Form 5 Chapter 8
Kinematics of Linear Motion
Fully-Worked Solutions

UPSKILL 8.1a

1 (a) $s = t^2 - 4t - 12$

$s = 3^2 - 4(3) - 12$

$s = -15$ m

(b) $s = 6^2 - 4(6) - 12$

$s = 0$ m

(c) $s = 7^2 - 4(7) - 12$

$s = 9$ m

2 (a) $s = 3$

$4t - t^2 = 3$

$t^2 - 4t + 3 = 0$

$(t-3)(t-1) = 0$

$t = 3$ or 1

(b) $s = -21$

$4t - t^2 = -21$

$t^2 - 4t - 21 = 0$

$(t-7)(t+3) = 0$

$t = 7$

(c) $s = 0$

$4t - t^2 = 0$

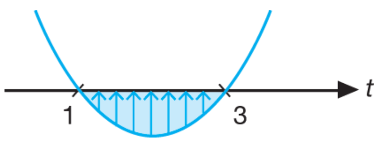
$t(4-t) = 0$

$t = 4$

3 (a) (i) $s < 0$

$t^2 - 4t + 3 < 0$

$(t-3)(t-1) < 0$

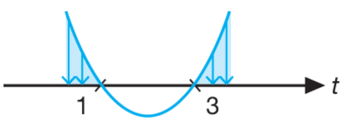


The range of values of t is $1 < t < 3$.

(ii) $s > 0$

$t^2 - 4t + 3 > 0$

$(t-3)(t-1) > 0$



The range of values of t is
$0 < t < 1$ or $t > 3$

(b) $t^2 - 4t + 3 = 0$

$(t-3)(t-1) = 0$

$t = 3$ or 1

4 (a) $s = t^2 - 7t + 10$

Initial displacement = 10 m

(b) $s = 28$

$t^2 - 7t + 10 = 28$

$t^2 - 7t - 18 = 0$

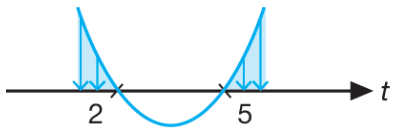
$(t-9)(t+2) = 0$

$t = 9$

(c) $s > 0$

$t^2 - 7t + 10 > 0$

$(t-5)(t-2) > 0$

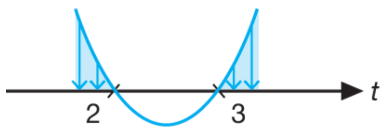


The range of values of t is $0 < t < 2$ or $t > 5$.

5 (a) $s > 0$

$t^2 - 5t + 6 > 0$

$(t-2)(t-3) > 0$



The range of values of t is $0 < t < 2$ or $t > 3$.

(b) Distance travelled during the 4th second

$= |s_4 - s_3|$

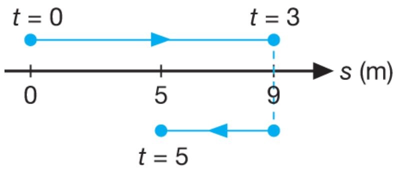
$= \left| 4^2 - 5(4) + 6 - (3^2 - 5(3) + 6) \right|$

$= |2 - 0|$

= 2 m

6 $s = 6t - t^2$

t (s)	0	1	2	3	4	5
s (m)	0	5	8	9	8	5



Total distance travelled
= 9 + 4
= 13 m

7 (a) $s = -20$

$16 - t^2 = -20$

$t^2 = 16 + 20$

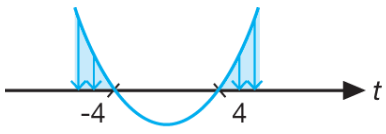
$t^2 = 36$

$t = 6$

(b) $s < 0$

$16 - t^2 < 0$

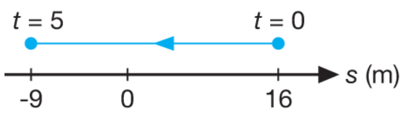
$t^2 > 16$



The range of values of t is $t > 4$.

(c) $s = 16 - t^2$

t (s)	0	1	2	3	4	5
s (m)	16	15	12	7	0	–9



Total distance travelled
= 9 + 16 = 25 m

8 (a) $s = 20 + 8t - t^2$

Distance travelled during the sixth second

$= |s_6 - s_5|$

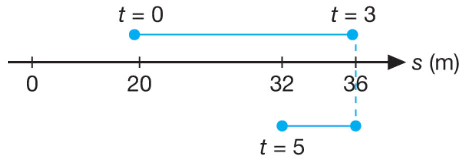
$= \left| 20 + 8(6) - 6^2 - \left[20 + 8(5) - 5^2\right] \right|$

$= |32 - 35|$

$= |-3| = 3$ m

(b) $s = 20 + 8t - t^2$

t	0	1	2	3	4	5	6
s	20	27	32	35	36	35	32



Total distance travelled

$= (36 - 20) + (36 - 32)$

$= 16 + 4$

= 20 m

9 (a) $s = 5 + 4t - t^2$

$s = 5 + 4(5) - 5^2 = 0$

(b) $|s_5 - s_4|$

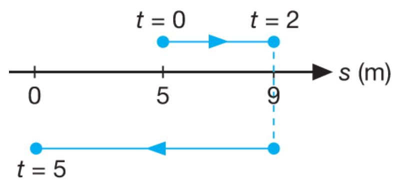
$= \left| \left[5 + 4(5) - 5^2\right] - \left[5 + 4(4) - 4^2\right] \right|$

$= |0 - 5|$

= 5 m

(c)

t (s)	0	1	2	3	4	5
s (m)	5	8	9	8	5	0



Total distance travelled
= 4+ 9
= 13 m

UPSKILL 8.1b

1 (a) $v = t^2 - 7t + 12$

When $v = 0$,

$t^2 - 7t + 12 = 0$

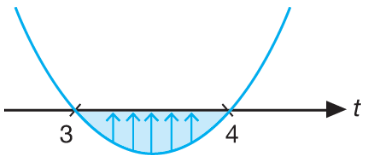
$(t - 4)(t - 3) = 0$

$t = 4$ or 3

(b) (i) $v < 0$

$t^2 - 7t + 12 < 0$

$(t - 4)(t - 3) < 0$

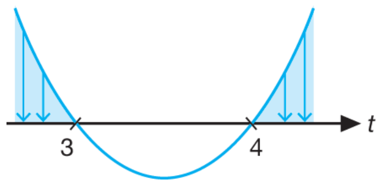


The range of values of t is $3 < t < 4$.

(ii) $v > 0$

$t^2 - 7t + 12 > 0$

$(t-4)(t-3) > 0$



The range of values of t is $0 < t < 3$ or $t > 4$.

UPSKILL 8.1c

1 (a) $a = 6 - 3t$

When the particle travelled at a uniform velocity,

$a = 0$

$6 - 3t = 0$

$t = 2$

(b) (i) When the particle accelerates,

$a > 0$

$6 - 3t > 0$

$-3t > -6$

$t < \frac{-6}{-3}$

$0 < t < 2$

(ii) When the particle decelerates,

$a < 0$

$6 - 3t < 0$

$-3t < -6$

$t > \frac{-6}{-3}$

$t > 2$

UPSKILL 8.2

1 (a) $s = t^2 + 5t - 36$

$v = \frac{ds}{dt}$

$v = 2t + 5$

When $t = 0$,

$v = 2(0) + 5 = 5$ m s^{-1}

(b) When $s = -22$,

$t^2 + 5t - 36 = -22$

$t^2 + 5t - 14 = 0$

$(t-2)(t+7) = 0$

$t = 2$

$v = 2(2) + 5 = 9$ m s^{-1}

(c) When $v = 13$,

$2t + 5 = 13$

$t = 4$

$s = 4^2 + 5(4) - 36$

$s = 0$

2 (a) $s = \frac{1}{3}t^3 - \frac{7}{2}t^2 + 10t$

$v = \frac{ds}{dt}$

$v = t^2 - 7t + 10$

When $v = 0$,

$t^2 - 7t + 10 = 0$

$(t-5)(t-2) = 0$

$t = 2$ or 5

(b) When $t = 2$,

$s = \frac{1}{3}(2)^3 - \frac{7}{2}(2)^2 + 10(2)$

$s = 8\frac{2}{3}$ m

When $t = 5$

$s = \frac{1}{3}(5)^3 - \frac{7}{2}(5)^2 + 10(5)$

$s = 4\frac{1}{6}$ m

Hence, the distance between the two points

$= 8\frac{2}{3} - 4\frac{1}{6}$

$= 4\frac{1}{2}$ m

3 (a) $v = t^2 + 3t - 10$

When $v = 8$,

$t^2 + 3t - 10 = 8$

$t^2 + 3t - 18 = 0$

$(t-3)(t+6) = 0$

$t = 3$

$a = \frac{dv}{dt}$

$a = 2t + 3$

$a = 2(3) + 3 = 9$ m s^{-2}

(b) When $v = 0$,
$$t^2 + 3t - 10 = 0$$
$$(t-2)(t+5) = 0$$
$$t = 2$$

$$a = 2(2) + 3 = 7 \text{ m s}^{-2}$$

4 $s = t^3 - 9t$
$$v = \frac{ds}{dt} = 3t^2 - 9$$
$$a = \frac{dv}{dt} = 6t$$

When $s = 0$,
$$t^3 - 9t = 0$$
$$t\left(t^2 - 9\right) = 0$$
$$t\left(t+3\right)\left(t-3\right) = 0$$
$$t = 3$$

$$v = 3(3)^2 - 9 = 18 \text{ m s}^{-1}$$
$$a = 6(3) = 18 \text{ m s}^{-2}$$

5 (a) $v = t^2 - 3t - 10$
$$a = \frac{dv}{dt} = 2t - 3$$

When $v = 0$,
$$t^2 - 3t - 10 = 0$$
$$(t-5)(t+2) = 0$$
$$t = 5$$

$$a = 2(5) - 3 = 7 \text{ m s}^{-2}$$

(b) When $a = 0$,
$$2t - 3 = 0$$
$$t = \frac{3}{2}$$

$$v = \left(\frac{3}{2}\right)^2 - 3\left(\frac{3}{2}\right) - 10 = -12.25 \text{ m s}^{-1}$$

6 (a) $s = t^3 - 9t^2 + 3$
$$v = \frac{ds}{dt} = 3t^2 - 18t$$
$$a = \frac{dv}{dt} = 6t - 18$$

When $t = 0$,
$$a = 6(0) - 18 = -18 \text{ m s}^{-2}$$

(b) When $a < 0$,
$$6t - 18 < 0$$
$$6t < 18$$
$$0 < t < 3$$

7 (a) $s = t^3 - 6t^2$
$$v = \frac{ds}{dt} = 3t^2 - 12t$$
$$a = \frac{dv}{dt} = 6t - 12$$

When $v = 0$,
$$3t^2 - 12t = 0$$
$$3t(t-4) = 0$$
$$t = 4$$

$$s = 4^3 - 6(4)^2 = -32 \text{ m}$$

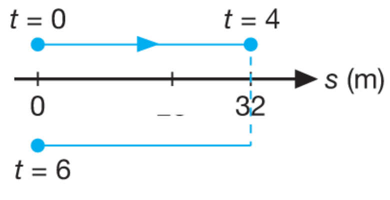
(b) $a > 0$
$$6t - 12 > 0$$
$$6t > 12$$
$$t > 2$$

8 (a) $s = 6t^2 - t^3$
$$v = \frac{ds}{dt} = 12t - 3t^2$$

When $v = 0$,
$$12t - 3t^2 = 0$$
$$3t(4-t) = 0$$
$$t = 4$$

(b)

t (s)	0	4	6
s (m)	0	32	0

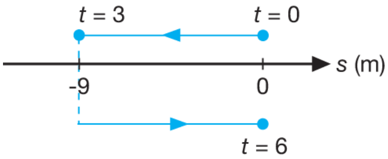


Total distance travelled
$= 32 + 32$
$= 64$ m

9 (a) $s = t^2 - 6t$
$$v = \frac{ds}{dt} = 2t - 6$$
When $v = 0$,
$$2t - 6 = 0$$
$$t = 3 \text{ s}$$

(b)

t (s)	0	3	6
s (m)	0	–9	0



Total distance travelled
= 9 + 9 = 18 m

10 (a) $s=4t^2-\frac{1}{3}t^3-15t$

$v=\frac{ds}{dt}=8t-t^2-15$

When $v=0$,

$8t-t^2-15=0$

$t^2-8t+15=0$

$(t-5)x-3)=0$

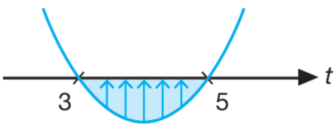
$t=5$ or 3

(b) $v>0$

$8t-t^2-15>0$

$t^2-8t+15<0$

$(t-3)\,(t-5)<0$



The range of values of t is $3<t<5$.

11 (a) $s=4t^2-\frac{4}{3}t^3$

$v=\frac{ds}{dt}=8t-4t^2$

When $v=0$,

$8t-4t^2=0$

$4t(2-t)=0$

$t=2$

$s=4(2)^2-\frac{4}{3}(2)^3$

$s=5\frac{1}{3}$ m

(b) $s=0$

$4t^2-\frac{4}{3}t^3=0$

$12t^2-4t^3=0$

$4t^2(3-t)=0$

$t=3$

$v=8(3)-4(3)^2=-12$ m s^{-1}

12 (a) $s=15t-7t^2-\frac{1}{3}t^3$

$v=\frac{ds}{dt}=15-14t-t^2$

When $v=0$,

$15-14t-t^2=0$

$t^2+14t-15=0$

$(t+15)(t-1)=0$

$t=1$

(b) When the displacement of the particle is a maximum,

$\frac{ds}{dt}=0$

$15-14t-t^2=0$

$t^2+14t-15=0$

$(t+15)(t-1)=0$

$t=1$

$\frac{d^2s}{dt^2}=-14-2t$

When $t=1$,

$\frac{d^2s}{dt^2}=-14-2(1)=-16$ (negative)

Hence, s is a maximum.

Maximum displacement

$=15(1)-7(1)^2-\frac{1}{3}(1)^3$

$=7\frac{2}{3}$ m

UPSKILL 8.3

1 $v=3t^2-2t+1$

$s=\int v\,dt$

$s=\int\left(3t^2-2t+1\right)dt$

$s=t^3-t^2+t+c$

When $t=0, s=0$. Thus, c = 0.

$s=t^3-t^2+t$

Distance travelled during the fifth second

$=|s_5-s_4|$

$=\left|5^3-5^2+5-\left(4^3-4^2+4\right)\right|$

$=|105-52|$

$=53$ m

2 $v = 3 + 2t$

$s = \int (3 + 2t)\, dt$

$s = 3t + t^2 + c$

When $t = 0$, $s = 2$. Thus, $c = 2$.

$s = 3t + t^2 + 2$

Distance travelled during the fourth second

$= | s_4 - s_3 |$

$= |12 + 16 + 2 - (9 + 9 + 2)|$

$= |30 - 20|$

$= 10$ m

3 $v = 12t - 3t^2$

$s = \int v\, dt$

$s = \int (12t - 3t^2)\, dt$

$s = 6t^2 - t^3 + c$

When $t = 0$, $s = 1$. Thus, $c = 0$.

$s = 6t^2 - t^3$

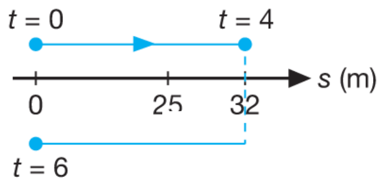
When $v = 0$,

$12t - 3t^2 = 0$

$3t(4 - t) = 0$

$t = 4$

t (s)	0	4	6
s (m)	0	32	0



Total distance travelled

$= 32 + 32$

$= 64$ m

4 (a) $v = 2t - 6$

$s = \int (2t - 6)\, dt$

$s = t^2 - 6t + c$

When $t = 0$, $s = 0$. Thus, $c = 0$.

$s = t^2 - 6t$

When $v = 0$,

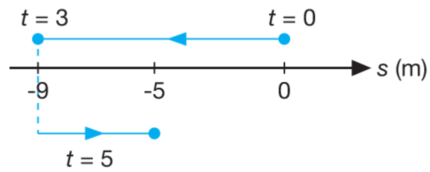
$2t - 6 = 0$

$t = 3$

$s = 3^2 - 6(3) = -9$ m

(b)

t (s)	0	3	5
s (m)	0	–9	–5



Total distance travelled

= 9 + 4

= 13 m

5 $v = 4 + 3t - t^2$

$s = \int (4 + 3t - t^2)\, dt$

$s = 4t + \frac{3t^2}{2} - \frac{t^3}{3} + c$

When $t = 0$, $s = 0$. Thus, $c = 0$.

$s = 4t + \frac{3t^2}{2} - \frac{t^3}{3}$

When the displacement is a maximum,

$v = \frac{ds}{dt} = 0$

$4 + 3t - t^2 = 0$

$t^2 - 3t - 4 = 0$

$(t - 4)(t + 1) = 0$

$t = 4$

$\frac{d^2s}{dt^2} = 3 - 2t$

When $t = 4$, $\frac{d^2s}{dt^2} = 3 - 2(4) = -5$ (negative)

Hence, the maximum displacement

$= 4(4) + \frac{3(4)^2}{2} - \frac{4^3}{3}$

$= 18\frac{2}{3}$ m

6 $a = 6 - 3t$

$v = \int (6 - 3t)\, dt$

$v = 6t - \frac{3t^2}{2} + c$

When $t = 0$, $v = -9$.

Hence, $v = 6t - \frac{3t^2}{2} - 9$

When the velocity of the particle is a maximum,

$\frac{dv}{dt} = a = 0$

$6 - 3t = 0$

$t = 2$

$\frac{d^2v}{dt^2} = -3$ (Negative)

Hence, the maximum velocity

$= 6(2) - \frac{3(2)^2}{2} - 9$

$= -3 \text{ m s}^{-1}$

7 (a) $v = 12t - 3t^2 - 7$

$a = \frac{dv}{dt} = 12 - 6t$

When $v = 2$,

$12t - 3t^2 - 7 = 2$

$3t^2 - 12t + 9 = 0$

$t^2 - 4t + 3 = 0$

$(t - 3)(t - 1) = 0$

$t = 3$ or 1

When $t = 3$, $a = 12 - 6(3) = -6 \text{ m s}^{-1}$

When $t = 1$, $a = 12 - 6(1) = 6 \text{ m s}^{-1}$

(b) When the velocity of the particle is a maximum,

$\frac{dv}{dt} = a = 0$

$12 - 6t = 0$

$t = 2$

$\frac{d^2v}{dt^2} = -6$ (negative)

Hence, the maximum velocity

$= 12(2) - 3(2)^2 - 7$

$= 5 \text{ m s}^{-1}$

8 (a) $a = 6t + 5$

$v = \int a \, dt$

$v = \int (6t + 5) \, dt$

$v = 3t^2 + 5t + c$

When $t = 0, v = -8$. Thus, $c = -8$.

$v = 3t^2 + 5t - 8$

When $t = 3$,

$v = 3(3)^2 + 5(3) - 8 = 34 \text{ m s}^{-1}$

(b) $s = \int v \, dt$

$s = \int \left(3t^2 + 5t - 8\right) dt$

$s = t^3 + \frac{5t^2}{2} - 8t + c$

When $t = 0$, $s = 0$.

Hence, $s = t^3 + \frac{5t^2}{2} - 8t$

When $v = 0$,

$3t^2 + 5t - 8 = 0$

$(t - 1)(3t + 8) = 0$

$t = 1$

When $t = 1$,

$s = 1^3 + \frac{5(1)^2}{2} - 8(1) = -4\frac{1}{2}$ m

9 (a) $a = 3 - 2t$

$v = \int (3 - 2t) \, dt$

$v = 3t - t^2 + c$

When $t = 0, v = -2$. Thus, $c = -2$

$v = 3t - t^2 - 2$

When $v = 0$,

$3t - t^2 - 2 = 0$

$t^2 - 3t + 2 = 0$

$(t - 2)(t - 1) = 0$

$t = 2$ or 1

(b) $s = \int v \, dt$

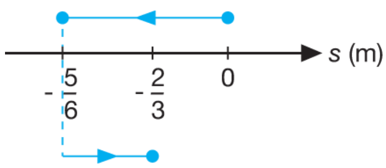
$s = \int \left(3t - t^2 - 2\right) dt$

$s = \frac{3t^2}{2} - \frac{t^3}{3} - 2t + c$

When $t = 0, s = 0$. Thus, $c = 0$

$s = \frac{3t^2}{2} - \frac{t^3}{3} - 2t$

t (s)	0	1	2
s (m)	0	$-\frac{5}{6}$	$-\frac{2}{3}$



Total distance travelled

$= \frac{5}{6} + \left(\frac{5}{6} - \frac{2}{3}\right)$

$= 1$ m

10 $a = 4t - 8$

$v = \int a\, dt$

$v = \int (4t - 8)\, dt$

$v = 2t^2 - 8t + c$

When $t = 0$, $v = 6$. Thus, $c = 6$

Hence, $v = 2t^2 - 8t + 6$

$s = \int v\, dt$

$s = \int \left(2t^2 - 8t + 6\right) dt$

$s = \frac{2t^3}{3} - 4t^2 + 6t + c$

When $t = 0, s = 0$. Thus, $c = 0$

$s = \frac{2t^3}{3} - 4t^2 + 6t$

When $v = 0$,

$2t^2 - 8t + 6 = 0$

$t^2 - 4t + 3 = 0$

$(t - 1)(t - 3) = 0$

$t = 1$ or 3

When $t = 1$,

$s = \frac{2(1)^3}{3} - 4(1)^2 + 6(1) = 2\frac{2}{3}$ m

When $t = 3$,

$s = \frac{2(3)^3}{3} - 4(3)^2 + 6(3) = 0$ m

Distance of PQ = $2\frac{2}{3}$ m

UPSKILL 8.4

1 `(a) $v = ht - kt^2$

$s = \int \left(ht - kt^2\right) dt$

$s = \frac{ht^2}{2} - \frac{kt^3}{3} + c$

When $t = 0, s = 0$. Thus, $c = 0$.

$s = \frac{ht^2}{2} - \frac{kt^3}{3}$

$|s_1 - s_0| = \frac{13}{3}$

$\frac{h}{2} - \frac{k}{3} = \frac{13}{3}$

$3h - 2k = 26 \ \ldots (1)$

$a = \frac{dv}{dt} = h - 2kt$

When $t = 3$, $a = -2$

$h - 2k(3) = -2$

$h = 6k - 2 \ \ldots (2)$

Substitute (2) into (1) :

$3(6k - 2) - 2k = 26$

$18k - 6 - 2k = 26$

$16k = 32$

$k = 2$

From (2) :

$h = 6(2) - 2 = 10$

(b) When h = 10 and k = 2,

$v = ht - kt^2$

$v = 10t - 2t^2$

$s = \frac{10t^2}{2} - \frac{2t^3}{3}$

$s = 5t^2 - \frac{2t^3}{3}$

When v = 12,

$10t - 2t^2 = 12$

$2t^2 - 10t + 12 = 0$

$t^2 - 5t + 6 = 0$

$(t - 2)(t - 3) = 0$

$t = 2$ or $t = 3$

Hence, the time taken by the particle to travel from point A to point B is 1 second.

(c) For point B,

$s = 5(3)^2 - \frac{2(3)^3}{3}$

$s = 27$ m

When $v = 0$,

$10t - 2t^2 = 0$

$2t(5 - t) = 0$

$t = 5$

For point C,

$$s = 5(5)^2 - \frac{2(5)^3}{3}$$

$$s = 41\frac{2}{3} \text{ m}$$

Distance of $BC = 41\frac{2}{3} - 27 = 14\frac{2}{3}$ m

2 (a) $v_M = 2t - 6$

$$s_M = \int (2t - 6)\, dt$$

$$s_M = t^2 - 6t$$

$$v_N = 5 - t$$

$$s_N = \int (5 - t)\, dt$$

$$s_N = 5t - \frac{t^2}{2} + c$$

When $t = 0$, $s_N = 8$, Thus, $c = 8$

$$s_N = 5t - \frac{t^2}{2} + 8$$

When the particles M and N meet,

$$t^2 - 6t = 5t - \frac{t^2}{2} + 8$$

$$2t^2 - 12t = 10t - t^2 + 16$$

$$3t^2 - 22t - 16 = 0$$

$$(t - 8)(3t + 2) = 0$$

$$t = 8$$

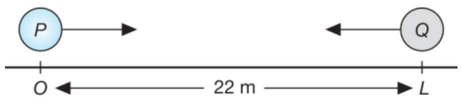
(b) When $t = 8$,

$$s_N = 5(8) - \frac{8^2}{2} + 8$$

$$s_N = 16 \text{ m}$$

Summative Practice 8

1



(a) $s_Q = 6t^3 - 2t$

$$v_Q = \frac{ds_Q}{dt} = 18t^2 - 2$$

When $t = 0$, $v_Q = 18(0)^2 - 2 = -2$

Hence, the initial velocity of particle Q is -2 m s^{-1}.

(b) When particle Q reverses its direction,

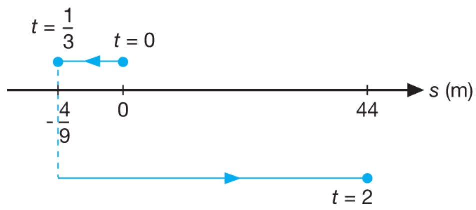
$$v_Q = 0$$

$$18t^2 - 2 = 0$$

$$t^2 = \frac{1}{9}$$

$$t = \frac{1}{3}$$

t (s)	0	$\frac{1}{3}$	2
s_Q (m)	0	$-\frac{4}{9}$	44



Hence, the total distance travelled by particle Q in the first 2 seconds

$$= \frac{4}{9} + \frac{4}{9} + 44$$

$$= 44\frac{8}{9} \text{ m}$$

(c) $v_P = 18t^2 + 20$

$$s_P = \int v_P\, dt$$

$$s_P = \int (18t^2 + 20)\, dt$$

$$s_P = 6t^3 + 20t + c$$

When $t = 0$,

s_P (from point O) = 0.

Thus, $s_P = 6t^3 + 20t$

$$s_Q = 6t^3 - 2t + 22$$

When $t = 0$, the distance of particle Q from point O is 22 m.

When particles P and Q meet,

$$s_P = s_Q$$

$$6t^3 + 20t = 6t^3 - 2t + 22$$

$$22t = 22$$

$$t = 1$$

Hence, when particles P and Q meet, the distance of both particles from point O

$= 6(1)^3 + 20(1)$

$= 26$ m

2 (a) $v = pt^2 - 6t$

$a = \frac{dv}{dt}$

$a = 2pt - 6$

When $t = 5$, $a = 34$

$34 = 2p(5) - 6$

$40 = 10p$

$p = 4$

(b) $v = 4t^2 - 6t$

When the velocity of the particle decreases,

$\frac{dv}{dt} < 0$

$8t - 6 < 0$

$8t < 6$

$t < \frac{3}{4}$

Hence, the required range of values of t is $0 < t < \frac{3}{4}$.

(c) $v = 0$

$4t^2 - 6t = 0$

$2t(2t - 3) = 0$

$t = 0$ or $\frac{3}{2}$

Hence, the time when the particle stops momentarily is $1\frac{1}{2}$ s.

(d) $s = \int v\,dt$

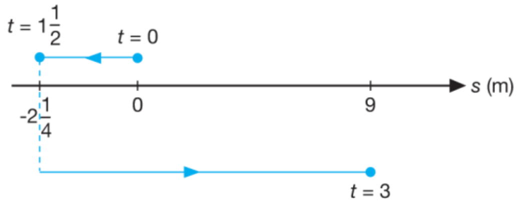
$s = \int (4t^2 - 6t)\,dt$

$s = \frac{4t^3}{3} - \frac{6t^2}{2} + c$

When $t = 0, s = 0$. Thus, $c = 0$.

$\therefore s = \frac{4t^3}{3} - 3t^2$

t (s)	0	$1\frac{1}{2}$	3
s (m)	0	$-2\frac{1}{4}$	9



The total distance travelled in the first 3 seconds

$= 2\frac{1}{4} + 2\frac{1}{4} + 9$

$= 13\frac{1}{2}$ m

3 (a) $v = -t^2 + 2t + 8$

When $t = 0$, $v = 8$

Hence, initial velocity of the particle is 8 m s^{-1}.

(b) When the particle stops momentarily,

$v = 0$

$-t^2 + 2t + 8 = 0$

$t^2 - 2t - 8 = 0$

$(t + 2)(t - 4) = 0$

$t = -2$ or 4

$t = -2$ is not accepted

$\therefore t = 4$

(c) When the velocity of the particle is a maximum,

$\frac{dv}{dt} = 0$

$-2t + 2 = 0$

$t = 1$

When $t = 1$,

$v_{max} = -(1)^2 + 2(1) + 8 = 9\text{ m s}^{-1}$

(d) $s = \int v\,dt$

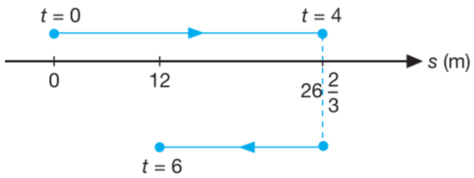
$s = \int (-t^2 + 2t + 8)\,dt$

$s = -\frac{t^3}{3} + t^2 + 8t + c$

When $t = 0, s = 0$. Thus, $c = 0$.

$\therefore s = -\frac{t^3}{3} + t^2 + 8t$

t (s)	0	4	6
s (m)	0	$26\frac{2}{3}$	12



Hence, the total distance travelled in the first 6 seconds

$$= 26\frac{2}{3} + \left(26\frac{2}{3} - 12\right) = 41\frac{1}{3} \text{ m}$$

4 (a) When $t = 0$, $a = 4 - 2(0) = 4$

Hence, the initial acceleration of the particle is 4 m s^{-2}.

(b) $v = \int a\, dt$

$v = \int (4 - 2t)\, dt$

$v = 4t - \frac{2t^2}{2} + c$

When $t = 0, v = 12$. Hence, $c = 12$

$v = 4t - t^2 + 12$

When the velocity of the particle is a maximum,

$\frac{dv}{dt} = 0$

$4 - 2t = 0$

$t = 2$

$\frac{d^2v}{dt^2} = -2$ (Negative)

$\therefore v_{max} = 4(2) - 2^2 + 12$

$= 16 \text{ m s}^{-1}$

(c) When $v = 0$,

$4t - t^2 + 12 = 0$

$t^2 - 4t - 12 = 0$

$(t + 2)(t - 6) = 0$

$t = -2$ or 6

$t = -2$ is not accepted

$\therefore t = 6$ s

(d) $s = \int v\, dt$

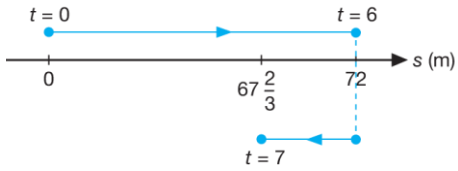
$s = \int \left(4t - t^2 + 12\right) dt$

$s = \frac{4t^2}{2} - \frac{t^3}{3} + 12t + c$

When $t = 0, s = 0$. Thus, $c = 0$

$s = 2t^2 - \frac{t^3}{3} + 12t$

t (s)	0	6	7
s (m)	0	72	$67\frac{2}{3}$



Hence, the total distance travelled in the first 7 seconds

$= 72 + \left(72 - 67\frac{2}{3}\right)$

$= 76\frac{1}{3}$ m

5 (a) $a = \frac{dv}{dt} = -4 \text{ m s}^{-2}$

(b) When $v = 0$,

$6 - 4t = 0$

$t = 1.5$ s

(c) $s = \int v\, dt$

$s = \int (6 - 4t)\, dt$

$s = 6t - 2t^2 + c$

When $t = 0, s = 0$. Then, $c = 0$

$s = 6t - 2t^2$

When $s = -8$ (at point P),

$6t - 2t^2 = -8$

$2t^2 - 6t - 8 = 0$

$t^2 - 3t - 4 = 0$

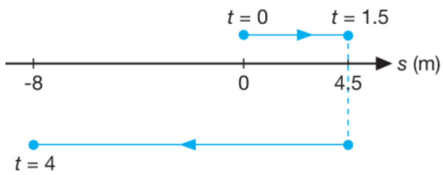
$(t + 1)(t - 4) = 0$

$\therefore t = 4$

When $t = 4$, $v = 6 - 4(4) = -10 \text{ m s}^{-1}$

(d)

t (s)	0	1.5	4
s (m)	0	4.5	–8



Hence, the total distance travelled in the first 5 seconds
$= 4.5 + 4.5 + 8 = 17$ m

6 (a) When $a = 0$, $7 - 4t = 0$

$$t = \frac{7}{4} \text{ s}$$

(b) At maximum velocity,

$$\frac{dv}{dt} = 0$$
$$a = 0$$
$$t = \frac{7}{4}$$
$$v = \int a\, dt$$
$$v = \int (7 - 4t)\, dt$$
$$v = 7t - 2t^2 + c$$

When $t = 0, v = 4$, $\therefore c = 4$

$\therefore v = 7t - 2t^2 + 4$

$$\frac{d^2v}{dt^2} = -4 \ (< 0)$$

Hence, $v_{max} = 7\left(\frac{7}{4}\right) - 2\left(\frac{7}{4}\right)^2 + 4$

$$= 10\frac{1}{8} \text{ m s}^{-1}$$

(c) When $v = 0$,

$$7t - 2t^2 + 4 = 0$$
$$2t^2 - 7t - 4 = 0$$
$$(2t + 1)(t - 4) = 0$$
$$t = -\frac{1}{2} \text{ or } 4$$

$t = -\frac{1}{2}$ is not accepted

$\therefore t = 4$ s

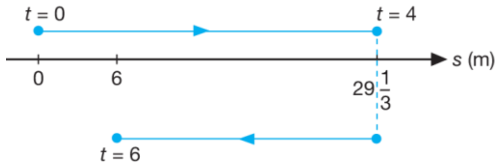
(d) $s = \int v\, dt$

$$s = \int (7t - 2t^2 + 4)\, dt$$
$$s = \frac{7t^2}{2} - \frac{2t^3}{3} + 4t + c$$

When $t = 0, s = 0$. Thus, $c = 0$

$$s = \frac{7t^2}{2} - \frac{2t^3}{3} + 4t$$

t (s)	0	4	6
s (m)	0	$29\frac{1}{3}$	6



Hence, the total distance travelled in the first 6 seconds

$$= 29\frac{1}{3} + \left(29\frac{1}{3} - 6\right)$$
$$= 52\frac{2}{3} \text{ m}$$

7 (a) $v = \int a\, dt$

$$v = \int (8 - 2t)\, dt$$
$$v = 8t - \frac{2t^2}{2} + c$$
$$v = 8t - t^2 + c$$

When $t = 0$, $v = -12$. Hence, $c = -12$.

$\therefore v = 8t - t^2 - 12$

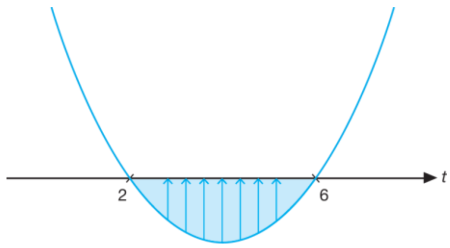
When the velocity of the particle is a maximum,

$$\frac{dv}{dt} = 0$$
$$8 - 2t = 0$$
$$t = 4$$

$$v_{max} = 8(4) - 4^2 - 12 = 4 \text{ ms}^{-1}$$

(b) When the particle moves to the right,

$$v > 0$$
$$8t - t^2 - 12 > 0$$
$$t^2 - 8t + 12 < 0$$
$$(t - 2)(t - 6) < 0$$



Hence, the range of values of t is $2 < t < 6$.

(c) $s = \int v\, dt$

$$s = \int \left(8t - t^2 - 12\right) dt$$

$$s = \frac{8t^2}{2} - \frac{t^3}{3} - 12t + c$$

$$s = 4t^2 - \frac{t^3}{3} - 12t + c$$

When $t = 0, s = 0$. Thus, $c = 0$.

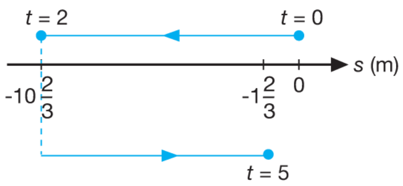
$$\therefore s = 4t^2 - \frac{t^3}{3} - 12t$$

When the particle reverses its direction,

$$v = 0$$
$$8t - t^2 - 12 = 0$$
$$t^2 - 8t + 12 = 0$$
$$(t-2)(t-6) = 0$$
$$t = 2 \text{ or } 6$$

Since $t \leq 5$, we use $t = 2$.

t (s)	0	2	5
s (m)	0	$-10\frac{2}{3}$	$-1\frac{2}{3}$



Hence, the total distance travelled in the first 5 seconds

$$= 10\frac{2}{3} + \left(10\frac{2}{3} - 1\frac{2}{3}\right)$$

$$= 19\frac{2}{3} \text{ m}$$

8 (a) When $t = 0$, $v = 15 + 7(0) - 4(0)^2$
$= 15 \text{ cm s}^{-1}$

(b) $a = \frac{dv}{dt} = 7 - 8t$

When $t = 0, a = 7 - 8(0)$
$a = 7 \text{ cm s}^{-2}$

(c) $\frac{dv}{dt} = 7 - 8t$

When the velocity of the particle is a maximum,

$$\frac{dv}{dt} = 0$$
$$7 - 8t = 0$$
$$t = \frac{7}{8}$$

$$\frac{d^2v}{dt^2} = -8 \text{ (Negative)}$$

Hence, the maximum velocity

$$= 15 + 7\left(\frac{7}{8}\right) - 4\left(\frac{7}{8}\right)^2$$

$$= 18\frac{1}{16} \text{ cm s}^{-1}$$

(d) $s = \int v\, dt$

$$s = \int (15 + 7t - 4t^2)\, dt$$

$$s = 15t + \frac{7t^2}{2} - \frac{4t^3}{3} + c$$

When $t = 0$, $s = 0$. Thus, $c = 0$

$$\therefore s = 15t + \frac{7t^2}{2} - \frac{4t^3}{3}$$

When $v = 0$,

$$15 + 7t - 4t^2 = 0$$
$$4t^2 - 7t - 15 = 0$$
$$(4t+5)(t-3) = 0$$
$$t = -\frac{5}{4} \text{ or } 3$$

$t = -\frac{5}{4}$ is not accepted

$\therefore t = 3$

When $t = 3$,

$$s = 15(3) + \frac{7(3)^2}{2} - \frac{4(3)^3}{3} = 40\frac{1}{2} \text{ cm}$$

9 (a) At point A,

$$v = 0$$
$$12 + 4t - t^2 = 0$$
$$t^2 - 4t - 12 = 0$$
$$(t+2)(t-6) = 0$$
$$t = -2 \text{ or } 6$$

$t = -2$ is not accepted

$\therefore t = 6$

$$a = \frac{dv}{dt} = 4 - 2t$$

When $t = 6, a = 4 - 2(6) = -8 \text{ m s}^{-2}$

(b) When velocity of the particle is a maximum,

$$\frac{dv}{dt} = 0$$
$$4 - 2t = 0$$
$$t = 2$$

$$\frac{d^2v}{dt^2} = -2 \ (< 0)$$

$$\therefore v_{max} = 12 + 4(2) - 2^2 = 16 \text{ m s}^{-1}$$

(c) $s = \int v \, dt$

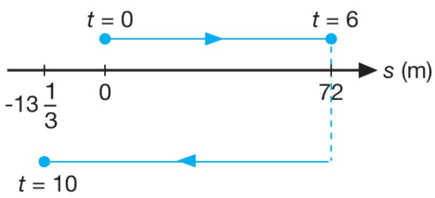
$$s = \int \left(12 + 4t - t^2\right) dt$$

$$s = 12t + 2t^2 - \frac{t^3}{3} + c$$

When $t = 0, s = 0$. Thus, $c = 0$.

$$\therefore s = 12t + 2t^2 - \frac{t^3}{3}$$

t (s)	0	6	10
s (m)	0	72	$-13\frac{1}{3}$



Hence, the total distance travelled in the first 10 seconds

$$= 72 + 72 + 13\frac{1}{3}$$

$$= 157\frac{1}{3} \text{ m}$$

10 (a) $v = -10 + 7t - t^2$

When $t = 0, v = -10$

Hence, the initial velocity is -10 m s^{-1}.

(b) When the velocity of the particle is a maximum,

$$\frac{dv}{dt} = 0$$

$$7 - 2t = 0$$

$$t = 3.5$$

When $t = 3.5$,

$$v = -10 + 7(3.5) - (3.5)^2$$

$$v = 2.25$$

$$\frac{d^2v}{dt^2} = -2 \ (< 0)$$

Hence, the maximum velocity is 2.25 m s^{-1}.

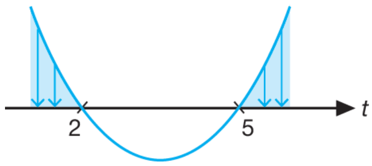
(c) When the particle moves to the right,

$$v > 0$$

$$-10 + 7t - t^2 > 0$$

$$t^2 - 7t + 10 < 0$$

$$(t - 2)(t - 5) < 0$$



The range of values of t is $2 < t < 5$.

(d) When $v = 0$,

$$-10 + 7t - t^2 = 0$$

$$t^2 - 7t + 10 = 0$$

$$(t - 2)(t - 5) = 0$$

$$t = 2 \text{ or } 5$$

Hence, the particle is at instantaneous rest for the second time when $t = 5$.

$$s = \int v \, dt$$

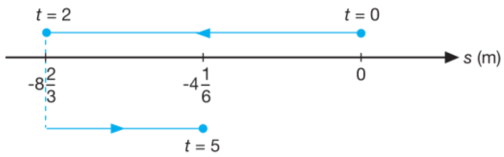
$$s = \int \left(-10 + 7t - t^2\right) dt$$

$$s = -10t + \frac{7t^2}{2} - \frac{t^3}{3} + c$$

When $t = 0, s = 0$. Thus, $c = 0$.

$$\therefore s = -10t + \frac{7t^2}{2} - \frac{t^3}{3}$$

t (s)	0	2	5
s (m)	0	$-8\frac{2}{3}$	$-4\frac{1}{6}$



Hence, the total distance travelled in the first 5

$$= 8\frac{2}{3} + \left(8\frac{2}{3} - 4\frac{1}{6}\right)$$

$$= 8\frac{2}{3} + 4\frac{1}{2}$$

$$= 13\frac{1}{6} \text{ m}$$

SPM Spot

1 (a) $a = 6 - 4t$

$v = \int (6 - 4t)\ dt$

$v = 6t - 2t^2 + c$

When $t = 0$, $v = 36$. Thus, $c = 36$.

$v = 6t - 2t^2 + 36$

When $t = 2$, $v = 6(2) - 2(2)^2 + 36$

$v = 40 \text{ m s}^{-1}$

(b) At maximum velocity,

$\frac{dv}{dt} = 0$

$a = 0$

$6 - 4t = 0$

$t = \frac{3}{2}$

Maximum velocity

$= 6\left(\frac{3}{2}\right) - 2\left(\frac{3}{2}\right)^2 + 36$

$= 40.5 \text{ m s}^{-1}$

(c) $s = \int v\ dt$

$s = \int \left(6t - 2t^2 + 36\right)\ dt$

$s = 3t^2 - \frac{2t^3}{3} + 36t + c$

When $t = 0$, $s = 0$. Thus, $c = 0$.

$s = 3t^2 - \frac{2t^3}{3} + 36t$

When the particle stops momentarily, $v = 0$.

$6t - 2t^2 + 36 = 0$

$t^2 - 3t - 18 = 0$

$(t + 3)(t - 6) = 0$

$t = -3$ or $t = 6$

$t = -3$ is not accepted.

$\therefore t = 6$

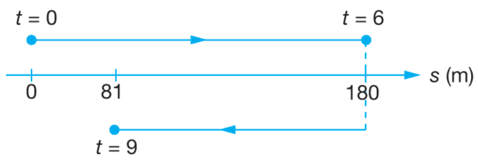
When $t = 6$,

$s = 3(6)^2 - \frac{2(6)^3}{3} + 36(6)$

$s = 180$ m

(d)

t (s)	0	6	9
s (m)	0	180	81



Total distance in the first 9 seconds
= 180 + (180 – 81)
= 279 m

Average speed

$= \frac{279}{9}$

$= 31 \text{ m s}^{-1}$

2 (a) $a_A = 6t + 8$

$v_A = \int (6t + 8)\ dt$

$= \frac{6t^2}{2} + 8t + c$

$= 3t^2 + 8t + c$

When $t = 0$, $v_A = 5$, Thus, $c = 5$.

$\therefore v_A = 3t^2 + 8t + 5$

$a_B = 6t - 2$

$v_B = \int (6t - 2)\ dt$

$= \frac{6t^2}{2} - 2t + k$

$= 3t^2 - 2t + k$

When $t = 0$, $v_B = 4$, Thus, $k = 4$.

$\therefore v_B = 3t^2 - 2t + 4$

(b) $s_A = \int \left(3t^2 + 8t + 5\right)\ dt$

$= \frac{3t^3}{3} + \frac{8t^2}{2} + 5t + h$

$= t^3 + 4t^2 + 5t + h$

When $t = 0$, $s_A = 0$. Thus $h = 0$.

$\therefore s_A = t^3 + 4t^2 + 5t$

$s_B = \int \left(3t^2 - 2t + 4\right)\ dt$

$= \frac{3t^3}{3} - \frac{2t^2}{2} + 4t + u$

$= t^3 - t^2 + 4t + u$

When $t = 0$, $s_B = 18$. Thus, $u = 18$.

$\therefore\ s_B = t^3 - t^2 + 4t + 18$

(c) When the particles A and B collide,

$s_A = s_B$

$$t^3 + 4t^2 + 5t = t^3 - t^2 + 4t + 18$$
$$5t^2 + t - 18 = 0$$
$$(5t - 9)(t + 2) = 0$$
$$t = \frac{9}{5} \text{ or } t = -2$$
$t = -2$ is not accepted.

Hence, $t = 1\frac{4}{5}$.

(d) Distance travelled by particle *A* when the particles *A* and *B* collide

$$= \left(\frac{9}{5}\right)^3 + 4\left(\frac{9}{5}\right)^2 + 5\left(\frac{9}{5}\right)$$
$$= 27\frac{99}{125} \text{ m}$$

