function is $g(x) = -x^2 - 8x - 12$.
(d) When the curve is reflected in the y-axis, its function is $g(x) = x^2 - 8x + 12$.

6 $B(x) = (x+2)(3x+6)$

$= 3x^2 + 12x + 12$

$B(x) = 300$

$3x^2 + 12x + 12 = 300$

$x^2 + 4x + 4 = 100$

$x^2 + 4x - 96 = 0$

7 $L(x) = \frac{1}{2}(4x+8)(2x+6)$

$= \frac{1}{2}\left(8x^2 + 40x + 48\right)$

$= 4x^2 + 20x + 24$

$L(x) = 80$

$4x^2 + 20x + 24 = 80$

$4x^2 + 20x - 56 = 0$

$x^2 + 5x - 14 = 0$

8 $L(x) = \frac{1}{2}(5x + 2 + 3x)(4x)$

$= \frac{1}{2}(8x+2)(4x)$

$= 2x(8x+2)$

$= 16x^2 + 4x$

$L(x) = 80$

$16x^2 + 4x - 80 = 0$

$4x^2 + x - 20 = 0$

9 $V(x) = (x+4)(5)(2x)$

$= 10x(x+4)$

$= 10x^2 + 40x$

$V(x) = 600$

$10x^2 + 40x = 600$

$x^2 + 4x - 60 = 0$

10 Swee Ling's age 5 years ago = $x - 5$.

If Swee Ling's age 5 years ago is half of her mother's age, then her mother's age is $2(x-5)$.

$h(x) = 2(x-5)(x-5)$

$= 2(x^2 - 10x + 25)$

$= 2x^2 - 20x + 50$

$h(x) = 1\ 250$

$2x^2 - 20x + 50 = 1\ 250$

$2x^2 - 20x - 1\ 200 = 0$

$x^2 - 10x - 600 = 0$

11 $3x^2 - 5x - 2 = 0$

(a) LHS

$= 3(2)^2 - 5(2) - 2$

= RHS

Thus, x = 2 is not a root for $3x^2 - 5x - 2 = 0$.

(b) LHS

$= 3(1)^2 - 5(1) - 2$

= –4

≠ RHS

Thus, x = 1 is not a root of $3x^2 - 5x - 2 = 0$.

(c) LHS

$= 3\left(-\frac{1}{3}\right)^2 - 5\left(-\frac{1}{3}\right) - 2$

$= 0$

= RHS

Thus, $x = -\frac{1}{3}$ is a root of $3x^2 - 5x - 2 = 0$.

12 $-2x^2 + 3x - 1 = 0$

(a) LHS

$= -2(3)^2 + 3(3) - 1$

$= -10$

≠ RHS

Thus, $x = 3$ is not a root of $-2x^2 + 3x - 1 = 0$.

(b) LHS

$= -2(1)^2 + 3(1) - 1$

= 0

= RHS

Thus, $x = 1$ is a root of $-2x^2 + 3x - 1 = 0$.

(c) LHS

$= -2\left(\frac{1}{2}\right)^2 + 3\left(\frac{1}{2}\right) - 1$

$= 0$

$=$ RHS

Thus, $x = \frac{1}{2}$ is a root of $-2x^2 + 3x - 1 = 0$.

13 (a) $3p^2 - 5p = 0$

$p(3p-5) = 0$

$p = 0$ or $p = \frac{5}{3}$

(b) $5q^2 + 45q = 0$

$5q(q+9) = 0$

$q = 0$ or -9

(c) $16r^2 - 25 = 0$

$(4r+5)(4r-5) = 0$

$r = -\frac{5}{4}$ or $r = \frac{5}{4}$

(d) $36s^2 - 16 = 0$

$4(9s^2 - 4) = 0$

$9s^2 - 4 = 0$

$(3s+2)(3s-2) = 0$

$s = -\frac{2}{3}$ or $\frac{2}{3}$

(e) $12t^2 - 28t + 15 = 0$

$(2t-3)(6t-5) = 0$

$t = \frac{3}{2}$ or $t = \frac{5}{6}$

(f) $8m^2 - 51m + 18 = 0$

$(m-6)(8m-3) = 0$

$m = 6$ or $m = \frac{3}{8}$

(g) $6u^2 + 5u - 6 = 0$

$(3u-2)(2u+3) = 0$

$u = \frac{2}{3}$ or $u = -\frac{3}{2}$

(h) $10v^2 - 7v - 12 = 0$

$(2v-3)(5v+4) = 0$

$v = \frac{3}{2}$ or $v = -\frac{4}{5}$

(i) $-12w^2 - 11w + 36 = 0$

$12w^2 + 11w - 36 = 0$

$(3w-4)(4w+9) = 0$

$w = \frac{4}{3}$ or $w = -\frac{9}{4}$

14 (a) $-3z^2 = 4 - 13z$

$3z^2 - 13z + 4 = 0$

$(z-4)(3z-1) = 0$

$z = 4$ or $z = \frac{1}{3}$

(b) $(2z+1)^2 = 16$

$4z^2 + 4z + 1 = 16$

$4z^2 + 4z - 15 = 0$

$(2z-3)(2z+5) = 0$

$z = \frac{3}{2}$ or $z = -\frac{5}{2}$

(c) $3f + 1 = \frac{7}{f-1}$

$(3f+1)(f-1) = 7$

$3f^2 - 2f - 1 - 7 = 0$

$3f^2 - 2f - 8 = 0$

$(f-2)(3f+4) = 0$

$f = 2$ or $f = -\frac{4}{3}$

(d) $g - 1 = \frac{g+20}{6g}$

$6g^2 - 6g = g + 20$

$6g^2 - 7g - 20 = 0$

$(2g-5)(3g+4) = 0$

$g = \frac{5}{2}$ or $-\frac{4}{3}$

(e) $(h-3)(h+2) = \frac{1}{2}h(h-3)$

$2(h^2 - h - 6) = h^2 - 3h$

$2h^2 - 2h - 12 = h^2 - 3h$

$h^2 + h - 12 = 0$

$(h-3)(h+4) = 0$

$h = 3$ or $h = -4$

(f) $\frac{j-1}{6}-\frac{2j-1}{5j}=0$

$\frac{5j(j-1)-6(2j-1)}{30j}=0$

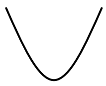
$\frac{5j^2-5j-12j+6}{60j}=0$

$5j^2-17j+6=0$

$(j-3)(5j-2)=0$

$j=3$ or $j=\frac{2}{5}$

15 (a) $y=f(x)=2x^2+2$

Since the coefficient of x^2 is positive, the shape of its curve is .

At the y-axis, $x=0$.

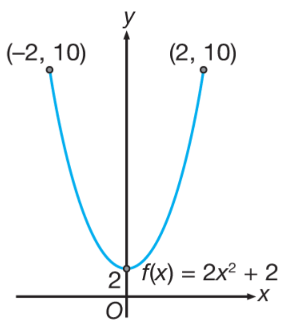
$y=2(0)^2+2=2$

Thus, the curve will intersect the x-axis at the point (0, 2).

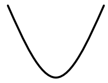
When $x=-2$, $y=2(-2)^2+2=10$

When $x=2$, $y=2(2)^2+2=10$

Thus, the curve will intersect the y-axis at the point $(-2, 10)$ and (2, 10).



(b) $y=g(x)=x^2+4x+3$

Since the coefficient of x^2 is positive, the shape of its curve is .

At the y-axis, x = 0.

$y=0^2+4(0)+3=3$

Thus, the curve will intersect the y-axis at the point (0, 3).

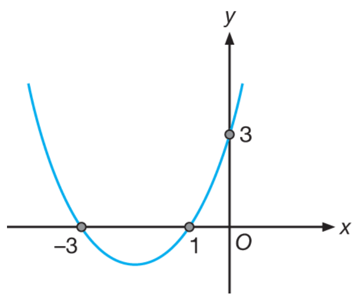
At the x-axis, y = 0.

$x^2+4x+3=0$

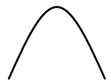
$(x+1)(x+3)=0$

$x=-1$ or $x=-3$

Thus, the curve will intersect the x-axis at the points $(-1, 0)$ and $(-3, 0)$.



(c) $y=h(x)=-x^2-2x+15$

Since the coefficient of x^2 is negative, the shape of the curve is .

At the y-axis, x = 0.

Thus, the curve will intersect the y-axis at the point (0, 3).

At the x-axis, y = 0.

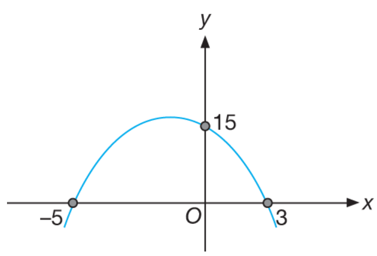
$-x^2-2x+15=0$

$x^2+2x-15=0$

$(x-3)(x+5)=0$

$x=3$ or $x=-5$

Thus, the curve will intersect the x-axis at the points $(-5, 0)$ and (3, 0).



16 (a) Area of the shaded region = 28 cm^2
Area of *ABCD* – Area of *BAR* = 28

$$8(x+2)-\frac{1}{2}x(x+2)=28$$
$$8x+16-\frac{1}{2}x^2-x=28$$
$$16x+32-x^2-2x=56$$
$$x^2-14x+24=0$$

(b) $x^2-14x+24=0$
$(x-12)(x-2)=0$
$x=12$ or $x=2$
$x=12$ is not accepted because x has to be less than 8.
$\therefore x=2$

(c) Area of *BAR* = $\frac{1}{2}x(x+2)$
$=\frac{1}{2}(2)(2+2)$
$=4$ cm^2

17 (a) Area of the triangle *ABC* = 70 cm^2

$$\frac{1}{2}(4x)(12-x)=70$$
$$24x-2x^2-70=0$$
$$2x^2-24x+70=0$$
$$x^2-12x+35=0$$

(b) $x^2-12x+35=0$
$(x-5)(x-7)=0$
$x=5$ or $x=7$

(c) The smaller value of x is 5.
$AB=12-x=12-5=7$ cm
$BC=4x=4(5)=20$ cm

Using the Pythagoras' Theorem,
$AC=\sqrt{7^2+20^2}=\sqrt{449}=21.19$ cm

18 (a) Area of the trapezium = 99 cm^2

$$\frac{1}{2}(2y+y+7)(2y-1)=99$$
$$(3y+7)(2y-1)=198$$
$$6y^2-3y+14y-7=198$$
$$6y^2+11y-205=0$$

(b) $6y^2+11y-205=0$
$(y-5)(6y+41)=0$
$y=5$ or $y=-\frac{41}{6}$
$y=-\frac{41}{6}$ is not accepted.
$\therefore y=5$

(c) $UV=2y=2(5)=10$ cm
$XW=y+7=5+7=12$ cm
$VW=2y-1=2(5)-1=9$ cm

19 (a) Area of the shaded region = 120 cm^2

$$(t+8)(t-6)=120$$
$$t^2+2t-48-120=0$$
$$t^2+2t-168=0$$

(b) $t^2+2t-168=0$
$(t-12)(t+14)=0$
$t=12$ or $t=-14$
$t=-14$ is not accepted.
$\therefore t=12$

(c) $QC=t+8=12+8=20$ cm

20 (a) Area of the L shape = 30 cm^2
Area of the first rectangle + Area of the second triangle = 30

$$6x+x(7-x)=30$$
$$6x+7x-x^2=30$$
$$13x-x^2-30=0$$
$$x^2-13x+30=0$$

(b) $x^2-13x+30=0$
$(x-10)(x-3)=0$
$x=10$ or $x=3$
$x=10$ is not accepted because x cannot be greater than 6.
$\therefore x=3$

21 Distance = Speed × Time

$$J(x)=3x(x-8)+(3x+5)(x-9)$$
$$J(x)=3x^2-24x+3x^2-22x-45$$
$$J(x)=6x^2-46x-45$$

$$J(x)=95$$
$$6x^2-46x-45=95$$
$$6x^2-46x-140=0$$
$$3x^2-23x-70=0$$

$(x-10)(3x+7)=0$

$x=10$ or $x=-\frac{7}{3}$

$x=-\frac{7}{3}$ is not accepted because the question states that x has to be positive.
$\therefore x=10$

22 $W(x)=(x+5)(x-4)+(x+10)(x-5)$
$+(x-4)(x-6)$
$=x^2+x-20+x^2+5x-50+$
$x^2-10x+24$
$=3x^2-4x-46$

$W(x)=214$
$3x^2-4x-46=214$
$3x^2-4x-260=0$
$(x-10)(3x+26)=0$

$x=10$ or $x=-\frac{26}{3}$

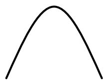
$x=-\frac{26}{3}$ is not accepted because the question states that x has to be positive.
$\therefore x=10$

Summative Practice 1

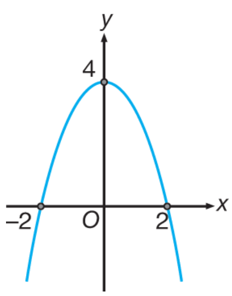
Multiple-Choice Questions

1 $2y^2+ky-12=0$
It is given that –4 is one of the root.
$2(-4)^2+k(-4)-12=0$
$-4k+20=0$
$-4k=-20$
$k=5$
Answer: D

2 Let the age of Sazali be x.
Sazali's sister (i.e. Tina) = x + 3
$x(x+3)=70$
$x^2+3x-70=0$
$(x-7)(x+10)=0$
$x=7$ or $x=-10$
$x=-10$ is not accepted.
$\therefore x=7$
Answer: A

3 $y=f(x)=-x^2+4$
Since the coefficient of x^2 is negative, the shape of the graph is  .
At the y-axis, x = 0.
$y=-0^2+4=4$
Thus the curve will intersect the y-axis at the point (0, 4).

At the x-axis, y = 0.
$-x^2+4=0$
$x^2=4$
$x=\pm 2$
Thus, the curve will intersect the x-axis at the points (–2, 0) and (2, 0).



Answer: A

4 Surface area of the sphere = 616 cm^2

$$4\pi j^2 = 616$$
$$4 \times \frac{22}{7} \times (x+3)^2 = 616$$
$$(x+3)^2 = \frac{616 \times 7}{4 \times 22}$$
$$x^2 + 6x + 9 = 49$$
$$x^2 + 6x - 40 = 0$$
$$(x-4)(x+10) = 0$$
$x = 4$ or $x = -10$

$x = -10$ is not accepted.

$\therefore x = 4$

Answer: A

5 Let *AB* = *x* cm

Height of the triangle = *x* + 5

Area of the triangle = 75 cm^2

$$\frac{1}{2}(x)(x+5) = 75$$
$$x(x+5) = 150$$
$$x^2 + 5x - 150 = 0$$
$$(x-10)(x+15) = 0$$
$x = 10$ or $x = -15$

$x = -15$ is not accepted.

$\therefore x = 10$

Answer: D

Structured Questions

1 (a)
$$m - 1 = \frac{6-m}{2m}$$
$$2m(m-1) = 6 - m$$
$$2m^2 - 2m = 6 - m$$
$$2m^2 - m - 6 = 0$$
$$(m-2)(2m+3) = 0$$
$m = 2$ or $m = -\frac{3}{2}$

(b)
$$\frac{4}{16c+9} = \frac{1}{c(c+4)}$$
$$4c(c+4) = 16c + 9$$
$$4c^2 + 16c - 16c - 9 = 0$$
$$4c^2 - 9 = 0$$
$$(2c+3)(2c-3) = 0$$
$c = -\frac{3}{2}$ or $\frac{3}{2}$

(c)
$$\frac{p(5p+4)}{3} = 2 - p$$
$$5p^2 + 4p = 6 - 3p$$
$$5p^2 + 7p - 6 = 0$$
$$(5p-3)(p+2) = 0$$
$p = \frac{3}{5}$ or $p = -2$

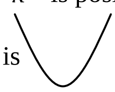
(d)
$$\frac{3f-5}{2} = -\frac{3f-1}{f}$$
$$f(3f-5) = -2(3f-1)$$
$$3f^2 - 5f = -6f + 2$$
$$3f^2 + f - 2 = 0$$
$$(3f-2)(f+1) = 0$$
$f = \frac{2}{3}$ or $f = -1$

(e)
$$\frac{3w(w+1)}{2} = 6 - w$$
$$3w(w+1) = 2(6-w)$$
$$3w^2 + 3w = 12 - 2w$$
$$3w^2 + 5w - 12 = 0$$
$$(3w-4)(w+3) = 0$$
$w = \frac{4}{3}$ or $w = -3$

(f)
$$\frac{z(z+4)-9}{z-3} = 2$$
$$z^2 + 4z - 9 = 2(z-3)$$
$$z^2 + 4z - 9 = 2z - 6$$
$$z^2 + 4z - 2z - 9 + 6 = 0$$
$$z^2 + 2z - 3 = 0$$
$$(z-1)(z+3) = 0$$
$z = 1$ or $z = -3$

2 (a) $y = x^2 + 6x + 8$

Since the coefficient of x^2 is positive,

The shape of the graph is  .

At the *y*-axis, *x* = 0.

$y = 0^2 + 6(0) + 8$

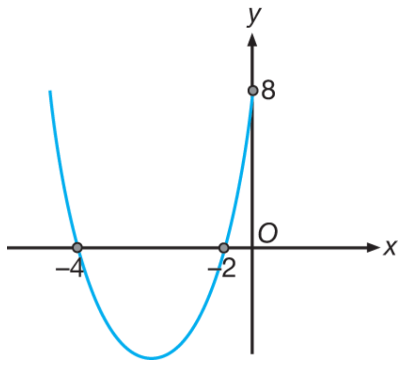
$y = 8$

Thus, the curve will intersect the *y*-axis at the point (0, 8).

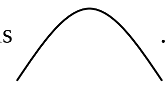
At the x-axis, $y = 0$.

$$y = 0$$
$$x^2 + 6x + 8 = 0$$
$$(x+4)(x+2) = 0$$
$$x = -4 \text{ or } x = -2$$

Thus, the curve will intersect the x-axis at the points (–4, 0) and (–2, 0).



(b) $y = -x^2 + 2x + 3$

Since the coefficient of x^2 is negative, the shape of the graph is  .

At the y-axis, $x = 0$.

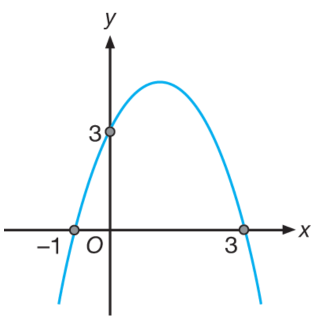
$$y = -0^2 + 2(0) + 3$$
$$y = 3$$

Thus, the curve will intersect the y-axis at the point (0, 3).

At the x-axis, $y = 0$

$$-x^2 + 2x + 3 = 0$$
$$x^2 - 2x - 3 = 0$$
$$(x-3)(x+1) = 0$$
$$x = 3 \text{ or } x = -1$$

Thus, the curve will intersect the y-axis at the points (–1, 0) and (3, 0).



3 (a) Volume of tank $= 4\frac{1}{2}$ m^3

$$(x)(2)(x) = \frac{9}{2}$$
$$2x^2 = \frac{9}{2}$$
$$x^2 = \frac{9}{4}$$
$$x = \frac{3}{2}$$

Hence, the width of the tank is 1.5 m.

(b) Volume of water

$$= 2x\left(x - \frac{6}{5}\right)$$
$$= 2(1.5)(1.5 - 1.2)$$
$$= 0.9 \text{ m}^3$$

4 (a)

$$(3x+10)\left(\frac{1}{5}x + \frac{7}{4}\right) = 150$$
$$(3x+10)\left(\frac{4x+35}{20}\right) = 150$$
$$(3x+10)(4x+35) = 150(20)$$
$$12x^2 + 105x + 40x + 350 = 3\ 000$$
$$12x^2 + 145x - 2\ 650 = 0$$
$$(x-10)(12x+265) = 0$$
$$x = 10 \text{ or } x = -\frac{265}{12}$$

$x = -\frac{265}{12}$ is not accepted.

$\therefore x = 10$

(b) Average speed $= 3(10) + 10 = 40$ km h^{-1}

5 (a)

$$(x+4)^2 = x(8x+2)$$
$$x^2 + 8x + 16 = 8x^2 + 2x$$
$$7x^2 - 6x - 16 = 0$$

(b) $7x^2 - 6x - 16 = 0$

$$(x-2)(7x+8) = 0$$
$$x = 2 \text{ or } x = -\frac{8}{7}$$

$x = -\frac{8}{7}$ is not accepted.

$\therefore x = 2$

(c) (i) Length of the side of the square
$= 2 + 4$
$= 6$ cm

(ii) For the rectangle,
length = 8(2) + 2 = 18 cm,
width = 2 cm

6 (a) Area of the shaded region
= Area of the rectangle *PQRS* – Area of ΔFSM – Area of ΔQPF

$= 20(2)(x+6) - \frac{1}{2}x(x+6)$

$- \frac{1}{2} \times 2(x+6)(20-x)$

$= 40x + 240 - \frac{1}{2}x^2 - 3x - (14x - x^2 + 120)$

$= 40x + 240 - \frac{1}{2}x^2 - 3x - 14x + x^2 - 120$

$= \frac{1}{2}x^2 + 23x + 120$

Area of the shaded region = 168 cm^2

$\frac{1}{2}x^2 + 23x + 120 = 168$

$\frac{1}{2}x^2 + 23x - 48 = 0$

$x^2 + 46x - 96 = 0$

(b) $x^2 + 46x - 96 = 0$
$(x-2)(x+48) = 0$
$x = 2$ or $x = -48$
$x = -48$ is not accepted.
$\therefore x = 2$

7 (a) Using the Pythagoras' Theorem,
$(2x-2)^2 + (2x)^2 = (2x+2)^2$
$4x^2 - 8x + 4 + 4x^2 = 4x^2 + 8x + 4$
$4x^2 - 16x = 0$
$4x(x-4) = 0$
$x = 0$ or $x = 4$
$x = 0$ is not accepted.
$\therefore x = 4$

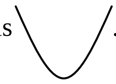
(b) $AB = 2(4) - 2 = 6$ cm
$BC = 2(4) = 8$ cm
$AC = 2(4) + 2 = 10$ cm
Perimeter of $\Delta ABC = 6 + 8 + 10 = 24$ cm
Area of $\Delta ABC = \frac{1}{2} \times 6 \times 8 = 24$ cm^2

8 It is given that Latifah's age is *x* years old.
$h(x) = x + (x-2) + x^2$
$h(x) = x^2 + 2x - 2$

$h(x) = 33$
$x^2 + 2x - 2 = 33$
$x^2 + 2x - 35 = 0$
$(x-5)(x+7) = 0$
$x = 5$ or $x = -7$
$x = -7$ is not accepted.
Hence, Latifah's age is 5 years old.

SPM Spot

1 Since $a > 0$, the shape of the curve

is  .

y-intercept = 9

At the *x*-axis, $y = 0$.
$x^2 - 9 = 0$
$x^2 = 9$
$x = \pm 3$
x-intercepts = -3 and 3.

Answer: B

2 $\frac{3k^2 - 7}{k-1} = 2$
$3k^2 - 7 = 2(k-1)$
$3k^2 - 7 = 2k - 2$
$3k^2 - 2k - 5 = 0$
$(3k-5)(k+1) = 0$
$k = \frac{5}{3}$ or $k = -1$

3 Volume of the cuboid = 840 cm^3
$(x+5)(7)(3x-11) = 840$
$(x+5)(3x-11) = 120$
$3x^2 + 4x - 55 = 120$
$3x^2 + 4x - 175 = 0$
$(x-7)(3x+25) = 0$
$x = 7$ or $x = -\frac{25}{3}$
$x = -\frac{25}{3}$ is not accepted.
$\therefore x = 7$