Form 5 Chapter 7
Linear Programming
Fully-Worked Solutions

UPSKILL 7.1

1 (a) $1500x + 900y \leq 45\,000 \Rightarrow 5x + 3y \leq 150$

(b) $y - x \leq 10$

(c) $y \geq \frac{1}{10}x$

2 (a) $x \geq 10$

(b) $y \geq 2x$

(c) $8x + 12y \leq 12 \times 60 \Rightarrow 2x + 3y \leq 180$

3 (a) $x + y \leq 90$

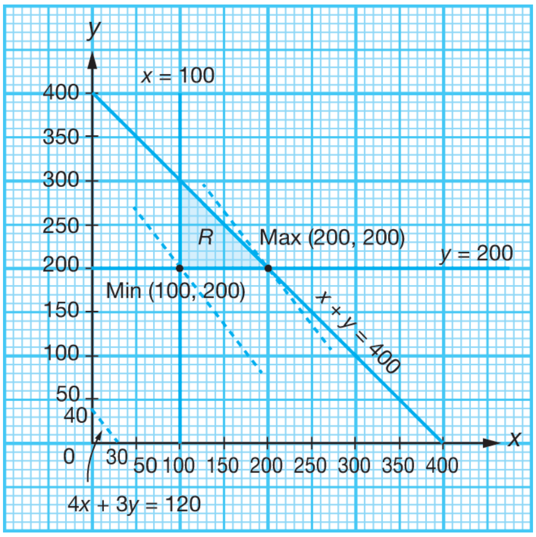
(b) $x \leq 2y$

(c) $y - x \leq 10$

UPSKILL 7.2

1 (a) $x \geq 100$, $y \geq 200$, $x + y \leq 400$

(b)



(c) Commission = $4x + 3y$

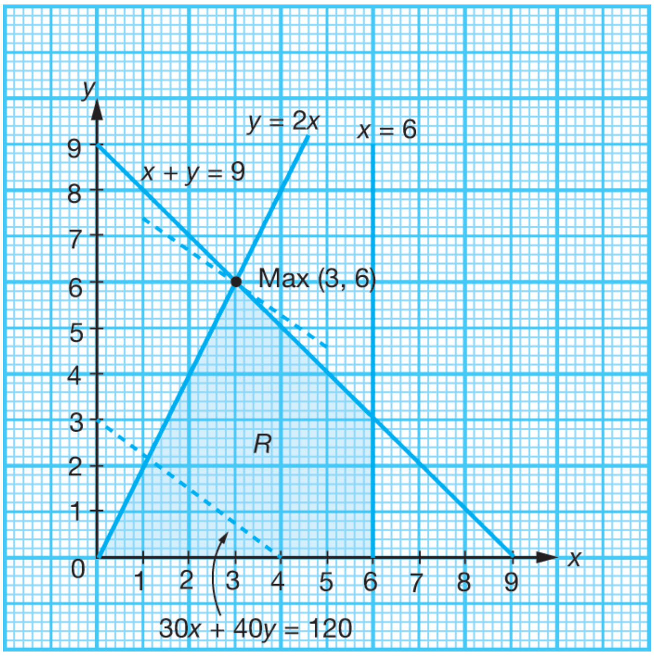
Draw the straight line $4x + 3y = 120$

The optimal point (minimum) is (100, 200).
Minimum commission= $4(100) + 3(200) = \text{RM1 000}$

The optimal point (maximum) is (200, 200).
Maximum commission = $4(200) + 3(200) = \text{RM1 400}$

2 (a) $x \leq 6$, $x + y \leq 9$, $y \leq 2x$

(b)



(c) Monthly fees = $30x + 40y$

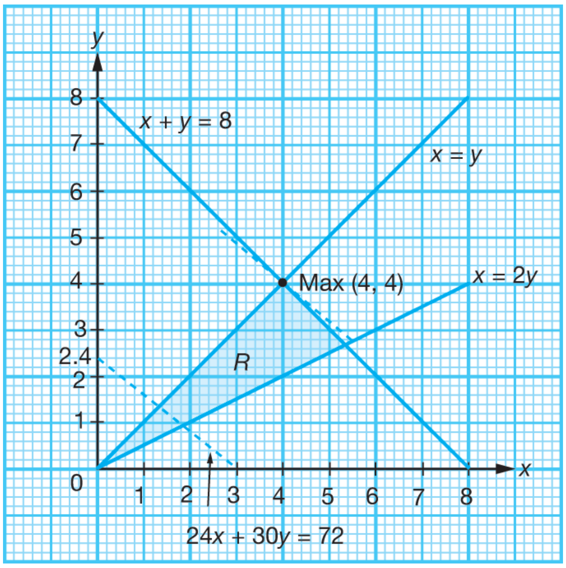
Draw the straight line $30x + 40y = 120$

The optimal point is (3, 6).

Maximum fee = $30(3) + 40(6) = \text{RM}330$

3 (a) $x + y \leq 8$, $x \geq y$, $x \leq 2y$

(b)



(c) Wage = $24x+30y$

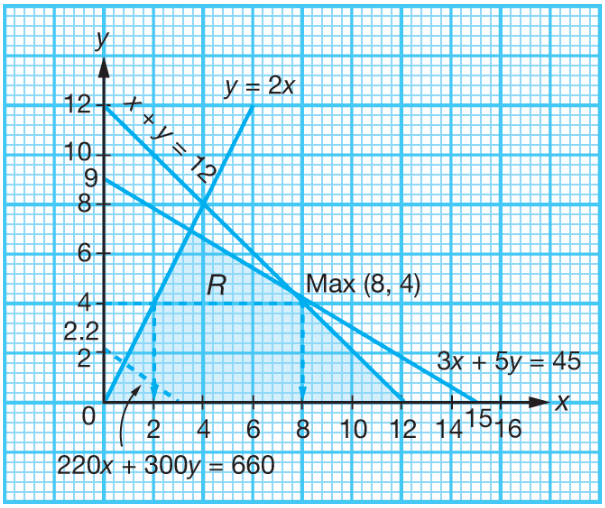
Draw the straight line $24x+30y=72$

The optimal point is (4, 4).

Maximum wage = $24(4)+30(4)=\text{RM}216$

4 (a) $x+y\le 12,\ y\le 2x,\ 3x+5y\le 45$

(b)



(c) (i) When $y = 4$,

x (minimum) = 2 cars

x (maximum) = 8 cars

(ii) Profits = $220x+300y$

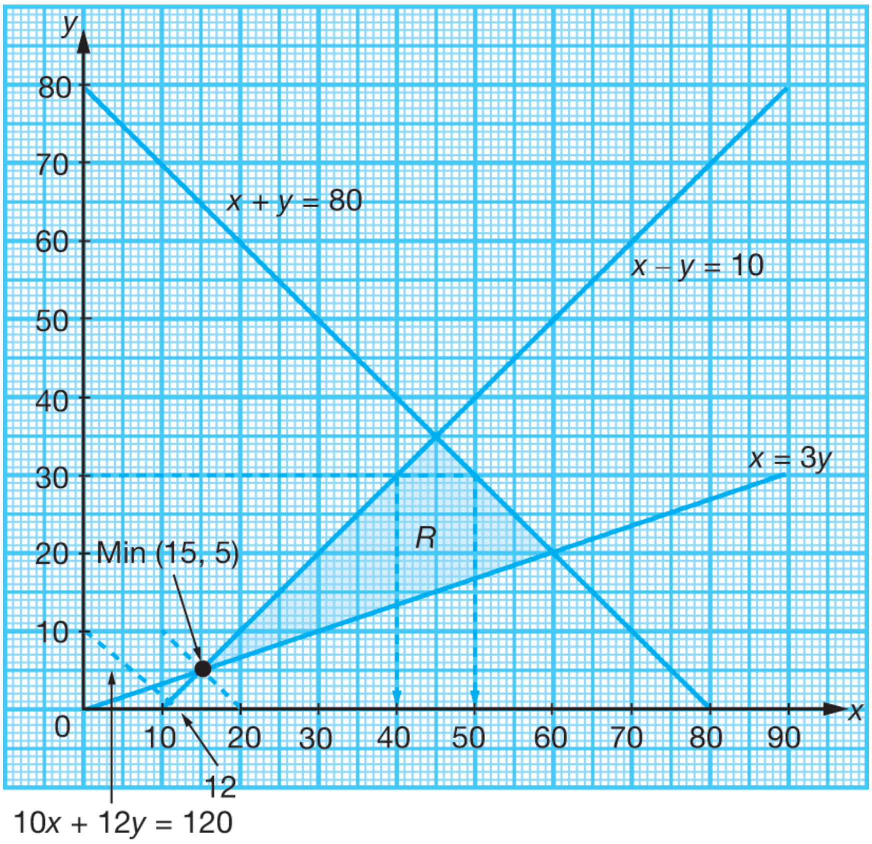
Draw the straight line $220x+300y=660$

The optimal point is (8, 4).

Maximum profit = $220(8)+300(4)=\text{RM}2\ 960$

5 (a) $x + y \leq 80$, $x \leq 3y$, $x - y \geq 10$

(b)



(c) (i) When $y = 30$, $40 \leq x \leq 50$.

(ii) Cost = $10x + 12y$

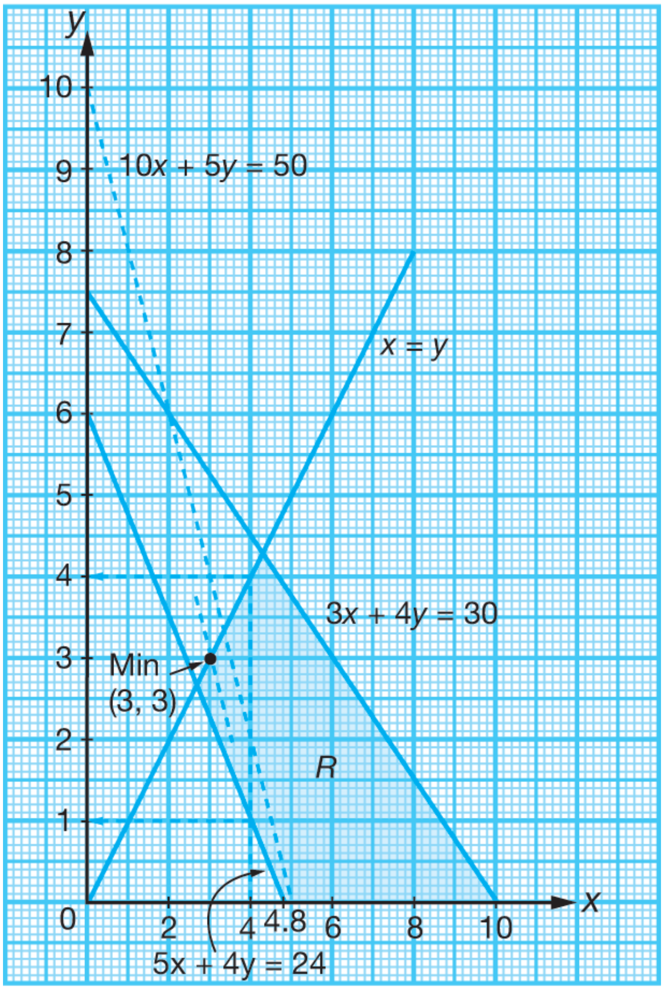
Draw the straight line $10x + 12y = 120$

The optimal point (minimum) is (15, 5).

Minimum cost = $10(15) + 12(5) = \text{RM}210$

6 (a) $60x+80y \le 10\times 60 \Rightarrow 3x+4y \le 30,$
$75x+60y \ge 6\times 60 \Rightarrow 5x+4y \le 24,$
$x \ge y$

(b)



(c) (i) Wage = $10x+5y$
Draw the straight line $5x+4y=24$
The optimal point (minimum) is (3, 3).

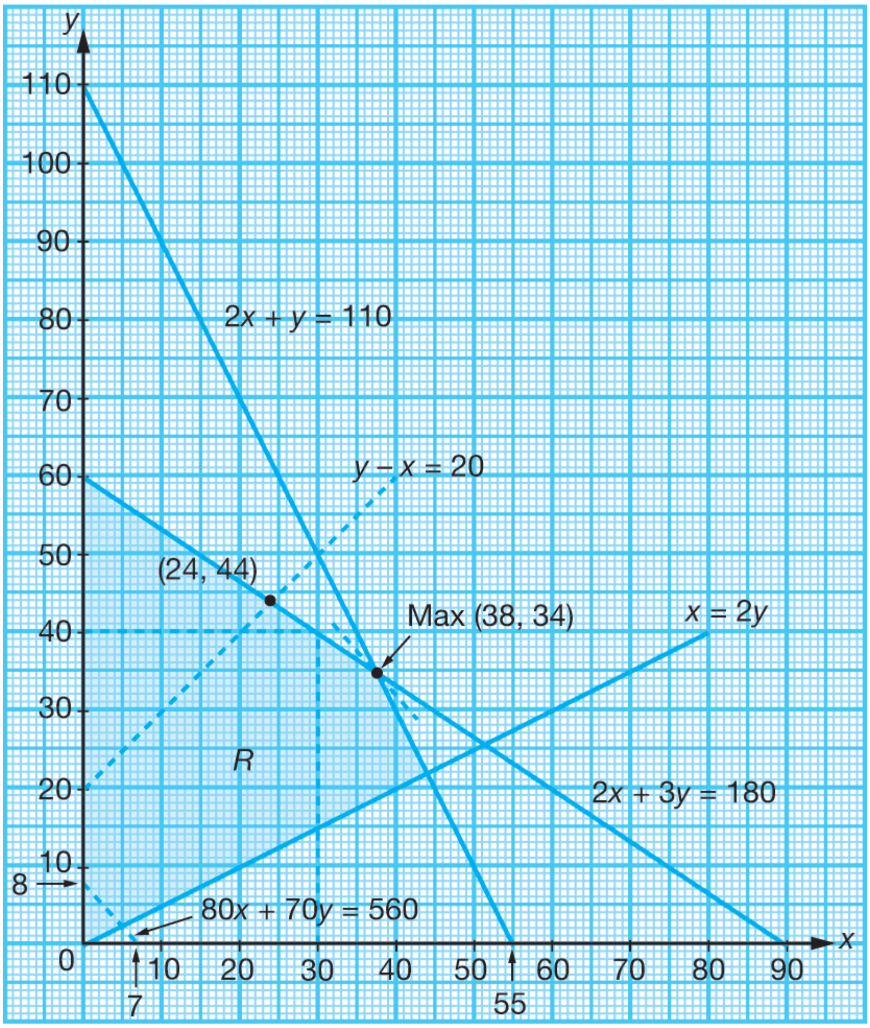
Minimum wage = $10x+5y$ = $10(3)+5(3)=\text{RM}45$

(ii) When x = 4, y (minimum) = 1
Minimum wage = $10(4)+5(1)=\text{RM}45$

When x = 4, y (maximum) = 4
Maximum wage = $10(4)+5(4)=\text{RM}60$

7 (a) $80x + 120y \leq 7\ 200 \Rightarrow 2x + 3y \leq 180,$
$60x + 30y \leq 3\ 300 \Rightarrow 2x + y \leq 110,$
$x \leq 2y$

(b)



(c) (i) Profits = $80x + 70y$
Draw the straight line $80x + 70y = 560$
The optimal point is (38, 34).

Maximum profit = $80(38) + 70(34) = 5\ 420$ sen = RM54.20

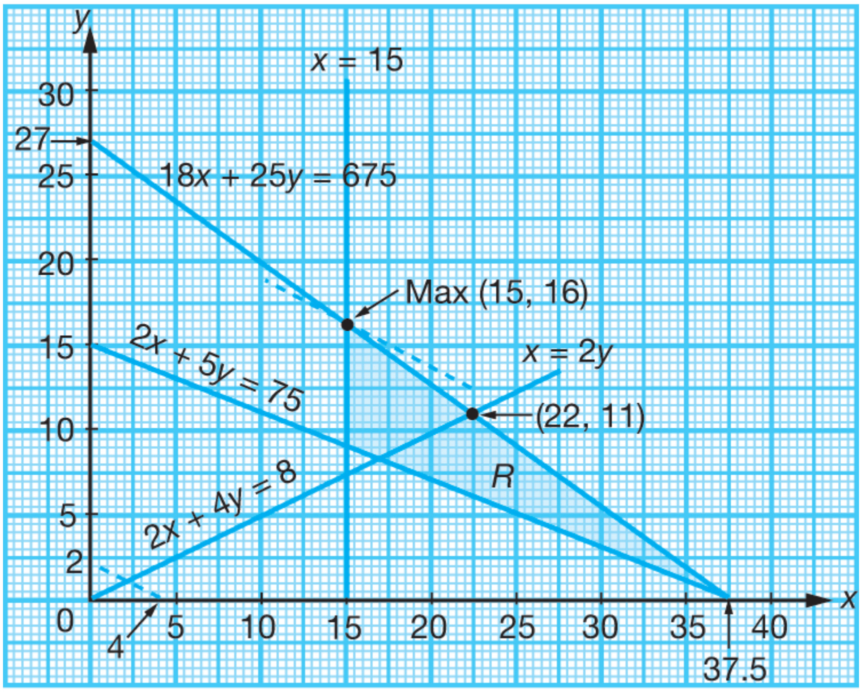
(ii) Draw the straight line $y - x = 20$
x (maximum) = 24 cakes
y (maximum) = 44 buns

8 (a) $180x + 250y \leq 6\,750 \Rightarrow 18x + 25y \leq 675,$

$60\,000x + 150\,000y \leq 2\,250\,000 \Rightarrow 2x + 5y \leq 75,$

$x \geq 15$

(b)



(c) (i) Profits = 20 000x + 40 000y

Draw the straight line 20 000x + 40 000y = 80 000

$2x + 4y = 8$

The optimal point is (15, 16).

Maximum profit = 20 000(15)+ 40 000(16) = RM940 000

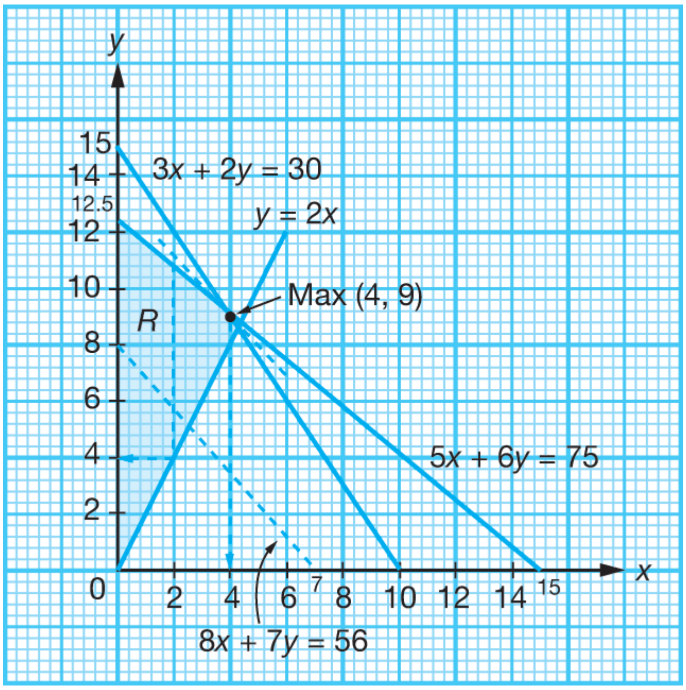
(ii) Draw the straight line $x = 2y$.

x (maximum) = 22 houses,

y (maximum) = 11 houses

9 (a) $50x + 60y \leq 750 \Rightarrow 5x + 6y \leq 75$,
$60x + 40y \leq 10 \times 60 \Rightarrow 3x + 2y \leq 30$,
$y \geq 2x$

(b)



(c) (i) Profits = $8x + 7y$
Draw the straight line $8x + 7y = 56$
The optimal point is (4, 9).

Maximum profit = $8(4) + 7(9) = \text{RM}95$

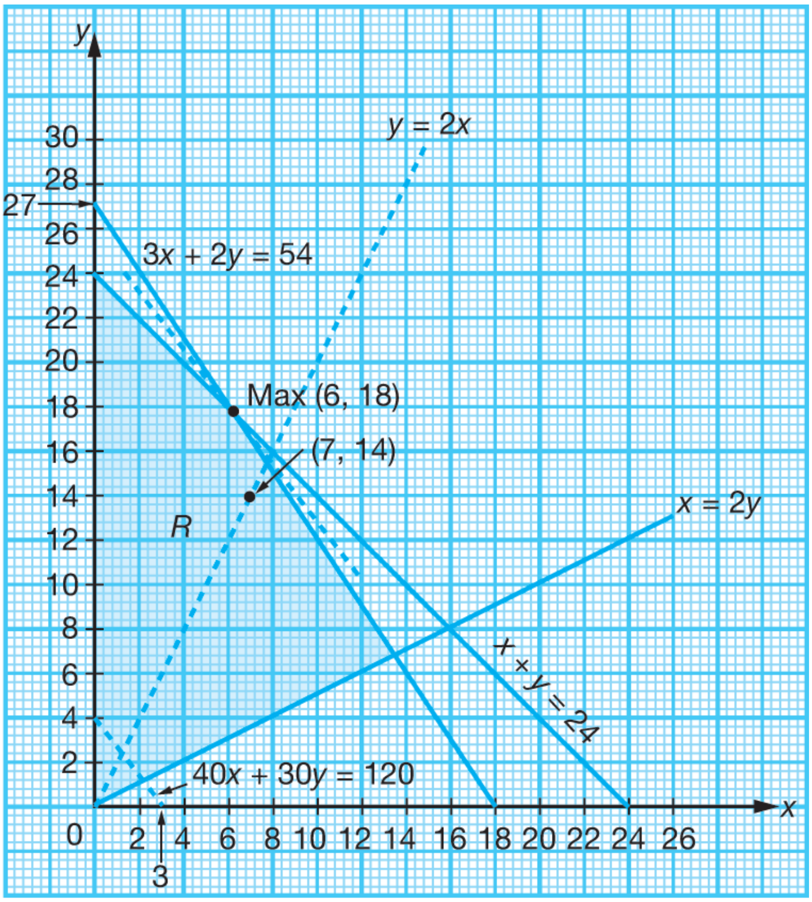
(ii) x (maximum) = 4 baskets

(iii) When x = 2, y (minimum) = 4 floor mats

Summative Practice 7

1 (a) $6x+4y\le 108 \Rightarrow 3x+2y\le 54$,
$x+y\le 24$, $2y\ge x$

(b)



(c) (i) Profits = $40x + 30y$
Draw the straight line $40x+30y=120$
The optimal point is (6, 18).

Maximum profit = 40(6) + 30(18) = RM780

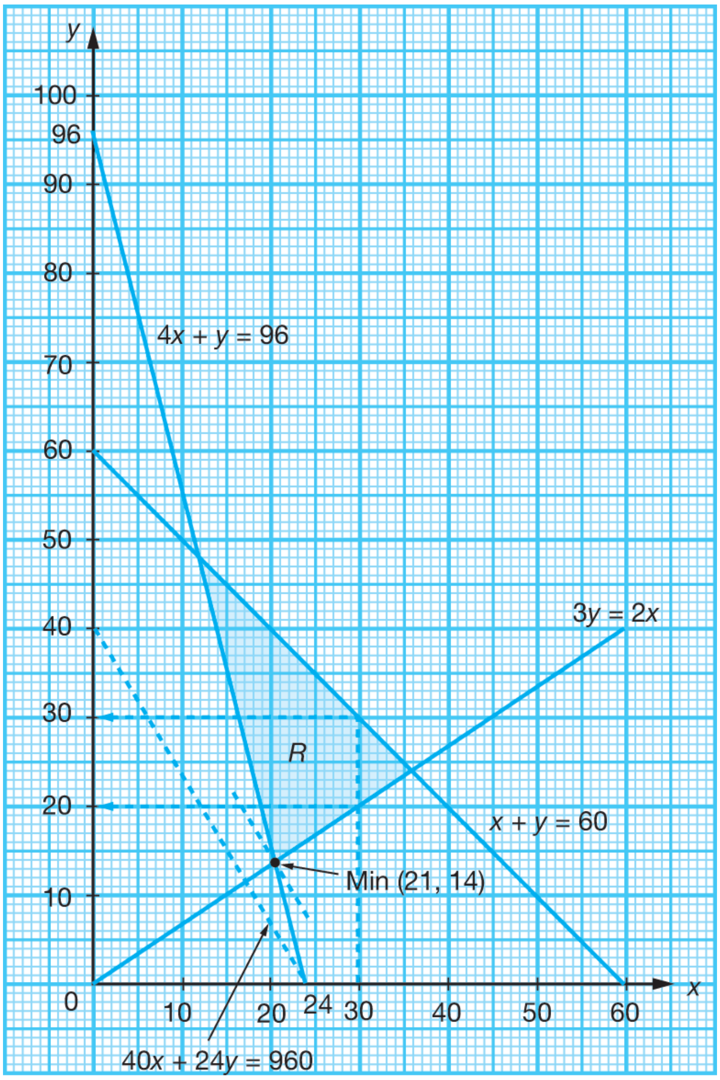
(ii) Draw the straight line $y=2x$
x (maximum) = 7 type P racquets
y (maximum) = 14 type Q racquets

2 (a) $2x+2y \leq 120 \Rightarrow x+y \leq 60$,

$4x+y \geq 96$

$x \leq \frac{60}{100}(x+y) \Rightarrow x \leq \frac{3}{5}(x+y) \Rightarrow 5x \leq 3x+3y \Rightarrow 3y \geq 2x$

(b)



(c) (i) Cost = $40x+24y$

Draw the straight line $40x+24y=960$

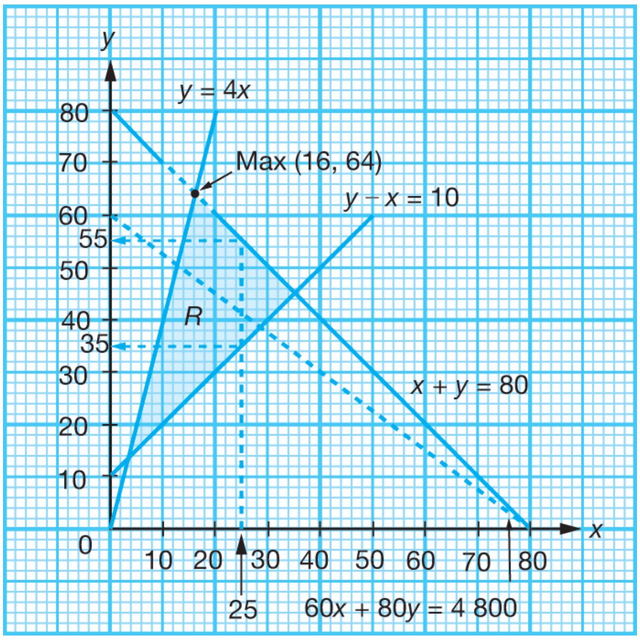
The optimal point (minimum) is (21, 14)

Minimum cost = $40(21)+24(14) = \text{RM1 176}$

(ii) When x = 30, $20 \leq y \leq 30$

3 (a) $x + y \leq 80$, $y \leq 4x$, $y - x \geq 10$

(b)



(c) (i) When $x = 25$, $35 \leq y \leq 55$

(ii) Allocation = $60x + 80y$

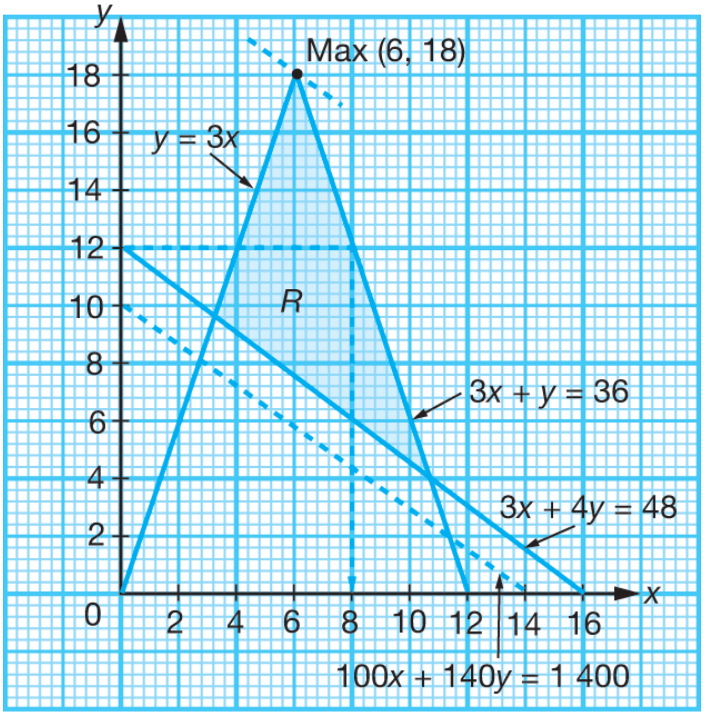
Draw the straight line $60x + 80y = 4\ 800$

The optimal point is (16, 64).

Maximum allocation = $60x + 80y = 60(16) + 80(64)$ = RM6 080

4 (a) $60x + 20y \leq 12 \times 60 \Rightarrow 3x + y \leq 36,$
$30x + 40y \geq 8 \times 60 \Rightarrow 3x + 4y \geq 48,$
$y \leq 3x$

(b)



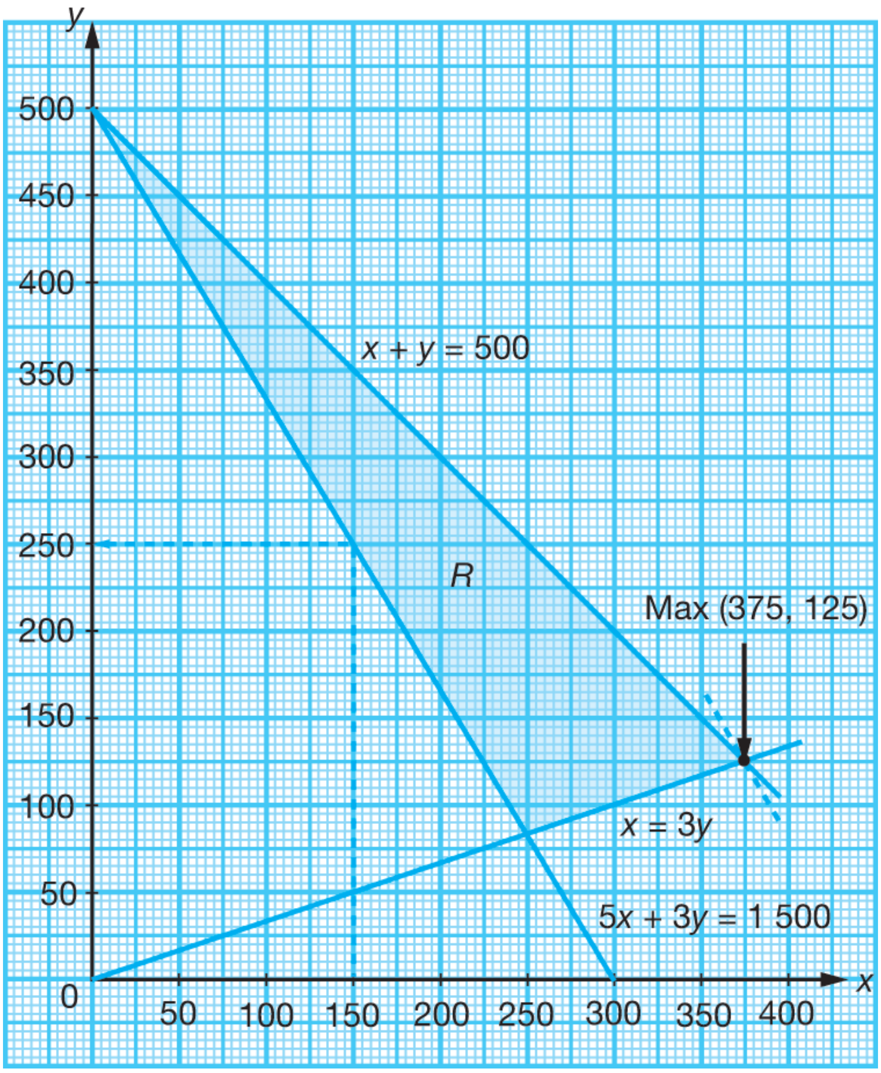
(c) (i) When $y = 12$, x (maximum) = 8 trophies

(ii) Profits = $100x + 140y$
Draw the straight line $100x + 140y = 1\ 400$
The optimal point is (6, 18).

Maximum profit = 100(6) + 140(18) = RM3 120

5 (a) $x + y \leq 500$, $x \leq 3y$, $5x + 3y \geq 1\,500$

(b)



(c) (i) If $x = 150$, y (minimum) = 250 boxes of pocket files

(ii) Profits = $5x + 3y$

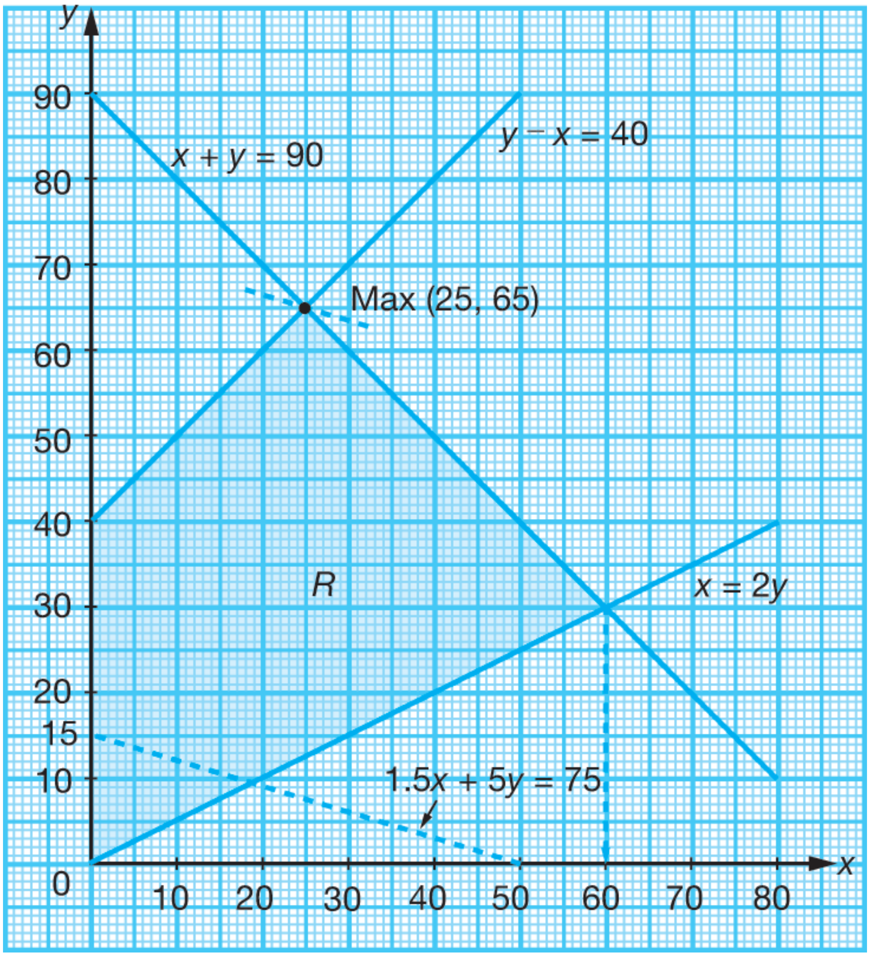
Draw the straight line $5x + 3y = 1\,500$

The optimal point is (375, 125)

Maximum profit = 5(375) + 3(125) = RM2 250

6 (a) $x + y \leq 90$, $x \leq 2y$, $y - x \leq 40$

(b)



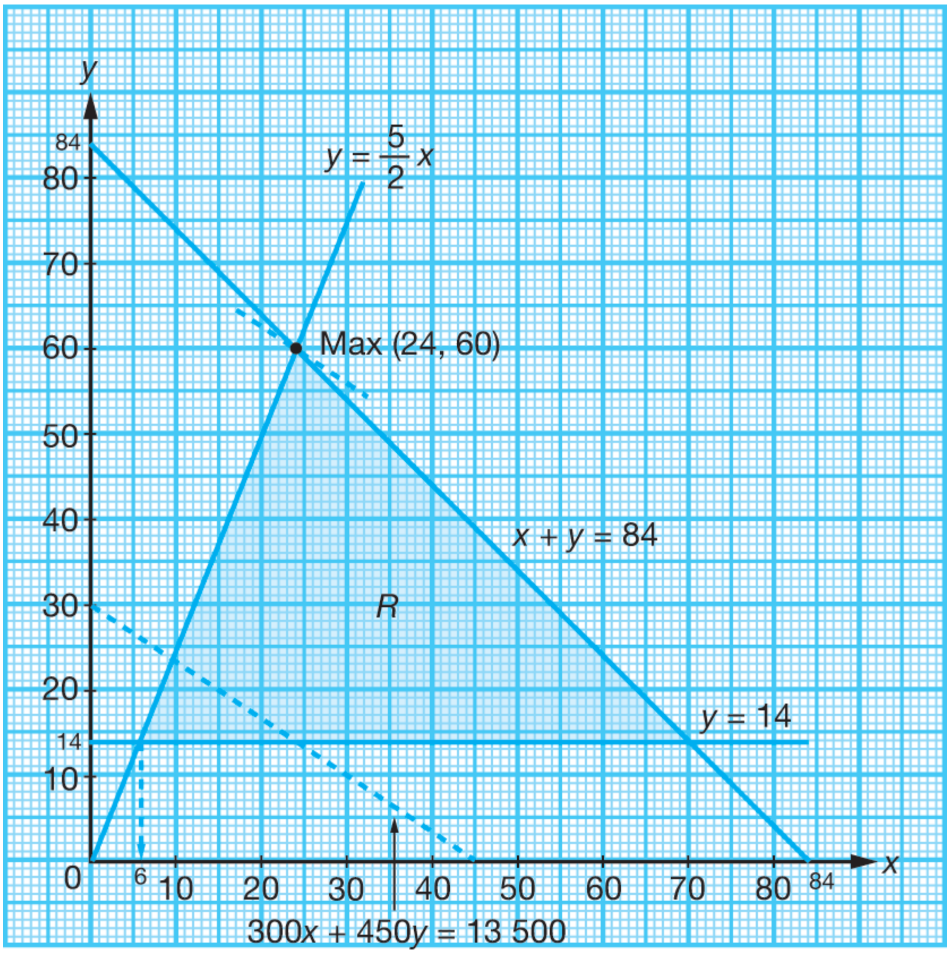
(c) (i) x (maximum) = 60 ballpoint pens

(ii) Cost = $1.5x + 5y$
Draw the straight line $1.5x + 5y = 75$
The optimal point is (25, 65).

Maximum cost = 1.5(25) + 5(65) = RM362.50

7 (a) $x + y \leq 84$, $y \geq 14$, $y \leq \frac{5}{2}x$

(b)



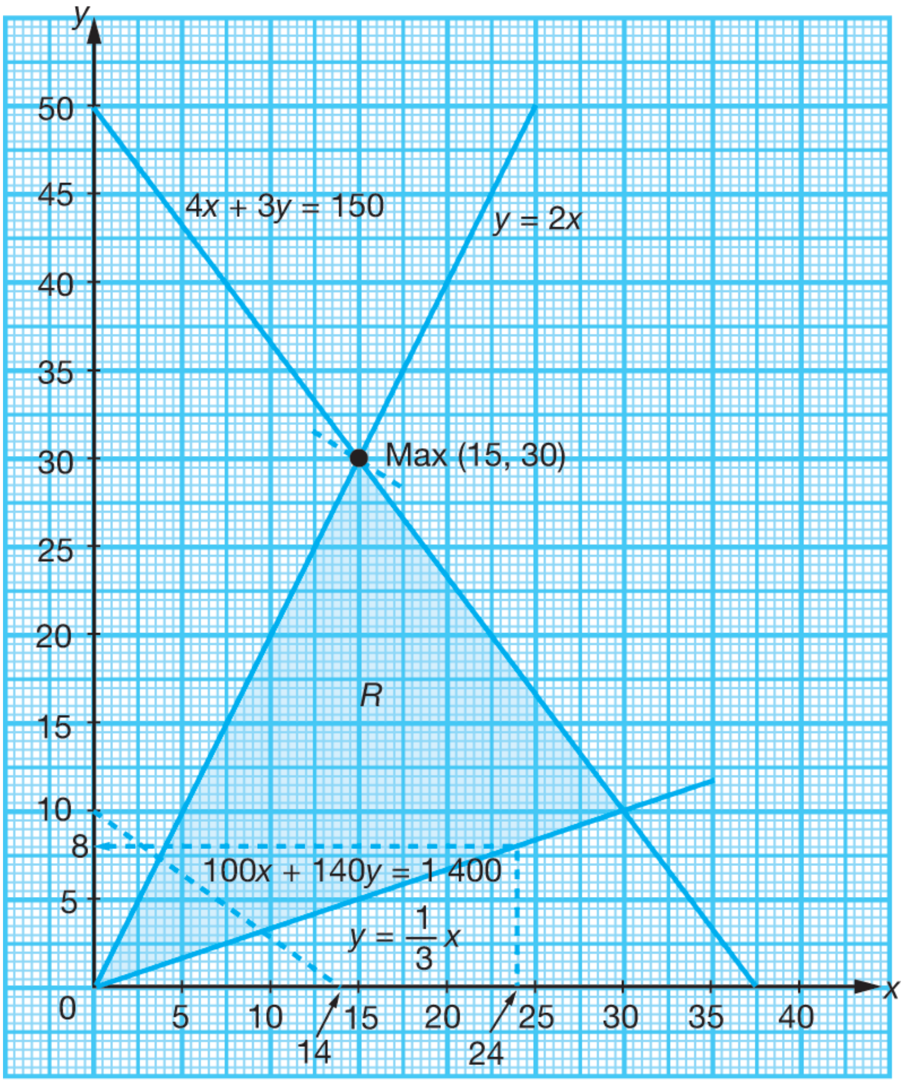
(c) (i) x (minimum) = 6 course P participants

(ii) Fees = $300x + 450y$
Draw the straight line $300x + 450y = 13\ 500$
The optimal point is (24, 60).

Maximum fee = 300(24) + 450(60) = RM34 200

8 (a) $y \le 2x$, $y \ge \frac{1}{3}x$, $4x + 3y \le 150$

(b)



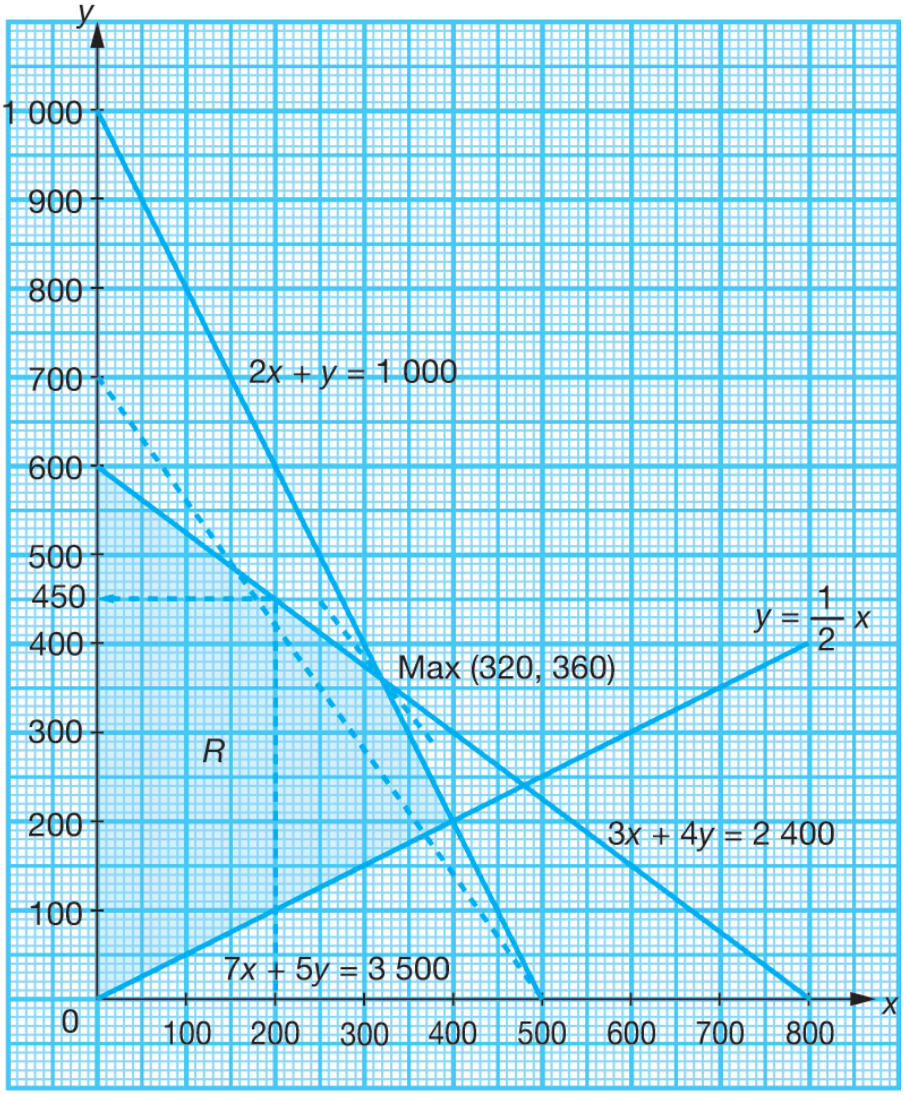
(c) (i) If $x = 24$, y (minimum) = 8 tins of *satin-glo* paints

(ii) Expenditure = $100x + 140y$
Draw the straight line $100x + 140y = 1\ 400$
The optimal point is (15, 30).

Maximum expenditure = 100(15) + 140(30) = RM5 700

9 (a) $3x + 4y \leq 2\,400$, $2x + y \leq 1\,000$, $y \geq \frac{1}{2}x$

(b)



(c) (i) If $x = 200$, y (maximum) = 450 chairs

(ii) Profits = $7x + 5y$
Draw the straight line $7x + 5y = 3\,500$
The optimal point is (320, 360).

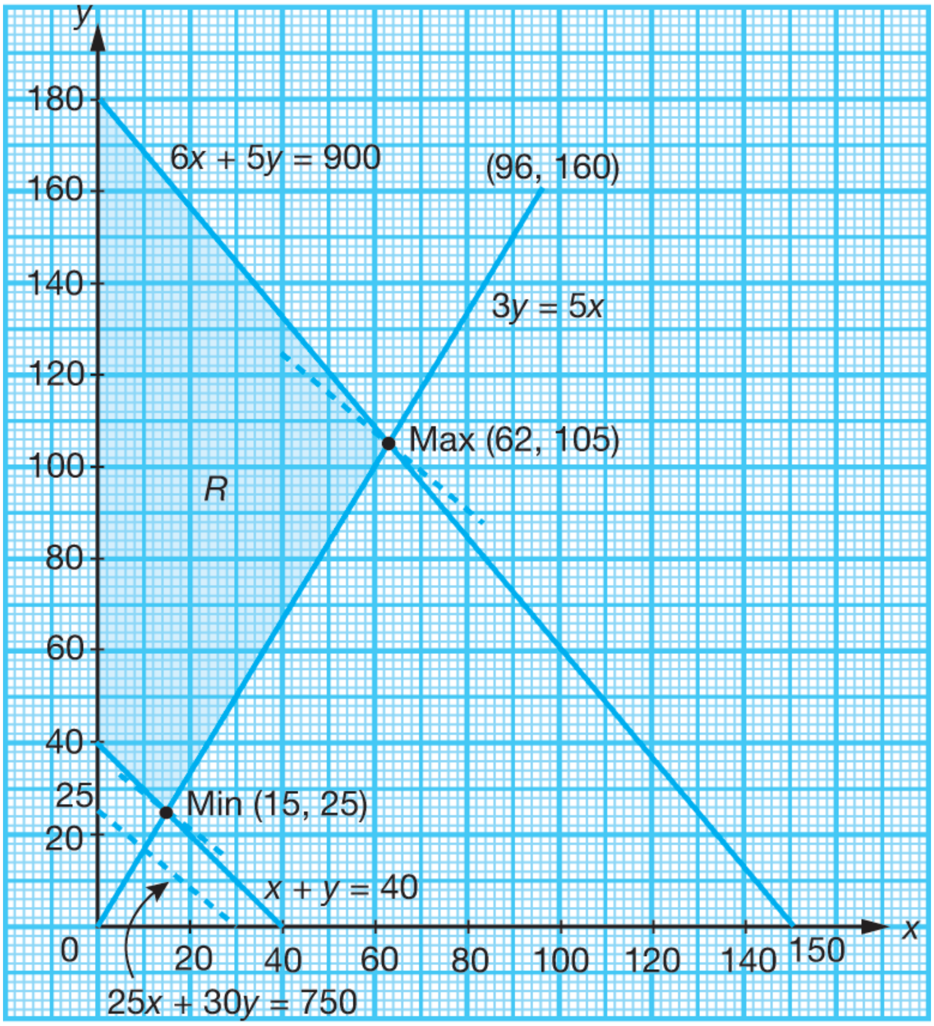
Maximum profit = 7(320) + 5(360) = RM4 040

10 (a) $x + y \geq 40$,

$96x + 80y \leq 240 \times 60 \Rightarrow 6x + 5y \leq 900$,

$\frac{x}{y} \leq \frac{3}{5} \Rightarrow 3y \geq 5x$

(b)



(c) Sales = $25x + 30y$

Draw the straight line $25x + 30y = 750$

The optimal point (minimum) is (15, 25).

Minimum sale = 25(15) + 30(25) = RM1 125

The optimal point (maximum) is (62, 105).

Maximum sale = 25(62) + 30(105) = RM4 700

RM1 125 $\leq$ Total sales $\leq$ RM4 700

SPM Spot

1 (a) I $x + y \geq 40$

x	0	40
y	40	0

II $6x + 5y \leq 15(60)$ = $6x + 5y \leq 900$

x	0	150
y	180	0

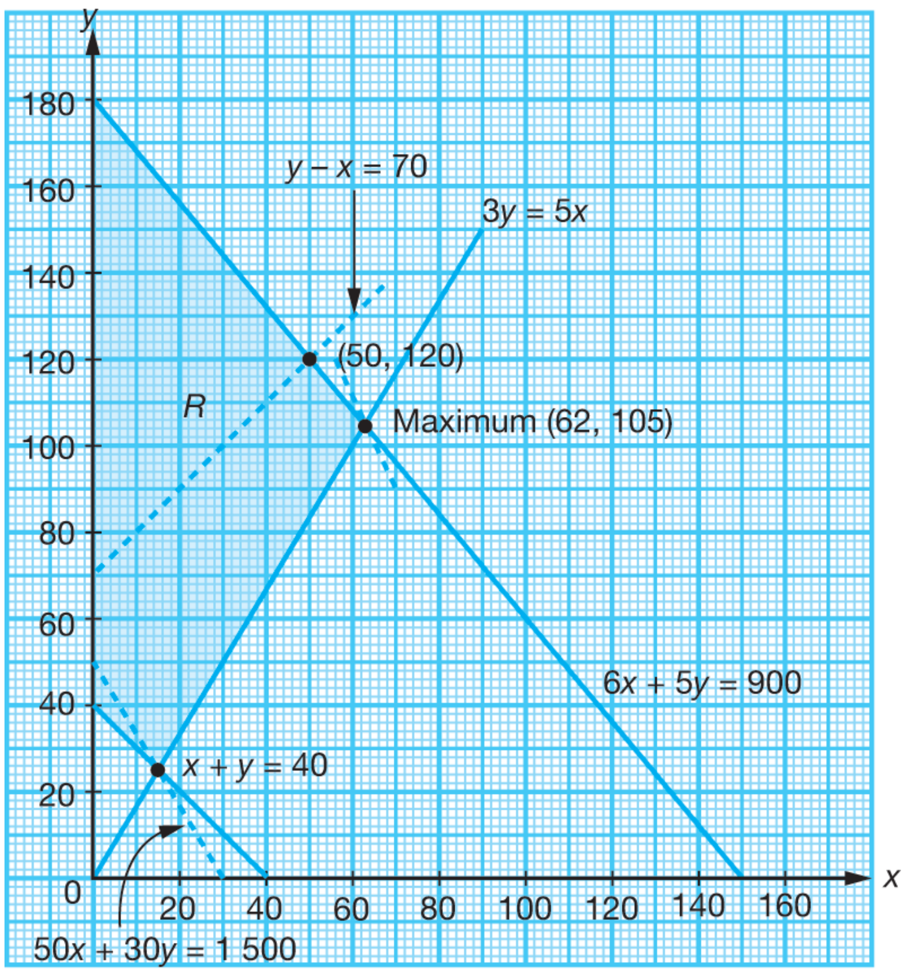
III $\frac{x}{y} \leq \frac{3}{5}$

$5x \leq 3y$

$3y \geq 5x$

x	0	90
y	0	150

(b)



(c) (i) Total sales = $50x + 30y$

Draw the straight line $50x + 30y = 1\ 500$

x	0	30
y	50	0

The optimal point is (62, 105).

Both of the x-coordinate and the y-coordinate must be integer.

Maximum sales = 50(62) + 30(105) = RM6 250

(ii) If the number pairs of the type S shoes sold exceeds the number pairs of the type the L by 70, draw the straight line $y - x = 70$.

$x_{\text{maximum}} = 50$ and $y_{\text{maximum}} = 120$

2 (a) I $x + y \leq 80$

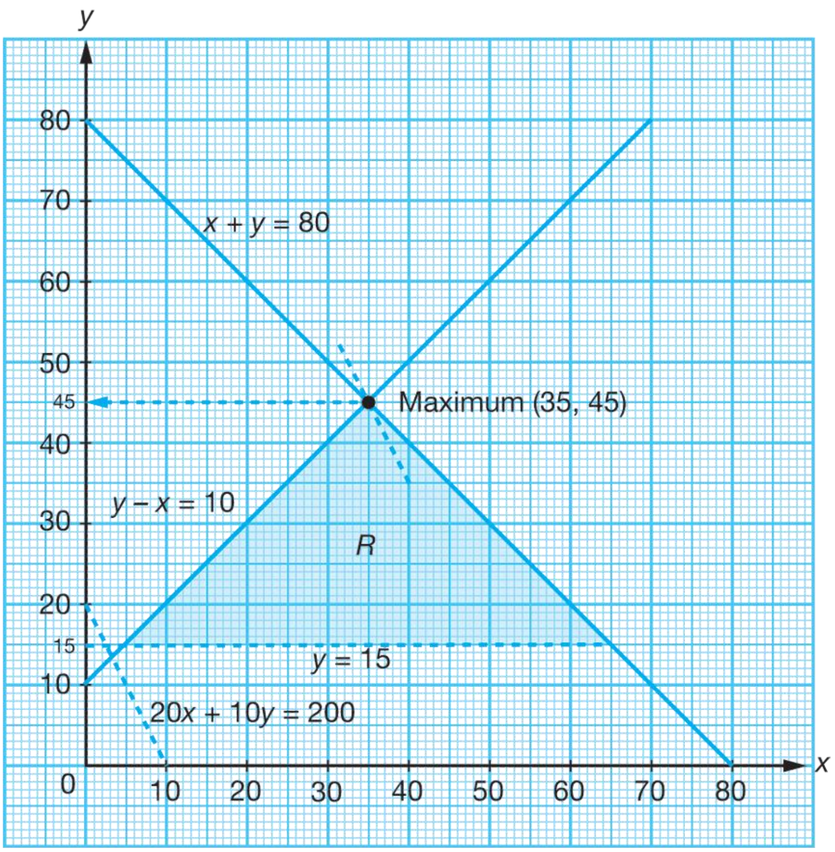
x	0	80
y	80	0

II $y - x \leq 10$

x	0	70
y	10	80

III $y > 15$

(b)



(c) (i) The maximum number of students from school B = 45

(ii) Fees = $20x + 10y$

Draw the straight line $20x + 10y = 200$.

From the graph, the optimal point is (35, 45).

Maximum fee
$= 20(35) + 10(45)$
= RM1 150

3 (a) I $x + y < 6$

x	0	6
y	6	0

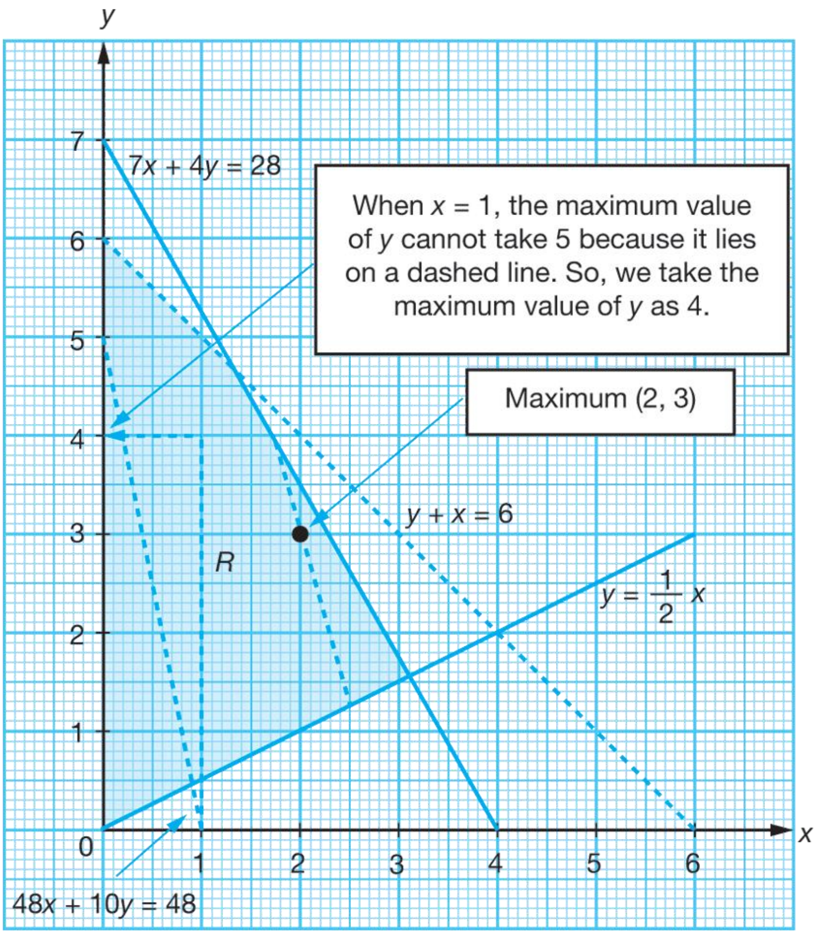
II $x \leq 2y \Rightarrow y \geq \frac{1}{2}x$

x	0	6
y	0	3

III $700x + 400y \leq 2\,800$
$7x + 4y \leq 28$

x	0	4
y	7	0

(b)



(c) (i) When $x = 1$, the maximum value of y is 4 vans.

(ii) Number of passengers = $48x + 10y$

Draw the straight line $48x + 10y = 48$
From the graph, the optimal point is (2, 3).
Hence, the total number of teachers that can be accommodated
= 48(2) + 10(3)
= 126

x	0	1
y	4.8	0