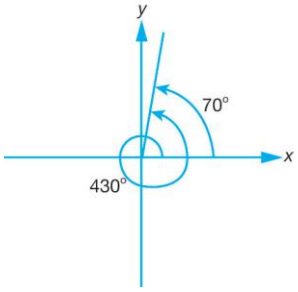
Form 5 Chapter 6
Trigonometric Functions
Fully-Worked Solutions

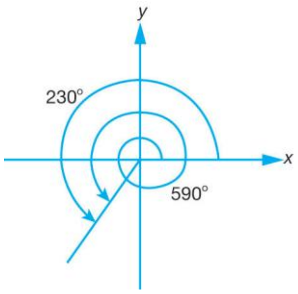
UPSKILL 6.1

1 (a)



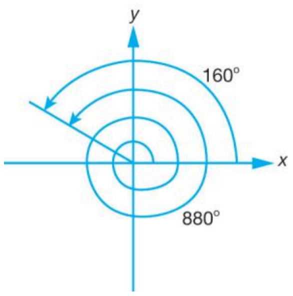
First quadrant

(b)



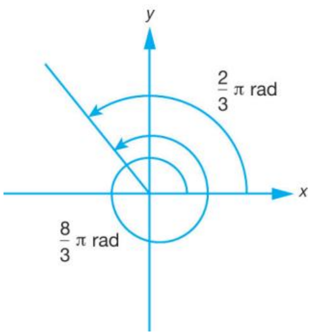
Third quadrant

(c)



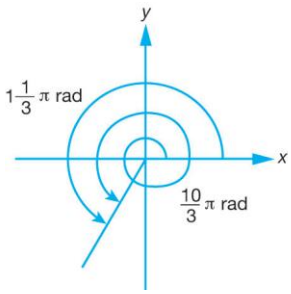
Second quadrant

(d)



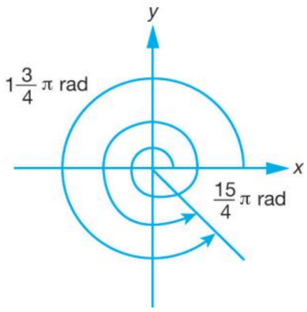
Second quadrant

(e)



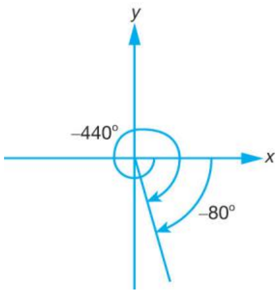
Third quadrant

(f)



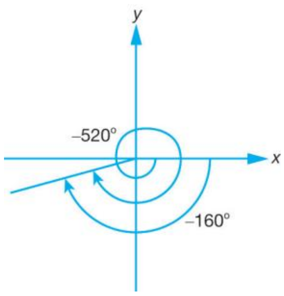
Fourth quadrant

2 (a)



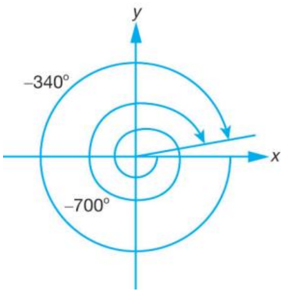
Fourth quadrant

(b)



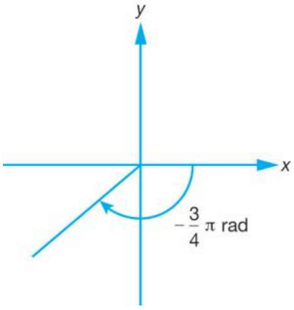
Third quadrant

(c)



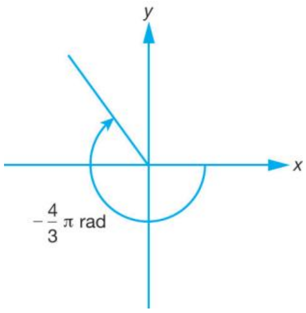
First quadrant

(d)



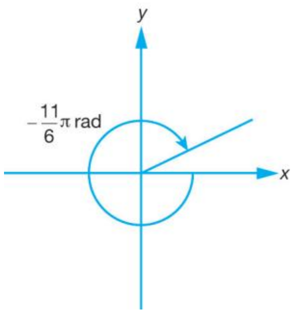
Third quadrant

(e)



Second quadrant

(f)



First quadrant

UPSKILL 6.2

1 (a) $\tan 15^\circ = \dfrac{\sin 15^\circ}{\cos 15^\circ} = \dfrac{0.2588}{0.9659} = 0.2679$

(b) $\cot 15^\circ = \dfrac{\cos 15^\circ}{\sin 15^\circ} = \dfrac{0.9659}{0.2588} = 3.7322$

(c) $\sec 15^\circ = \dfrac{1}{\cos 15^\circ} = \dfrac{1}{0.9659} = 1.0353$

(d) $\operatorname{cosec} 15^\circ = \dfrac{1}{\sin 15^\circ} = \dfrac{1}{0.2588} = 3.8640$

2 (a) $\tan \dfrac{2}{3}\pi = \dfrac{\sin \frac{2}{3}\pi}{\cos \frac{2}{3}\pi} = \dfrac{0.8660}{-0.5} = -1.732$

(b) $\cot \dfrac{2}{3}\pi = \dfrac{\cos \frac{2}{3}\pi}{\sin \frac{2}{3}\pi} = \dfrac{-0.5}{0.8660} = -0.5774$

(c) $\sec \dfrac{2}{3}\pi = \dfrac{1}{\cos \frac{2}{3}\pi} = \dfrac{1}{-0.5} = -2$

(d) $\operatorname{cosec} \dfrac{2}{3}\pi = \dfrac{1}{\sin \frac{2}{3}\pi} = \dfrac{1}{0.8660} = 1.1547$

3 (a) $\tan 30^\circ = \dfrac{\sin 30^0}{\cos 30^\circ} = \dfrac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \dfrac{1}{\sqrt{3}}$

(b) $\cot 30^\circ = \dfrac{\cos 30^\circ}{\sin 30^\circ} = \dfrac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3}$

(c) $\sec 30^\circ = \dfrac{1}{\cos 30^\circ} = \dfrac{1}{\frac{\sqrt{3}}{2}} = \dfrac{2}{\sqrt{3}}$

(d) $\operatorname{cosec} 30^\circ = \dfrac{1}{\sin 30^\circ} = \dfrac{1}{\frac{1}{2}} = 2$

4 (a) $\cos 150^\circ = -\cos(180^\circ - 150^\circ)$

$= -\cos 30^\circ$

$= -\dfrac{\sqrt{3}}{2}$

(b) $\sin 225^\circ = -\sin(225^\circ - 180^\circ)$
$= -\sin 45^\circ$
$= -\dfrac{1}{\sqrt{2}}$

(c) $\tan 240^\circ = \tan(240^\circ - 180^\circ)$
$= \tan 60^\circ$
$= \sqrt{3}$

(d) $\cos\left(\dfrac{7}{4}\pi\right) = \cos\left(\dfrac{7}{4} \times 180^\circ\right)$
$= \cos 315^\circ$
$= \cos(360^\circ - 315^\circ)$
$= \cos 45^\circ$
$= \dfrac{1}{\sqrt{2}}$

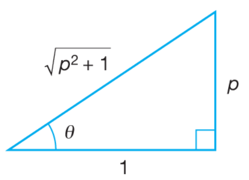
(e) $\text{cosec } 135^\circ$
$= \dfrac{1}{\sin 135^\circ}$
$= \dfrac{1}{\sin(180^\circ - 135^\circ)}$
$= \dfrac{1}{\sin 45^\circ}$
$= \dfrac{1}{\frac{1}{\sqrt{2}}}$
$= \sqrt{2}$

(f) $\sec 570^\circ$
$= \sec(570^\circ - 360^\circ)$
$= \sec(210^\circ)$
$= \dfrac{1}{\cos 210^\circ}$
$= \dfrac{1}{-\cos(210^\circ - 180^\circ)}$
$= \dfrac{1}{-\cos 30^\circ}$
$= -\dfrac{1}{\frac{\sqrt{3}}{2}}$
$= -\dfrac{2}{\sqrt{3}}$

(g) $\cot 870^\circ$
$= \cot(870^\circ - 720^\circ)$
$= \cot 150^\circ$
$= \dfrac{1}{-\tan(180^\circ - 150^\circ)}$
$= \dfrac{1}{-\tan(30^\circ)}$
$= -\dfrac{1}{\frac{1}{\sqrt{3}}}$
$= -\sqrt{3}$

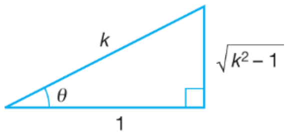
(h) $\text{cosec}\left(-\dfrac{11}{3}\pi\right)$
$= \text{cosec}\left(-\dfrac{11}{3} \times 180^\circ\right)$
$= \text{cosec}(-660^\circ)$
$= \text{cosec}(-660^\circ + 720^\circ)$
$= \text{cosec}(60^\circ)$
$= \dfrac{1}{\sin 60^\circ}$
$= \dfrac{1}{\frac{\sqrt{3}}{2}}$
$= \dfrac{2}{\sqrt{3}}$

5 (a) $\text{cosec } \theta = \dfrac{1}{\sin \theta}$
$= \dfrac{1}{\frac{p}{\sqrt{p^2+1}}}$
$= \dfrac{\sqrt{p^2+1}}{p}$



(b) $\cos(-\theta)$
$= \cos \theta$
$= \dfrac{1}{\sqrt{p^2+1}}$

6



(a) $\sin(-\theta) = -\sin\theta$

$= -\dfrac{\sqrt{k^2 - 1}}{k}$

(b) $\sec(-\theta) = \dfrac{1}{\cos(-\theta)}$

$= \dfrac{1}{\cos\theta}$

$= \dfrac{1}{\frac{1}{k}}$

$= k$

(c) $\cot(-\theta) = \dfrac{1}{\tan(-\theta)}$

$= \dfrac{1}{-\tan\theta}$

$= -\dfrac{1}{\sqrt{k^2 - 1}}$

7 (a) $\sin 2\theta = 0.5278$

Basic ∠ = 31.86°

$2\theta = 31.86^{\circ}, 148.14^{\circ}, 391.86^{\circ}, 508.14^{\circ}$

$\theta = 15.93^{\circ}, 74.07, 195.93^{\circ}, 254.07^{\circ}$

(b) $\cos 2\theta = -0.4630$

Basic ∠ = 62.42°

$2\theta = 117.58^{\circ}, 242.42o, 477.58^{\circ}, 602.42^{\circ}$

$\theta = 58.79^{\circ}, 121.21^{\circ}, 238.79^{\circ}, 301.21^{\circ}$

(c) $\tan 2\theta = -0.4287$

Basic ∠ $= 23.20^{\circ}$

$2\theta = 156.80^{\circ}, 336.80^{\circ}, 516.80^{\circ}, 696.80^{\circ}$

$\theta = 78.40^{\circ}, 168.40^{\circ}, 258.40^{\circ}, 348.40^{\circ}$

(d) $\sin 3\theta = -0.4479$

Basic ∠ = 26.61°

$3\theta = 206.61^{\circ}, 333.39^{\circ}, 566.61^{\circ},$

$693.39^{\circ}, 926.61^{\circ}, 1\ 053.39^{\circ}$

$\theta = 68.87^{\circ}, 111.13^{\circ}, 188.87^{\circ},$

$231.13^{\circ}, 308.87^{\circ}, 351.13^{\circ}$

(e) $\cos 3\theta = 0.5358$

Basic ∠ $= 57.60^{\circ}$

$3\theta = 57.60, 302.40, 417.60,$

$662.40^{\circ}, 777.60^{\circ}, 1\ 022.40^{\circ}$

$\theta = 19.20^{\circ}, 100.80^{\circ}, 139.20^{\circ},$

$220.80^{\circ}, 259.20^{\circ}, 340.80^{\circ}$

(f) $\tan 3\theta = 1.5849$

Basic ∠ = 57.75°

$3\theta = 57.75^{\circ}, 237.75^{\circ}, 417.75^{\circ},$

$597.75^{\circ}, 777.75^{\circ}, 957.75^{\circ}$

$\theta = 19.25^{\circ}, 79.25^{\circ}, 139.25^{\circ},$

$199.25^{\circ}, 259.25^{\circ}, 319.25^{\circ}$

8 (a) $\sin x = \cos 65^{\circ}$

$\sin x = \sin(90^{\circ} - 65^{\circ})$

$\sin x = \sin 25^{\circ}$

Basic ∠ = 25°

$x = 25^{\circ}, 155^{\circ}$

(b) $\cos x = \sin 47^{\circ}$

$\cos x = \cos(90^{\circ} - 47^{\circ})$

$\cos x = \cos 43^{\circ}$

Basic ∠ = 43°

$x = 43^{\circ}, 317^{\circ}$

(c) $\tan x = \cot 83^{\circ}$

$\tan x = \tan(90^{\circ} - 83^{\circ})$

$\tan x = \tan 7^{\circ}$

Basic ∠ = 7°

$x = 7^{\circ}, 187^{\circ}$

(d) $\sec x = \text{cosec } 56^\circ$

$\sec x = \sec(90^\circ - 56^\circ)$

$\sec x = \sec 34^\circ$

Basic $\angle = 34^\circ$

$x = 34^\circ,\ 326^\circ$

(e) $\sec x = -\text{cosec } 48^\circ$

$\sec x = -\sec(90^\circ - 48^\circ)$

$\sec x = -\sec 42^\circ$

Basic $\angle = 42^\circ$

$x = 138^\circ,\ 222^\circ$

(f) $\sin 2x = -\cos 66^\circ$

$\sin 2x = -\sin(90^\circ - 66^\circ)$

$\sin 2x = -\sin 24^\circ$

Basic $\angle = 24^\circ$

$2x = 204^\circ,\ 336^\circ, 564^\circ, 696^\circ$

$x = 102^\circ, 168^\circ, 282^\circ, 348^\circ$

(g) $\cos 2x = -\sin 72^\circ$

$\cos 2x = -\cos(90^\circ - 72^\circ)$

$\cos 2x = -\cos 18^\circ$

Basic $\angle = 18^\circ$

$2x = 162^\circ, 198^\circ, 522^\circ, 558^\circ$

$x = 81^\circ, 99^\circ, 261^\circ, 279^\circ$

(h) $\tan 3x = \cot 57^\circ$

$\tan 3x = \tan(90^\circ - 57^\circ)$

$\tan 3x = \tan 33^\circ$

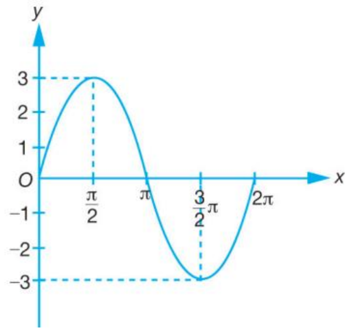
Basic $\angle = 33^\circ$

$3x = 33^\circ, 213^\circ, 393^\circ, 573^\circ, 753^\circ, 933^\circ$

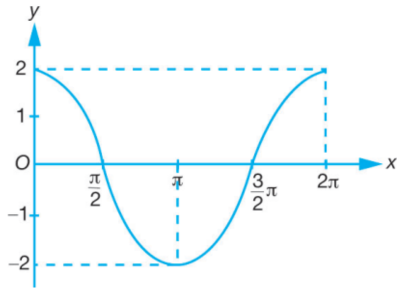
$x = 11^\circ, 71, 131^\circ, 191^\circ, 251^\circ, 311^\circ$

UPSKILL 6.3a

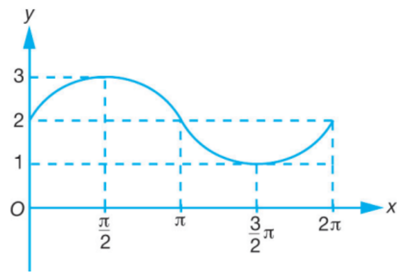
1 (a)



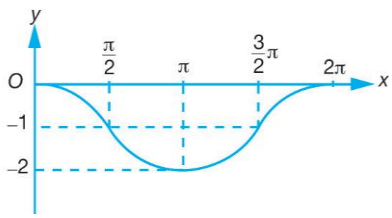
(b)



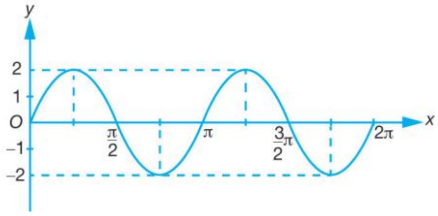
(c)



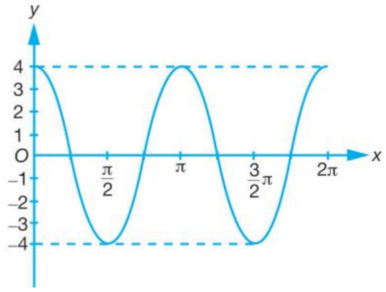
(d)



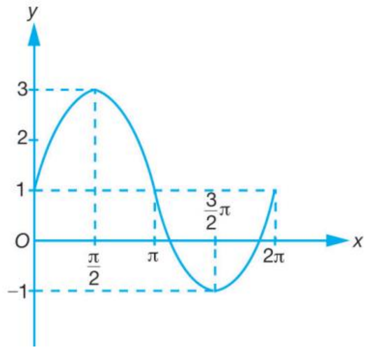
2 (a)



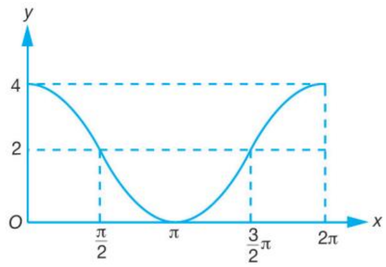
(b)



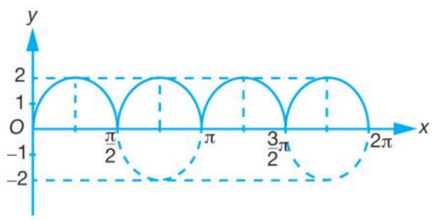
(c)



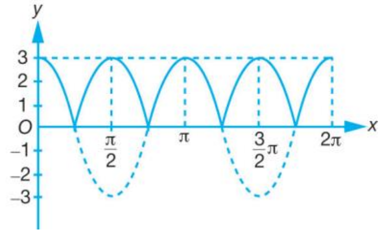
(d)



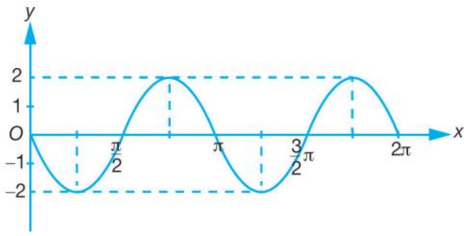
(e)



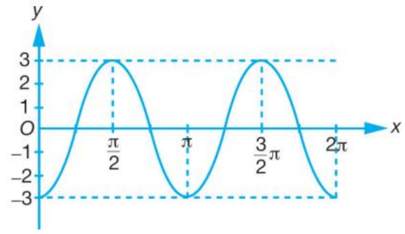
(f)



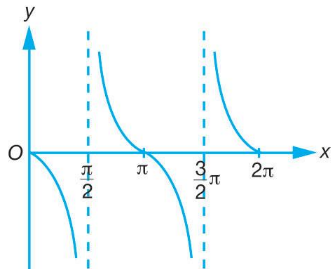
(g)



(h)

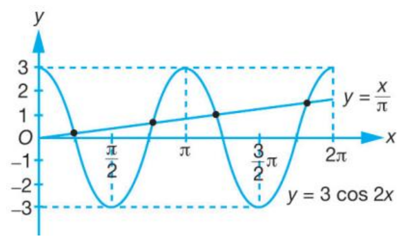


(i)



UPSKILL 6.3b

1

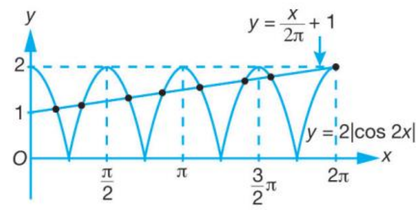


$3\pi \cos 2x = x$

$3 \cos 2x = \dfrac{x}{\pi}$

Number of solutions
= Number of points of intersection
= 4

2



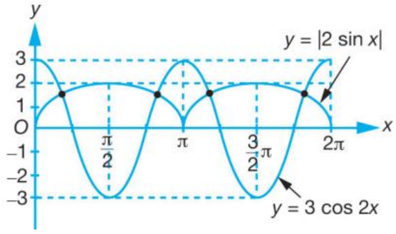
$$2\pi\left(2|\cos 2x|-1\right)=x$$

$$2|\cos 2x|-1=\frac{x}{2\pi}$$

$$2|\cos 2x|=\frac{x}{2\pi}+1$$

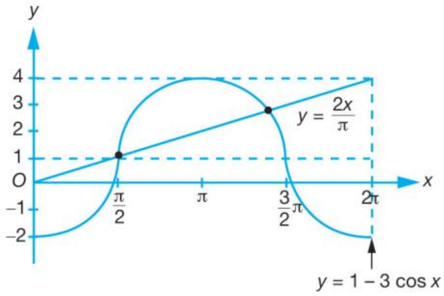
Number of solutions
= Number of points of intersection
= 8

3



Number of solutions
= Number of points of intersection
= 4

4 (a)



(b) $\pi - 3\pi\cos x = 2x$

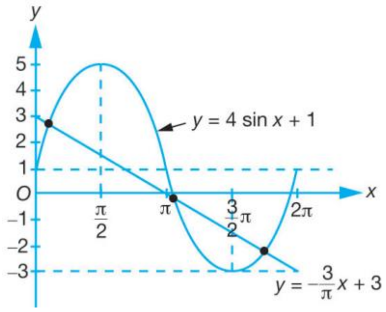
$$\pi\left(1-3\cos x\right)=2x$$

$$1-3\cos x=\frac{2x}{\pi}$$

Sketch the straight line $y=\frac{2x}{\pi}$

(c) Number of solutions
= Number of points of intersection
= 2

5 (a)



(b) $4\pi\sin x = 2\pi - 3x$

$$4\sin x=\frac{2\pi-3x}{\pi}$$

$$4\sin x+1=\frac{2\pi-3x}{\pi}+1$$

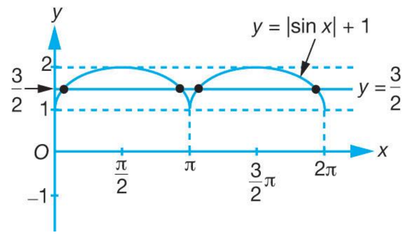
$$4\sin x+1=2-\frac{3}{\pi}x+1$$

$$4\sin x+1=3-\frac{3}{\pi}x$$

Sketch the straight line $y=-\frac{3}{\pi}x+3$

(c) Number of solutions
= Number of points of intersection
= 3

6 (a)



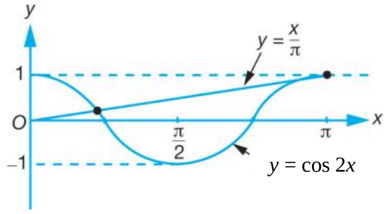
(b) $|2\sin x|+2=3$

$$|\sin x|+1=\frac{3}{2}$$

Sketch the straight line $y=\frac{3}{2}$

(c) Number of solutions
= Number of points of intersection
= 4

7 (a)



(b) $\pi \cos 2x - x = 0$

$\pi \cos 2x = x$

$\cos 2x = \dfrac{x}{\pi}$

Sketch the straight line $y = \dfrac{x}{\pi}$

Number of solutions
= Number of points of intersection
= 2

UPSKILL 6.4

1 (a) LHS

$= \dfrac{\tan^2 \theta}{1 + \tan^2 \theta}$

$= \dfrac{\tan^2 \theta}{\sec^2 \theta}$

$= \dfrac{\dfrac{\sin^2 \theta}{\cos^2 \theta}}{\dfrac{1}{\cos^2 \theta}}$

$= \sin^2 \theta$

= RHS

(b) LHS

$= \dfrac{\sec \theta}{\sec \theta - \cos \theta}$

$= \dfrac{\dfrac{1}{\cos \theta}}{\dfrac{1}{\cos \theta} - \cos \theta}$

$= \dfrac{\dfrac{1}{\cos \theta}}{\dfrac{1 - \cos^2 \theta}{\cos \theta}}$

$= \dfrac{1}{\sin^2 \theta}$

$= \operatorname{cosec}^2 \theta$

= RHS

(c) LHS

$= \sin \theta \tan \theta + \cos \theta$

$= \sin \theta \left(\dfrac{\sin \theta}{\cos \theta} \right) + \cos \theta$

$= \dfrac{\sin^2 \theta + \cos^2 \theta}{\cos \theta}$

$= \dfrac{1}{\cos \theta}$

$= \sec \theta$

= RHS

(d) LHS

$= \tan \theta + \cot \theta$

$= \dfrac{\sin \theta}{\cos \theta} + \dfrac{\cos \theta}{\sin \theta}$

$= \dfrac{\sin^2 \theta + \cos^2 \theta}{\sin \theta \cos \theta}$

$= \dfrac{1}{\sin \theta \cos \theta}$

$= \operatorname{cosec} \theta \sec \theta$

= RHS

(e) LHS

$= (\sin \theta + \cos \theta)^2 + (\sin \theta - \cos \theta)^2$

$= \sin^2 \theta + \cos^2 \theta + 2 \sin \theta \cos \theta +$

$\sin^2 \theta + \cos^2 \theta - 2 \sin \theta \cos \theta$

$= 1 + 1$

= 2

= RHS

(f) LHS

$= \dfrac{\operatorname{cosec} \theta}{\operatorname{cosec} \theta - \sin \theta}$

$= \dfrac{\dfrac{1}{\sin \theta}}{\dfrac{1}{\sin \theta} - \sin \theta}$

$= \dfrac{\dfrac{1}{\sin \theta}}{\dfrac{1 - \sin^2 \theta}{\sin \theta}}$

$= \dfrac{1}{\cos^2 \theta}$

$= \sec^2 \theta$

= RHS

(g) LHS

$= \dfrac{\sin \theta}{\operatorname{cosec} \theta - \cot \theta}$

$= \dfrac{\sin \theta}{\dfrac{1}{\sin \theta} - \dfrac{\cos \theta}{\sin \theta}}$

$= \dfrac{\sin \theta}{\dfrac{1 - \cos \theta}{\sin \theta}}$

$= \dfrac{\sin^2 \theta}{1 - \cos \theta}$

$= \dfrac{1 - \cos^2 \theta}{1 - \cos \theta}$

$$=\frac{(1+\cos\theta)(1-\cos\theta)}{1-\cos\theta}$$
$$=1+\cos\theta$$
= RHS

(h) LHS
$$=\frac{1-\sin\theta}{\cos\theta}+\frac{\cos\theta}{1-\sin\theta}$$
$$=\frac{(1-\sin\theta)^2+\cos^2\theta}{\cos\theta\,(1-\sin\theta)}$$
$$=\frac{1-2\sin\theta+\sin^2\theta+\cos^2\theta}{\cos\theta\,(1-\sin\theta)}$$
$$=\frac{1-2\sin\theta+1}{\cos\theta\,(1-\sin\theta)}$$
$$=\frac{2-2\sin\theta}{\cos\theta\,(1-\sin\theta)}$$
$$=\frac{2(1-\sin\theta)}{\cos\theta\,(1-\sin\theta)}$$
$$=\frac{2}{\cos\theta}$$
$$=2\sec\theta$$
= RHS

(i) LHS
$$=\cot^2\theta-\cos^2\theta$$
$$=\frac{\cos^2\theta}{\sin^2\theta}-\cos^2\theta$$
$$=\frac{\cos^2\theta-\cos^2\theta\sin^2\theta}{\sin^2\theta}$$
$$=\frac{\cos^2\theta\,(1-\sin^2\theta)}{\sin^2\theta}$$
$$=\frac{\cos^2\theta}{\sin^2\theta}\times\frac{\cos^2\theta}{1}$$
$$=\cos^2\theta\cot^2\theta$$
= RHS

(j) LHS
$$=\frac{1}{1+\cos\theta}+\frac{1}{1-\cos\theta}$$
$$=\frac{1-\cos\theta+1+\cos\theta}{(1+\cos\theta)(1-\cos\theta)}$$
$$=\frac{2}{1-\cos^2\theta}$$
$$=\frac{2}{\sin^2\theta}$$
$$=2\,\mathrm{cosec}^2\,\theta$$
= RHS

(k) LHS
$$=\frac{1-\tan^2\theta}{1+\tan^2\theta}$$
$$=\frac{1-\tan^2\theta}{\sec^2\theta}$$
$$=\frac{1-\frac{\sin^2\theta}{\cos^2\theta}}{\frac{1}{\cos^2\theta}}$$
$$=\frac{\frac{\cos^2\theta-\sin^2\theta}{\cos^2\theta}}{\frac{1}{\cos^2\theta}}$$
$$=\cos^2\theta-\sin^2\theta$$
= RHS

(l) LHS
$$=\sec^2\theta+\cot^2\theta$$
$$=1+\tan^2\theta+\mathrm{cosec}^2 x-1$$
$$=\tan^2\theta+\mathrm{cosec}^2\theta$$
= RHS

UPSKILL 6.5

1 (a) $\sin 34^\circ\cos 46^\circ+\cos 34^\circ\sin 46^\circ$
$$=\sin\,(34^\circ+46^\circ)$$
$$=\sin\,80^\circ$$

(b) $\sin 53^\circ\cos 23^\circ-\cos 53^\circ\sin 23^\circ$
$$=\sin\,(53^\circ-23^\circ)$$
$$=\sin 30^\circ$$

(c) $\cos 63^\circ\,\cos 48^\circ+\sin\,63^\circ\sin 48^\circ$
$$=\cos\,(63^\circ-48^\circ)$$
$$=\cos 15^\circ$$

(d) $\cos 65^\circ\,\cos 35^\circ-\sin\,65^\circ\sin 35^\circ$
$$=\cos\,(65^\circ+35^\circ)$$
$$=\cos 100^\circ$$

(e) $\dfrac{\tan 27^\circ+\tan 78^\circ}{1-\tan 27^\circ\,\tan 78^\circ}$
$$=\tan\,(27^\circ+78^\circ)$$
$$=\tan\,105^\circ$$

(f) $\dfrac{\tan 92^\circ - \tan 26^\circ}{1 + \tan 92^\circ \tan 26^\circ}$

$= \tan(92^\circ - 26^\circ)$

$= \tan 66^\circ$

2 (a) $2 \sin 51^\circ \cos 51^\circ$

$= \sin 2(51^\circ)$

$= \sin 102^\circ$

(b) $\cos^2 62^\circ - \sin^2 62^\circ$

$= \cos 2(62^\circ)$

$= \cos 124^\circ$

(c) $2\cos^2 110^\circ - 1$

$= \cos 2(110^\circ)$

$= \cos 220^\circ$

(d) $1 - 2\sin^2 85^\circ$

$= \cos 2(85^\circ)$

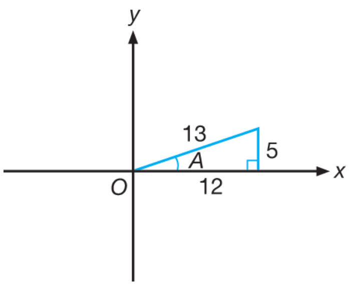
$= \cos 170^\circ$

(e) $\dfrac{2\tan 76^\circ}{1 - \tan^2 76^\circ}$

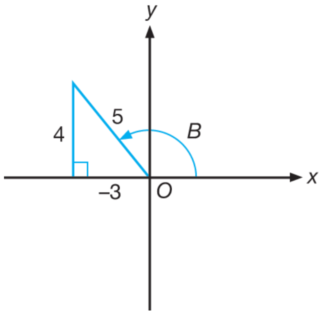
$= \tan 2(76^\circ)$

$= \tan 152^\circ$

3



$\sin A = \dfrac{5}{13}$, $\cos A = \dfrac{12}{13}$, $\tan A = \dfrac{5}{12}$



$\sin B = \dfrac{4}{5}$, $\cos B = -\dfrac{3}{5}$, $\tan B = -\dfrac{4}{3}$

(a) $\sin (A + B)$

$= \sin A \cos B + \cos A \sin B$

$= \left(\dfrac{5}{13}\right)\left(-\dfrac{3}{5}\right) + \left(\dfrac{12}{13}\right)\left(\dfrac{4}{5}\right)$

$= \dfrac{33}{65}$

(b) $\cos (A + B)$

$= \cos A \cos B - \sin A \sin B$

$= \left(\dfrac{12}{13}\right)\left(-\dfrac{3}{5}\right) - \left(\dfrac{5}{13}\right)\left(\dfrac{4}{5}\right)$

$= -\dfrac{56}{65}$

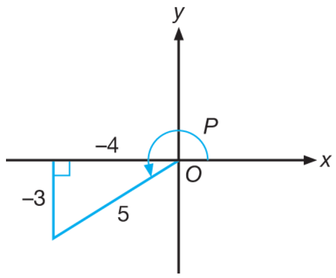
(c) $\tan (A + B)$

$= \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$

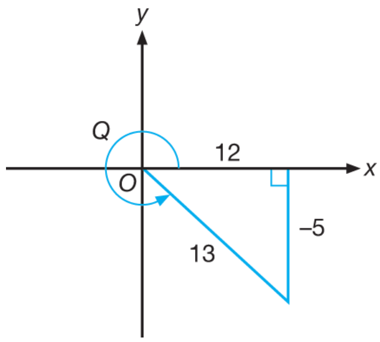
$= \dfrac{\dfrac{5}{12} + \left(-\dfrac{4}{3}\right)}{1 - \left(\dfrac{5}{12}\right)\left(-\dfrac{4}{3}\right)}$

$= \dfrac{-\dfrac{11}{12}}{\dfrac{14}{9}} = -\dfrac{33}{56}$

4



$\sin P = -\frac{3}{5}, \quad \cos P = -\frac{4}{5}, \quad \tan P = \frac{3}{4}$



$\sin Q = -\frac{5}{13}, \quad \cos Q = \frac{12}{13}, \quad \tan Q = -\frac{5}{12}$

(a) $\sin\ (P - Q)$

$= \sin P \cos Q - \cos P \sin Q$

$= \left(-\frac{3}{5}\right)\left(\frac{12}{13}\right) - \left(-\frac{4}{5}\right)\left(-\frac{5}{13}\right)$

$= -\frac{56}{65}$

(b) $\cos\ (P - Q)$

$= \cos P \cos Q + \sin P \sin Q$

$= \left(-\frac{4}{5}\right)\left(\frac{12}{13}\right) + \left(-\frac{3}{5}\right)\left(-\frac{5}{13}\right)$

$= -\frac{33}{65}$

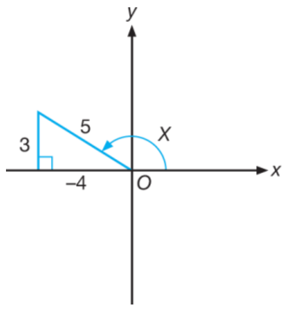
(c) $\tan\ (P - Q)$

$= \frac{\tan P - \tan Q}{1 + \tan P \tan Q}$

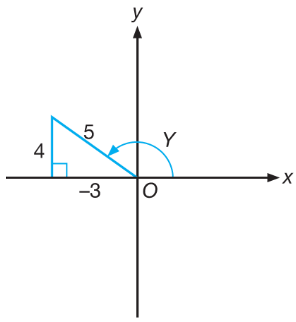
$= \frac{\frac{3}{4} - \left(-\frac{5}{12}\right)}{1 + \left(\frac{3}{4}\right)\left(-\frac{5}{12}\right)}$

$= \frac{\frac{7}{6}}{\frac{11}{16}} = \frac{56}{33}$

5



$\sin X = \frac{3}{5}, \quad \cos X = -\frac{4}{5}, \quad \tan X = -\frac{3}{4}$



$\sin Y = \frac{4}{5}, \quad \cos Y = -\frac{3}{5}, \quad \tan Y = -\frac{4}{3}$

(a) $\cos 2X = 1 - 2\sin^2 X = 1 - 2\left(\frac{3}{5}\right)^2 = \frac{7}{25}$

(b) $\sin\ (2X + Y)$

$= \sin 2X \cos Y + \cos 2X \sin Y$

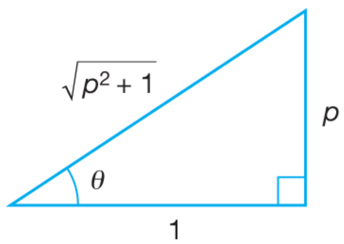
$= 2\sin X \cos X\ (\cos Y) + \cos 2X \sin Y$

$= 2\left(\frac{3}{5}\right)\left(-\frac{4}{5}\right)\left(-\frac{3}{5}\right) + \frac{7}{25}\left(\frac{4}{5}\right)$

$= \frac{72}{125} + \frac{28}{125}$

$= \frac{4}{5}$

6



(a) $\sin\theta = \frac{p}{\sqrt{p^2 + 1}}$

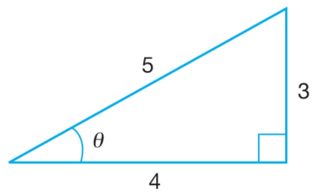
(b) $\cos(-\theta) = \cos\theta = \dfrac{1}{\sqrt{p^2+1}}$

(c) $\cos 2\theta = 1 - 2\sin^2\theta$

$= \dfrac{p^2 + 1 - 2p^2}{p^2+1}$

$= \dfrac{1-p^2}{1+p^2}$

7



(a) $\cos 2\theta = 2\cos^2\theta - 1$

$= 2\left(\dfrac{4}{5}\right)^2 - 1$

$= \dfrac{7}{25}$

(b) $\sec(180^\circ - \theta)$

$= \dfrac{1}{\cos(180^\circ - \theta)}$

$= \dfrac{1}{\cos 180^\circ \cos\theta + \sin 180^\circ \sin\theta}$

$= \dfrac{1}{-\cos\theta + 0}$

$= -\dfrac{1}{\frac{4}{5}}$

$= -\dfrac{5}{4}$

(c) $\tan(90^\circ - \theta)$

$= \cot\theta$

$= \dfrac{1}{\tan\theta}$

$= \dfrac{1}{\frac{3}{4}}$

$= \dfrac{4}{3}$

8 (a) $\cos 2A = -\dfrac{7}{25}$

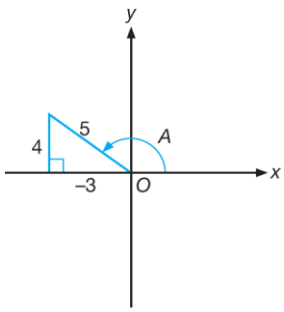
$1 - 2\sin^2 A = -\dfrac{7}{25}$

$2\sin^2 A = \dfrac{32}{25}$

$\sin^2 A = \dfrac{16}{25}$

$\sin A = \dfrac{4}{5}$

(b)



$\cos A = -\dfrac{3}{5}$

(c) $\tan A = -\dfrac{4}{3}$

9 (a) LHS

$= (\sin\theta + \cos\theta)^2$

$= \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta$

$= 1 + \sin 2\theta$

= RHS

(b) LHS

$= \cos^4\theta - \sin^4\theta$

$= (\cos^2\theta + \sin^2\theta)(\cos^2\theta - \sin^2\theta)$

$= (1)(\cos 2\theta)$

$= \cos 2\theta$

= RHS

(c) LHS

$= \tan\theta + \cot\theta$

$= \dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{\sin\theta}$

$= \dfrac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$

$= \dfrac{1}{\sin\theta\cos\theta}$

$= \dfrac{2}{2\sin\theta\cos\theta}$

$= \frac{2}{\sin 2\theta}$
$= 2 \text{ cosec } 2\theta$
= RHS

(d) LHS

$= \frac{2\sin\theta}{2\cos\theta - \sec\theta}$

$= \frac{2\sin\theta}{2\cos\theta - \frac{1}{\cos\theta}}$

$= \frac{2\sin\theta}{\frac{2\cos^2\theta - 1}{\cos\theta}}$

$= \frac{2\sin\theta\cos\theta}{\cos 2\theta}$

$= \frac{\sin 2\theta}{\cos 2\theta}$

$= \tan 2\theta$
= RHS

(e) LHS

$= \frac{1-\tan^2\theta}{1+\tan^2\theta}$

$= \frac{1-\frac{\sin^2\theta}{\cos^2\theta}}{\sec^2\theta}$

$= \frac{\frac{\cos^2\theta - \sin^2\theta}{\cos^2\theta}}{\frac{1}{\cos^2\theta}}$

$= \cos 2\theta$
= RHS

10 (a) LHS

$= \frac{\sin 2\theta}{1-\cos 2\theta}$

$= \frac{2\sin\theta\cos\theta}{1-(1-2\sin^2\theta)}$

$= \frac{2\sin\theta\cos\theta}{2\sin^2\theta}$

$= \frac{\cos\theta}{\sin\theta}$

$= \cot\theta$
= RHS

(b) LHS

$= \frac{1+\cos 2\theta}{1-\cos 2\theta}$

$= \frac{1+(2\cos^2\theta - 1)}{1-(1-2\sin^2\theta)}$

$= \frac{2\cos^2\theta}{2\sin^2\theta}$

$= \frac{\cos^2\theta}{\sin^2\theta}$

$= \cot^2\theta$
= RHS

(c) LHS

$= \frac{\sin\theta + \sin 2\theta}{1+\cos\theta+\cos 2\theta}$

$= \frac{\sin\theta + 2\sin\theta\cos\theta}{1+\cos\theta + 2\cos^2\theta - 1}$

$= \frac{\sin\theta(1+2\cos\theta)}{\cos\theta(1+2\cos\theta)}$

$= \frac{\sin\theta}{\cos\theta}$

$= \tan\theta$
= RHS

11 (a) LHS

$= \frac{2(\cos\theta+\sin\theta)}{\sin 2\theta + \cos 2\theta + 1}$

$= \frac{2(\cos\theta+\sin\theta)}{\sin 2\theta + (2\cos^2\theta - 1) + 1}$

$= \frac{2(\cos\theta+\sin\theta)}{2\sin\theta\cos\theta + 2\cos^2\theta}$

$= \frac{2(\cos\theta+\sin\theta)}{2\cos\theta(\sin\theta+\cos\theta)}$

$= \frac{1}{\cos\theta}$

$= \sec\theta$
= RHS

(b) LHS

$= \frac{\sin\theta}{1-\cos 2\theta} + \frac{\cos\theta}{1+\cos 2\theta}$

$= \frac{\sin\theta}{1-(1-2\sin^2\theta)} + \frac{\cos\theta}{1+(2\cos^2\theta - 1)}$

$= \frac{\sin\theta}{2\sin^2\theta} + \frac{\cos\theta}{2\cos^2\theta}$

$= \frac{1}{2\sin\theta} + \frac{1}{2\cos\theta}$

$= \dfrac{\cos\theta + \sin\theta}{2\sin\theta\cos\theta}$

$= \dfrac{\cos\theta + \sin\theta}{\sin 2\theta}$

= RHS

(c) LHS

$= \tan 2\theta\ (2\cos\theta - \sec\theta)$

$= \dfrac{\sin 2\theta}{\cos 2\theta}\left(2\cos\theta - \dfrac{1}{\cos\theta}\right)$

$= \dfrac{\sin 2\theta}{\cos 2\theta}\left(\dfrac{2\cos^2\theta - 1}{\cos\theta}\right)$

$= \dfrac{2\sin\theta\cos\theta}{\cos 2\theta}\left(\dfrac{\cos 2\theta}{\cos\theta}\right)$

$= 2\sin\theta$

= RHS

12 (a) LHS

$= \dfrac{\cos(A+B)}{\sin A\cos B}$

$= \dfrac{\cos A\cos B - \sin A\sin B}{\sin A\cos B}$

$= \dfrac{\cos A\cos B}{\sin A\cos B} - \dfrac{\sin A\sin B}{\sin A\cos B}$

$= \dfrac{\cos A}{\sin A} - \dfrac{\sin B}{\cos B}$

$= \cot A - \tan B$

= RHS

(b) LHS

$= \dfrac{\cos(A-B) - \cos(A+B)}{\sin(A+B) + \sin(A-B)}$

$= \dfrac{\cos A \text{ kos } B + \sin A\sin B - (\cos A\cos B - \sin A\sin B)}{\sin A\cos B + \cos A\sin B + \sin A\cos B - \cos A\sin B}$

$= \dfrac{\sin A\sin B + \sin A\sin B}{\sin A\cos B + \sin A\cos B}$

$= \dfrac{2\sin A\sin B}{2\sin A\cos B}$

$= \dfrac{\sin B}{\cos B}$

$= \tan B$

= RHS

13 (a) LHS

$= \dfrac{1-\cos\theta}{\sin\theta}$

$= \dfrac{1-\left(1-2\sin^2\frac{\theta}{2}\right)}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}$

$= \dfrac{2\sin^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}$

$= \dfrac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}}$

$= \tan\dfrac{\theta}{2}$

(b) LHS

$= \dfrac{1-\cos\theta}{1+\cos\theta}$

$= \dfrac{1-\left(1-2\sin^2\frac{\theta}{2}\right)}{1+\left(2\cos^2\frac{\theta}{2}-1\right)}$

$= \dfrac{2\sin^2\frac{\theta}{2}}{2\cos^2\frac{\theta}{2}}$

$= \tan^2\dfrac{\theta}{2}$

= RHS

UPSKILL 6.6a

1 (a) $\cot\theta = -2\cos\theta$

$\dfrac{\cos\theta}{\sin\theta} = -2\cos\theta$

$\cos\theta = -2\sin\theta\cos\theta$

$\cos\theta + 2\sin\theta\cos\theta = 0$

$\cos\theta\,(1+2\sin\theta) = 0$

$\text{kos }\theta = 0 \text{ or } \sin\theta = -\dfrac{1}{2}$

When $\cos\theta = 0$,

$\theta = 90^{\circ}, 270^{\circ}$

When $\sin\theta = -\dfrac{1}{2}$,

Basic $\angle = 30^{\circ}$

$\theta = 210^o, 330^o$

$\therefore \theta = 90^o, 210^o, 270^o, 330^o$

(b) $3\sin\theta = \tan\theta$

$3\sin\theta = \dfrac{\sin\theta}{\cos\theta}$

$3\sin\theta\cos\theta = \sin\theta$

$3\sin\theta\cos\theta - \sin\theta = 0$

$\sin\theta(3\cos\theta - 1) = 0$

$\sin\theta = 0$ or $\cos\theta = \dfrac{1}{3}$

When $\sin\theta = 0$,

$\theta = 0^o, 180^o, 360^o$

When $\cos\theta = \dfrac{1}{3}$,

Basic $\angle$ = 70.53^o

$\theta = 70.53^o,\ 289.47^o$

$\therefore \theta = 0^o, 70.53^o, 180^o, 289.47^o, 360^o$

(c) $3\sec\theta = 4\cos\theta$

$\dfrac{3}{\cos\theta} = 4\cos\theta$

$3 = 4\cos^2\theta$

$\cos^2\theta = \dfrac{3}{4}$

$\cos\theta = \pm\dfrac{\sqrt{3}}{2}$

$\theta = 30^o,\ 150^o,\ 210^o,\ 330^o$

(d) $16\tan\theta = \text{kot}\ \theta$

$16\tan\theta = \dfrac{1}{\tan\theta}$

$\tan^2\theta = \dfrac{1}{16}$

$\tan\theta = \pm\dfrac{1}{4}$

$\therefore\ \theta = 14.04^o, 165.96^o, 194.04^o, 345.96^o$

2 (a) $3\sin^2\theta - 2\sin\theta - 1 = 0$

$(\sin\theta - 1)(3\sin\theta + 1) = 0$

$\sin\theta = 1$ or $\sin\theta = -\dfrac{1}{3}$

When $\sin\theta = 1$,

$\theta = 90^o$

When $\sin\theta = -\dfrac{1}{3}$,

Basic $\angle$ = 19.47^o

$\theta = 199.47^o,\ 340.53^o$

$\therefore\ \theta = 90^o,\ 199.47^o\ 340.53^o$

(b) $2\sin\theta = \text{cosec}\ \theta + 1$

$2\sin\theta = \dfrac{1}{\sin\theta} + 1$

$2\sin^2\theta = 1 + \sin\theta$

$2\sin^2\theta - \sin\theta - 1 = 0$

$(\sin\theta - 1)(2\sin\theta + 1) = 0$

$\sin\theta = 1$ or $\sin\theta = -\dfrac{1}{2}$

When $\sin\theta = 1$,

$\theta = 90^o$

When $\sin\theta = -\dfrac{1}{2}$,

Basic $\angle$ = 30^o

$\theta = 210^o, 330^o$

$\therefore\ \theta = 90^o, 210^o, 330^o$

(c) $3\cos^2\theta + \sin\theta = 1$

$3(1 - \sin^2\theta) + \sin\theta - 1 = 0$

$3 - 3\sin^2\theta + \sin\theta - 1 = 0$

$3\sin^2\theta - \sin\theta - 2 = 0$

$(\sin\theta - 1)(3\sin\theta + 2) = 0$

$\sin\theta = 1$ or $\sin\theta = -\dfrac{2}{3}$

When $\sin\theta = 1$,

$\theta = 90^o$

When $\sin\theta = -\dfrac{2}{3}$,

Basic $\angle$ = 41.81^o

$\theta = 221.81^o, 318.19^o$

$\therefore\ \theta = 90^o,\ 221.81^o, 318.19^o$

(d) $5\sin^2\theta = 2(1 + \cos\theta)$

$5(1 - \cos^2\theta) = 2 + 2\cos\theta$

$5 - 5\cos^2\theta - 2\cos\theta - 2 = 0$

$5\cos^2\theta + 2\cos\theta - 3 = 0$

$(5\cos\theta - 3)(\cos\theta + 1) = 0$

$\cos\theta = \dfrac{3}{5}$ or $\cos\theta = -1$

When $\cos\theta = \dfrac{3}{5}$,

Basic $\angle$ = 53.13^o

$\theta = 53.13^o$ or 306.87^o

When $\cos\theta = -1$,

$\theta = 180^o$

$\therefore\ \theta = 53.13^o,\ \theta = 180^o,\ 306.87^o$

(e) $2\sec\theta = 1 + \cos\theta$

$\frac{2}{\cos\theta} = 1 + \cos\theta$

$2 = \cos\theta + \cos^2\theta$

$\cos^2\theta + \cos\theta - 2 = 0$

$(\cos\theta - 1)(\cos\theta + 2) = 0$

$\cos\theta = 1$ or $\cos\theta = -2$

$\cos\theta = -2$ does not have solution because the minimum value of $\cos\theta = -1$.

When $\cos\theta = 1$,

$\theta = 0^o, 360^o$

(f) $2\cot\theta = \tan\theta + 1$

$\frac{2}{\tan\theta} = \tan\theta + 1$

$2 = \tan^2\theta + \tan\theta$

$\tan^2\theta + \tan\theta - 2 = 0$

$(\tan\theta - 1)(\tan\theta + 2) = 0$

$\tan\theta = 1$ or $\tan\theta = -2$

When $\tan\theta = 1$,

$\theta = 45^o, 225^o$

When $\tan\theta = -2$,

Basic $\angle$ = 63.43^o

$\theta = 116.57^o, 296.57^o$

$\therefore\ \theta = 45^o, 116.57^o, 225^o, 296.57^o$

(g) $3\sin\theta + 1 = 2\operatorname{cosec}\theta$

$3\sin\theta + 1 = \frac{2}{\sin\theta}$

$3\sin^2\theta + \sin\theta - 2 = 0$

$(3\sin\theta - 2)(\sin\theta + 1) = 0$

$\sin\theta = \frac{2}{3}$ or $\sin\theta = -1$

When $\sin\theta = \frac{2}{3}$,

Basic $\angle$ = 41.81^o

$\theta = 41.81^o, 138.19^o$

When $\sin\theta = -1$,

$\theta = 270^o$

$\therefore\ 41.81^o, 270^o, 138.19^o$

3 (a) $3\sec^2\theta = 5(1 + \tan\theta)$

$3(1 + \tan^2\theta) = 5 + 5\tan\theta$

$3 + 3\tan^2\theta - 5\tan\theta - 5 = 0$

$3\tan^2\theta - 5\tan\theta - 2 = 0$

$(\tan\theta - 2)(3\tan\theta + 1) = 0$

$\tan\theta = 2$ or $\tan\theta = -\frac{1}{3}$

When $\tan\theta = 2$,

Basic $\angle$ = 63.43^o

$\theta = 63.43^o, 243.43^o$

When $\tan\theta = -\frac{1}{3}$,

Basic $\angle$ = 18.43^o

$\theta = 161.57^o, 341.57^o$

$\therefore\ \theta = 63.43^o, 161.57^o, 243.43^o, 341.57^o$

(b) $2\cot^2\theta + 8 = 7\operatorname{cosec}\theta$

$2(\operatorname{cosec}^2\theta - 1) + 8 - 7\operatorname{cosec}\theta = 0$

$2\operatorname{cosec}^2\theta - 2 + 8 - 7\operatorname{cosec}\theta = 0$

$2\operatorname{cosec}^2\theta - 7\operatorname{cosec}\theta + 6 = 0$

$(\operatorname{cosec}\theta - 2)(2\operatorname{cosec}\theta - 3) = 0$

$\operatorname{cosec}\theta = 2$ or $\operatorname{cosec}\theta = \frac{3}{2}$

When $\operatorname{cosec}\theta = 2$,

$\frac{1}{\sin\theta} = 2$

$\sin\theta = \frac{1}{2}$

$\theta = 30^o, 150^o$

When $\operatorname{cosec}\theta = \frac{3}{2}$,

$\frac{1}{\sin\theta} = \frac{3}{2}$

$$\sin\theta = \frac{2}{3}$$

Basic $\angle$ = 41.81^{o}

$$\theta = 41.81^{o}, 138.19^{o}$$

$$\therefore\ \theta = 30^{o}, 41.81^{o}, 150^{o}, 138.19^{o}$$

4 (a)
$$4\sin\theta = \sec\theta$$
$$4\sin\theta = \frac{1}{\cos\theta}$$
$$2\,(2\sin\theta\cos\theta) = 1$$
$$2\sin 2\theta = 1$$
$$\sin 2\theta = \frac{1}{2}$$

Basic $\angle$ = 30^{o}

$$2\theta = 30^{o}, 150^{o}, 390^{o}, 510^{o}$$
$$\therefore\ \theta = 15^{o}, 75^{o}, 195^{o}, 255^{o}$$

(b)
$$\sin 2\theta + \sin\theta = 0$$
$$2\sin\theta\cos\theta + \sin\theta = 0$$
$$\sin\theta\,(2\cos\theta + 1) = 0$$
$$\sin\theta = 0 \text{ or } \cos\theta = -\frac{1}{2}$$

When $\sin\theta = 0$,
$$\theta = 0^{o}, 180^{o}, 360^{o}$$

When $\cos\theta = -\frac{1}{2}$

Basic $\angle$ = 60^{o}

$$\theta = 120^{o}, 240^{o}$$
$$\therefore\ \theta = 0^{o}, 120^{o}, 180^{o}, 240^{o}, 360^{o}$$

(c)
$$2\sin\theta = \tan\theta$$
$$2\sin\theta = \frac{\sin\theta}{\cos\theta}$$
$$2\sin\theta\cos\theta = \sin\theta$$
$$2\sin\theta\cos\theta - \sin\theta = 0$$
$$\sin\theta\,(2\cos\theta - 1) = 0$$
$$\sin\theta = 0 \text{ or } \cos\theta = \frac{1}{2}$$

When $\sin\theta = 0$,
$$\theta = 0^{o}, 180^{o}, 360^{o}$$

When $\cos\theta = \frac{1}{2}$,
$$\theta = 60^{o}, 300^{o}$$
$$\therefore\ \theta = 0^{o}, 60^{o}, 180^{o}, 300^{o}, 360^{o}$$

5 (a)
$$3\cos 2\theta + \sin\theta - 2 = 0$$
$$3\left(1 - 2\sin^{2}\theta\right) + \sin\theta - 2 = 0$$
$$3 - 6\sin^{2}\theta + \sin\theta - 2 = 0$$
$$6\sin^{2}\theta - \sin\theta - 1 = 0$$
$$(2\sin\theta - 1)(3\sin\theta + 2) = 0$$
$$\sin\theta = \frac{1}{2} \text{ or } \sin\theta = -\frac{2}{3}$$

When $\sin\theta = \frac{1}{2}$,
$$\theta = 30^{o}, 150^{o}$$

When $\sin\theta = -\frac{1}{3}$,

Basic $\angle$ = 19.47^{o}

$$\theta = 199.47^{o}, 340.53^{o}$$
$$\therefore\ \theta = 30^{o}, 150^{o}, 199.47^{o}, 340.53^{o}$$

(b)
$$3\cos 2\theta + 8\sin\theta + 5 = 0$$
$$3\left(1 - 2\sin^{2}\theta\right) + 8\sin\theta + 5 = 0$$
$$3 - 6\sin^{2}\theta + 8\sin\theta + 5 = 0$$
$$6\sin^{2}\theta - 8\sin\theta - 8 = 0$$
$$3\sin^{2}\theta - 4\sin\theta - 4 = 0$$
$$(\sin\theta - 2)(3\sin\theta + 2) = 0$$
$$\sin\theta = 2 \text{ or } \sin\theta = -\frac{2}{3}$$

$\sin\theta = 2$ does not have solution because the maximum value of sin θ is 1.

$$\sin\theta = -\frac{2}{3}$$

Basic $\angle$ = 41.81^{o}

$$\theta = 221.81^{o} \text{ or } 318.19^{o}$$

(c)
$$\cos 2\theta + \cos\theta = 0$$
$$2\cos^{2}\theta - 1 + \cos\theta = 0$$
$$2\cos^{2}\theta + \cos\theta - 1 = 0$$
$$(2\cos\theta - 1)(\cos\theta + 1) = 0$$
$$\cos\theta = \frac{1}{2} \text{ or } \cos\theta = -1$$

When $\cos\theta = \frac{1}{2}$,
$$\theta = 60^{o}, 300^{o}$$

When $\cos\theta = -1$,
$$\theta = 180^{o}$$
$$\therefore\ \theta = 60^{o}, 180^{o}, 300^{o}$$

(d) $6\cos 2\theta - 17\cos\theta + 12 = 0$

$6(2\cos^2\theta - 1) - 17\cos\theta + 12 = 0$

$12\cos^2\theta - 6 - 17\cos\theta + 12 = 0$

$12\cos^2\theta - 17\cos\theta + 6 = 0$

$(4\cos\theta - 3)(3\cos\theta - 2) = 0$

$\cos\theta = \frac{3}{4}$ or $\cos\theta = \frac{2}{3}$

When $\cos\theta = \frac{3}{4}$,

Basic $\angle$ = 41.41^o

$\theta = 41.41^o,\ 318.59^o$

When $\cos\theta = \frac{2}{3}$,

Basic $\angle$ = 48.19^o

$\theta = 48.19^o,\ 311.81^o$

$\therefore\ \theta = 41.41^o,\ 48.19^o,\ 311.81^o\ 318.59^o$

6 (a) $\cos(\theta + 60^o) = \sin\theta$

$\cos\theta\cos 60^o - \sin\theta\sin 60^o = \sin\theta$

$\frac{1}{2}\cos\theta - \frac{\sqrt{3}}{2}\sin\theta = \sin\theta$

$\frac{1}{2}\cos\theta = \left(1 + \frac{\sqrt{3}}{2}\right)\sin\theta$

$(2 + \sqrt{3})\sin\theta = \cos\theta$

$\frac{\sin\theta}{\cos\theta} = \frac{1}{(2+\sqrt{3})}$

$\tan\theta = 0.2679$

Basic $\angle$ $= 15^o$

$\theta = 15^o,\ 195^o$

(b) $2\cos(\theta + 30^o) = \sin\theta$

$2(\cos\theta\cos 30^o - \sin\theta\sin 30^o) = \sin\theta$

$2\left(\frac{\sqrt{3}}{2}\cos\theta - \frac{1}{2}\sin\theta\right) = \sin\theta$

$\sqrt{3}\cos\theta - \sin\theta = \sin\theta$

$\sqrt{3}\cos\theta = 2\sin\theta$

$\frac{\sin\theta}{\cos\theta} = \frac{\sqrt{3}}{2}$

$\tan\theta = \frac{\sqrt{3}}{2}$

Basic $\angle$ = 40.89^o

$\theta = 40.89^o,\ 220.89^o$

(c) $4\sin(\theta - 30^o) = \cos\theta$

$4(\sin\theta\cos 30^o - \cos\theta\sin 30^o) = \cos\theta$

$4\left(\frac{\sqrt{3}}{2}\sin\theta - \frac{1}{2}\cos\theta\right) = \text{kos}\,\theta$

$2\sqrt{3}\sin\theta - 2\cos\theta = \cos\theta$

$2\sqrt{3}\sin\theta = 3\,\text{kos}\,\theta$

$\frac{\sin\theta}{\cos\theta} = \frac{3}{2\sqrt{3}}$

$\tan\theta = 0.8660$

Basic $\angle$ = 40.89^o

$\theta = 40.89^o,\ 220.89^o$

UPSKILL 6.6b

1 $I = A\sin 120\pi t$

$120\pi t = 2\pi$

$t = \frac{2\pi}{120\pi}$

$t = \frac{1}{60}$

Period = $\frac{1}{60}$ second

2 $y = f(x) = 6 + 2\sin\frac{\pi}{6}t$

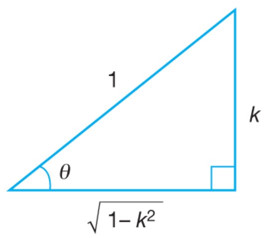
$\frac{\pi}{6}t = 2\pi$

$t = 12$

Period = 12 hours

Summative Practice 6

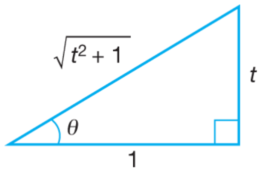
1



(a) $\tan\theta = \frac{k}{\sqrt{1-k^2}}$

(b) $\sec(-\theta) = \frac{1}{\cos(-\theta)} = \frac{1}{\cos\theta} = \frac{1}{\sqrt{1-k^2}}$

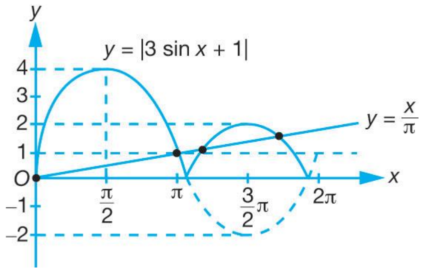
2



(a) $\cot(-\theta) = \dfrac{1}{\tan(-\theta)} = -\dfrac{1}{\tan\theta} = -\dfrac{1}{t}$

(b) $\cos(90^\circ - \theta) = \sin\theta = \dfrac{t}{\sqrt{t^2+1}}$

3 (a)



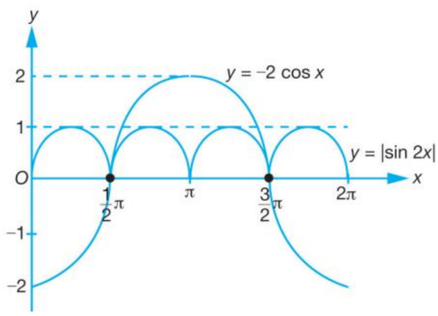
(b) $\pi|3\sin x + 1| - x = 0$

$|3\sin x + 1| = \dfrac{x}{\pi}$

Sketch the straight line $y = \dfrac{x}{\pi}$

Number of solutions
= Number of points of intersection
= 4

4 (a)



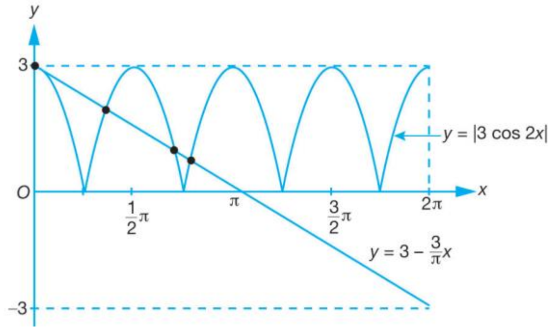
(b) $|\sin 2x| + 2\cos x = 0$

$|\sin 2x| = -2\cos x$

Sketch the straight line $y = |\sin 2x|$

Number of solutions
= Number of points of intersection
= 2

5 (a)



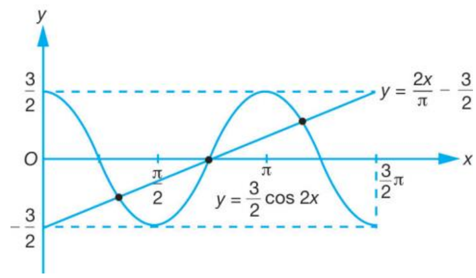
(b) $3 - |3\cos 2x| = \dfrac{3}{\pi}x$

$|3\cos 2x| = 3 - \dfrac{3}{\pi}x$

Sketch the straight line $y = 3 - \dfrac{3}{\pi}x$

Number of solutions
= Number of points of intersection
= 4

6 (a)



(b) $\left(\dfrac{4}{3\pi}\right)x - \cos 2x = 1$

$\cos 2x = \left(\dfrac{4}{3\pi}\right)x - 1$

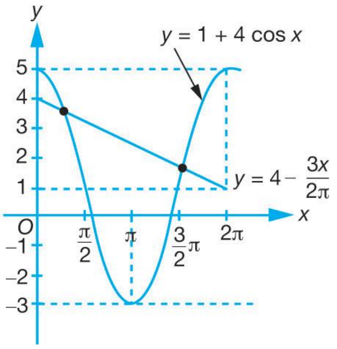
$\dfrac{3}{2}\cos 2x = \dfrac{3}{2}\left[\left(\dfrac{4}{3\pi}\right)x - 1\right]$

$\dfrac{3}{2}\cos 2x = \left(\dfrac{2}{\pi}\right)x - \dfrac{3}{2}$

Sketch the straight line $y = \left(\dfrac{2}{\pi}\right)x - \dfrac{3}{2}$

Number of solutions
= Number of points of intersection
= 3

7 (a)



(b) $4\pi \cos x = 3\pi - \frac{3}{2}x$

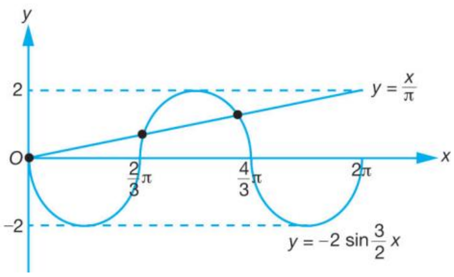
$4 \cos x = 3 - \frac{3}{2}\left(\frac{x}{\pi}\right)$

$1 + 4 \cos x = 4 - \frac{3}{2}\left(\frac{x}{\pi}\right)$

Sketch the straight line $y = 4 - \frac{3x}{2\pi}$

Number of solutions
= Number of points of intersection
= 2

8 (a)



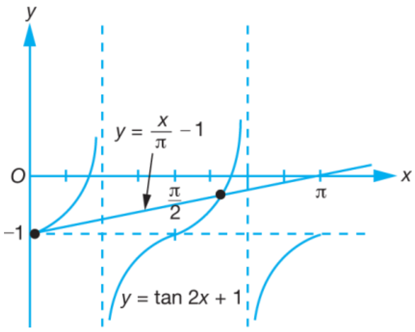
(b) $\frac{x}{\pi} + 2 \sin \frac{3}{2}x = 0$

$\frac{x}{\pi} = -2 \sin \frac{3}{2}x$

Sketch the straight line $y = \frac{x}{\pi}$

Number of solutions
= Number of points of intersection
= 3

9 (a)



(b) $\pi \tan 2x - x = 0$

$\pi \tan 2x = x$

$\tan 2x = \frac{x}{\pi}$

$\tan 2x - 1 = \frac{x}{\pi} - 1$

Sketch the straight line $y = \frac{x}{\pi} - 1$

Number of solutions
= Number of points of intersection
= 2

10 (a) LHS

$= (\tan x + \sec x)^2$

$= \left(\frac{\sin x}{\cos x} + \frac{1}{\cos x}\right)^2$

$= \left(\frac{\sin x + 1}{\cos x}\right)^2$

$= \frac{(\sin x + 1)^2}{\cos^2 x}$

$= \frac{(\sin x + 1)^2}{1 - \sin^2 x}$

$= \frac{(1 + \sin x)(1 + \sin x)}{(1 + \sin x)(1 - \sin x)}$

$= \frac{1 + \sin x}{1 - \sin x}$

= RHS

(b) LHS

$= \sin x + \operatorname{cosec} x \cos^2 x$

$= \sin x + \frac{\cos^2 x}{\sin x}$

$= \frac{\sin^2 x + \cos^2 x}{\sin x}$

$= \frac{1}{\sin x}$

$= \operatorname{cosec} x$

= RHS

(c) LHS

$$= \frac{\cos x}{1-\tan x} - \frac{\sin x}{\cot x - 1}$$

$$= \frac{\cos x}{1-\frac{\sin x}{\cos x}} - \frac{\sin x}{\frac{\cos x}{\sin x} - 1}$$

$$= \frac{\cos x}{\frac{\cos x - \sin x}{\cos x}} - \frac{\sin x}{\frac{\cos x - \sin x}{\sin x}}$$

$$= \frac{\cos^2 x}{\cos x - \sin x} - \frac{\sin^2 x}{\cos x - \sin x}$$

$$= \frac{\cos^2 x - \sin^2 x}{\cos x - \sin x}$$

$$= \frac{(\cos x + \sin x)(\cos x - \sin x)}{\text{kos } x - \sin x}$$

$$= \cos x + \sin x$$

= RHS

11 (a) LHS

$$= \tan\theta \sec\theta - \sin\theta$$

$$= \left(\frac{\sin\theta}{\cos\theta}\right)\left(\frac{1}{\cos\theta}\right) - \sin\theta$$

$$= \left(\frac{\sin\theta}{\cos^2\theta}\right) - \sin\theta$$

$$= \frac{\sin\theta - (\sin\theta)(\cos^2\theta)}{\cos^2\theta}$$

$$= \frac{\sin\theta(1-\cos^2\theta)}{\cos^2\theta}$$

$$= \frac{\sin\theta(\sin^2\theta)}{\cos^2\theta}$$

$$= \frac{\sin^3\theta}{\cos^2\theta}$$

$$= \sin^3\theta \sec^2\theta$$

= RHS

(b) LHS

$$= \frac{1+\cos\theta}{\sin\theta} + \frac{\sin\theta}{1+\cos\theta}$$

$$= \frac{(1+\cos\theta)^2 + \sin^2\theta}{\sin\theta(1+\cos\theta)}$$

$$= \frac{1+2\cos\theta + \cos^2\theta + \sin^2\theta}{\sin\theta(1+\cos\theta)}$$

$$= \frac{1+2\cos\theta+1}{\sin\theta(1+\cos\theta)}$$

$$= \frac{2+2\cos\theta}{\sin\theta(1+\cos\theta)}$$

$$= \frac{2(1+\cos\theta)}{\sin\theta(1+\cos\theta)}$$

$$= \frac{2}{\sin\theta}$$

$$= 2\operatorname{cosec}\theta$$

= RHS

(c) LHS

$$= \frac{\sec\theta + \operatorname{cosec}\theta}{\tan\theta + \cot\theta}$$

$$= \frac{\frac{1}{\cos\theta} + \frac{1}{\sin\theta}}{\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}}$$

$$= \frac{\frac{\sin\theta + \cos\theta}{\cos\theta\sin\theta}}{\frac{\sin^2\theta + \cos^2\theta}{\cos\theta\sin\theta}}$$

$$= \frac{\sin\theta + \cos\theta}{1}$$

$$= \sin\theta + \cos\theta$$

= RHS

12 (a) LHS

$$= \cot x - \tan x$$

$$= \frac{\cos x}{\sin x} - \frac{\sin x}{\cos x}$$

$$= \frac{\cos^2 x - \sin^2 x}{\sin x \cos x}$$

$$= \frac{\cos 2x}{\sin x \cos x}$$

$$= \frac{2\cos 2x}{2\sin x\cos x}$$

$$= \frac{2\cos 2x}{\sin 2x}$$

$$= 2\cot 2x$$

= RHS

(b) LHS

$$= \cot x - 2\cot 2x$$

$$= \frac{\cos x}{\sin x} - 2\left(\frac{\cos 2x}{\sin 2x}\right)$$

$$= \frac{\cos x}{\sin x} - 2\left(\frac{\cos 2x}{2\sin x\cos x}\right)$$

$$= \frac{\cos x}{\sin x} - \frac{\cos 2x}{\sin x\cos x}$$

$$= \frac{\cos^2 x - (\cos^2 x - \sin^2 x)}{\sin x\cos x}$$

$$= \frac{\sin^2 x}{\sin x \cos x}$$

$$= \frac{\sin x}{\cos x}$$

$= \tan x$
= RHS

13 (a) LHS

$$= \frac{\tan 2x}{1 + \sec 2x}$$

$$= \frac{\frac{\sin 2x}{\cos 2x}}{1 + \frac{1}{\cos 2x}}$$

$$= \frac{\frac{\sin 2x}{\cos 2x}}{\frac{\cos 2x + 1}{\cos 2x}}$$

$$= \frac{\sin 2x}{\cos 2x + 1}$$

$$= \frac{\sin 2x}{2\cos^2 x - 1 + 1}$$

$$= \frac{2\sin x \cos x}{2\cos^2 x}$$

$$= \frac{\sin x}{\cos x}$$

$= \tan x$
= RHS

(b) LHS

$= \operatorname{cosec} 2x + \cot 2x$

$$= \frac{1}{\sin 2x} + \frac{\cos 2x}{\sin 2x}$$

$$= \frac{1 + \cos 2x}{\sin 2x}$$

$$= \frac{1 + 2\cos^2 x - 1}{2\sin x \cos x}$$

$$= \frac{2\cos^2 x}{2\sin x \cos x}$$

$$= \frac{\cos x}{\sin x}$$

$= \cot x$
= RHS

14 (a) LHS

$= \cot x - \cot 2x$

$$= \frac{\cos x}{\sin x} - \frac{\cos 2x}{\sin 2x}$$

$$= \frac{\cos x}{\sin x} - \frac{\cos 2x}{2\sin x \cos x}$$

$$= \frac{2\cos^2 x - \cos 2x}{2\sin x \cos x}$$

$$= \frac{2\cos^2 x - (2\cos^2 x - 1)}{2\sin x \cos x}$$

$$= \frac{1}{2\sin x \cos x}$$

$$= \frac{1}{\sin 2x}$$

$= \operatorname{cosec} 2x$
= RHS

(b) LHS

$$= \operatorname{cosec} x - \frac{2\cos x \cos 2x}{\sin 2x}$$

$$= \frac{1}{\sin x} - \frac{2\cos x(\cos 2x)}{2\sin x \cos x}$$

$$= \frac{2\cos x - 2\cos x \cos 2x}{2\sin x \cos x}$$

$$= \frac{2\cos x(1 - \cos 2x)}{2\sin x \cos x}$$

$$= \frac{1 - \cos 2x}{\sin x}$$

$$= \frac{1 - (1 - 2\sin^2 x)}{\sin x}$$

$$= \frac{2\sin^2 x}{\sin x}$$

$= 2\sin x$
= RHS

15 (a) LHS

$= \tan 2\theta\,(2\cos\theta - \sec\theta)$

$$= \frac{\sin 2\theta}{\cos 2\theta}\left(2\cos\theta - \frac{1}{\cos\theta}\right)$$

$$= \frac{\sin 2\theta}{\cos 2\theta}\left(\frac{2\cos^2\theta - 1}{\cos\theta}\right)$$

$$= \frac{\sin 2\theta}{\cos 2\theta}\left(\frac{\cos 2\theta}{\cos\theta}\right)$$

$$= \frac{2\sin\theta\cos\theta}{\cos\theta}$$

$= 2\sin\theta$
= RHS

(b) LHS

$= \dfrac{\cos (A-B)}{\sin (A+B)}$

$= \dfrac{\cos A \cos B + \sin A \sin B}{\sin A \cos B + \cos A \sin B}$

$= \dfrac{\dfrac{\cos A \cos B}{\cos A \cos B} + \dfrac{\sin A \sin B}{\cos A \cos B}}{\dfrac{\sin A \cos B}{\cos A \cos B} + \dfrac{\cos A \sin B}{\cos A \cos B}}$

$= \dfrac{1 + \tan A \tan B}{\tan A + \tan B}$

= RHS

16 (a) LHS

$= 2\sin(\theta + 45^\circ)\cos(\theta + 45^\circ)$

$= 2(\sin\theta \cos 45^\circ + \cos\theta \sin 45^\circ)(\cos\theta \cos 45^\circ - \sin\theta \sin 45^\circ)$

$= 2\left(\dfrac{\sin\theta}{\sqrt{2}} + \dfrac{\cos\theta}{\sqrt{2}}\right)\left(\dfrac{\cos\theta}{\sqrt{2}} - \dfrac{\sin\theta}{\sqrt{2}}\right)$

$= 2\left(\dfrac{\sin\theta + \cos\theta}{\sqrt{2}}\right)\left(\dfrac{\cos\theta - \sin\theta}{\sqrt{2}}\right)$

$= \cos^2\theta - \sin^2\theta$

$= \cos 2\theta$

= RHS

(b) LHS

$= 2\cos(\theta + 45^\circ)\cos(\theta - 45^\circ)$

$= 2(\cos\theta \cos 45^\circ - \sin\theta \sin 45^\circ)(\cos\theta \cos 45^\circ + \sin\theta \sin 45^\circ)$

$= 2\left(\dfrac{\cos\theta}{\sqrt{2}} - \dfrac{\sin\theta}{\sqrt{2}}\right)\left(\dfrac{\cos\theta}{\sqrt{2}} + \dfrac{\sin\theta}{\sqrt{2}}\right)$

$= 2\left(\dfrac{\cos\theta - \sin\theta}{\sqrt{2}}\right)\left(\dfrac{\cos\theta + \sin\theta}{\sqrt{2}}\right)$

$= \cos^2\theta - \sin^2\theta$

$= \cos 2\theta$

= RHS

17 (a) LHS

$= (\sec x - \tan x)^2$

$= \left(\dfrac{1}{\cos x} - \dfrac{\sin x}{\cos x}\right)^2$

$= \left(\dfrac{1 - \sin x}{\cos x}\right)^2$

$= \dfrac{(1 - \sin x)^2}{\cos^2 x}$

$= \dfrac{(1 - \sin x)^2}{1 - \sin^2 x}$

$= \dfrac{(1 - \sin x)^2}{(1 + \sin x)(1 - \sin x)}$

$= \dfrac{1 - \sin x}{1 + \sin x}$

= RHS

(b) LHS

$= \dfrac{\cos^2 x - \cos 2x}{\sin^2 x + \cos 2x}$

$= \dfrac{\cos^2 x - (\cos^2 x - \sin^2 x)}{\sin^2 x + \cos^2 x - \sin^2 x}$

$= \dfrac{\sin^2 x}{\cos^2 x}$

$= \tan^2 x$

= RHS

18 (a) $\cos 30^\circ = \sqrt{1 - \sin^2 30^\circ} = \sqrt{1 - h^2}$

$\sin 40^\circ = \sqrt{1 - \cos^2 40^\circ} = \sqrt{1 - k^2}$

$\sin 70^\circ$

$= \sin(30^\circ + 40^\circ)$

$= \sin 30^\circ \text{ kos } 40^\circ + \cos 30^\circ \sin 40^\circ$

$= hk + \sqrt{1 - h^2} \times \sqrt{1 - k^2}$

(b) $\cos 80^\circ = 2\cos^2 40^\circ - 1 = 2k^2 - 1$

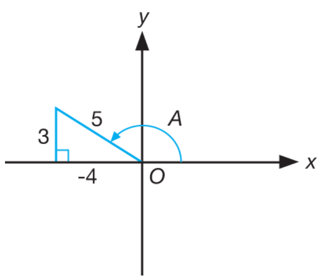
(c) $\cos 40^\circ = 2\cos^2 20^\circ - 1$

$k = 2\cos^2 20^\circ - 1$

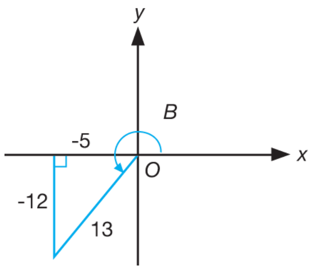
$\dfrac{k+1}{2} = \cos^2 20^\circ$

$\cos 20^\circ = \sqrt{\dfrac{k+1}{2}}$

19



$\tan A = -\dfrac{3}{4}$



$$\tan B = \frac{12}{5}$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$= \frac{-\frac{3}{4} + \frac{12}{5}}{1 - \left(-\frac{3}{4}\right)\left(\frac{12}{5}\right)}$$

$$= \frac{\frac{33}{20}}{\frac{14}{5}}$$

$$= \frac{33}{56}$$

20 (a) $8\tan\theta = 3\cos\theta$

$$\frac{8\sin\theta}{\cos\theta} = 3\cos\theta$$

$$8\sin\theta = 3\cos^2\theta$$

$$8\sin\theta = 3(1-\sin^2\theta)$$

$$8\sin\theta = 3 - 3\sin^2\theta$$

$$3\sin^2\theta + 8\sin\theta - 3 = 0$$

$$(3\sin\theta - 1)(\sin\theta + 3) = 0$$

$$\sin\theta = \frac{1}{3} \text{ or } \sin\theta = -3$$

$\sin\theta = -3$ is not accepted because the minimum value of $\sin\ \theta$ is -1.

When $\sin\theta = \frac{1}{3}$,

Basic $\angle = 19.47^{o}$

$$\theta = 19.47^{o}, 160.53^{o}$$

(b) $2\tan\theta + 5\sin\theta = 0$

$$\frac{2\sin\theta}{\cos\theta} + 5\sin\theta = 0$$

$$2\sin\theta + 5\sin\theta\cos\theta = 0$$

$$\sin\theta(2 + 5\cos\theta) = 0$$

$$\sin\theta = 0 \text{ or } \cos\theta = -\frac{2}{5}$$

When $\sin\theta = 0$,

$$\theta = 0^{o}, 180^{o}, 360^{o}$$

When $\cos\theta = -\frac{2}{5}$,

Basic $\angle = 66.42^{o}$

$$\theta = 113.58^{o}, 246.42^{o}$$

$$\theta = 0^{o}, 113.58^{o}, 180^{o}, 246.42^{o}, 360^{o}$$

(c) $3(\cos\theta - \sin\theta) = \sin\theta$

$$3\cos\theta - 3\sin\theta = \sin\theta$$

$$3\cos\theta = 4\sin\theta$$

$$\frac{\sin\theta}{\cos\theta} = \frac{3}{4}$$

$$\tan\theta = 0.75$$

Basic $\angle = 36.87^{o}$

$$\theta = 36.87^{o}, 216.87^{o}$$

21 (a) $2\sin\theta = \operatorname{cosec}\theta$

$$2\sin\theta = \frac{1}{\sin\theta}$$

$$\sin^2\theta = \frac{1}{2}$$

$$\sin\theta = \pm\frac{1}{\sqrt{2}}$$

$$\theta = 45^{o}, 135^{o}, 225^{o}, 315^{o}$$

(b) $\tan\theta = 3\cot\theta$

$$\frac{\sin\theta}{\cos\theta} = \frac{3\cos\theta}{\sin\theta}$$

$$\sin^2\theta = 3\cos^2\theta$$

$$\frac{\sin^2\theta}{\cos^2\theta} = 3$$

$$\tan^2\theta = 3$$

$$\tan\theta = \pm\sqrt{3}$$

$$\theta = 60^{o}, 120^{o}, 240^{o}, 300^{o}$$

(c) $4\cos\theta = 3\sec\theta$

$$4\cos\theta = \frac{3}{\cos\theta}$$

$$\cos^2\theta = \frac{3}{4}$$

$$\cos\theta = \pm\frac{\sqrt{3}}{2}$$

Basic $\angle = 30^{o}$

$$\theta = 30^{o}, 150^{o}, 210^{o}, 330^{o}$$

22 (a) $2\sec\theta - \cos\theta = 1$

$\frac{2}{\cos\theta} - \cos\theta - 1 = 0$

$2 - \cos\theta^2 - \cos\theta = 0$

$\cos^2\theta + \cos\theta - 2 = 0$

$(\cos\theta - 1)(\cos\theta + 2) = 0$

$\cos\theta = 1$ or $\cos\theta = -2$

$\cos\theta = -2$ does not have solution because the minimum value of $\cos\theta$ is -1.

When $\cos\theta = 1$,

$\theta = 0^o, 360^o$

(b) $\cos^2\theta + 7\sin^2\theta + \sin\theta = 2$

$1 - \sin^2\theta + 7\sin^2\theta + \sin\theta - 2 = 0$

$6\sin^2\theta + \sin\theta - 1 = 0$

$(3\sin\theta - 1)(2\sin\theta + 1) = 0$

$\sin\theta = \frac{1}{3}$ or $\sin\theta = -\frac{1}{2}$

When $\sin\theta = \frac{1}{3}$,

Basic $\angle$ = 19.47^o

$\theta = 19.47^o, 160.53^o$

When $\sin\theta = -\frac{1}{2}$,

$\theta = 210^o, 330^o$

$\therefore\ \theta = 19.47^o, 160.53^o, 210^o, 330^o$

(c) $\sin\theta(\sin\theta + 1) + \cos\theta(\cos\theta - 2) = 1$

$\sin^2\theta + \sin\theta + \cos^2\theta - 2\cos\theta - 1 = 0$

$1 + \sin\theta - 2\cos\theta - 1 = 0$

$\sin\theta = 2\cos\theta$

$\frac{\sin\theta}{\cos\theta} = 2$

$\tan\theta = 2$

Basic $\angle$ = $63.43^o, 243.43^o$

23 (a) $4\tan 2x = \text{kot } x$

$4\left(\frac{2\tan x}{1 - \tan^2 x}\right) = \frac{1}{\tan x}$

$8\tan^2 x = 1 - \tan^2 x$

$9\tan^2 x = 1$

$\tan^2 x = \frac{1}{9}$

$\tan x = \pm\frac{1}{3}$

Basic $\angle$ = 18.43^o

$x = 18.43^o, 161.57^o$

$198.43^o, 341.57^o$

(b) $2\tan\theta = 3\tan(45^o - \theta)$

$2\tan\theta = \frac{3\tan 45^o - 3\tan\theta}{1 + \tan 45^o \tan\theta}$

$2\tan\theta = \frac{3 - 3\tan\theta}{1 + \tan\theta}$

$2\tan\theta + 2\tan^2\theta = 3 - 3\tan\theta$

$2\tan^2\theta + 5\tan\theta - 3 = 0$

$(2\tan\theta - 1)(\tan\theta + 3) = 0$

$\tan\theta = \frac{1}{2}$ or $\tan\theta = -3$

When $\tan\theta = \frac{1}{2}$,

Basic $\angle$ = 26.57^o

$\theta = 26.57^o, 206.57^o$

When $\tan\theta = -3$,

Basic $\angle$ = 71.57^o

$\theta = 108.43^o, 288.43^o$

$\theta = 26.57^o, 108.43^o, 206.57^o, 288.43^o$

24 $5\sin x\cos x - \sin x = 0$

$\sin x(5\cos x - 1) = 0$

$\sin x = 0$ or $\cos x = \frac{1}{5}$

When $\sin x = 0$,

$x = 0^o, 180^o, 360^o$

When $\cos x = \frac{1}{5}$,

Basic $\angle$ = 78.46^o

$x = 78.46^o, 281.54^o$

$\therefore\ x = 0^o, 78.46^o, 180^o, 281.54^o, 360^o$

25 (a) $4\cos 2x + 2\sin x = 3$

$4(1 - 2\sin^2 x) + 2\sin x - 3 = 0$

$4 - 8\sin^2 x + 2\sin x - 3 = 0$

$8\sin^2 x - 2\sin x - 1 = 0$

$(2\sin x - 1)(4\sin x + 1) = 0$

$\sin x = \frac{1}{2}$ or $\sin x = -\frac{1}{4}$

When $\sin x = \frac{1}{2}$,

$x = 30^{o}, 150^{o}$

When $\sin x = -\frac{1}{4}$,

Basic $\angle$ = 14.48^{o}

$x = 194.48^{o}, 345.52^{o}$

$\therefore\ x = 30^{o}, 150^{o}, 194.48^{o}, 345.52^{o}$

(b) $3\cos 2x - 10\cos x + 7 = 0$

$3\left(2\cos^2 x - 1\right) - 10\cos x + 7 = 0$

$6\cos^2 x - 3 - 10\cos x + 7 = 0$

$6\cos^2 x - 10\cos x + 4 = 0$

$3\cos^2 x - 5\cos x + 2 = 0$

$(\cos x - 1)(3\cos x - 2) = 0$

$\cos x = 1$ or $\cos x = \frac{2}{3}$

When $\cos x = 1$, $x = 0^{o}, 360^{o}$

When $\cos x = \frac{2}{3}$,

Basic $\angle$ = 48.19^{o}

$x = 48.19^{o}, 311.81^{o}$

$\therefore\ x = 0^{o}, 48.19^{o}, 311.81^{o}, 360^{o}$

26 (a) $\sin\theta = \cos\left(\theta + 30^{o}\right)$

$\sin\theta = \cos\theta \text{ kos } 30^{o} - \sin\theta \sin 30^{o}$

$\sin\theta = \frac{\sqrt{3}}{2}\cos\theta - \frac{1}{2}\sin\theta$

$2\sin\theta = \sqrt{3}\cos\theta - \sin\theta$

$2\sin\theta = \sqrt{3}\cos\theta - \sin\theta$

$\frac{\sin\theta}{\cos\theta} = \frac{\sqrt{3}}{3}$

$\tan\theta = 0.5774$

Basic $\angle = 30^{o}$

$\theta = 30^{o}, 210^{o}$

(b) $\cos\theta = 4\cos\left(\theta - 60^{o}\right)$

$\cos\theta = 4\left(\cos\theta\cos 60^{o} + \sin\theta\sin 60^{o}\right)$

$\cos\theta = 4\left(\frac{\cos\theta}{2} + \frac{\sqrt{3}}{2}\sin\theta\right)$

$\cos\theta = 2\cos\theta + 2\sqrt{3}\sin\theta$

$-\cos\theta = 2\sqrt{3}\sin\theta$

$-\frac{1}{2\sqrt{3}} = \frac{\sin\theta}{\text{kos }\theta}$

$\tan\theta = -0.2886$

Basic $\angle$ = 16.10^{o}

$\theta = 163.90^{o}, 343.90^{o}$

27 (a) $8\sin x + 3\sec x = 0$

$8\sin x + \frac{3}{\cos x} = 0$

$8\sin x\cos x + 3 = 0$

$4\left(2\sin x\cos x\right) + 3 = 0$

$4\sin 2x = -3$

$\sin 2x = -\frac{3}{4}$

Basic $\angle$ = 48.59^{o}

$2x = 228.59^{o}, 311.41^{o}$

$x = 114.30, 155.70^{o}$

(b) $3\cos^2 x - 3\sin^2 x - 8\sin x\cos x = 0$

$(\cos x - 3\sin x)(3\text{ kos } x + \sin x) = 0$

$\cos x - 3\sin x = 0$ or

$3\cos x + \sin x = 0$

When

$\cos x - 3\sin x = 0$,

$\cos x = 3\sin x$

$\frac{\sin x}{\cos x} = \frac{1}{3}$

$\tan x = \frac{1}{3}$

Basic $\angle$ = 18.43^{o}

$x = 18.43^{o}$

When

$3\cos x + \sin x = 0$,

$3\cos x = -\sin x$

$\frac{\sin x}{\cos x} = -3$

$\tan x = -3$

Basic $\angle$ = 71.57^{o}

$x = 108.43^{o}$

$\therefore\ x = 18.43^{o}, 108.43^{o}$

28 (a) LHS

$= \cot x \sin 2x$

$= \left(\frac{\cos x}{\sin x}\right)(2\sin x \cos x)$

$= 2\cos^2 x$

$= \cos 2x + 1$ ←

$\cos 2x = 2\cos^2 x - 1$

$2\cos^2 x = \cos 2x + 1$

= RHS

(b) $\cot x \sin 2x = \frac{2}{3}$

$\cos 2x + 1 = \frac{2}{3}$

$\cos 2x = -\frac{1}{3}$

Basic ∠ = 70.53^{o}

$2x = 109.47^{o}, 250.53^{o},$

$469.47^{o}, 610.53^{o}$

$x = 54.74^{o}, 125.27^{o},$

$234.74^{o}, 305.27^{o}$

29 (a) (i) LHS

$= 2\sin(x + 45^{o})\sin(x - 45^{o})$

$= 2(\sin x \cos 45^{o} + \cos x \sin 45^{o})$
$(\sin x \cos 45^{o} - \cos x \sin 45^{o})$

$= 2\left(\frac{\sin x}{\sqrt{2}} + \frac{\cos x}{\sqrt{2}}\right)\left(\frac{\sin x}{\sqrt{2}} - \frac{\cos x}{\sqrt{2}}\right)$

$= (\sin x + \cos x)(\sin x - \cos x)$

$= \sin^2 x - \cos^2 x$

$= -\cos 2x$

= RHS

(ii) $2\sin(x + 45^{o})\sin(x - 45^{o}) = \frac{1}{2}$

$-\cos 2x = \frac{1}{2}$

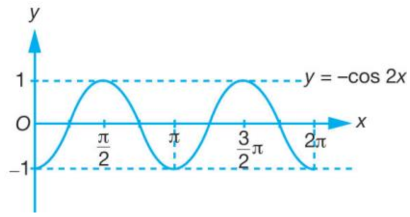
$\cos 2x = -\frac{1}{2}$

Basic ∠ = 60^{o}

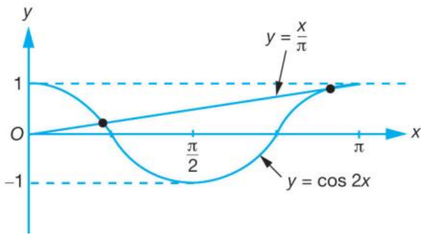
$2x = 120^{o}, 240^{o}, 480^{o}, 600^{o}$

$x = 60^{o}, 120^{o}, 240^{o}, 300^{o}$

(b)



30 (a)



(b) $1 - 2\sin^2 x - \frac{x}{\pi} = 0$

$\cos 2x = \frac{x}{\pi}$

Sketch the straight line $y = \frac{x}{\pi}$

Number of solutions
= Number of points of intersection
= 2

31 (a) LHS

$= \sec^2 x - \tan^2 x - 2\cos^2 x$

$= 1 + \tan^2 x - \tan^2 x - 2\cos^2 x$

$= 1 - 2\cos^2 x$

$= -(2\cos^2 x - 1)$

$= -\cos 2x$

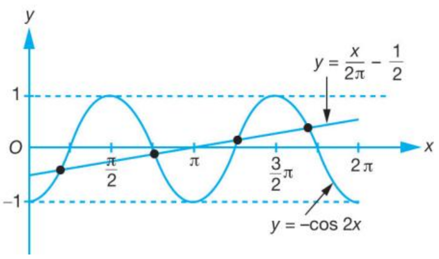
= RHS

(b) $2(\sec^2 x - \tan^2 x - 2\cos^2 x) = \frac{x}{\pi} - 1$

$-2\cos 2x = \frac{x}{\pi} - 1$

$-\cos 2x = \frac{x}{2\pi} - \frac{1}{2}$

Sketch the straight line $y = \frac{x}{2\pi} - \frac{1}{2}$



Number of solutions
= Number of points of intersection
= 4

32 (a) LHS

$$= \frac{2 \cot x}{2 - \text{cosek}^2 x}$$

$$= \frac{\frac{2 \cos x}{\sin x}}{2 - \frac{1}{\sin^2 x}}$$

$$= \frac{\frac{2 \cos x}{\sin x}}{\frac{2\sin^2 x - 1}{\sin^2 x}}$$

$$= \frac{2 \cos x}{\sin x} \times \frac{\sin^2 x}{2\sin^2 x - 1}$$

$$= \frac{2 \sin x \cos x}{2 \sin^2 x - 1}$$

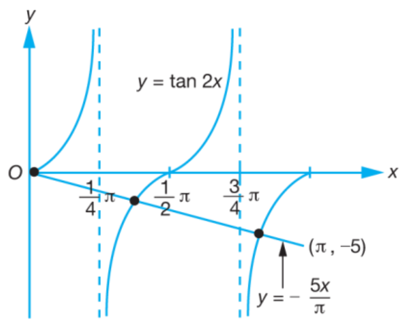
$$= \frac{\sin 2x}{-(1 - 2\sin^2 x)}$$

$$= \frac{\sin 2x}{-\cos 2x}$$

$$= -\tan 2x$$

= RHS

(b) (i)



(ii) $$\frac{2 \cot x}{2 - \text{cosec}^2 x} - \frac{5x}{\pi} = 0$$

$$-\tan 2x = \frac{5x}{\pi}$$

$$\tan 2x = -\frac{5x}{\pi}$$

Sketch the straight line $y = -\frac{5x}{\pi}$.

Number of solutions
= Number of points of intersection
= 3

33 (a) LHS

$$= \tan 2x\,(2 \cos x - \sec x)$$

$$= \frac{\sin 2x}{\cos 2x}\left(2 \cos x - \frac{1}{\cos x}\right)$$

$$= \frac{2 \sin x \cos x}{\cos 2x}\left(\frac{2 \cos^2 x - 1}{\cos x}\right)$$

$$= \frac{2 \sin x}{\text{kos } 2x}(\cos 2x)$$

$$= 2 \sin x$$

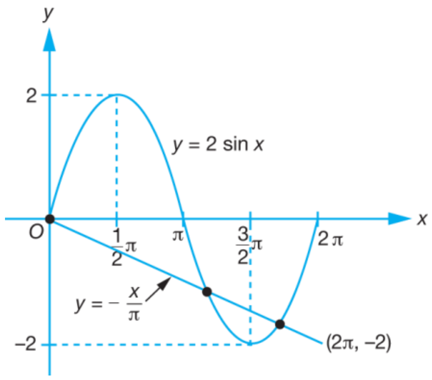
= RHS

(b) $$\pi \tan 2x\,(2 \cos x - \sec x) + x = 0$$

$$\tan 2x\,(2 \cos x - \sec x) = -\frac{x}{\pi}$$

$$2 \sin x = -\frac{x}{\pi}$$

Sketch the straight line $y = -\frac{x}{\pi}$



Number of solutions
= Number of points of intersection
= 3

34 (a) LHS

$$= \frac{\sin 2x}{2 \cos^2 x + \cot^2 x - \text{cosec}^2 x}$$

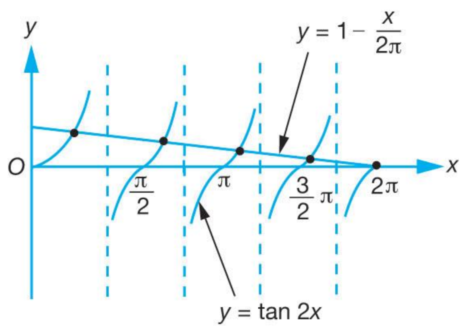
$$= \frac{\sin 2x}{2 \cos^2 x - 1}$$

$$= \frac{\sin 2x}{\cos 2x}$$

$$= \tan 2x$$

= RHS

(b)



(c) $\dfrac{\sin 2x}{2\cos^2 x + \cot^2 x - \text{cosec}^2 x} + \dfrac{x}{2\pi} = 1$

$\tan 2x = 1 - \dfrac{x}{2\pi}$

Sketch the straight line $y = 1 - \dfrac{x}{2\pi}$.

Number of solutions
= Number of points of intersection
= 5

35 (a) LHS

$= 2\cot x \sin^2 x$

$= 2\left(\dfrac{\cos x}{\sin x}\right)\left(\sin^2 x\right)$

$= 2\sin x \cos x$

$= \sin 2x$

= RHS

(b) $4\cot x \sin^2 x = \sqrt{3}$

$2\left(2\cot x \sin^2 x\right) = \sqrt{3}$

$2\left(\sin 2x\right) = \sqrt{3}$

$\sin 2x = \dfrac{\sqrt{3}}{2}$

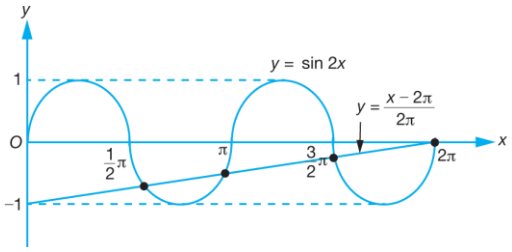
Basic $\angle$ = 60°

$2x = 60^\circ, 120^\circ, 420^\circ, 480^\circ$

$x = 30^\circ, 60^\circ, 210^\circ, 240^\circ$

$x = \dfrac{1}{6}\pi, \dfrac{1}{3}\pi, \dfrac{7}{6}\pi, \dfrac{4}{3}\pi$ rad

(c) (i)



Sketch the straight line $y = \dfrac{x - 2\pi}{2\pi}$

Number of solutions
= Number of points of intersection
= 4

36 $e = 0.014\cos\left(2\pi f t\right)$

$2\pi f t = 2\pi$

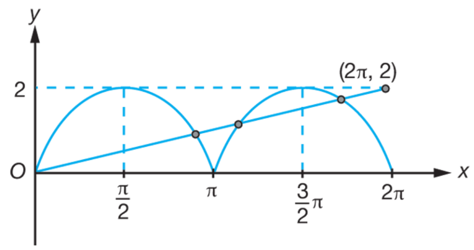
$f t = 1$

$t = \dfrac{1}{f}$

Period = $\dfrac{1}{f} = \dfrac{1}{950\,000} = 1.053 \times 10^{-6}$ s

SPM Spot

1 (a), (b)



Number of solutions
= Number of points of intersection
= 4

2 (a) LHS

$= \dfrac{1 + \sin A}{\cos A} + \dfrac{\cos A}{1 + \sin A}$

$= \dfrac{(1 + \sin A)^2 + \cos^2 A}{\cos A\,(1 + \sin A)}$

$= \dfrac{1 + 2\sin A + \sin^2 A + \cos^2 A}{\cos A\,(1 + \sin A)}$

$= \dfrac{2 + 2\sin A}{\cos A\,(1 + \sin A)}$

$= \dfrac{2\,(1 + \sin A)}{\cos A\,(1 + \sin A)}$

$= \dfrac{2}{\cos A}$

$= 2\sec A$

= RHS

(b) $4\cos 2\theta + \sin\theta = -3$

$$4\left(1 - 2\sin^2\theta\right) + \sin\theta + 3 = 0$$

$$4 - 8\sin^2\theta + \sin\theta + 3 = 0$$

$$8\sin^2\theta - \sin\theta - 7 = 0$$

$$(\sin\theta - 1)(8\sin\theta + 7) = 0$$

$$\sin\theta = 1 \text{ or } \sin\theta = -\frac{7}{8}$$

When $\sin\theta = 1$,

$\theta = 90^\circ$

When $\sin\theta = -\frac{7}{8}$,

Basic $\angle$ = 61.04°

$\theta = 241.04^\circ,\ 298.96^\circ$

Hence, $\theta = 90^\circ,\ 241.04^\circ,\ 298.96^\circ$