Form 5 Chapter 1
Circular Measure
Fully-Worked Solutions

UPSKILL 1.1

1 (a) $53.6^{o} = 53.6 \times \frac{3.142}{180} = 0.9356 \text{ rad}$

(b) $126.4^{o} = 126.4 \times \frac{3.142}{180} = 2.2064 \text{ rad}$

(c) $37^{o}38' = 37\frac{38}{60} \times \frac{3.142}{180} = 0.6569 \text{ rad}$

(d) $93^{o}20' = 93\frac{20}{60} \times \frac{3.142}{180} = 1.6292 \text{ rad}$

(e) $176^{o}30' = 176\frac{30}{60} \times \frac{3.142}{180} = 3.0809 \text{ rad}$

2 (a) $30^{o} = 30 \times \frac{\pi}{180} = \frac{\pi}{6} \text{ rad}$

(b) $45^{o} = 45 \times \frac{\pi}{180} = \frac{\pi}{4} \text{ rad}$

(c) $60^{o} = 60 \times \frac{\pi}{180} = \frac{\pi}{3} \text{ rad}$

(d) $120^{o} = 120 \times \frac{\pi}{180} = \frac{2}{3}\pi \text{ rad}$

(e) $300^{o} = 300 \times \frac{\pi}{180} = \frac{5}{3}\pi \text{ rad}$

3 (a) $0.3574 \text{ rad} = 0.3574 \times \frac{180}{3.142} = 20.47^{o}$

(b) $0.8121 \text{ rad} = 0.8121 \times \frac{180}{3.142} = 46.52^{o}$

(c) $1.0438 \text{ rad} = 1.0438 \times \frac{180}{3.142} = 59.80^{o}$

(d) $1.1693 \text{ rad} = 1.1693 \times \frac{180}{3.142} = 66.99^{o}$

(e) $1.2325 \text{ rad} = 1.2325 \times \frac{180}{3.142} = 70.61^{o}$

4 (a) $\frac{5}{12}\pi = \frac{5}{12}\pi \times \frac{180}{\pi} = 75^{o}$

(b) $\frac{7}{4}\pi = \frac{7}{4}\pi \times \frac{180}{\pi} = 315^{o}$

(c) $\frac{28}{9}\pi = \frac{28}{9}\pi \times \frac{180}{\pi} = 560^{o}$

(d) $\frac{13}{4}\pi = \frac{13}{4}\pi \times \frac{180}{\pi} = 585^{o}$

(e) $\frac{18}{5}\pi = \frac{18}{5}\pi \times \frac{180}{\pi} = 648^{o}$

UPSKILL 1.2a

1 (a) $s = 11 \times 2 = 22 \text{ cm}$

(b) $s = 7 \times \left(160 \times \frac{3.142}{180}\right) = 19.55 \text{ cm}$

(c) $\theta = 2 \times 3.142 - 1.2 = 5.084 \text{ rad}$
$s = 8 \times 5.084 = 40.672 \text{ cm}$

(d) $\theta = 2 \times 3.142 - 4.5 = 1.784 \text{ rad}$
$s = 4 \times 1.784 = 7.136 \text{ cm}$

2 (a) $\theta = \frac{15}{4} \text{ rad}$

$\alpha = 2 \times 3.142 - \frac{15}{4} = 2.534 \text{ rad}$

(b) $\alpha = 2 \times 3.142 - 4.2 = 2.084 \text{ rad}$

$r = \frac{10.42}{2.084} = 5 \text{ cm}$

3 Perimeter of *ABCD* = 40

$6 + 6 + \frac{2}{3}OA + \frac{2}{3}(OA + 6) = 40$

$12 + \frac{4}{3}OA + 4 = 40$

$\frac{4}{3}OA = 40 - 16$

$\frac{4}{3}OA = 24$

$OA = 24 \times \frac{3}{4}$

$OA = 18 \text{ cm}$

4 (a) $r = \frac{s}{\theta}$

$r = \frac{12}{1.2}$

$r = 10 \text{ cm}$

(b) $\angle HOK = \frac{s}{r}$

$\angle HOK = \frac{36}{10} = 3.6 \text{ rad}$

5 (a) Let $PS = x$ cm

$$\frac{\text{Arc length } SR}{\text{Arc length } PQ} = \frac{3}{2}$$

$$\frac{\frac{\pi}{3}(14+x)}{\frac{\pi}{3}(14)} = \frac{3}{2}$$

$$14 + x = \frac{3}{2} \times 14$$

$$x = 21 - 14$$

$PS = 7$ cm

(b) Arc length RS

$$= 21 \times \frac{3.142}{3}$$

$= 21.994$ cm

6 (a) $\angle AOM = \frac{12}{18} = \frac{2}{3}$ rad

(b) Perimeter of the shaded region

$$= 18 + 18 + \left(\frac{\pi}{2} - \frac{2}{3}\right)(18)$$

$$= (24 + 9\pi) \text{ cm}$$

7 Perimeter of the minor sector POQ = Major arc length PQ

$$r + r + r\alpha = (2\pi - \alpha)r$$

$$2r + r\alpha = (2\pi r - \alpha r)$$

$$2 + \alpha = 2\pi - \alpha$$

$$2\alpha = 2\pi - 2$$

$\alpha = \pi - 1$ [Shown]

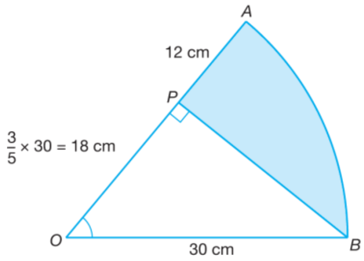
$$\angle POQ = 3.142 - 1 = 2.142 \text{ rad}$$

$$= 2.142 \times \frac{180}{3.142}$$

$= 122.71^\circ$

$= 122^\circ 43'$

8



$$\cos\theta = \frac{18}{30}$$

$$\theta = 53.13^\circ = 53.13 \times \frac{3.142}{180}$$

$= 0.9274$ rad

Arc length $AB = 30\ (0.9274)$

$= 27.822$ cm

$PB = \sqrt{30^2 - 18^2} = 24$ cm

Perimeter of the shaded region

$= PB + AP +$ Arc length AB

$= 24 + 12 + 27.822$

$= 63.82$ cm

9 $PQ = \sqrt{14^2 + 14^2} = 19.7990$ cm

Arc length QS

$$= \frac{3.142}{4} \times 19.7990$$

$= 15.5521$ cm

Length of OS

$= 19.7990 - 14$

$= 5.799$ cm

Perimeter of the shaded region

$= OQ + OS +$ Arc length QS

$= 14 + 5.799 + 15.5521$

$= 35.35$ cm

10 (a) $\cos\angle PBA = \dfrac{24}{30}$

$\angle PBA = 36.87^{o} = 36.87 \times \dfrac{3.142}{180}$

$= 0.6436$ rad

$\angle PBQ = 2 \times 0.6436 = 1.2872$ rad

(b) $AP = AQ = \sqrt{30^2 - 24^2} = 18$ cm

Arc length $PQ = 24 \times 1.2872$

$= 30.8928$ cm

Perimeter of the shaded region

$= 18 + 18 + 30.8928$

$= 66.90$ cm

UPSKILL 1.2b

1 (a) Length of the chord $PR = 12$ cm

$2r\sin\dfrac{\theta}{2} = 12$

$2(10)\sin\dfrac{\theta}{2} = 12$

$\sin\dfrac{\theta}{2} = \dfrac{3}{5}$

$\dfrac{\theta}{2} = 36.87^{o}$

$\theta = 73.74^{o}$

$\theta = 73.74 \times \dfrac{3.142}{180} = 1.2872$ rad

(b) Perimeter of the shaded segment

$= 10 \times 1.2872 + 12 = 24.87$ cm

2 (a) Length of chord $PQ = 10$ cm

$2r\sin\dfrac{\theta}{2} = 10$

$2r\sin\dfrac{120^{o}}{2} = 10$

$2r \times 0.8660 = 10$

$r = 5.774$ cm

(b) Major arc length PRQ

$= 5.774 \times \left(240 \times \dfrac{3.142}{180}\right) = 24.19$ cm

3 (a) (i) $\angle POT = 2(3.142) - 5 = 1.284$ rad

(ii) 1.284 rad $= 1.284 \times \dfrac{180}{3.142} = 73.56^{o}$

(b) Perimeter of the shaded segment

$= 2(10)\sin\left(\dfrac{73.56}{2}\right)^{o} + (10 \times 1.284)$

$= 24.81$ cm

UPSKILL 1.2c

1 (a) Using the cosine rule,

$4^2 = 7^2 + 5^2 - 2(7)(5)\cos\angle PRQ$

$\cos\angle PRQ = \dfrac{7^2 + 5^2 - 4^2}{2(7)(5)}$

$\cos\angle PRQ = \dfrac{7^2 + 5^2 - 4^2}{2(7)(5)}$

$\cos\angle PRQ = 0.82857$

$\angle PRQ = 34.05^{o}$

$\angle PRQ = 34.05 \times \dfrac{3.142}{180}$

$\angle PRQ = 0.5944$ rad

(b) Perimeter of the shaded region

$= PS + PQ +$ Arc length SQ

$= 2 + 4 + 5(0.5944)$

$= 8.972$ cm

UPSKILL 1.3a

1 (a) $A = \dfrac{1}{2}(12)^2(3) = 216$ cm^2

(b) $A = \dfrac{1}{2}(10)^2\left(120 \times \dfrac{3.142}{180}\right)$

$A = \dfrac{1}{2}(100)(2.09467)$

$A = 104.73$ cm^2

(c) $\theta = 2 \times 3.142 - 5 = 1.284$ rad

$A = \dfrac{1}{2}(12)^2(1.284)$

$A = 92.448$ cm^2

2 (a) $A = \dfrac{1}{2}r^2\theta$

$48 = \dfrac{1}{2}(12)^2\theta$

$48 = 72\theta$

$\theta = \dfrac{2}{3}$ rad

(b) $A = \frac{1}{2}r^2\theta$

$\frac{27}{2}\pi = \frac{1}{2}r^2\left(\frac{\pi}{3}\right)$

$\frac{27}{2} = \frac{1}{2}r^2\left(\frac{1}{3}\right)$

$r^2 = 81$

$r = 9$ cm

3 $\angle POQ = \frac{7}{10} = 0.7$ rad

$\angle QOR = 3.142 - 0.7 = 2.442$ rad

Area of the shaded region

$= \frac{1}{2}(10)^2(2.442)$

$= 122.1$ cm^2

4 (a) $\frac{\text{Arc length } BC}{\text{Arc length } AD} = \frac{3}{2}$

$\frac{9}{6\theta} = \frac{3}{2}$

$\theta = 1$ rad

(b) $OB = \frac{9}{1} = 9$ cm

Area of the shaded region

$= \frac{1}{2}(9)^2(1) - \frac{1}{2}(6)^2(1)$

$= 22.5$ cm^2

5 Area of sector ORS – Area of sector OPQ = 24

$\frac{1}{2}(5k)^2\left(\frac{3}{4}\right) - \frac{1}{2}(3k)^2\left(\frac{3}{4}\right) = 24$

$75k^2 - 27k^2 = 192$

$48k^2 = 192$

$k^2 = 4$

$k = 2$

6 $\dfrac{\frac{1}{2}(18)^2\left(\frac{2}{3}\right)}{\frac{1}{2}r^2\left(\frac{2}{3}\right) - \frac{1}{2}(18)^2\left(\frac{2}{3}\right)} = \frac{9}{7}$

$\frac{(18)^2(2)}{r^2(2) - (18)^2(2)} = \frac{9}{7}$

$\frac{648}{2r^2 - 648} = \frac{9}{7}$

$9(2r^2 - 648) = 4536$

$18r^2 - 5832 = 4536$

$r^2 = 576$

$r = 24$

7 (a) $\tan \angle XOT = \frac{14}{4}$

$\angle XOT = 74.05°$

$\angle XOT = 74.05 \times \frac{3.142}{180}$

$\angle XOT = 1.29266$

$\angle XOT = 1.2927$

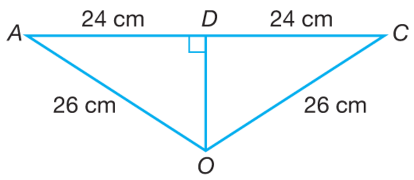
$\therefore \angle XOY = 2 \times 1.29270 = 2.5853$ rad

(b) Area of the shaded region

$= 2 \times \left(\frac{1}{2} \times 4 \times 14\right) - \frac{1}{2}(4)^2(2.5853)$

$= 35.3176$ cm^2 $= 35.32$ cm^2

8 (a)



$\sin \angle AOD = \frac{24}{26}$

$\angle AOD = 67.38°$

$\angle AOC = 67.38° \times 2$

$\angle AOC = 134.76°$

$\angle AOC = 134.76 \times \frac{3.142}{180}$

$\angle AOC = 2.3523$ rad

(b) Area of the shaded region

$= \frac{1}{2}(26)^2(2 \times 3.142 - 2.3523)$

$= 1\ 328.91$ cm^2

UPSKILL 1.3b

1 (a) Area of the shaded segment

$= \frac{1}{2}r^2(\theta - \sin\theta)$

$= \frac{1}{2}(20)^2(1.5 - \sin 85.93^{o})$

$= 100.5\text{ cm}^2$

(b) $\theta = 2 \times 3.142 - 4.9 = 1.384$ rad

$\theta = 1.384 \times \frac{180}{3.142} = 79.29^{o}$

Area of the shaded segment

$= \frac{1}{2}r^2(\theta - \sin\theta)$

$= \frac{1}{2}(12)^2(1.384 - \sin 79.29^{o})$

$= 28.90\text{ cm}^2$

UPSKILL 1.3c

1 (a) $\frac{\pi}{3} = \frac{3.142}{180} = 1.04733\text{ rad} = 60^{o}$

$\cos 60^{o} = \frac{r}{20}$

$r = 10$

(b) Major arc length PT

$= 10\left(3.142 - \frac{3.142}{3}\right)$

$= 20.9467$ cm

$= 20.95$ cm

(c) $\sin 60^{o} = \frac{PR}{20}$

$PR = 20 \times \sin 60^{o} = 17.3205$ cm

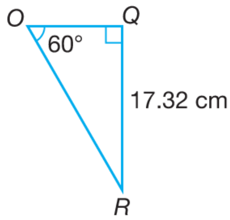
Area of the shaded region

$= \frac{1}{2} \times 20^2 \times 1.04733 - \frac{1}{2} \times 10 \times 17.3205$

$= 209.466 - 86.6025$

$= 122.86\text{ cm}^2$

2 (a)



$PR = 2(20)\sin\frac{120^{o}}{2} = 34.6410$ cm

$QR = \frac{34.6410}{2} = 17.32$ cm

$\tan 60^{o} = \frac{17.32}{r}$

$r = \frac{17.32}{\tan 60^{o}}$

$r = 10$

(b) $\angle PQR = 120 \times \frac{3.142}{180} = 2.094667$ rad

Area of the shaded region

$= \frac{1}{2}(20)^2\left(120 \times \frac{3.142}{180} - \sin 120^{o}\right)$

$= 245.73\text{ cm}^2$

$= 245.7\text{ cm}^2$

(c) Perimeter of the shaded region

$= 2 \times 20 \times \sin\frac{120}{2} + 20\left(120 \times \frac{3.142}{180}\right)$

$= 34.6410 + 41.8933$

$= 76.53$ cm

3 (a) $0.9\text{ rad} = 0.9 \times \frac{180}{3.142}$

$= 51.56^{o}$

$\cos 51.56^{o} = \frac{OD}{16}$

$OD = 9.9471$ cm

$\sin 51.56^{o} = \frac{AD}{16}$

$AD = 12.5322$ cm

Perimeter of the region X

$= (16 - 9.9471) + 12.5322 + 16(0.9)$

$= 32.9851$ cm

$= 32.99$ cm

(b) $\tan 51.56^{o} = \dfrac{AC}{16}$

$AC = 20.1581$ cm

Area of the region Y

$= \dfrac{1}{2} \times 16 \times 20.1581 - \dfrac{1}{2} \times 16^2 \times 0.9$

$= 46.0648$ cm^2

$= 46.06$ cm^2

4 (a) Area of the sector KOL

$= \dfrac{1}{2} \times \left(2 \times \sqrt{3}\right)^2 \times \dfrac{3.142}{3}$

$= 6.284$ cm^2

(b) Area of the sector $PAQB$

$= \dfrac{1}{2} \times \left(\sqrt{3}\right)^2 \times \left(\dfrac{2}{3} \times 3.142\right)$

$= 3.142$ cm^2

(c) Area of the quadrilateral $OAPB$

= Area of $\Delta OPA \times 2$

$= 2\left(\dfrac{1}{2} \times \sqrt{3} \times \sqrt{3} \times \sin 120^{o}\right) \times 2$

$= 2.59808$ cm^2

(d) Area of the sector OKL

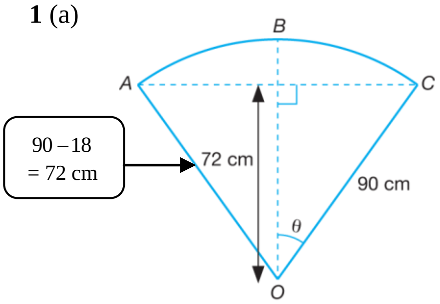
$= \dfrac{1}{2} \times \left(2\sqrt{3}\right)^2 \times \dfrac{3.142}{3}$

$= 6.284$ cm^2

Area of the shaded region
= Area of the sector OKL – Area of the quadrilateral $OAPB$ – Area of the sector $PAQB$
= 6.284 – 2.59808 – 3.142
= 0.5439 cm^2

UPSKILL 1.4

1 (a)



$\cos \theta = \dfrac{72}{90} = 0.8$

$\theta = 36.87^{o}$

$\theta = 36.87 \times \dfrac{3.142}{180}$

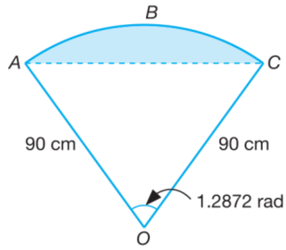
$\theta = 0.6436$ rad

$\angle AOC = 2\theta = 2 \times 0.6463 = 1.2872$ rad

$= 1.2872 \times \dfrac{180}{3.142}$

$= 73.74^{o}$

(b)



Cross-sectional area above the water surface

$= \dfrac{1}{2}(90)^2\left(1.2872 - \sin 73.74^{o}\right)$

$= 1\ 325.16$ cm^2

2 (a) Radius of the sector $OAPB$

= Slant height of the cone

$= \sqrt{6^2 + 5^2}$

$= \sqrt{61}$

= 7.8102 cm

(b) Arc length APB

= Circumference of the circular base of the cone

$= 2 \times 3.142 \times 5$

= 31.42 cm

(c) $\theta = \dfrac{s}{r} = \dfrac{31.42}{7.8102} = 4.0229$ rad

(d) Area of the sector $OAPB$

$= \dfrac{1}{2}(7.8102)^2(4.0229)$

$= 122.7$ cm^2

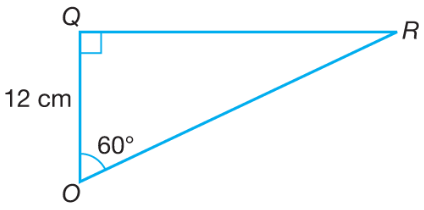
Summative Practice 1

1 (a) Since the radii of a circle are always the same, $OP = OQ = OR = 12$ cm
Since the opposite side of a rhombus are the same, then $PQ = OR$ and $RQ = OP$.
Therefore, ΔOPQ and ΔOQR are equilateral triangles such that
$\angle POR = \angle POQ + \angle QOR = 60^o + 60^o = 120^o$

Hence, $\angle POR = 120 \times \frac{\pi}{180} = \frac{2}{3}\pi$ rad

$\angle POR = \frac{2}{3} \times 3.142 = 2.09467$ rad

(b)



$\cos 60^o = \frac{12}{OR}$

$\frac{1}{2} = \frac{12}{OR}$

$OR = 12 \times 2$

$OR = 24$ cm

Hence, arc length JLK

$= 24 \times \frac{2}{3}(3.142)$

$= 50.27$ cm

(c) Area of the shaded segment

$= \frac{1}{2}(24)^2(2.09467 - \sin 120^o)$

$= 353.8$ cm^2

2 (a) 1 rad $= \frac{180}{3.142} = 57.29^o$

In ΔOQR,

$\cos 57.29^o = \frac{10}{OR}$

$OR = \frac{10}{\cos 57.29^o}$

$= 18.50525$ cm

$\sin 57.29^o = \frac{RQ}{18.50525}$

$RQ = 18.52525 \times \sin 57.29^o$

$= 15.5875$ cm

Perimeter of the shaded region A
= Arc length $PQ + PR + QR$
= 10(1) + (18.50525 – 10) + 15.5872
= 10 + 8.50525 + 15.5872
= 34.09 cm

(b) Area of the shaded region B
= Area of the sector ORS – Area of ΔOQR

$= \frac{1}{2} \times (18.50535)^2 \times 1 - \frac{1}{2} \times 10 \times 15.57062$

$= 93.37$ cm^2

3 (a) $\angle COQ = 1.982$ rad $= 1.982 \times \frac{180}{3.142}$

$= 113.55^o$

$\angle CAO = \angle COA = 3.142 - 1.982$

$= 1.16$ rad

(b) $\angle CAO = \angle COA = 180^o - 113.55^o$

$= 66.45^o$

$\angle ACO = 180^o - 66.45^o - 66.45^o = 47.10^o$

In ΔACO, using the sine rule,

$\frac{AO}{\sin 47.10^o} = \frac{5}{\sin 66.45^o}$

$AO = \frac{5}{\sin 66.45^o} \times \sin 47.10^o$

$AO = 3.9955$ cm
$AR = AQ = 5 + 3.9955 = 8.996$ cm

(c) $CR = AR - AC = 8.996 - 5 = 3.996$ cm
Arc length $CQ = 1.982 \times 5$
$= 9.91$ cm
Arc length $RQ = 8.996 \times 1.16$
$= 10.4354$ cm

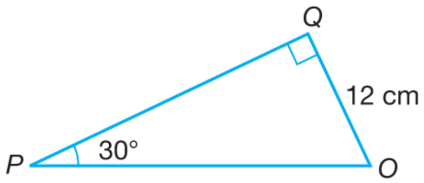
Perimeter of the shaded region
= CR + Arc length CQ + Arc length RQ
= 3.996 + 9.91 + 10.4354
= 24.34 cm

(d) Thus, the area of the shaded region
= Area of the sector ARQ – Area of the sector OCQ – Area of ΔACO

$= \frac{1}{2}(8.996)^2(1.16) - \frac{1}{2}(5)^2(1.982) - \frac{1}{2}(5)(5)\sin 47.10^o$

= 46.9385 – 24.775 – 9.1568
= 13.01 cm^2

4 (a)



$\sin 30^o = \dfrac{12}{PO}$

$PO = \dfrac{12}{\sin 30^o}$

$= 24$ cm

Length of POM = 24 + 12

= 36 cm

Arc length AMB

$= 36 \times \dfrac{3.142}{3}$

= 37.70 cm

(b) In ΔPOQ, $\tan 30^o = \dfrac{12}{PQ}$

$PQ = \dfrac{12}{\tan 30^o}$

= 20.78 cm

Area of the shaded region

= Area of sector $PAMB$ – 2 × Area of ΔPOQ – Area major sector QMR

$= \dfrac{1}{2}(36)^2\left(\dfrac{3.142}{3}\right) - 2\left(\dfrac{1}{2} \times 12 \times 20.7846\right)$

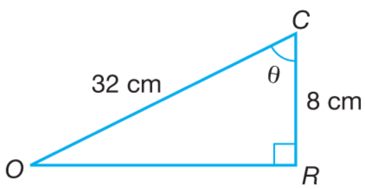
$-\dfrac{1}{2}(12)^2\left(240 \times \dfrac{3.142}{180}\right)$

= 678.672 – 249.4152 – 301.632

= 127.6248 cm^2

= 127.62 cm^2

5 (a)



$\cos \theta = \dfrac{8}{32}$

$\theta = 75.5225^o$

$\theta = 75.5225 \times \dfrac{3.142}{180}$

$\theta = 1.318$ rad [Shown]

(b) $OR = \sqrt{32^2 - 8^2} = 30.9839$ cm

$\sin \angle COR = \dfrac{8}{32}$

$\angle COR = 14.48^o$

Thus, $\angle ROP = 90^o + 14.48^o = 104.48^o$

$104.48^o = 104.48 \times \dfrac{3.142}{180} = 1.8238$ rad

Therefore, arc length PR

= 12(1.8238) = 21.89 cm

(c) Area of the shaded region

= Area of the trapezium $POCQ$ – Area of the sector POR – Area of the sector CRQ

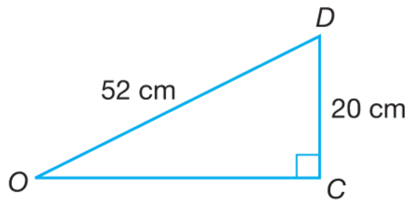
$= \dfrac{1}{2}(12 + 20)(30.9839) - \dfrac{1}{2}(12)^2(1.8238) -$

$\dfrac{1}{2}(20)^2(1.318)$

= 495.7424 – 131.3136 – 263.6

= 100.83 cm^2

6 (a)



$\sin \angle AOB = \sin \angle DCO = \dfrac{20}{52}$

$\angle AOB = 22.62^o$

$\angle AOB = 22.62 \times \dfrac{3.142}{180}$

$\angle AOB = 0.3948$ rad

(b) $\tan \angle DCO = \dfrac{20}{OC}$

$OC = \dfrac{20}{\tan 22.62^o}$

$OC = \dfrac{20}{0.41667}$

$OC = 47.9996$

Perimeter of the shaded region

= Arc length DT + DA + TB + Arc length AB

$= 20\left(90 \times \dfrac{3.142}{180}\right) + 52 + (104 - 20 - 48) +$

104(0.3948)

= 31.42 + 52 + 36 + 41.06

= 160.48 cm

(c) Area of the shaded region
= Area of the sector OAB – Area of ΔOCD – Area of quadrant CDT

$= \frac{1}{2}(104)^2(0.3948) - \frac{1}{2}(48)(20) - \frac{1}{2}(20)^2\left(90 \times \frac{3.142}{180}\right)$

$= 2135.08 - 480 - 314.2$

= 1 340.88 cm^2

7 (a) $\theta = 360^o - 80^o = 280^o$

$280^o = 280 \times \frac{3.142}{180}$

= 4.8876 rad
= 4.888 rad

(b) Area of the shaded major sector

$OAC = 244\frac{17}{45}$ cm^2.

$\frac{1}{2}r^2(4.88755) = 244\frac{17}{45}$

$r^2 = 100$

$r = 10$

Hence, the radius of the circle is 10 cm.

(c) Perimeter of the shaded segment
= 10 + 10 + 10(4.888)
= 68.88 cm

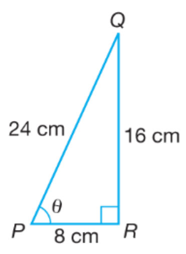
(d) Acute $\angle AOC = 80 \times \frac{3.142}{180} = 1.3964$ rad

Area of the shaded segment ABC

$= \frac{1}{2}(10)^2(1.3964 - \sin 80^o)$

= 20.58 cm^2

8



(a) $\tan\theta = \frac{16}{8}$

$\theta = 63.43^o$

$\theta = 63.43 \times \frac{3.142}{180}$

$\theta = 1.1072$ rad

(b) $\sin 63.43^o = \frac{16}{PQ}$

$PQ = \frac{16}{\sin 63.43^o} = 17.8893$ cm

Perimeter of the shaded region
= Arc length CB + Arc length QB + QC

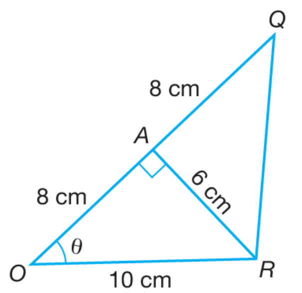
= 24(1.1072) + 16$\left(\frac{3.142}{2}\right)$ + (24 – 17.8893)
= 26.5728 + 25.136 + 6.1107
= 57.82 cm

(c) Area of the shaded region
= Area of the sector PCB – Area of ΔPOQ – Area of the quadrant QOB

$= \frac{1}{2}(24)^2(1.1072) - \frac{1}{2}(8)(16) - \frac{1}{2} \times 16^2 \times \left(\frac{3.142}{2}\right)$

= 318.87 – 64 – 201.088 = 53.78 cm^2

9 (a)



$\cos\theta = \frac{8}{10}$

$\theta = 36.87^o$

$\theta = 36.87 \times \frac{3.142}{180}$

$\theta = 36.87 \times \frac{3.142}{180}$

$\theta = 0.6436$ rad

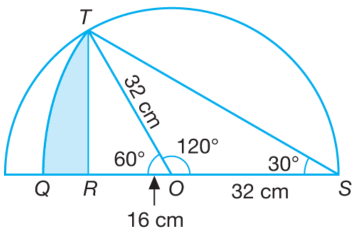
(b) Perimeter of the shaded region
= QR + RP + Arc length PQ
= 10 + 6 + 16(0.6436)
= 26.30 cm

(c) Area of the shaded region
= Area of sector OPQ – Area of ΔORQ

$= \frac{1}{2}(16)^2(0.6436) - \frac{1}{2} \times 16 \times 6$

= 34.38 cm^2

10 (a)



$\cos \angle TOR = \frac{16}{32}$

$\angle TOR = 60^o$

$\angle TOR = \frac{3.142}{3} = 1.0473$ rad

(b) $TS^2 = 32^2 + 32^2 - 2(32)(32) \cos 120^o$

$TS^2 = 3072$

$TS = 55.4256$ cm

Arc length TQ

$= 55.4256 \left(\frac{3.142}{6}\right)$

$= 29.02455$

$= 29.02$ cm

(c) Area of the shaded region

= Area of sector STR – Area of ΔTOS – Area of ΔTRO

$= \frac{1}{2}(55.4256)^2\left(\frac{3.142}{6}\right) -$

$\frac{1}{2}(32)(32)\sin 120^o - \frac{1}{2}(32)(16)\sin 60^o$

$= 804.3512 - 443.4050 - 221.7025$

$= 139.2$ cm^2

11 (a) Length of the arc of $BE = \frac{4}{3}\pi$

$2 \times \angle BAE = \frac{4}{3} \times 3.142$

$\angle BAE = \frac{1}{2} \times \frac{4}{3} \times \pi$

$= \frac{2}{3}\pi$ rad

$= \frac{2}{3} \times 3.142$

$= 2.0947$ rad

(b) $\angle BAE = 120^o$

$\angle OAE = 180^o - 120^o$

$= 60^o$

$\angle AOE = \angle AEO = 60^o = 1.0473$ rad

Area of sector BOD

$= \frac{1}{2} \times 8^2 \times 1.0473$

$= 33.5136$ cm^2

$= 33.51$ cm^2

(c) Area of the sector BAE

$= \frac{1}{2} \times 4^2 \times 2.0947$

$= 16.7576$ cm^2

Area of ΔAOE

$= \frac{1}{2} \times 4(4) \times \sin 60^o$

$= 6.9282$ cm^2

Area of the shaded region

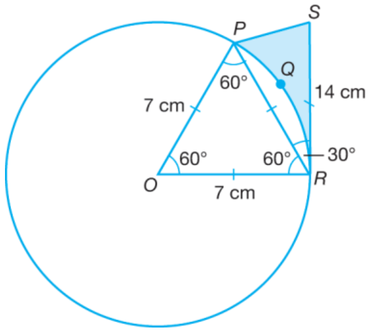
= Area of sector BOD – Area of the sector BAE – Area of ΔAOE

$= 33.5136 - 16.7576 - 6.9282$

$= 9.8278$ cm^2

$= 9.828$ cm^2

12 (a)



$\angle POR = \frac{3.142}{3} = 1.0473$ cm

Area of the segment PQR

$= \frac{1}{2}(7)^2\left(1.0473 - \sin 60^o\right)$

$= 4.4412$ cm^2

$= 4.441$ cm^2

(b) Area of ΔPRS

$= \frac{1}{2} \times 7 \times 7 \times \sin 30^o$

$= 12.25$ cm^2

Area of the shaded region

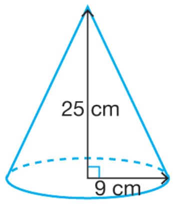
= Area of ΔPRS – Area of the segment PQR

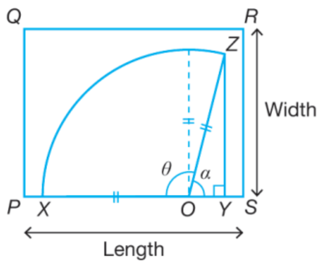
$= 12.25 - 4.4412$

$= 7.8088$ cm^2

$= 7.809$ cm^2

13





Radius of sector = Slant height of the cone

$= \sqrt{9^2 + 25^2}$

$= \sqrt{706}$

= 26.57 cm

Arc length of sector = Circumference of the cone

$= 2 \times 3.142 \times 9$

$= 56.556$ cm

$\theta = \frac{s}{r} = \frac{56.556}{26.57} = 2.1286$ rad

$\alpha = 3.142 - 2.1286 = 1.0134$ rad

1.0134 rad = $1.0134 \times \frac{180}{3.142}$

= 58.06°

$\sin \alpha = \frac{ZY}{r}$

$\sin 58.06 = \frac{ZY}{26.57}$

$ZY = 22.55$ cm

Hence, the minimum width of the cardboard is 22.55 cm.

$\cos \alpha = \frac{OY}{r}$

$\cos 58.06^{\circ} = \frac{OY}{26.57}$

$OY = 14.06$ cm

Hence, the minimum length of the cardboard

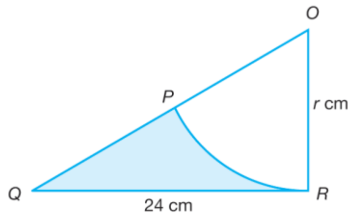
= $XO + OY$

= 26.57 + 14.06

= 40.63 cm

SPM Spot

1



(a) Area of ΔOQR = 84 cm^2

$\frac{1}{2} \times 24 \times r = 84$

$r = 7$ cm

$\tan \angle POR = \frac{24}{7}$

$\angle POR = 73.74^{\circ}$

$\angle POR = 73.74 \times \frac{3.142}{180}$

$\angle POR = 1.2872$ rad

(b) Area of the shaded region

$= 84 - \frac{1}{2} \times 7^2 \times 1.2872$

= 84 – 31.53

= 52.46 cm^2

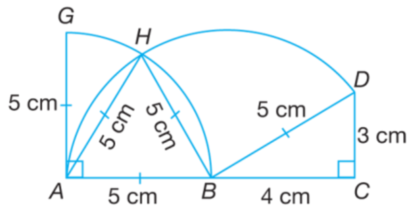
2 (a) $\sin \angle CBD = \frac{3}{5}$

$\angle CBD = 36.8699^{\circ}$

$= 36.8699^{\circ} \times \frac{3.142}{180^{\circ}}$

$= 0.6436$ rad

(b)



For the arc *AHD*,

$BD = BH = AB = 5$ cm

For the quadrant *AGHB*,

$AG = AH = AB = 5$ cm

Thus, *AH* = *BH* = *AB* = 5 cm

Therefore, ΔABH is an equilateral triangle.

Hence, $\angle ABH = 60^{\circ}$

(c) $\angle ABH = 60^{o} = \frac{\pi}{3}$ rad

Hence, $\angle HBD = \pi - \frac{\pi}{3} - 0.6436$ rad

= 1.4511 rad

$\angle GAH = \frac{\pi}{2} - \frac{\pi}{3} = \frac{\pi}{6}$ rad

Arc length *GH*

$= 5 \times \frac{\pi}{6}$

$= \frac{5\pi}{6}$ cm

Arc length *HD*

= 5×1.4511

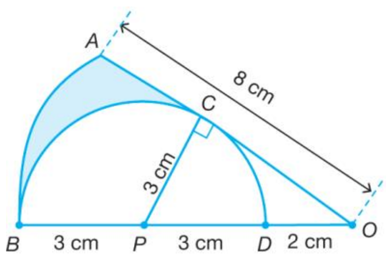
= 7.2555 cm

Hence, the sum of the arc length *GH* and the arc length *HD*

$= \frac{5(3.142)}{6} + 7.2555$

= 9.874 cm

3



(a) In ΔPOC,

$\sin \angle COP = \frac{3}{5}$

$\angle COP = 36.87^{o}$

$\angle COP = 36.87 \times \frac{3.142}{180}$

= 0.6436 rad

$\therefore \angle AOB = \angle COP = 36.87^{o}$

= 0.6436 rad

(b) In ΔPOC,

$\angle CPD = 180^{o} - 90^{o} - 36.87^{o}$

$= 53.13^{o}$

$\therefore \angle BPC = 180^{o} - 53.13^{o} = 126.87^{o}$

$= 126.87 \times \frac{3.142}{180}$

= 2.2146 rad

In ΔOPC, using the Pythagoras' Theorem,

$OC = \sqrt{5^2 - 3^3} = 4$ cm

Arc length *BC*

= 3(2.2146) = 6.6438 cm

Arc length *AB*

= 8(0.6436) = 5.1488 cm

Hence, the perimeter of the shaded region

= *AC* + Arc length *BC* + Arc length *AB*

= (8 – 4) + 6.6438 + 5.1488

= 15.79 cm

(c) Area of the shaded region

= Area of sector *OAB* –Area of ΔPOC – Area of sector *BPC*

$= \frac{1}{2}(8)^2(0.6436) - \frac{1}{2}(3)(4) - \frac{1}{2}(3)^2(2.2146)$

= 20.5952 – 6 – 9.9657

= 4.630 cm^2