Form 4 Chapter 9
Solution of Triangles
Fully-Worked Solutions

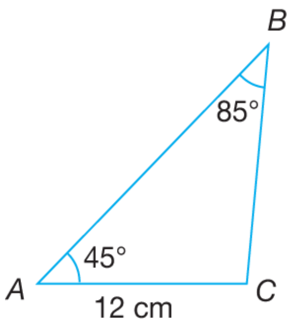
UPSKILL 9.1a

1 $\dfrac{AC}{\sin 40^o} = \dfrac{13}{\sin 33^o}$

$AC = \dfrac{13}{\sin 33^o} \times \sin 40^o$

$AC = 15.34 \text{ cm}$

2



(a) $\dfrac{BC}{\sin 40^o} = \dfrac{12}{\sin 85^o}$

$BC = \dfrac{12}{\sin 85^o} \times \sin 45^o$

$BC = 8.518 \text{ cm}$

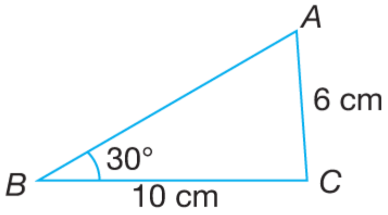
(b) $\angle ACB = 180^o - 45^o - 85^o = 50^o$

$\dfrac{AB}{\sin 50^o} = \dfrac{12}{\sin 85^o}$

$AB = \dfrac{12}{\sin 85^o} \times \sin 50^o$

$AB = 9.228 \text{ cm}$

3



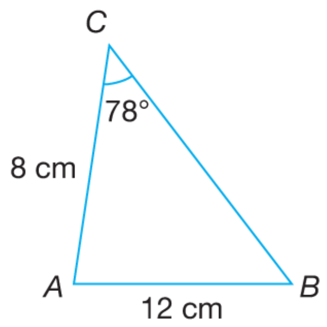
$\dfrac{\sin \angle BAC}{10} = \dfrac{\sin 30^o}{6}$

$\sin \angle BAC = \dfrac{\sin 30^o}{6} \times 10$

$\sin \angle BAC = 0.83333$

$\angle BAC = 56.44^o$

4



$\dfrac{\sin \angle ABC}{8} = \dfrac{\sin 78^o}{12}$

$\sin \angle ABC = \dfrac{\sin 78^o}{12} \times 8$

$\sin \angle ABC = 0.6521$

$\angle ABC = 40.70^o$

5 $\angle PQR = 180^o - 66^o - 72^o = 42^o$

$\dfrac{PQ}{\sin 72^o} = \dfrac{7}{\sin 66^o}$

$PQ = \dfrac{7}{\sin 66^o} \times \sin 72^o$

$PQ = 7.287 \text{ cm}$

$\dfrac{PR}{\sin 42^o} = \dfrac{7}{\sin 66^o}$

$PR = \dfrac{7}{\sin 66^o} \times \sin 42^o$

$PR = 5.127 \text{ cm}$

6 $\angle RPQ = 180^o - 42^o - 100^o = 38^o$

$\dfrac{PR}{\sin 100^o} = \dfrac{5}{\sin 42^o}$

$PR = \dfrac{5}{\sin 42^o} \times \sin 100^o$

$PR = 7.359 \text{ cm}$

$\dfrac{QR}{\sin 38^o} = \dfrac{5}{\sin 42^o}$

$QR = \dfrac{5}{\sin 42^o} \times \sin 38^o$

$QR = 4.600 \text{ cm}$

7 $\frac{\sin \angle ABC}{12} = \frac{\sin 65^o}{14}$

$\sin \angle ABC = \frac{\sin 65^o}{14} \times 12$

$\sin \angle ABC = 0.77684$

$\angle ABC = 50.97^o$

$\angle ACB = 180^o - 65^o - 50.97^o = 64.03^o$

$\frac{AB}{\sin 64.03^o} = \frac{14}{\sin 65^o}$

$AB = \frac{14}{\sin 65^o} \times \sin 64.03^o$

$AB = 13.89 \text{ cm}$

8 $\frac{\sin \angle BAC}{2} = \frac{\sin 35^o}{8}$

$\sin \angle BAC = \frac{\sin 35^o}{8} \times 2$

$\sin \angle BAC = 0.14339$

$\angle BAC = 8.24^o$

$\angle ACB = 180^o - 8.24^o - 35^o = 136.76^o$

$\frac{AB}{\sin 136.76^o} = \frac{8}{\sin 35^o}$

$AB = \frac{8}{\sin 35^o} \times \sin 136.76^o$

$AB = 9.555 \text{ cm}$

9 $\frac{\sin \angle PRQ}{13} = \frac{\sin 19^o}{5}$

$\sin \angle PRQ = \frac{\sin 19^o}{5} \times 13$

Basic $\angle$ = 57.83

$\angle PRQ = 180^o - 57.83^o$

$\angle PRQ = 122.17^o$

10 (a) In ΔACD,

$\angle ADC = 180^o - 70^o - 60^o = 50^o$

$\frac{AC}{\sin 50^o} = \frac{20}{\sin 60^o}$

$AC = \frac{20}{\sin 60^o} \times \sin 50^o$

$AC = 17.69 \text{ cm}$

(b) In ΔABC,

$\frac{\sin \angle ABC}{17.69} = \frac{\sin 120^o}{19}$

$\sin \angle ABC = \frac{\sin 120^o}{19} \times 17.69$

$\sin \angle ABC = 0.80632$

$\angle ABC = 53.74^o$

11 (a) In ΔPQR,

$\frac{PQ}{\sin 60^o} = \frac{14}{\sin 50^o}$

$PQ = \frac{14}{\sin 50^o} \times \sin 60^o$

$PQ = 15.83 \text{ cm}$

(b) In ΔPRS,

$\frac{\sin \angle RSP}{14} = \frac{\sin 120^o}{20}$

$\sin \angle RSP = \frac{\sin 120^o}{20} \times 14$

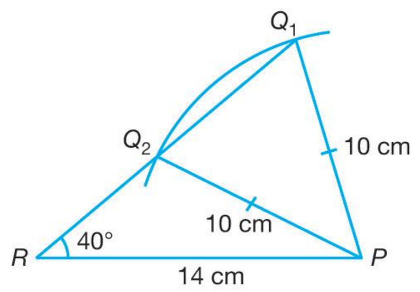
$\sin \angle RSP = 0.60622$

$\angle RSP = 37.32^o$

UPSKILL 9.1b

1 (a) Not ambiguous
(b) Ambiguous

2 (a)



(b) $\frac{\sin \angle Q}{14} = \frac{\sin 40^o}{10}$

$\sin \angle Q = \frac{\sin 40^o}{10} \times 14$

Basic $\angle$ = 64.15^o

$\angle PQ_1R = 64.15^o$

$\angle PQ_2R = 180^o - 64.15^o = 115.85^o$

In ΔPQ_1R,

$\angle RPQ_1 = 180^o - 40^o - 64.15^o$

$= 75.85^o$

$$\frac{RQ_1}{\sin 75.85^o} = \frac{10}{\sin 40^o}$$

$$RQ_1 = \frac{10}{\sin 40^o} \times \sin 75.85^o$$

$$RQ_1 = 15.09 \text{ cm}$$

In ΔPQ_2R,

$\angle RPQ_2 = 180^o - 40^o - 115.85^o$

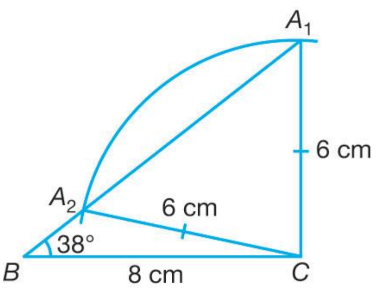
$= 24.15^o$

$$\frac{RQ_2}{\sin 24.15^o} = \frac{10}{\sin 40^o}$$

$$RQ_2 = \frac{10}{\sin 40^o} \times \sin 24.15^o$$

$$RQ_2 = 6.365 \text{ cm}$$

3 (a)



(b) $\frac{\sin \angle A}{8} = \frac{\sin 38^o}{6}$

$\sin \angle A = \frac{\sin 38^o}{6} \times 8$

$\sin \angle A = 0.82088$

Basic $\angle$ = 55.17^o

$\angle BA_1C = 55.17^o$

$\angle BA_2C = 124.83^o$

In ΔBA_1C,

$\angle BCA_1 = 180^o - 38^o - 55.17^o$

$= 86.83^o$

$$\frac{BA_1}{\sin 86.83^o} = \frac{6}{\sin 38^o}$$

$$BA_1 = \frac{6}{\sin 38^o} \times \sin 86.83^o$$

$$BA_1 = 9.731 \text{ cm}$$

In ΔBA_2C,

$\angle BCA_2 = 180^o - 38^o - 124.83^o$

$= 17.17^o$

$$\frac{BA_2}{\sin 17.17^o} = \frac{6}{\sin 38^o}$$

$$BA_2 = 2.877 \text{ cm}$$

4 (a) $\frac{\sin \angle ABC}{12} = \frac{\sin 35^o}{8}$

$\sin \angle ABC = \frac{\sin 35^o}{8} \times 12$

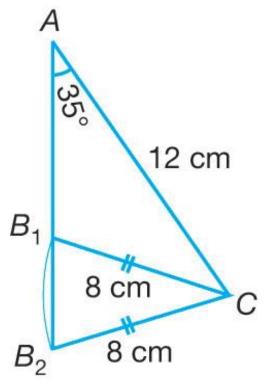
$\sin \angle ABC = 0.86036$

Basic $\angle$ = 59.36^o

$\angle AB_1C = 180^o - 29.36^o$

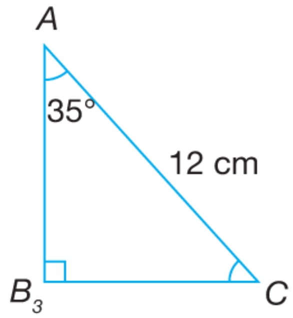
$= 120.64^o$

(b)



(c) Acute angle AB_2C = 59.36^o

(d)



$\sin 35^o = \frac{B_3C}{12}$

$B_3C = 6.883 \text{ cm}$

UPSKILL 9.1c

1 (a) $\angle BCD = 180^o - 65^o - 80^o = 35^o$

In ΔBCD,

$$\frac{BD}{\sin 35^o} = \frac{11}{\sin 65^o}$$

$$BD = \frac{11}{\sin 65^o} \times \sin 35^o$$

$$BD = 6.962 \text{ cm}$$

(b) In ΔABD,

$$\frac{\sin \angle DAB}{6.962} = \frac{\sin 115^o}{9}$$

$$\sin \angle DAB = \frac{\sin 115^o}{9} \times 6.962$$

$$\angle DAB = 44.51^o$$

UPSKILL 9.2

1 $PR^2 = 12^2 + 15^2 - 2(12)(15) \cos 53^o$

$$PR^2 = 152.3466$$

$$PR = 12.34 \text{ cm}$$

$$\frac{\sin \angle RPQ}{12} = \frac{\sin 53^o}{12.34}$$

$$\sin \angle RPQ = 0.77663$$

$$\angle RPQ = 50.95^o$$

$$\angle PRQ = 180^o - 53^o - 50.95^o = 76.05^o$$

2 $XY^2 = 9^2 + 6^2 - 2(9)(6) \text{ kos} 140^o$

$$XY^2 = 199.7328$$

$$XY = 14.13 \text{ cm}$$

$$\frac{\sin \angle YXZ}{6} = \frac{\sin 140^o}{14.13}$$

$$\sin \angle YXZ = 0.27295$$

$$\angle YXZ = 15.84^o$$

$$\angle XYZ = 180^o - 140^o - 15.84^o = 24.16^o$$

3 $10^2 = 15^2 + 18^2 - 2(15)(18) \cos \angle LKM$

$$\cos \angle LKM = \frac{15^2 + 18^2 - 10^2}{2(15)(18)}$$

$$\cos \angle LKM = 0.83148$$

$$\angle LKM = 33.75^o$$

$$\frac{\sin \angle LMK}{15} = \frac{\sin 33.75^o}{10}$$

$$\sin \angle LMK = \frac{\sin 33.75^o}{10} \times 15$$

$$\angle LMK = 56.44^o$$

$$\angle KLM = 180^o - 33.75^o - 56.44^o$$

$$= 89.81^o$$

4 $14^2 = 11^2 + 8^2 - 2(11)(8) \cos \angle PQR$

$$\cos \angle PQR = \frac{11^2 + 8^2 - 14^2}{2(11)(8)}$$

$$\cos \angle PQR = -0.0625$$

$$\angle PQR = 93.58^o$$

$$\frac{\sin \angle QPR}{8} = \frac{\sin 93.58}{14}$$

$$\sin \angle QPR = 0.5703$$

$$\angle QPR = 34.77^o$$

$$\angle PRQ = 180^o - 93.58^o - 34.77^o$$

$$\angle PRQ = 51.65^o$$

5 (a) In ΔABP,

$$\angle ABP = 180^o - 38^o - 78^o = 64^o$$

$$\frac{BP}{\sin 38^o} = \frac{5}{\sin 64^o}$$

$$BP = 3.4249 \text{ cm}$$

(b) In ΔCBP,

$$\angle CBP = 180^o - 64^o = 116^o$$

$$CP^2 = 7^2 + 3.4249^2 - 2(7)(3.4249) \cos 116^o$$

$$CP = 9.0415 \text{ cm}$$

6 (a) In ΔABD,

$$6^2 = 10^2 + 7^2 - 2(10)(7)\cos \angle ABD$$

$$\cos \angle ABD = \frac{10^2 + 7^2 - 6^2}{2(10)(7)}$$

$$\angle ABD = 36.18^{o}$$

(b) In ΔDBP,

$$\angle DBP = 180^{o} - 36.18^{o} = 143.82^{o}$$

$$\frac{\sin \angle DPB}{7} = \frac{\sin 143.82}{16}$$

$$\sin \angle DPB = 0.25827$$

$$\angle DPB = 14.97^{o}$$

UPSKILL 9.3

1 Area of $\Delta ABC = \frac{1}{2} \times 9 \times 11 \times \sin 43^{o}$

$$= 33.76 \text{ cm}^2$$

2 $\angle ABC = 180^{o} - 48^{o} - 24^{o}$

$$= 108^{o}$$

Area of $\Delta ABC = \frac{1}{2} \times 12 \times 8 \times \sin 108^{o}$

$$= 45.65 \text{ cm}^2$$

3 (a) $3^2 = 2^2 + 4^2 - 2(2)(4)\cos \angle BAC$

$$\cos \angle BAC = \frac{2^2 + 4^2 - 3^2}{2(2)(4)}$$

$$\cos \angle BAC = 0.6875$$

$$\angle BAC = 46.57^{o}$$

(b) Area of ΔABC

$$= \frac{1}{2} \times 2 \times 4 \times \sin 46.57^{o}$$

$$= 2.905 \text{ cm}^2$$

4 $s = \frac{4+6+9}{2} = 9.5$

Area of ΔABC

$$= \sqrt{9.5(9.5-4)(9.5-6)(9.5-9)}$$

$$= \sqrt{9.5(5.5)(3.5)(0.5)}$$

$$= \sqrt{91.4375}$$

$$= 9.562 \text{ cm}^2$$

5 Area of ΔABC = 20 cm^2

$$\frac{1}{2} \times 7 \times 10 \times \sin \theta = 20$$

$$\sin \theta = 0.57143$$

$$\theta = 34.85^{o}$$

6 Area of ΔABC = 43 cm^2

$$\frac{1}{2} \times 8 \times 13 \times \sin \angle BAC = 43$$

$$\sin \angle BAC = 0.8269$$

$$\text{Basic } \angle = 55.78^{o}$$

$$\angle BAC = 124.22^{o}$$

UPSKILL 9.4

1 (a) In ΔPQS,

$$QS^2 = 18^2 + 16^2 - 2(18)(16)\cos 43^{o}$$

$$QS^2 = 1587403$$

$$QS = 12.599 \text{ cm}$$

(b) In ΔSQR,

$$\frac{\sin \angle SQR}{20} = \frac{\sin 20^{o}}{12.599}$$

$$\sin \angle SQR = \frac{\sin 20^{o}}{12.599} \times 20$$

$$\sin \angle SQR = 0.5429$$

$$\text{Basic } \angle = 32.88^{o}$$

$$\angle SQR = 180^{o} - 32.88^{o}$$

$$= 147.12^{o}$$

(c) In ΔQSR,

$$\angle QSR = 180^{o} - 20^{o} - 147.12^{o}$$

$$= 12.88^{o}$$

Area of the whole diagram

$$= \frac{1}{2}(18)(16)\sin 43^{o} + \frac{1}{2}(12.599)(20)\sin 12.88^{o}$$

$$= 98.21 + 28.08$$

$$= 126.29 \text{ cm}^2$$

2 (a) In ΔPQS ,

$$\frac{\sin \angle PSQ}{8} = \frac{\sin 35^o}{7}$$

$$\sin \angle PSQ = \frac{\sin 35^o}{7} \times 8$$

$$\sin \angle PSQ = 0.65552$$

$$\angle PSQ = 40.96^o$$

$$\angle PQS = 180^o - 40.96^o - 35^o$$

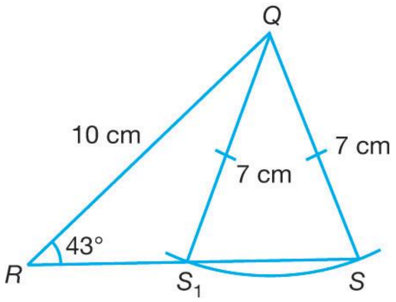
$$= 104.04^o$$

Area of ΔPQS

$$= \frac{1}{2} \times 8 \times 7 \times \sin 104.04^o$$

= 27.16 cm^2

(b)



$$\frac{\sin \angle S}{10} = \frac{\sin 43^o}{7}$$

$$\sin \angle S = \frac{\sin 43^o}{7} \times 10$$

$$\sin \angle S = 0.97428$$

$$\angle S = 76.98^o \text{ or } 103.02^o$$

Thus, $\angle RS_1Q = 103.02^o$

$$\angle RQS_1 = 180^o - 43^o - 103.02^o$$

$$= 33.98^o$$

$$\frac{RS_1}{\sin 33.98^o} = \frac{7}{\sin 43^o}$$

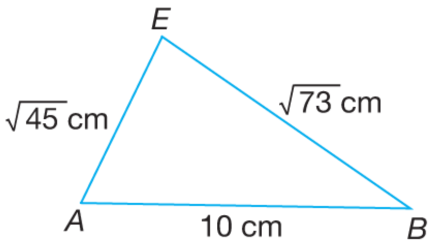
$$RS_1 = \frac{7}{\sin 43^o} \times \sin 33.98^o$$

$$RS_1 = 5.737 \text{ cm}$$

3 (a) $AE = \sqrt{6^2 + 3^2} = \sqrt{45}$ cm

$BE = \sqrt{8^2 + 3^2} = \sqrt{73}$ cm

$AB = DC = 10$



$$10^2 = 45 + 73 - 2\sqrt{45}\sqrt{73} \cos \angle AEB$$

$$\cos \angle AEB = \frac{45 + 73 - 100}{2\sqrt{45}\sqrt{73}}$$

$$\cos \angle AEB = 0.15703$$

$$\angle AEB = 80.97^o$$

(b) Area of ΔAEB

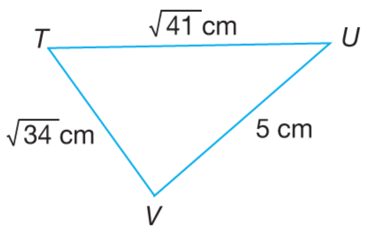
$$= \frac{1}{2} \times \sqrt{45} \times \sqrt{73} \sin 80.97^o$$

= 28.30 cm^2

4 $TV = \sqrt{3^2 + 5^2} = \sqrt{34}$

$UV = \sqrt{3^2 + 4^2} = 5$

$TU = \sqrt{5^2 + 4^2} = \sqrt{41}$



(a) $5^2 = 34 + 41 - 2\sqrt{34}\sqrt{41} \cos \angle UTV$

$$\cos \angle UTV = \frac{34 + 41 - 25}{2\sqrt{34}\sqrt{41}}$$

$$\cos \angle UTV = 0.66959$$

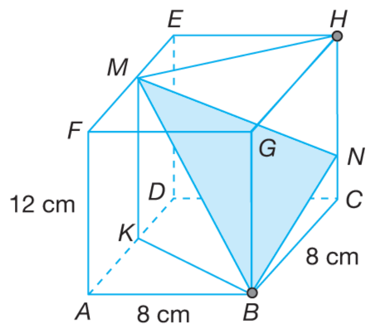
$$\angle UTV = 47.96^o$$

(b) Area of ΔUTV

$$= \frac{1}{2}\sqrt{34}\sqrt{41} \times \sin 47.96^o$$

= 13.86 cm^2

5



$KB^2 = 4^2 + 8^2 = 80$

$MB^2 = KB^2 + MK^2 = 80 + 12^2$

$MB = \sqrt{224}$ cm

$NC = \frac{1}{4}(12) = 3$ cm

$NB^2 = NC^2 + BC^2 = 9 + 64 = 73$

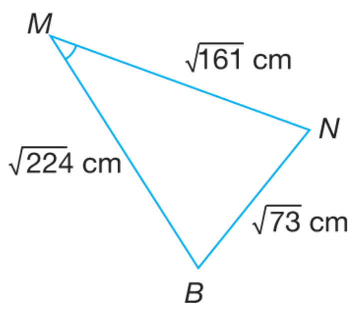
$NB = \sqrt{73}$ cm

$MH^2 = 4^2 + 8^2 = 80$

$HN = \frac{3}{4}(12) = 9$ cm

$MN^2 = MH^2 + HN^2 = 80 + 9^2 = 161$

$MN = \sqrt{161}$ cm



$73 = 224 + 161 - 2\sqrt{224}\sqrt{161}\cos\angle MBN$

$\cos\angle MBN = \frac{224 + 161 - 73}{2\sqrt{224}\sqrt{161}}$

$\cos\angle MBN = 0.821462$

$\angle MBN = 34.77^\circ$

Area of ΔMNB

$= \frac{1}{2} \times \sqrt{224} \times \sqrt{161} \times \sin 34.77^\circ$

$= 54.15\text{ cm}^2$

Summative Practice 9

1 (a) Area of $\Delta DBC = 29\text{ cm}^2$

$\frac{1}{2} \times 6 \times 10 \times \sin\angle DCB = 29$

$\sin\angle DCB = 0.96667$

Basic $\angle = 75.16^\circ$

$\angle DCB = 180^\circ - 75.16^\circ$

$\angle DCB = 104.84^\circ$ (obtuse)

(b) $BD^2 = 6^2 + 10^2 - 2(6)(10)\cos 104.84^\circ$

$BD^2 = 166.73448$

$BD = 12.91$ cm

(c) In ΔABD,

$\frac{\sin\angle A}{12.91} = \frac{\sin 45^\circ}{9.5}$

$\sin\angle A = \frac{\sin 45^\circ}{9.5} \times 12.91$

$\sin\angle A = 0.96092$

$\angle A = 73.93^\circ$ ← Quadrant I

$\therefore \angle DAB = 73.93^\circ$

and $\angle DA'B = 180^\circ - 73.93^\circ$

$= 106.07^\circ$ ← Quadrant II

Thus, $\angle ABD = 180^\circ - 45^\circ - \angle DAB$

$= 180^\circ - 45^\circ - 73.93^\circ$

$= 61.07^\circ$

Thus,

$\angle A'BD = 180^\circ - 45^\circ - \angle DA'B$

$= 180^\circ - 45^\circ - 106.07^\circ$

$= 28.93^\circ$

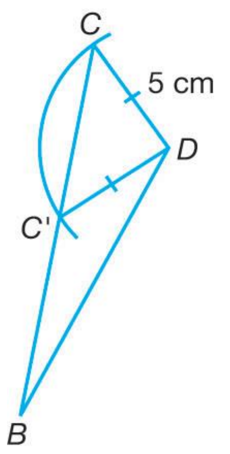
2 (a) (i) In ΔABC,

$$BC^2 = 16^2 + 7^2 - 2(16)(7)\cos 55^\circ$$
$$= 176.5189$$
$$BC = \sqrt{176.5189} = 13.29 \text{ cm}$$

(ii) In ΔCBD,

$$\frac{\sin \angle CBD}{5} = \frac{\sin 115^\circ}{13.29}$$
$$\sin \angle CBD = \frac{\sin 115^\circ}{13.29} \times 5$$
$$\sin \angle CBD = 0.34097$$
$$\angle CBD = 19.94^\circ$$

(b) (i)



(ii) In ΔBCD,

$$\angle BCD = 180^\circ - 115^\circ - 19.94^\circ = 45.06^\circ$$

In $\Delta DCC'$,

$$\angle DC'C = \angle DCC' = 45.06^\circ$$
$$\therefore \angle DC'B = 180^\circ - 45.06^\circ = 134.94^\circ$$

In $\Delta C'DB$,

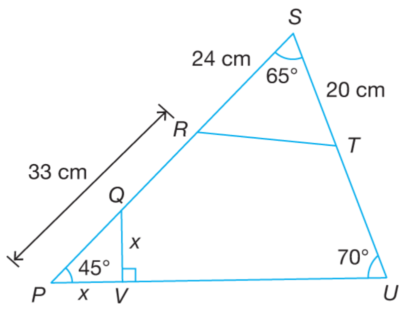
$$\frac{BD}{\sin 134.94^\circ} = \frac{5}{\sin 19.94^\circ}$$
$$BD = \frac{5 \times \sin 134.94^\circ}{\sin 19.94^\circ}$$
$$BD = 10.38 \text{ cm}$$

$\therefore$ Area of $\Delta BC'D$

$$= \frac{1}{2} \times 5 \times 10.38 \times \sin 25.12^\circ$$
$$= 11.02 \text{ cm}^2$$

$\angle C'DB = 180^\circ - 19.94^\circ - 134.94^\circ = 25.12^\circ$

3



(a) (i) In ΔRST,

$$RT^2 = 24^2 + 20^2 - 2(24)(20)\text{kos } 65^\circ$$
$$RT^2 = 570.2865$$
$$RT = 23.88 \text{ cm}$$

(ii) $\angle SUP = 180^\circ - 45^\circ - 65^\circ = 70^\circ$

In ΔPSU,

$$\frac{SU}{\sin 45^\circ} = \frac{57}{\sin 70^\circ}$$
$$SU = \frac{57}{\sin 70^\circ} \times \sin 45^\circ$$
$$SU = 42.89 \text{ cm}$$

$$\therefore TU = 42.89 - 20 = 22.89 \text{ cm}$$

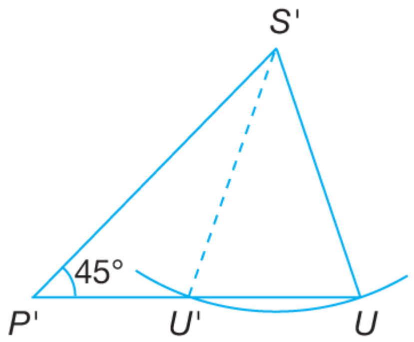
(b) Area of ΔPVQ

$$= \frac{1}{2} \times \text{Area of } \Delta RST$$
$$= \frac{1}{2} \times \left(\frac{1}{2} \times 24 \times 20 \times \sin 65^\circ\right)$$
$$= 108.7569 \text{ cm}^2$$

Hence, $\frac{1}{2}x^2 = 108.7569$

$$x^2 = 217.5139$$
$$x = 14.75$$
$$PV = 14.75 \text{ cm}$$

(c)



4 (a) In ΔEHN,

$NH^2 = 8^2 + 7^2 - 2(8)(7)\cos 67^\circ$

$NH^2 = 69.2381$

$NH = 8.321$ cm

(b) In ΔEHN,

$$\frac{\sin \angle ENH}{7} = \frac{\sin 67^\circ}{8.321}$$

$$\sin \angle ENH = \frac{\sin 67^\circ}{8.321} \times 7$$

$\sin \angle ENH = 0.7744$

$\angle ENH = 50.75^\circ$

In ΔHGN,

$NG^2 = 3^2 + 8.321^2 - 2(3)(8.321)\cos 50.75^\circ$

$NG^2 = 46.6506$

$NG = 6.830$ cm

(c) In ΔHGN,

$$\frac{\sin \angle NGH}{8.321} = \frac{\sin 50.75^\circ}{6.830}$$

$\sin \angle NGH = 0.9434$

Basic $\angle = 70.64^\circ$

$\angle NGH = 180^\circ - 70.64^\circ$

$= 109.36^\circ$ (obtuse)

(d) Area of ΔHGN

$$= \frac{1}{2} \times 3 \times 8.321 \times \sin 50.75^\circ$$

$= 9.666$ cm^2

5 (a) (i) In ΔABC,

$$\frac{BC}{\sin 35^\circ} = \frac{7}{\sin 40^\circ}$$

$$BC = \frac{7}{\sin 40^\circ} \times \sin 35^\circ$$

$= 6.246$ cm

(ii) $\angle ECD = \angle ACB$

$= 180^\circ - 35^\circ - 40^\circ$

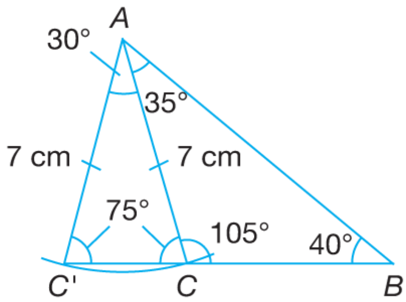
$= 105^\circ$

In ΔECD,

$DE^2 = 11^2 + 4^2 - 2(11)(4)\cos 105^\circ$

$DE = 12.64$ cm

(b) (i)



(ii) $\angle AC'B = \angle ACC'$

$= 180^\circ - 105^\circ$

$= 75^\circ$

(iii) $$\frac{C'B}{\sin 65^\circ} = \frac{7}{\sin 40^\circ}$$

$$C'B = \frac{7}{\sin 40^\circ} \times \sin 65^\circ$$

$= 9.8698$ cm

Area of $\Delta AC'B$

$$= \frac{1}{2} \times 7 \times 9.8698 \times \sin 75^\circ$$

$= 33.37$ cm^2

6 (a) (i) In ΔPQR,

$$\frac{\sin \angle PRQ}{9} = \frac{\sin 80.94^\circ}{10}$$

$\sin \angle PRQ = 0.88877$

$\angle PRQ = 62.72^\circ$

(ii) $\angle TQR = 180^\circ - \angle TRQ - \angle QTR$

$= 180^\circ - 62.72^\circ - 62.72^\circ$

$= 54.56^\circ$

In ΔQRT,

$TR^2 = 6^2 + 6^2 - 2(6)(6)\cos 54.56^\circ$

$TR^2 = 30.25079$

$TR = 5.50$ cm

(iii) $\angle PQT = 80.94^\circ - \angle TQR$

$= 80.94^\circ - 54.56^\circ$

$= 26.38^\circ$

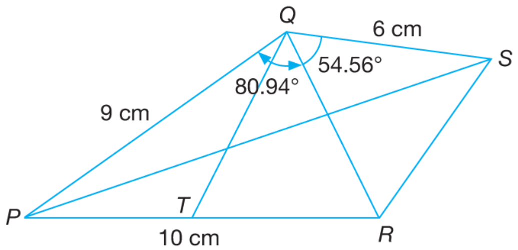
Area of ΔPQT

$= \frac{1}{2} \times 9 \times 6 \times \sin 26.38^\circ$

$= 12.00$ cm^2

(b) $\angle SQR = \angle TQR = 54.56^\circ$

$\therefore \angle PQS = 80.94^\circ + 54.56^\circ = 135.5^\circ$



In ΔPQS,

$PS^2 = 9^2 + 6^2 - 2(9)(6)\cos 135.5^\circ$

$PS^2 = 194.0310$

$PS = 13.93$ cm

7 (a) (i) Area of $\Delta ABC = 112$ cm^2

$\frac{1}{2} \times 18 \times BC \times \sin 34^\circ = 112$

$BC = 22.25$ cm

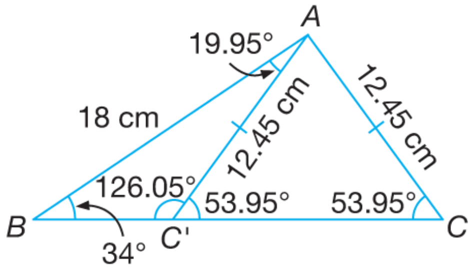
(ii) $AC^2 = 18^2 + 22.25^2 - 2(18)(22.25)\cos 34^\circ$

$AC = 12.45$ cm

(iii) $\frac{\sin \angle ACB}{18} = \frac{\sin 34^\circ}{12.45}$

$\angle ACB = 53.95^\circ$

(b) (i)



(ii) Area of $\Delta ABC'$

$= \frac{1}{2} \times 18 \times 12.45 \times \sin 19.95^\circ$

$= 38.23$ cm^2

8 (a) (i) In ΔPQR,

$$10^2 = 8^2 + 14^2 - 2(8)(14)\cos\angle QPR$$

$$\cos\angle QPR = \frac{8^2 + 14^2 - 10^2}{2(8)(14)}$$

$$\angle QPR = 44.42^\circ$$

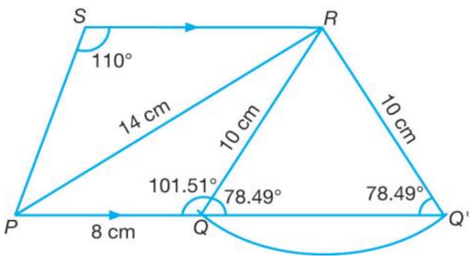
(ii) In ΔPSR,

$$\frac{PS}{\sin 44.42^\circ} = \frac{14}{\sin 110^\circ}$$

$$PS = \frac{14}{\sin 110^\circ} \times \sin 44.42^\circ$$

$$PS = 10.43 \text{ cm}$$

(b) (i)



In ΔPQR,

$$\frac{\sin\angle PQR}{14} = \frac{\sin 44.42^\circ}{10}$$

$$\sin\angle PQR = \frac{\sin 44.42^\circ}{10} \times 14$$

Basic $\angle = 78.49^\circ$

$$\angle PQR = 180^\circ - 78.49^\circ$$

$$= 101.51^\circ \text{ (obtuse)}$$

(ii) $\angle QRQ' = 180^\circ - 78.49^\circ - 78.49^\circ$

$$= 23.02^\circ$$

Area of $\Delta QQ'R$

$$= \frac{1}{2} \times 10 \times 10 \times \sin 23.02^\circ$$

$$= 19.55 \text{ cm}^2$$

9 (a) (i) In ΔABC,

$$\frac{BC}{\sin 101^\circ} = \frac{11.6}{\sin 34^\circ}$$

$$BC = \frac{11.6}{\sin 34^\circ} \times \sin 101^\circ$$

$$BC = 20.36 \text{ cm}$$

(ii) In ΔBED,

$$BD^2 = 17^2 + 9.2^2 - 2(17)(9.2)\cos 140^\circ$$

$$BD^2 = 613.2587$$

$$BD = 24.76 \text{ cm}$$

$$\therefore CD = BD - BC$$

$$= 24.76 - 20.36$$

$$= 4.40 \text{ cm}$$

(iii) $\angle ACB = 180^\circ - 34^\circ - 101^\circ$

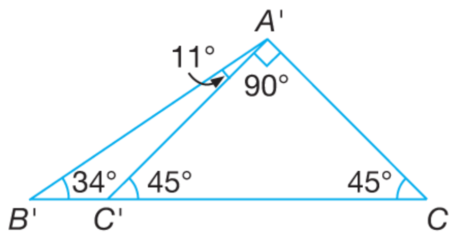
$$= 45^\circ$$

Area of ΔABC

$$= \frac{1}{2}(11.6)(20.36)\sin 45^\circ$$

$$= 83.50 \text{ cm}^2$$

(b) (i)



(ii) $\angle B'A'C' = 101^\circ - 90^\circ = 11^\circ$

10 (a) (i) $\angle SPQ = 180^o - 78^o - 32^o = 70^o$

In ΔPSQ,

$$\frac{QS}{\sin 70^o} = \frac{16}{\sin 32^o}$$

$QS = 28.37$ cm

(ii) In ΔSRQ,

$28.37^2 = 12^2 + 18^2 - 2(12)(18)\cos \angle QRS$

$$\cos \angle QRS = \frac{12^2 + 18^2 - 28.37^2}{2(12)(18)}$$

$\cos \angle QRS = -0.7798$

$\angle QRS = 141.24^o$

(iii) Area of $PQRS$

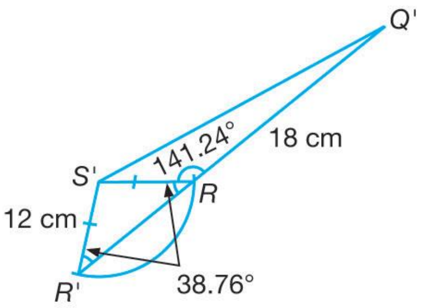
= Area of ΔPSQ + Area of ΔSRQ

$= \frac{1}{2} \times 16 \times 28.37 \times \sin 78^o +$

$\frac{1}{2} \times 12 \times 18 \times \sin 141.24^o$

$= 289.61$ cm^2

(b) (i)



(ii) $\angle S'R'Q' = \angle S'RR'$

$= 180^o - 141.24^o$

$= 38.76^o$

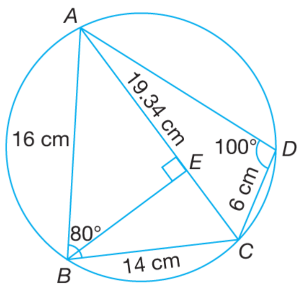
11 (a) (i) In ΔABC,

$AC^2 = 16^2 + 14^2 - 2(16)(14)\cos 80^o$

$AC^2 = 374.2056$

$AC = 19.34$ cm

(ii)



$\angle ADC = 180^o - \angle ABC$

$= 180^o - 80^o$

$= 100^o$

In ΔACD,

$$\frac{\sin \angle CAD}{6} = \frac{\sin 100^o}{19.34}$$

$\sin \angle CAD = 0.30552$

$\angle CAD = 17.79^o$

$\therefore \angle ACD = 180^o - 17.79^o - 100^o$

$= 62.21^o$

(b) (i) Area of ΔABC

$= \frac{1}{2} \times 16 \times 14 \times \sin 80^o$

$= 110.30$ cm^2

(ii) Area of $\Delta ABC = 110.30$ cm^2

$\frac{1}{2} \times AC \times BE = 110.30$

$\frac{1}{2} \times 19.34 \times BE = 110.30$

$BE = 11.41$ cm

Hence, the shortest distance from B to AC is 11.41 cm.

12 (a) (i) $BC^2 = 7^2 + 12^2 - 2(7)(12)\cos 115^\circ$

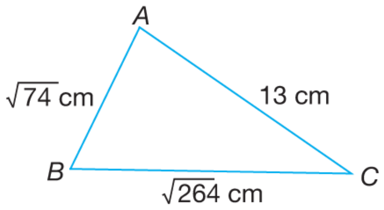
$BC^2 = 264.0$

$BC = \sqrt{264}$ cm

$BC = 16.25$ cm

(ii) $AB = \sqrt{5^2 + 7^2} = \sqrt{74}$ cm

$AC = 13$ cm



$264 = 74 + 13^2 - 2\sqrt{74} \times 13 \cos \angle BAC$

$\cos \angle BAC = \dfrac{74 + 13^2 - 264}{2\sqrt{74}\,(13)}$

$\angle BAC = 95.39^\circ$

(iii) Area of ΔBAC

$= \dfrac{1}{2}\sqrt{74}\,(13) \sin 95.39^\circ$

$= 55.67 \text{ cm}^2$

(b) $\dfrac{\sin \angle T}{16.25} = \dfrac{\sin 30^\circ}{9}$

$\sin \angle T = \dfrac{\sin 30^\circ}{9} \times 16.25$

$\sin \angle T = 0.9028$

$\angle T = 64.53^\circ$ or 115.47°

SPM Spot

1 (a) (i) In ΔAQB,

$AB = \sqrt{12^2 + 9^2} = \sqrt{225} = 15$ cm

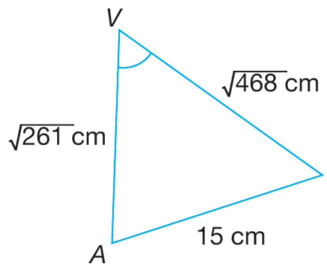
In ΔMPA, $MA^2 = 6^2 + 9^2 = 117$

In ΔVMA, $VA = \sqrt{12^2 + 117}$

$= \sqrt{261}$ cm

In ΔVMB, $VB = \sqrt{12^2 + 18^2}$

$= \sqrt{468}$ cm



$15^2 = 261 + 468 - 2\sqrt{261}\sqrt{468} \cos \angle AVB$

$2\sqrt{261}\sqrt{468} \cos \angle AVB = 261 + 468 - 225$

$\cos \angle AVB = \dfrac{261 + 468 - 225}{2\sqrt{261}\sqrt{468}}$

$\cos \angle AVB = 0.72104$

$\angle AVB = 43.86^\circ$

(ii) Area of ΔVAB

$= \dfrac{1}{2}\sqrt{261}\sqrt{468} \sin 43.86^\circ$

$= 121.1 \text{ cm}^2$

(b) $\dfrac{\sin \angle S}{18} = \dfrac{\sin 40^\circ}{12}$

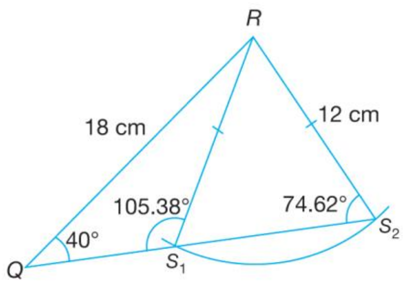
$\sin \angle S = \dfrac{\sin 40^\circ}{12} \times 18$

$\sin \angle S = 0.96418$

Basic $\angle = 74.62^\circ$

$\angle QS_2R = 74.62^\circ$ and

$\angle QS_1R = 180^\circ - 74.62^\circ = 105.38^\circ$



2 (a) In triangle *JNM*, using the cosine rule,

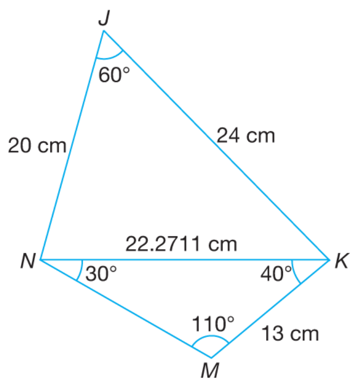
(i) $NK^2 = 20^2 + 24^2 - 2(20)(24)\cos 60^\circ$
$= 496$
$NK = 22.2711$ cm
$= 22.27$ cm

(ii) In triangle *NMK*, using the sine rule,

$$\frac{NM}{\sin 40^\circ} = \frac{22.2711}{\sin 110^\circ}$$

$$NM = \frac{22.2711}{\sin 110^\circ} \times \sin 40^\circ$$

$= 15.2343$ cm
$= 15.23$ cm



Area of triangle *JNK*

$= \frac{1}{2}(20)(24)\sin 60^\circ$

$= 207.85$ cm^2

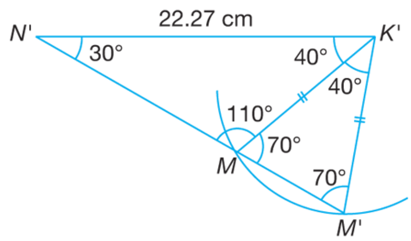
Area of triangle *NMK*

$= \frac{1}{2}(15.2343)(22.2711)\sin 30^\circ$

$= 84.82$ cm^2

Hence, the area of the quadrilateral *JKMN*
$= 207.85 + 84.82$
$= 292.67$ cm^2

(b)



$\angle N'K'M' = 80^\circ$