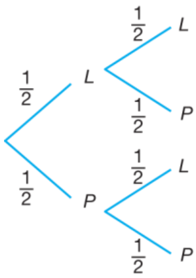
Form 5 Chapter 5
Probability Distribution
Fully-Worked Solutions

UPSKILL 5.1

1 (a)



Outcomes

LL

LP

PL

PP

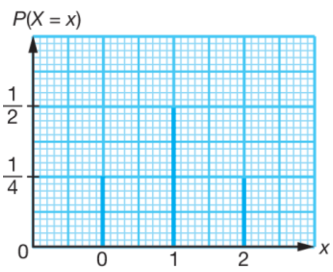
P – Female
L – Males

Outcome	*LL*	*LP*	*PL*	*PP*
$X = x$	0	1	1	2

The values that can be taken by X are 0, 1 and 2.

$X = x$	0	1	2
$P(X = x)$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

(b)



UPSKILL 5.2a

1 (a) $X = 0, 1, 2, 3, 4$
(b) $Y = 0, 1, 2, 3$
(c) $Z = 0, 1, 2$
(d) $W = 0, 1, 2, 3, 4, 5$

2 (a) $X = 0, 1, 2, 3$
(b) $X \sim B\ (n,\ p)$

$$X \sim B\left(3, \frac{1}{2}\right)$$

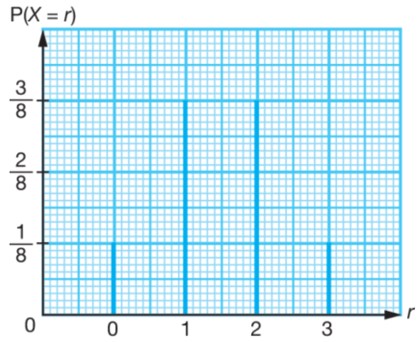
$$P(X = 0) = {}^3C_0\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^3 = \frac{1}{8}$$

$$P(X = 1) = {}^3C_1\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^2 = \frac{3}{8}$$

$$P(X = 2) = {}^3C_2\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^1 = \frac{3}{8}$$

$$P(X = 3) = {}^3C_3\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^0 = \frac{1}{8}$$

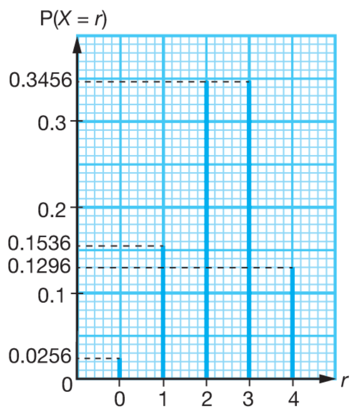
(c)



3 (a) X – Number of students who achieve distinction
$X \sim B(4, 0.6)$
$X = 0, 1, 2, 3, 4$

(b) $P(X = 0) = {}^4C_0(0.6)^0(0.4)^4 = 0.0256$
$P(X = 1) = {}^4C_1(0.6)^1(0.4)^3 = 0.1536$
$P(X = 2) = {}^4C_2(0.6)^2(0.4)^2 = 0.3456$
$P(X = 3) = {}^4C_3(0.6)^3(0.4)^1 = 0.3456$
$P(X = 4) = {}^4C_4(0.6)^4(0.4)^0 = 0.1296$

(c)



4 X – Number of males

$X \sim \text{B}\left(4, \frac{3}{5}\right)$

(a) $\text{P}(X = 2)$

$= {}^4C_2\left(\frac{3}{5}\right)^2\left(\frac{2}{5}\right)^2$

$= 0.3456$

(b) $\text{P}(X \geq 1)$

$= 1 - \text{P}(X = 0)$

$= 1 - {}^4C_0\left(\frac{3}{5}\right)^0\left(\frac{2}{5}\right)^4$

$= 1 - 0.0256$

$= 0.9744$

5 X – Number of students who fail Additional Mathematics

$X \sim \text{B}(10, 0.2)$

(a) $\text{P}(X = 0)$

$= {}^{10}C_0(0.2)^0(0.8)^{10}$

$= 0.1074$

(b) $\text{P}(X \leq 2)$

$= \text{P}(X = 0) + \text{P}(X = 1) + \text{P}(X = 2)$

$= 0.1074 +$

${}^{10}C_1(0.2)^1(0.8)^9 + {}^{10}C_2(0.2)^2(0.8)^8$

$= 0.1074 + 0.2684 + 0.3020$

$= 0.6778$

(c) $\text{P}(X \geq 2)$

$= 1 - \text{P}(X = 0) - \text{P}(X = 1)$

$= 1 - 0.1074 - 0.2684$

$= 0.6242$

6 $X - \text{B}(8, 0.25)$

(a) $\text{P}(X \leq 2)$

$= \text{P}(X = 0) + \text{P}(X = 1) + \text{P}(X = 2)$

$= {}^8C_0(0.25)^0(0.75)^8 + {}^8C_1(0.25)^1(0.75)^7$

$+ {}^8C_2(0.25)^2(0.75)^6$

$= 0.1001 + 0.2670 + 0.3115$

$= 0.6786$

(b) $\text{P}(X \geq 2)$

$= 1 - \text{P}(X = 0) - \text{P}(X = 1)$

$= 1 - 0.1001 + 0.2670$

$= 0.6329$

UPSKILL 5.2b

1 X – Number of workers who take leaves

$X \sim \text{B}(15, 0.2)$

(a) $\text{P}(X = 0)$

$= {}^{15}C_0(0.2)^0(0.8)^{15}$

$= 0.0352$

(b) Mean = $np = 15 \times \frac{1}{5} = 3$

Standard deviation

$= npq$

$= \sqrt{15 \times \frac{1}{5} \times \frac{4}{5}}$

$= 1.55$

2 X – Number of candidates who pass

$X \sim \text{B}(5, 0.7)$

(a) $\text{P}(X \geq 1)$

$= 1 - \text{P}(X = 0)$

$= 1 - {}^5C_0(0.7)^0(0.3)^5$

$= 0.99757$

(b) Mean = $100 \times 0.7 = 70$

Standard deviation

$= \sqrt{100 \times 0.7 \times 0.3}$

$= 4.58$

3 Mean = 2

$np = 2 \ \ldots (1)$

Standard deviation = $\frac{2\sqrt{2}}{\sqrt{5}}$

Variance = $\frac{4(2)}{5} = 1.6$

$npq = 1.6 \ \ldots (2)$

$\frac{(2)}{(1)}: \ \frac{npq}{np} = \frac{1.6}{2}$

$q = 0.8$

$p = 1 - 0.8$

$p = 0.2$

From (1) :

$np = 2$

$(0.2)n = 2$

$n = \frac{2}{0.2}$

$n = 10$

4 Mean = 40
$np = 40 \quad \ldots (1)$

Variance = 24
$npq = 24 \ldots (2)$

$$\frac{(2)}{(1)}: \frac{npq}{np} = \frac{24}{40}$$
$$q = \frac{3}{5}$$
$$p = 1 - \frac{3}{5}$$
$$p = \frac{2}{5}$$

From (1):
$np = 40$
$$\frac{2}{5}n = 40$$
$$n = 100$$

UPSKILL 5.2c

1 X – Number of star fruits that are rotten
$X \sim B(15, 0.05)$

(a) $P(X \geq 2)$
$= 1 - P(X = 0) - P(X = 1)$
$= 1 - {}^{15}C_0(0.05)^0(0.95)^{15} - {}^{15}C_1(0.05)^1(0.95)^{14}$
$= 1 - 0.4633 - 0.3658$
$= 0.1709$

(b)
$$P(X \geq 1) > 0.85$$
$$1 - P(X = 0) > 0.85$$
$$1 - {}^{n}C_0(0.05)^0(0.95)^n > 0.85$$
$$1 - 0.95^n > 0.85$$
$$1 - 0.85 > 0.95^n$$
$$0.15 > 0.95^n$$
$$0.95^n < 0.15$$
$$n \lg 0.95 < 0.15$$
$$-0.0223n < -0.8239$$
$$n > \frac{-0.8239}{-0.0223}$$
$$n > 36.95$$
Minimum number of $n = 37$

2 X – Number of LED lights that are defective
$X \sim B(n, 0.1)$

(a) Mean = 200 × 0.1 = 20
Standard deviation
$= \sqrt{200(0.1)(0.9)}$
$= 4.24$

(b)
$$P(X \geq 1) > 0.8$$
$$1 - P(X = 0) > 0.8$$
$$1 - {}^{n}C_0(0.1)^0(0.9)^n > 0.8$$
$$1 - 0.9^n > 0.8$$
$$1 - 0.8 > 0.9^n$$
$$0.2 > 0.9^n$$
$$0.9^n < 0.2$$
$$n \lg 0.9 < 0.2$$
$$-0.0458n < -0.6990$$
$$n > \frac{-0.6990}{-0.0458}$$
$$n > 15.26$$
Minimum number of $n = 16$

3 X – Number of times to score a strike
$$X \sim B\left(n, \frac{4}{5}\right)$$
$${}^{n}C_n\left(\frac{4}{5}\right)^n\left(\frac{1}{5}\right)^0 = \frac{256}{625}$$
$$\left(\frac{4}{5}\right)^n = \left(\frac{4}{5}\right)^4$$
$$n = 4$$

4 (a) X – Number of times to solve a word puzzle
$$X \sim B\left(7, \frac{3}{5}\right)$$
$P(X \geq 2)$
$= 1 - P(X = 0) - P(X = 1)$
$$= 1 - {}^{7}C_0\left(\frac{3}{5}\right)^0\left(\frac{2}{5}\right)^7 - {}^{7}C_1\left(\frac{3}{5}\right)^1\left(\frac{2}{5}\right)^6$$
$= 0.9812$

(b) $Y \sim B(4, 0.9812)$
$P(Y = 3)$
$= {}^{4}C_3(0.9812)^3(0.0188)^1$
$= 0.0710$

UPSKILL 5.3a

1 (a) $X \sim N(50, 10^2)$

(i) $$Z = \frac{60 - 50}{10} = 1$$

(ii) $$Z = \frac{25 - 50}{10} = -2.5$$

(b) (i) $Z = 2$

$$\frac{X - 50}{10} = 2$$
$$X = 70$$

(ii) $Z = -1.5$

$$\frac{X - 50}{10} = -1.5$$
$$X = 35$$

2 X – Volume, in m*l*, of a bottle of drink

$X \sim N(210, 10^2)$

(a) (i) $Z = \dfrac{220 - 210}{10} = 1$

(ii) $Z = \dfrac{200 - 210}{10} = -1$

(b) (i) $Z = 1.5$

$$\frac{X - 210}{10} = 1.5$$
$$X = 225 \text{ m}l$$

(ii) $Z = -0.5$

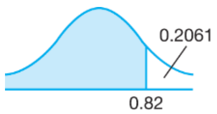
$$\frac{X - 210}{10} = -0.5$$
$$X = 205 \text{ m}l$$

UPSKILL 5.3b

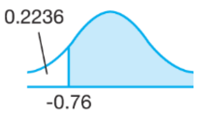
1 (a) $P(Z > 1.284) = 0.0996$

(b) $P(Z < -1.37) = 0.0853$

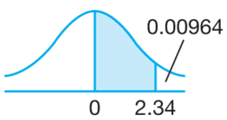
(c) $P(Z < 0.82)$
$= 1 - 0.2061$
$= 0.7939$



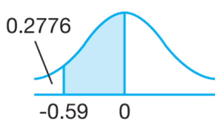
(d) $P(Z > -0.76) = 1 - 0.2236$
$= 0.7764$



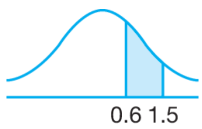
(e) $P(0 < Z < 2.34)$
$= 0.5 - 0.00964$
$= 0.49036$



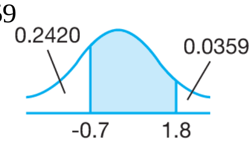
(f) $P(-0.59 < Z < 0)$
$= 0.5 - 0.2776$
$= 0.2224$



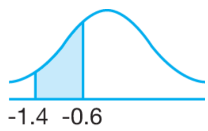
(g) $P(0.6 < Z < 1.5)$
$= 0.2743 - 0.0668$
$= 0.2075$



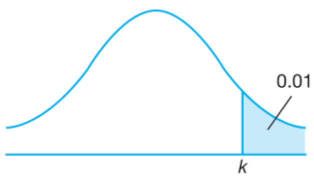
(h) $P(-0.7 < Z < 1.8)$
$= 1 - 0.2420 - 0.0359$
$= 0.7221$



(i) $P(-1.4 < Z < -0.6)$
$= 0.2743 - 0.0808$
$= 0.1935$

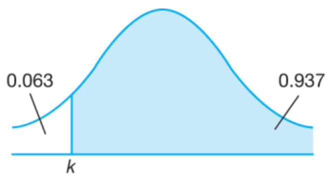


2 (a)



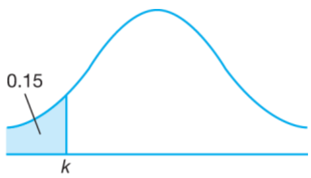
From the standard normal distribution table, $k = 2.326$.

(b)



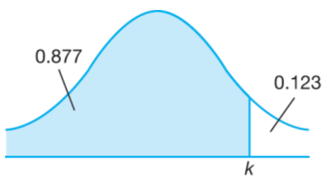
From the standard normal distribution table, $k = -1.53$.

(c)



From the standard normal distribution table, $k = -1.037$.

(d)



From the standard normal distribution table, $k = 1.16$.

UPSKILL 5.3c

1 X – Mass, in kg, of a concrete rod
$X \sim N(240, 10^2)$

(a) $P(X > 250) = P\left(Z > \frac{250-240}{10}\right)$
$= P(Z > 1)$
$= 0.1587$

(b) $P(X < 225) = P\left(Z < \frac{225-240}{10}\right)$
$= P(Z < -1.5)$
$= 0.0668$
Number of concrete rods
$= 0.0668 \times 200$
$= 13$

(c) $P(220 < X < 260)$
$= P\left(\frac{220-240}{10} < Z < \frac{260-240}{10}\right)$
$= P(-2 < Z < 2)$
$= 1 - 0.0228 - 0.0228$
$= 0.9544$
$= 95.44\%$

2 X – Lifespan, in hours, of a battery
$X \sim N(750, 50^2)$

(a) $P(X < 725) = P\left(Z < \frac{725-750}{50}\right)$
$= P(Z < -0.5)$
$= 0.3085$

(b) $P(X > 775) = P\left(Z > \frac{775-750}{50}\right)$
$= P(Z > 0.5)$
$= 0.3085$
Number of batteries
$= 0.3085 \times 800$
$= 247$

(c) $P(730 < X < 740)$
$= P\left(\frac{730-750}{50} < Z < \frac{740-750}{50}\right)$
$= P(-0.4 < Z < -0.2)$
$= 0.4207 - 0.3446$
$= 0.0761$
$= 7.61\%$

3 X – Additional Mathematics marks
$X \sim N(45, 10^2)$

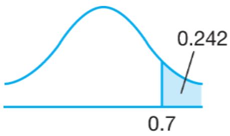
(a) $P(X < 50) = P\left(Z < \frac{50-45}{10}\right)$
$= P(Z < 0.5)$
$= 1 - 0.3085$
$= 0.6915$

(b) $P(45 < X < 55)$
$= P\left(\frac{45-45}{10} < Z < \frac{55-45}{10}\right)$
$= P(0 < Z < 1)$
$= 0.5 - 0.1587$
$= 0.3413$
Number of students
$= 0.3413 \times 200$
$= 68$

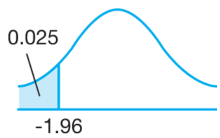
(c) $P(X > 38) = P\left(Z > \frac{38-45}{10}\right)$
$= P(Z > -0.7)$
$= 1 - 0.2420$
$= 0.758$
$= 75.8\%$

4 X – Length, in cm, of *siakap* fish
$X \sim N(55, 5^2)$

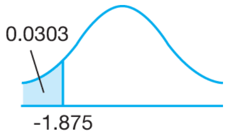
(a) $P(X > h) = 0.242$
$P\left(Z > \frac{h-55}{5}\right) = 0.242$
$\frac{h-55}{5} = 0.7$
$h = 58.5$



(b) $P(X < k) = 0.025$
$P\left(Z < \frac{k-55}{5}\right) = 0.025$
$\frac{k-55}{5} = -1.96$
$k = 45.2$

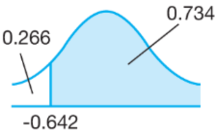


5 X – Science marks
$X \sim N(55, 8^2)$
(a) $P(X < m) = 0.0303$
$P\left(Z < \frac{m-55}{8}\right) = 0.0303$



$$\frac{m-55}{8}=-1.875$$
$$m=40$$
Minimum mark to pass = 40

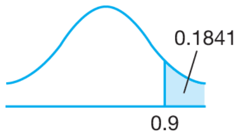
(b) $$\mathrm{P}(X>h)=0.734$$
$$\mathrm{P}\left(Z>\frac{h-55}{8}\right)=0.734$$



$$\frac{h-55}{8}=-0.642$$
$$h=49.9$$
Minimum mark to obtain a credit = 50

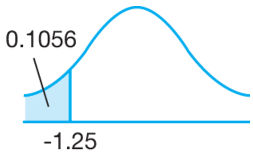
6 X – Height, in cm, of Year 1 pupils
$X \sim \mathrm{N}(120, 5^2)$

(a) $$\mathrm{P}(X>m)=0.1841$$
$$\mathrm{P}\left(Z>\frac{m-120}{5}\right)=0.1841$$



$$\frac{m-120}{5}=0.9$$
$$m=124.5$$

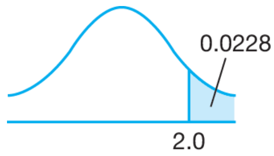
(b) $$\mathrm{P}(X<k)=0.1056$$
$$\mathrm{P}\left(Z<\frac{k-120}{5}\right)=0.1056$$



$$\frac{k-120}{5}=-1.25$$
$$k=113.75$$

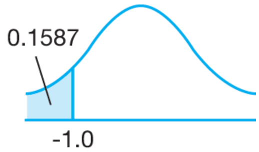
7 X – Mass, in g, of documents
$X \sim \mathrm{N}(\mu, \sigma^2)$

$$\mathrm{P}(X>5.4)=0.0228$$
$$\mathrm{P}\left(Z>\frac{5.4-\mu}{\sigma}\right)=0.0228$$



$$\frac{5.4-\mu}{\sigma}=2.0$$
$$5.4-\mu=2\sigma \quad \ldots (1)$$

$$\mathrm{P}(X<4.8)=0.1587$$
$$\mathrm{P}\left(Z<\frac{4.8-\mu}{\sigma}\right)=0.1587$$



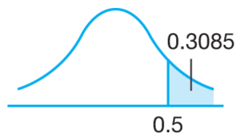
$$\frac{4.8-\mu}{\sigma}=-1.0$$
$$4.8-\mu=-\sigma \quad \ldots (2)$$

$(1)-(2):\ 0.6=3\sigma$
$$\sigma=0.2$$

From (1) :
$$5.4-\mu=2(0.2)$$
$$\mu=5$$

8 X – Mass, in g, of an egg
$X \sim \mathrm{N}(\mu, \sigma^2)$

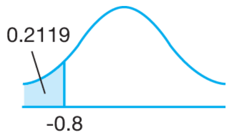
$$\mathrm{P}(X>55)=0.3085$$
$$\mathrm{P}\left(Z>\frac{55-\mu}{\sigma}\right)=0.3085$$



$$\frac{55-\mu}{\sigma}=0.5$$
$$55-\mu=0.5\sigma \quad \ldots (1)$$

$$P(X < 42) = 0.2119$$

$$P\left(Z < \frac{42 - \mu}{\sigma}\right) = 0.2119$$



$$\frac{42 - \mu}{\sigma} = -0.8$$

$$42 - \mu = -0.8\sigma \ \ldots (2)$$

$$(1) - (2): \ 13 = 1.3\sigma$$

$$\sigma = 10 \text{ g}$$

From (1):

$$55 - \mu = 0.5(10)$$

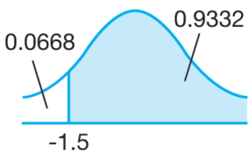
$$\mu = 50 \text{ g}$$

9 X – Diameter, in cm, of a polystyrene ball

$X \sim N(\mu, \sigma^2)$

$$P(X > 37) = 0.9332$$

$$P\left(Z > \frac{37 - \mu}{\sigma}\right) = 0.9332$$

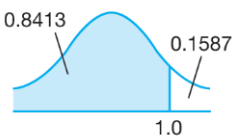


$$\frac{37 - \mu}{\sigma} = -1.5$$

$$37 - \mu = -1.5\sigma \ \ldots (1)$$

$$P(X < 42) = 0.8413$$

$$P\left(Z < \frac{42 - \mu}{\sigma}\right) = 0.8413$$



$$\frac{42 - \mu}{\sigma} = 1.0$$

$$42 - \mu = \sigma \ \ldots (2)$$

$$(2) - (1): \ 5 = 2.5\sigma$$

$$\sigma = 2$$

From (2):

$$42 - \mu = 2$$

$$\mu = 40$$

10 (a) $X \sim N(52, 3^2)$

$$Z = \frac{X - \mu}{\sigma}$$

$$-\frac{2}{3} = \frac{k - 52}{3}$$

$$k = 50$$

(b) $P(50 < X < 58)$

$$= P\left(\frac{50 - 52}{3} < Z < \frac{58 - 52}{3}\right)$$

$$= P(-0.667 < Z < 2)$$

$$= 1 - 0.2523 - 0.0228$$

$$= 0.7249$$

(c) $P(X < 55)$

$$= P\left(Z < \frac{55 - 52}{3}\right)$$

$$= P(Z < 1)$$

$$= 1 - 0.1587$$

$$= 0.8413$$

Number of students

$= 0.8413 \times 200$

$= 168$

11 X – Age of a teacher (in years)

$X \sim N(38, 4^2)$

$$P(30 < X < 44)$$

$$= P\left(\frac{30 - 38}{4} < Z < \frac{44 - 38}{4}\right)$$

$$= P(-2 < Z < 1.5)$$

$$= 1 - 0.0228 - 0.0668$$

$$= 0.9104$$

$$0.9104N = 102$$

$$N = 112$$

Hence, the total number of teachers is 112.

Summative Practice 5

1 X – Number of candidates who pass

$X \sim B(8, 0.7)$

(a) $P(X = 3) = {}^{8}C_3(0.7)^3(0.3)^5$

$$= 0.04668$$

(b) $P(X \leq 3)$

$= P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3)$

$= {}^8C_0(0.7)^0(0.3)^8 + {}^8C_1(0.7)^1(0.3)^7 + {}^8C_2(0.7)^2(0.3)^6 + 0.04668$

$= 0.05797$

2 X – Number of students who know how to swim

$X \sim B\left(5, \frac{1}{6}\right)$

(a) $P(X \geq 2)$

$= 1 - P(X = 0) - P(X = 1)$

$= 1 - {}^5C_0\left(\frac{1}{6}\right)^0\left(\frac{5}{6}\right)^5 - {}^5C_1\left(\frac{1}{6}\right)^1\left(\frac{5}{6}\right)^4$

$= 0.1962$

(b) Mean = $np = 1\,200 \times \frac{1}{6} = 200$

Standard deviation = $\sqrt{5 \times \frac{1}{6} \times \frac{5}{6}}$ = 12.91

3 X – Number of gastric patients who recover

(a) Mean = 90

$np = 90$

$150p = 90$

$p = \frac{3}{5}$

(b) Standard deviation

$= \sqrt{150 \times \frac{3}{5} \times \frac{2}{5}}$

$= 6$

(c) $P(X = 0) = {}^5C_0\left(\frac{3}{5}\right)^0\left(\frac{2}{5}\right)^5 = 0.01024$

4 X – Number of students who ride motorcycle to school

$X \sim B\left(8, \frac{1}{5}\right)$

(a) $P(X \geq 2)$

$= 1 - P(X = 0) - P(X = 1)$

$= 1 - {}^8C_0\left(\frac{1}{5}\right)^0\left(\frac{4}{5}\right)^8 - {}^8C_1\left(\frac{1}{5}\right)^1\left(\frac{4}{5}\right)^7$

$= 0.4967$

(b) Mean = $np = 1\,500 \times \frac{1}{5} = 300$

Standard deviation

$= \sqrt{1\,500 \times \frac{1}{5} \times \frac{4}{5}}$

$= 15.49$

5 X – Number of students who choose the Science stream

$X \sim B(10, 0.6)$

(a) $P(X = 4) = {}^{10}C_4(0.6)^4(0.4)^6 = 0.1115$

(b) $P(X \geq 9)$

$= P(X = 9) + P(X = 10)$

$= {}^{10}C_9(0.6)^9(0.4)^1 + {}^{10}C_{10}(0.6)^{10}(0.4)^0$

$= 0.0403 + 0.0060$

$= 0.0463$

6 (a) Mean = 15

$np = 15$ … (1)

Standard deviation = $\frac{3\sqrt{6}}{2}$

$\sqrt{npq} = \frac{3\sqrt{6}}{2}$

$npq = \frac{9(6)}{4}$

$npq = 13.5$ … (2)

$\frac{(2)}{(1)}$: $\frac{npq}{np} = \frac{13.5}{15}$

$q = 0.9$

$p = 1 - 0.9$

$p = 0.1$

From (1) :

$n(0.1) = 15$

$n = 150$

(b) $P(X \geq 1) = 1 - P(X = 0)$

$= 1 - {}^{10}C_0(0.1)^0(0.9)^{10}$

$= 0.6513$

7 X – Number of yellow marbles drawn

$X \sim B\left(8, \frac{2}{5}\right)$

(a) $P(X = 2) = {}^8C_2\left(\frac{2}{5}\right)^2\left(\frac{3}{5}\right)^6$

$= 0.2090$

(b) $P(X \geq 1) = 1 - P(X = 0)$

$= 1 - {}^8C_0\left(\frac{2}{5}\right)^0\left(\frac{3}{5}\right)^8$

$= 0.9832$

8 (a) P(success)
= P(all heads) + P(all tails)
$= \left(\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\right)+\left(\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\right)$
$= \frac{1}{4}$

(b) $X \sim B(10, 0.25)$
$P(X \geq 2)$
$= 1 - P(X = 0) - P(X = 1)$
$= 1 - {}^{10}C_0(0.25)^0(0.75)^{10} - {}^{10}C_1(0.25)^1(0.75)^9$
$= 1 - 0.05631 - 0.18771$
= 0.7560

(c) Mean = np = 10(0.25) = 2.5
Standard deviation
$= \sqrt{npq} = \sqrt{10(0.25)(0.75)} = 1.369$

9 (a) P(at least a tail and at least a head are obtained)
= 1 – P(all tails or all heads are obtained)
= $1 - \left(\frac{1}{2}\right)^3 - \left(\frac{1}{2}\right)^3 = \frac{3}{4}$

(b) $X \sim B(20, 0.75)$
Mean = np = 20(0.75) = 15
Standard deviation
$= \sqrt{npq} = \sqrt{20(0.75)(0.25)}$ = 1.936

10 X – Number of bottles that are cracked
$X \sim B(10, 0.08)$
(a) $P(X \geq 2)$
$= 1 - P(X = 0) - P(X = 1)$
$= 1 - {}^{10}C_0(0.08)^0(0.92)^{10}$
$- {}^{10}C_1(0.08)^1(0.92)^9$
= 0.1879

(b)
$$P(X \geq 1) > 0.95$$
$$1 - P(X = 0) > 0.95$$
$$1 - {}^{n}C_0(0.08)^0(0.92)^n > 0.95$$
$$1 - (0.92)^n > 0.95$$
$$0.05 > (0.92)^n$$
$$(0.92)^n < 0.05$$
$$n \lg 0.92 < \lg 0.05$$
$$-0.0362n < -1.3010$$
$$n > \frac{-1.3010}{-0.0362}$$
$$n > 35.94$$
Hence, the minimum number of bottles = 36

11 X – Number of graduates who could find a job
$X \sim B(8, 0.7)$
(a) $P(X \geq 7) = P(X = 7) + P(X = 8)$
$= {}^{8}C_7(0.7)^7(0.3)^1 + {}^{8}C_8(0.7)^8(0.3)^0$
$= 0.2553$

(b) Let Y – Number of graduates who could not find a job
$Y \sim B(8, 0.3)$
$P(Y \leq 2)$
$= P(Y = 0) + P(Y = 1) + P(Y = 2)$
$= {}^{8}C_0(0.3)^0(0.7)^8 + {}^{8}C_1(0.3)^1(0.7)^7 + {}^{8}C_3(0.3)^3(0.7)^5$
$= 0.5094$

12 (a) X – Number of questions that are guessed correctly
$X \sim B\left(75, \frac{1}{5}\right)$
(i) Mean = np = $75 \times \frac{1}{5} = 15$
(ii) Standard deviation
$= \sqrt{npq}$
$= \sqrt{75 \times \frac{1}{5} \times \frac{4}{5}}$
$= 3.464$

(b) Y – Number of questions that are guessed correctly for the balance 15
$Y \sim B\left(15, \frac{1}{5}\right)$
(i) $P(Y = 6) = {}^{15}C_6\left(\frac{1}{5}\right)^6\left(\frac{4}{5}\right)^9$
$= 0.04299$

(ii) $P(Y \geq 3)$
$= 1 - P(Y = 0) - P(Y = 1) - P(Y = 2)$
$= 1 - {}^{15}C_0\left(\frac{1}{5}\right)^0\left(\frac{4}{5}\right)^{15}$
$- {}^{15}C_1\left(\frac{1}{5}\right)^1\left(\frac{4}{5}\right)^{14} - {}^{15}C_2\left(\frac{1}{5}\right)^2\left(\frac{4}{5}\right)^{13}$
= 1 – 0.0352 – 0.1319 – 0.2309
= 0.6020

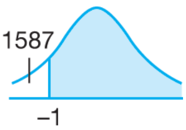
13 X – Thickness of a book, in cm

$X \sim N(5, 0.2^2)$

(a) $P(X > 4.8)$

$= P\left(Z > \frac{4.8-5}{0.2}\right)$

$= P(Z > -1)$ ←



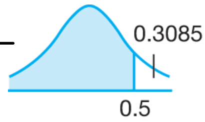
$= 1 - 0.1587$

$= 0.8413$

(b) $P(X < 5.1)$

$= P\left(Z < \frac{5.1-5}{0.2}\right)$

$= P(Z < 0.5)$ ←



$= 1 - 0.3085$

$= 0.6915$

$= 69.15\ \%$

(c) $P(4.6 < X < 5.4)$

$= P\left(\frac{4.6-5}{0.2} < Z < \frac{5.4-5}{0.2}\right)$

$= P(-2 < Z < 2)$

$= 1 - 0.0228 - 0.0228$

$= 0.9544$

Number of books

$= 0.9544 \times 10\ 000$

$= 9\ 544$

14 X – Mass of a box, in g

$X \sim N(30, \sigma^2)$

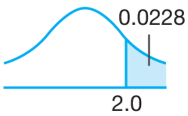
(a) $P(X > 40) = 0.0228$

$P\left(Z > \frac{40-30}{\sigma}\right) = 0.0228$

$P\left(Z > \frac{10}{\sigma}\right) = 0.0228$

$\frac{10}{\sigma} = 2.0$

$\sigma = 5$



(b) $P(X < 32)$

$= P\left(Z < \frac{32-30}{5}\right)$

$= P(Z < 0.4)$

$= 1 - 0.3446$

$= 0.6554$

15 X – Mass of a potato, in g

$X \sim N(100, 5^2)$

(a) $P(90 < X < 105)$

$= P\left(\frac{90-100}{5} < Z < \frac{105-100}{5}\right)$

$= P(-2 < Z < 1)$

$= 1 - 0.0228 - 0.1587$

$= 0.8185$

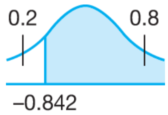
$= 81.85\%$

(b) $P(X > m) = 0.8$

$P\left(Z > \frac{m-100}{5}\right) = 0.8$

$\frac{m-100}{5} = -0.842$

$m = 95.79$



16 X – Mass of a student, in kg

$X \sim N(50, 8^2)$

(a) $P(45 < X < 60)$

$= P\left(\frac{45-50}{8} < X < \frac{60-50}{8}\right)$

$= P(-0.625 < Z < 1.25)$

$= 1 - 0.2660 - 0.1056$

$= 0.6284$

Number of students

$= 0.6284 \times 1500$

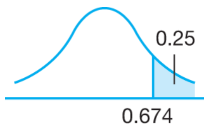
$= 943$

(b) $P(X > m) = 0.25$

$P\left(Z > \frac{m-50}{8}\right) = 0.25$

$\frac{m-50}{8} = 0.674$

$m = 55.39$



17 X – Number of soaps, in g

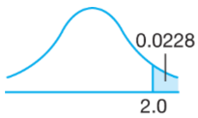
$X \sim N(\mu, \sigma^2)$

$P(X > 150) = 0.0228$

$P\left(Z > \frac{150-\mu}{\sigma}\right) = 0.0228$

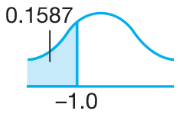
$\frac{150-\mu}{\sigma} = 2.0$

$150 - \mu = 2\sigma \quad \ldots (1)$



$P(X < 142.5) = 0.1587$

$P\left(Z < \frac{142.5 - \mu}{\sigma}\right) = 0.1587$



$\frac{142.5 - \mu}{\sigma} = -1.0$

$142.5 - \mu = -\sigma \ \ldots (2)$

$150 - \mu = 2\sigma$

$142.5 - \mu = -\sigma$

$(1) - (2): \ 7.5 = 3\sigma$

$\sigma = 2.5$

From (1) :

$150 - \mu = 2(2.5)$

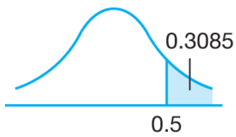
$\mu = 145$

18 X – Height of a tree, in m

$X \sim N(\mu, \sigma^2)$

$P(X > 32.5) = 0.3085$

$P\left(Z > \frac{32.5 - \mu}{\sigma}\right) = 0.3085$

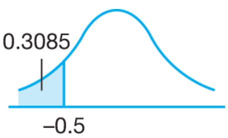


$\frac{32.5 - \mu}{\sigma} = 0.5$

$32.5 - \mu = 0.5\sigma \ \ldots (1)$

$P(X < 22.5) = 0.0668$

$P\left(Z < \frac{22.5 - \mu}{\sigma}\right) = 0.3085$



$\frac{22.5 - \mu}{\sigma} = -0.5$

$22.5 - \mu = -0.5\sigma \ \ldots (2)$

$(1) - (2): \ \sigma = 10$

From (1) :

$32.5 - \mu = 0.5(10)$

$\mu = 27.5$ m

19 X – Number of a pineapple, in kg

$X \sim N(1.3, 0.2^2)$

(a) P(grade A)

$= P(X > 1.4)$

$= P\left(Z > \frac{1.4 - 1.3}{0.2}\right)$

$= P(Z > 0.5)$

$= 0.3085$

(b) P(grade B)

$= P(1.2 < x \leq 1.4)$

$= P\left(\frac{1.2 - 1.3}{0.2} < Z < \frac{1.4 - 1.3}{0.2}\right)$

$= P(-0.5 < Z < 0.5)$

$= 1 - 0.3085 - 0.3085$

$= 0.383$

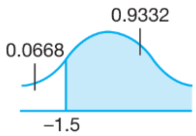
Hence, the number of grade B pineapples

$= 0.383 \times 1\,000$

$= 383$

(c) $P(X > m) = 93.32\%$

$P\left(Z > \frac{m - 1.3}{0.2}\right) = 0.9332$



$\frac{m - 1.3}{0.2} = -1.5$

$m = 1.0$

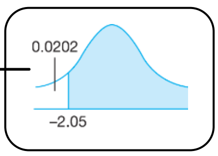
20 (a) X – Mass of a dragon fruit, in g

$X \sim N(550, 40^2)$

$P(X > 468)$

$= P\left(Z > \frac{468 - 550}{40}\right)$

$= P(Z > -2.05)$



$= 1 - 0.0202$

$= 0.9798$

(b) (i) Number of dragon fruits that have masses of more than 468 g

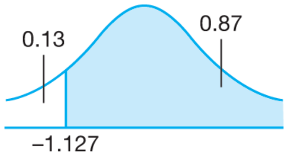
$= 0.9798 \times 400$

$= 391.92$

$= 392$ (correct to the nearest integer)

(ii) $\text{P}(X > m) = \dfrac{348}{400}$

$\text{P}\left(Z > \dfrac{m - 550}{40}\right) = 0.87$



$\dfrac{m - 550}{40} = -1.127$

$m = 504.92$

21 X – Time, in minutes, taken in a cross-country event

$X - \text{N}(24, 12^2)$

(a) $\text{P}(X > 36)$

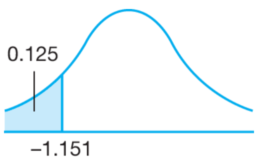
$= \text{P}\left(Z > \dfrac{36 - 24}{12}\right)$

$= \text{P}(Z > 1)$

$= 0.1587$

(b) $\text{P}(X < t) = \dfrac{100}{800}$

$\text{P}\left(Z < \dfrac{t - 24}{12}\right) = 0.125$



$\dfrac{t - 24}{12} = -1.151$

$t = 10.188$

22 X – Cumulative Grade Point Average

$X \sim N(2.7, 0.25^2)$

(a) $\text{P}(X > 3.1)$

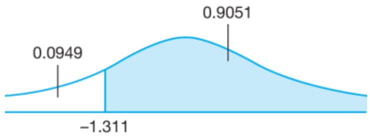
$= \text{P}\left(Z > \dfrac{3.1 - 2.7}{0.25}\right)$

$= \text{P}(Z > 1.6)$

$= 0.0548$

(b) $\text{P}(X > k) = 90.51\%$

$\text{P}\left(Z > \dfrac{k - 2.7}{0.25}\right) = 0.9051$



$\dfrac{k - 2.7}{0.25} = -1.311$

$k = 2.372$

23 X – Travelling time

$X \sim \text{N}(15, 4^2)$

(a) $\text{P}(X \le 20)$

$= \text{P}\left(Z \le \dfrac{20 - 15}{4}\right)$

$= \text{P}(Z \le 1.25)$ ←

$= 1 - 0.1056$

$= 0.8944$

(b) Y – Number of days not late for school

$Y \sim \text{N}(5, 0.8944)$

$\text{P}(Y = 5) = {}^5C_5 (0.8944)^5 (0.1056)^0$

$= 0.5723$

SPM Spot

1 (a) (i) X – Number of times the bus services reach the destination on time.

$X \sim B(6, 0.9)$

$P(X \geq 5) = P(X = 5) + P(X = 6)$

$= {}^6C_5(0.9)^5(0.1)^1 + {}^6C_6(0.9)^6(0.1)^0$

$= 0.354294 + 0.531441$

$= 0.8857$

(ii) Y – Number of times the bus company is awarded monthly excellent service

$Y \sim B(12, 0.8857, 0.1143)$

$P(Y > 10)$

$= P(Y = 11) + P(Y = 12)$

$= {}^{12}C_{11}(0.8857)^{11}(0.1143)^1 + {}^{12}C_{12}(0.8857)^{12}(0.1143)^0$

$= 0.36089 + 0.23304$

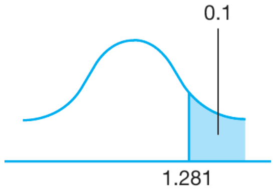
$= 0.5939$

(b) X – Mass of a Mathematics book, in g

$X \sim N(\mu, \sigma^2)$

$P(X > 850) = 0.1$

$$P\left(Z > \frac{850-\mu}{\sigma}\right) = 0.1$$

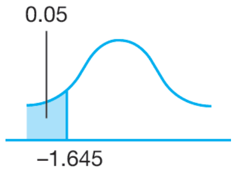


$$\frac{850-\mu}{\sigma} = 1.281$$

$850 - \mu = 1.281\sigma \quad \ldots (1)$

$P(X < 700) = 0.05$

$$P\left(Z < \frac{700-\mu}{\sigma}\right) = 0.05$$



$$\frac{700-\mu}{\sigma} = -1.645$$

$700 - \mu = -1.645\sigma \quad \ldots (2)$

$(1) - (2): 150 = 2.926\sigma$

$\sigma = 51.265$

From $(1): 850 - \mu = 1.281(51.265)$

$850 - \mu = 65.6705$

$\mu = 784.33$