Form 4 Chapter 8
Vectors
Fully-Worked Solutions

UPSKILL 8.1a

1 $\left|\overrightarrow{AB}\right| = \sqrt{3^2+3^2} = \sqrt{18} = 2\sqrt{2} = 4.243$ units

The direction of $\overrightarrow{AB}$ is due southwest.

UPSKILL 8.1b

1 (i) $\overrightarrow{AB} = \underset{\sim}{a}$

(ii) $\overrightarrow{RS} = -\underset{\sim}{a}$

(iii) $\overrightarrow{XY} = \underset{\sim}{b}$

(iv) $\overrightarrow{KL} = \underset{\sim}{c}$

(v) $\overrightarrow{PQ} = \underset{\sim}{d}$

(vi) $\overrightarrow{MN} = -\underset{\sim}{b}$

(vii) $\overrightarrow{VW} = -\underset{\sim}{c}$

(viii) $\overrightarrow{CD} = -\underset{\sim}{d}$

(b) (i) $\left|\overrightarrow{RS}\right| = \sqrt{3^2+4^2} = 5$ units

(ii) $\left|\overrightarrow{XY}\right| = 3$ units

(iii) $\left|\overrightarrow{KL}\right| = 4$ units

(iv) $\left|\overrightarrow{PQ}\right| = \sqrt{2^2+3^2} = \sqrt{13} = 3.606$ units

UPSKILL 8.1c

1 (a) $\overrightarrow{CD} = \frac{1}{2}\underset{\sim}{b}$

(b) $\overrightarrow{EF} = 2\underset{\sim}{b}$

(c) $\overrightarrow{GH} = -\frac{3}{2}\underset{\sim}{b}$

2 (a) $(h-4)\underset{\sim}{v} = (5h-k)\underset{\sim}{w}$

$h-4=0 \Rightarrow h=4$

$5h-k=0 \Rightarrow 20-k=0 \Rightarrow k=20$

(b) $(2h-4)\underset{\sim}{v} = (k-6h+3)\underset{\sim}{w}$

$2h-4=0 \Rightarrow h=2$

$k-6h+3=0$

$k-6(2)+3=0$

$k=9$

UPSKILL 8.2a

1 (a) $\underset{\sim}{a}+\underset{\sim}{b} = \overrightarrow{PR}$

(b) $\underset{\sim}{b}+\underset{\sim}{c} = \overrightarrow{QS}$

(c) $\overrightarrow{PQ} + \overrightarrow{QS} = \overrightarrow{PS}$

2 (a) $\overrightarrow{EH} + \overrightarrow{EF} = \overrightarrow{EG}$

(b) $\overrightarrow{FE} + \overrightarrow{FG} = \overrightarrow{FH}$

3 (a) $\overrightarrow{PQ} + \overrightarrow{QR} + \overrightarrow{RS} = \overrightarrow{PS}$

(b) $\overrightarrow{PR} + \overrightarrow{RS} + \overrightarrow{ST} = \overrightarrow{PT}$

(c) $\overrightarrow{PQ} + \overrightarrow{QR} + \overrightarrow{RS} + \overrightarrow{ST} = \overrightarrow{PT}$

4 (a) $\overrightarrow{ON} - \overrightarrow{MN} - \overrightarrow{LM}$

$= \overrightarrow{ON} + \overrightarrow{NM} + \overrightarrow{ML}$

$= \overrightarrow{OL}$

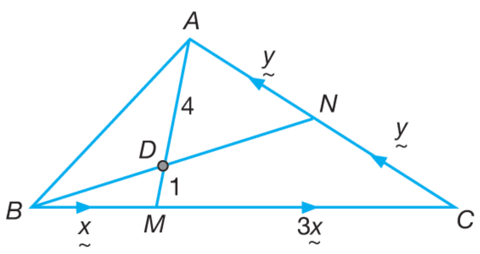
(b) $\overrightarrow{NJ} - \overrightarrow{KJ} - \overrightarrow{MK}$

$= \overrightarrow{NJ} + \overrightarrow{JK} + \overrightarrow{KM}$

$= \overrightarrow{NM}$

UPSKILL 8.2b

1



(a) $\overrightarrow{MA} = \overrightarrow{MC} + \overrightarrow{CA} = 3\underline{x} + 2\underline{y}$

(b) $\overrightarrow{MD} = \frac{1}{5}\overrightarrow{MA} = \frac{1}{5}(3\underline{x} + 2\underline{y}) = \frac{3}{5}\underline{x} + \frac{2}{5}\underline{y}$

(c) $\overrightarrow{BD} = \overrightarrow{BM} + \overrightarrow{MD}$

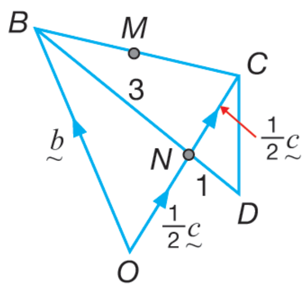
$= \underline{x} + \frac{3}{5}\underline{x} + \frac{2}{5}\underline{y}$

$= \frac{8}{5}\underline{x} + \frac{2}{5}\underline{y}$

(d) $\overrightarrow{BN} = \overrightarrow{BC} + \overrightarrow{CN} = 4\underline{x} + \underline{y}$

UPSKILL 8.2c

1



(a) (i) $\overrightarrow{OM} = \overrightarrow{OC} + \overrightarrow{CM}$

$= c + \frac{1}{2}\overrightarrow{CB}$

$= c + \frac{1}{2}\left(\overrightarrow{CO} + \overrightarrow{OB}\right)$

$= c + \frac{1}{2}(-\underline{c} + \underline{b})$

$= \frac{1}{2}\underline{c} + \frac{1}{2}\underline{b}$

$= \frac{1}{2}(\underline{c} + \underline{b})$ … (1)

(ii) $\frac{NB}{DB} = \frac{3}{4}$

$\overrightarrow{DB} = \frac{4}{3}\overrightarrow{NB}$

$= \frac{4}{3}\left(\overrightarrow{NO} + \overrightarrow{OB}\right)$

$= \frac{4}{3}\left(-\frac{1}{2}\underline{c} + \underline{b}\right)$

$= -\frac{2}{3}\underline{c} + \frac{4}{3}\underline{b}$

(b) $\overrightarrow{DC} = \overrightarrow{DB} + \overrightarrow{BC}$

$= -\frac{2}{3}\underline{c} + \frac{4}{3}\underline{b} + (-\underline{b} + \underline{c})$

$= \frac{1}{3}c + \frac{1}{3}b$

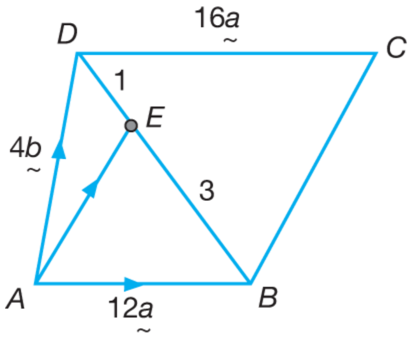
$= \frac{1}{3}(c + b)$

$= \frac{1}{3}\left(2\overrightarrow{OM}\right)$ — From (1) : $\underline{c} + \underline{b} = 2\overrightarrow{OM}$

$= \frac{2}{3}\overrightarrow{OM}$

Since $\overrightarrow{DC} = \frac{2}{3}\overrightarrow{OM}$, thus $\overrightarrow{DC}$ can be expressed as a scalar multiple of $\overrightarrow{OM}$. Hence, *DC* is parallel to *OM*.

2



(a) (i) $\overrightarrow{DB} = -4\underline{b} + 12\underline{a}$

(ii) $\overrightarrow{AE} = \overrightarrow{AD} + \frac{1}{4}\overrightarrow{DB}$

$= 4b + \frac{1}{4}(-4b + 12a)$

$= 4\underline{b} - \underline{b} + 3\underline{a}$

$= 3\underline{b} + 3\underline{a}$

$= 3(\underline{b} + \underline{a})$ … (1)

(b) $\overrightarrow{BC} = \overrightarrow{BD} + \overrightarrow{DC}$

$= 4\underline{b} - 12\underline{a} + \frac{4}{3}\left(\overrightarrow{AB}\right)$

$= 4\underline{b} - 12\underline{a} + \frac{4}{3}(12\underline{a})$

$= 4\underline{b} + 4\underline{a}$

$= 4(\underline{b} + \underline{a})$

$= 4\left(\frac{1}{3}\overrightarrow{AE}\right)$

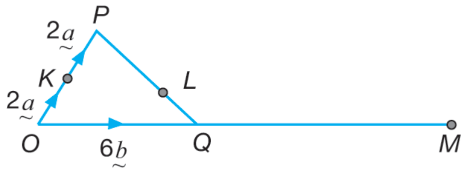
From (1) : $\underline{b} + \underline{c} = \frac{1}{3}\overrightarrow{AE}$

$= \frac{4}{3}\overrightarrow{AE}$

Since $\overrightarrow{BC} = \frac{4}{3}\overrightarrow{AE}$, thus $\overrightarrow{BC}$ can be expressed as a scalar multiple of $\overrightarrow{AE}$. Hence, *BC* is parallel to *AE*.

UPSKILL 8.2d

1



(a) $\overrightarrow{PQ} = \overrightarrow{PO} + \overrightarrow{OQ}$

$= -4\underline{a} + 6\underline{b}$

(b) $\overrightarrow{PL} = \frac{3}{5}\overrightarrow{PQ}$

$= \frac{3}{5}(-4\underline{a} + 6\underline{b})$

$= -\frac{12}{5}\underline{a} + \frac{18}{5}\underline{b}$

(c) $\overrightarrow{OM} = 3\overrightarrow{OQ}$

$= 3(6\underline{b})$

$= 18\underline{b}$

(d) $\overrightarrow{KL} = \overrightarrow{KP} + \overrightarrow{PL}$

$= 2\underline{a} - \frac{12}{5}\underline{a} + \frac{18}{5}\underline{b}$

$= -\frac{2}{5}\underline{a} + \frac{18}{5}\underline{b}$

(e) $\overrightarrow{KM} = \overrightarrow{KO} + \overrightarrow{OM}$

$= -2\underline{a} + 18\underline{b}$

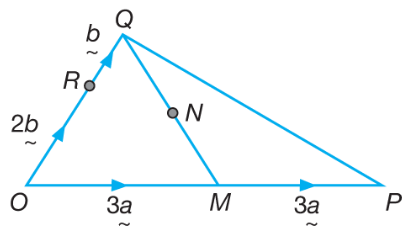
$\overrightarrow{KL} = -\frac{2}{5}\underline{a} + \frac{18}{b}\underline{b}$

$= \frac{1}{5}(-2\underline{a} + 18\underline{b})$

$= \frac{1}{5}\overrightarrow{KM}$

Since $\overrightarrow{KL} = \frac{1}{5}\overrightarrow{KM}$, $\overrightarrow{KL}$ can be expressed as a scalar multiple of $\overrightarrow{KM}$ and *K* is a common point. Thus, the points *K*, *L* and *M* are collinear.

2



(a) $\overrightarrow{PQ} = \overrightarrow{PO} + \overrightarrow{OQ}$

$= -6\underset{\sim}{a} + 3\underset{\sim}{b}$

(b) $\overrightarrow{QM} = \overrightarrow{QO} + \overrightarrow{OM}$

$= -3\underset{\sim}{b} + 3\underset{\sim}{a}$

(c) $\overrightarrow{ON} = \overrightarrow{OQ} + \overrightarrow{QN}$

$= 3\underset{\sim}{b} + \frac{1}{2}\overrightarrow{QM}$

$= 3\underset{\sim}{b} + \frac{1}{2}(-3\underset{\sim}{b} + 3\underset{\sim}{a})$

$= 3\underset{\sim}{b} - \frac{3}{2}\underset{\sim}{b} + \frac{3}{2}\underset{\sim}{a}$

$= \frac{3}{2}\underset{\sim}{b} + \frac{3}{2}\underset{\sim}{a}$

(d) $\overrightarrow{RN} = \overrightarrow{RQ} + \overrightarrow{QN}$

$= \underset{\sim}{b} - \frac{3}{2}\underset{\sim}{b} + \frac{3}{2}\underset{\sim}{a}$

$= -\frac{1}{2}\underset{\sim}{b} + \frac{3}{2}\underset{\sim}{a}$

(e) $\overrightarrow{RP} = \overrightarrow{RO} + \overrightarrow{OP}$

$= -2\underset{\sim}{b} + 6\underset{\sim}{a}$

$= 2(-\underset{\sim}{b} + 3\underset{\sim}{a})$ … (1)

$\overrightarrow{RN} = -\frac{1}{2}\underset{\sim}{b} + \frac{3}{2}\underset{\sim}{a}$

$= \frac{1}{2}(-\underset{\sim}{b} + 3\underset{\sim}{a})$

$= \frac{1}{2}\left(\frac{1}{2}\overrightarrow{RP}\right)$

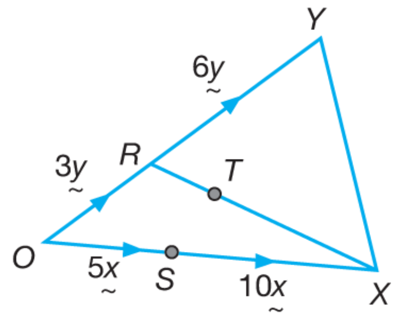
$= \frac{1}{4}\overrightarrow{RP}$

From (1) : $-\underset{\sim}{b} + 2\underset{\sim}{a} = \frac{1}{2}\overrightarrow{RP}$

Since $\overrightarrow{RN} = \frac{1}{4}\overrightarrow{RP}$, $\overrightarrow{RN}$ can be expressed as a scalar multiple of $\overrightarrow{RP}$ and R is a common point. Thus, the points R, N and P are collinear.

$RN : RP = 1 : 4$

3



(a) $\overrightarrow{RS} = \overrightarrow{RO} + \overrightarrow{OS}$

$= -3\underset{\sim}{y} + 5\underset{\sim}{x}$

(b) $\overrightarrow{RX} = \overrightarrow{RO} + \overrightarrow{OX}$

$= -3\underset{\sim}{y} + 15\underset{\sim}{x}$

(c) $\overrightarrow{SY} = \overrightarrow{SO} + \overrightarrow{OY}$

$= -5\underset{\sim}{x} + 9\underset{\sim}{y}$ … (1)

(d) $\overrightarrow{OT} = \overrightarrow{OR} + \overrightarrow{RT}$

$= 3\underset{\sim}{y} + \frac{1}{4}\overrightarrow{RX}$

$= 3\underset{\sim}{y} + \frac{1}{4}(-3\underset{\sim}{y} + 15\underset{\sim}{x})$

$= \frac{9}{4}\underset{\sim}{y} + \frac{15}{4}\underset{\sim}{x}$

(e) $\overrightarrow{ST} = \overrightarrow{SO} + \overrightarrow{OT}$

$= -5\underset{\sim}{x} + \frac{9}{4}\underset{\sim}{y} + \frac{15}{4}\underset{\sim}{x}$

$= -\frac{5}{4}\underset{\sim}{x} + \frac{9}{4}\underset{\sim}{y}$

$= \frac{1}{4}(-5\underset{\sim}{x} + 9\underset{\sim}{y})$

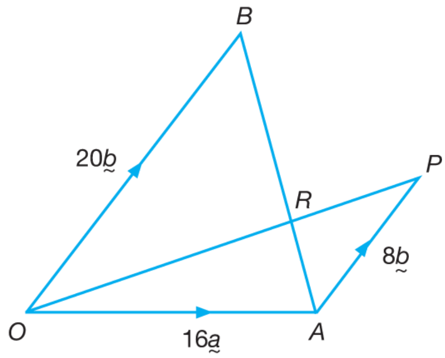
$= \frac{1}{4}\overrightarrow{SY}$

From (1) : $-5\underset{\sim}{x} + 9\underset{\sim}{y} = \overrightarrow{SY}$

Since $\overrightarrow{ST} = \frac{1}{4}\overrightarrow{SY}$, $\overrightarrow{ST}$ can be expressed as a scalar multiple of $\overrightarrow{SY}$ and S is a common point. Thus, the points S, T and Y are collinear.

UPSKILL 8.2e

1



(a) (i) $\overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{AP}$

$= 16\underline{a} + \frac{2}{5}\overrightarrow{OB}$

$= 16\underline{a} + \frac{2}{5}(20\underline{b})$

$= 16\underline{a} + 8\underline{b}$

(ii) $\overrightarrow{BA} = -20\underline{b} + 16\underline{a}$

(b) (i) $\overrightarrow{OR} = m\overrightarrow{OP}$

$= m(16\underline{a} + 8\underline{b})$

$= 16m\underline{a} + 8m\underline{b}$

(ii) $\overrightarrow{OR} = \overrightarrow{OB} + \overrightarrow{BR}$

$= 20\underline{b} + n\,\overrightarrow{BA}$

$= 20\underline{b} + n(-20\underline{b} + 16\underline{a})$

$= 20\underline{b} - 20n\underline{b} + 16n\underline{a}$

$= (20b - 20n)\underline{b} + 16n\underline{a}$

(c) $16m\underline{a} + 8m\underline{b} = (20 - 20n)\underline{b} + 16n\underline{a}$

Equating the coefficients of $\underline{a}$,

$16m = 16n$

$m = n \quad \dots (1)$

Equating the coefficients of $\underline{b}$,

$8m = 20 - 20n$

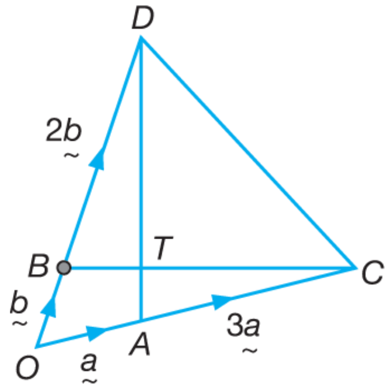
$8m = 20 - 20m$

$28m = 20$

$m = \frac{5}{7}$

From (1) : $n = m = \frac{5}{7}$

2



$\overrightarrow{AD} = -\underline{a} + 3\underline{b}$

$\overrightarrow{BC} = -\underline{b} + 4\underline{a}$

(a) $\overrightarrow{OT} = \overrightarrow{OA} + \overrightarrow{AT}$

$= \underline{a} + k\,\overrightarrow{AD}$

$= \underline{a} + k(-\underline{a} + 3\underline{b})$

$= (1 - k)\underline{a} + 3k\underline{b}$

(b) $\overrightarrow{OT} = \overrightarrow{OB} + \overrightarrow{BT}$

$= \underline{b} + t\,\overrightarrow{BC}$

$= \underline{b} + t(-\underline{b} + 4\underline{a})$

$= (1 - t)\underline{b} + 4t\underline{a}$

$(1 - k)\underline{a} + 3k\underline{b} = (1 - t)\underline{b} + 4t\underline{a}$

Equating the coefficients of $\underline{a}$,

$1 - k = 4t$

$k = 1 - 4t \quad \dots (1)$

Equating the coefficients of $\underline{b}$,

$3k = 1 - t \quad \dots (2)$

Substitute (1) into (2) :

$3(1 - 4t) = 1 - t$

$3 - 12t = 1 - t$

$11t = 2$

$t = \frac{2}{11}$

From (1) :

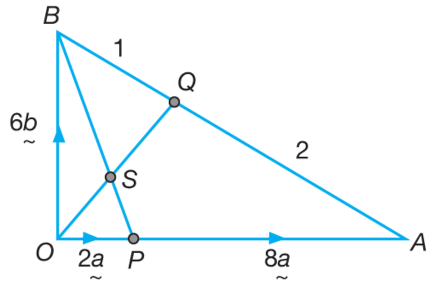
$k = 1 - 4\left(\frac{2}{11}\right) = \frac{3}{11}$

$\overrightarrow{OT} = (1-k)\underline{a} + 3k\underline{b}$

$\overrightarrow{OT} = \left(1-\frac{3}{11}\right)\underline{a} + 3\left(\frac{3}{11}\right)\underline{b}$

$\overrightarrow{OT} = \frac{8}{11}\underline{a} + \frac{9}{11}\underline{b}$

3



(a) (i) $\overrightarrow{BP} = -6\underline{b} + 2\underline{a}$

(ii) $\overrightarrow{OQ} = \overrightarrow{OB} + \overrightarrow{BQ}$

$= 6\underline{b} + \frac{1}{3}\overrightarrow{BA}$

$= 6\underline{b} + \frac{1}{3}(-6\underline{b} + 10\underline{a})$

$= 4\underline{b} + \frac{10}{3}\underline{a}$

(b) $\overrightarrow{OS} = \overrightarrow{OB} + \overrightarrow{BS}$

$h\overrightarrow{OQ} = 6\underline{b} + k\overrightarrow{BP}$

$h\left(4\underline{b} + \frac{10}{3}\underline{a}\right) = 6\underline{b} + k(-6\underline{b} + 2\underline{a})$

$4h\underline{b} + \frac{10}{3}h\underline{a} = 6\underline{b} - 6k\underline{b} + 2k\underline{a}$

$4h\underline{b} + \frac{10}{3}h\underline{a} = (6-6k)\underline{b} + 2k\underline{a}$

Equating the coefficients of $\underline{b}$,

$4h = 6 - 6k \quad \ldots (1)$

Equating the coefficients of $\underline{a}$,

$\frac{10}{3}h = 2k$

$10h = 6k$

$5h = 3k$

$6k = 10h \quad \ldots (2)$

Substitute (2) into (1) :

$4h = 6 - 10h$

$14h = 6$

$h = \frac{3}{7}$

From (2) :

$6k = 10\left(\frac{3}{7}\right)$

$k = \frac{5}{7}$

UPSKILL 8.3a

1 (a) $\overrightarrow{AB} = 5\underline{i}$

$\left|\overrightarrow{AB}\right| = 5$

(b) $\overrightarrow{CD} = 4\underline{i} + 3\underline{j}$

$\left|\overrightarrow{CD}\right| = \sqrt{4^2 + 3^2} = 5$

(c) $\overrightarrow{EF} = 5\underline{i} - 3\underline{j}$

$\left|\overrightarrow{EF}\right| = \sqrt{5^5 + (-3)^2} = \sqrt{34} = 5.831$

(d) $\overrightarrow{PQ} = -11\underline{i} - 5\underline{j}$

$\left|\overrightarrow{PQ}\right| = \sqrt{(-11)^2 + (-5)^2} = \sqrt{146} = 12.08$

UPSKILL 8.3b

1 (a) $|\underline{r}| = \sqrt{(-8)^2 + (-6)^2} = 10$

$\hat{\underline{r}} = \frac{1}{10}(-8\underline{i} - 6\underline{j}) = -\frac{4}{5}\underline{i} + \frac{3}{5}\underline{j}$

(b) $|\underline{s}| = \sqrt{(-8)^2 + 15^2} = 17$

$\hat{\underline{s}} = \frac{1}{17}\begin{pmatrix} -8 \\ 15 \end{pmatrix} = \begin{pmatrix} -\frac{8}{17} \\ \frac{15}{17} \end{pmatrix}$

UPSKILL 8.3c

1 (a) Given the point $A(-3, 4)$, then

$\overrightarrow{OA} = -3\underset{\sim}{i} + 4\underset{\sim}{j}$.

Given the point $B(-8, -6)$, then

$\overrightarrow{OB} = -8\underset{\sim}{i} - 6\underset{\sim}{j}$.

Using the concept of position vectors,

$$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$$
$$= -8\underset{\sim}{i} - 6\underset{\sim}{j} - \left(-3\underset{\sim}{i} + 4\underset{\sim}{j}\right)$$
$$= -5\underset{\sim}{i} - 10\underset{\sim}{j}$$

(b)
$$\overrightarrow{AC} = \frac{4}{5}\overrightarrow{AB}$$
$$\overrightarrow{OC} - \overrightarrow{OA} = \frac{4}{5}\left(-5\underset{\sim}{i} - 10\underset{\sim}{j}\right)$$
$$\overrightarrow{OC} - \left(-3\underset{\sim}{i} + 4\underset{\sim}{j}\right) = -4\underset{\sim}{i} - 8\underset{\sim}{j}$$
$$\overrightarrow{OC} + 3\underset{\sim}{i} - 4\underset{\sim}{j} = -4\underset{\sim}{i} - 8\underset{\sim}{j}$$
$$\overrightarrow{OC} = -7\underset{\sim}{i} - 4\underset{\sim}{j}$$

UPSKILL 8.3d

1 (a) $2\underline{a} = 2\left(-3\underline{i} + 4\underline{j}\right) = -6\underline{i} + 8\underline{j}$

$|2\underline{a}| = \sqrt{(-6)^2 + 8^2} = 10$

(b) $-3\underline{b} = -3\left(\underline{i} + 3\underline{j}\right) = -3\underline{i} - 9\underline{j}$

$|-3\underline{b}| = \sqrt{(-3)^2 + (-9)^2} = 3\sqrt{10} = 9.487$

2 (a) $2\underline{a} + 3\underline{b} = 2\begin{pmatrix}3\\4\end{pmatrix} + 3\begin{pmatrix}1\\-3\end{pmatrix} = \begin{pmatrix}9\\-1\end{pmatrix}$

(b) $3\underline{a} + 2\underline{c} = 3\begin{pmatrix}3\\4\end{pmatrix} + 2\begin{pmatrix}-2\\4\end{pmatrix} = \begin{pmatrix}5\\20\end{pmatrix}$

(c) $\underline{a} - \underline{b} + \underline{c} = \begin{pmatrix}3\\4\end{pmatrix} - \begin{pmatrix}1\\-3\end{pmatrix} + \begin{pmatrix}-2\\4\end{pmatrix} = \begin{pmatrix}0\\11\end{pmatrix}$

(d) $2\underline{a} - 3\underline{b} + 3\underline{c} = 2\begin{pmatrix}3\\4\end{pmatrix} - 3\begin{pmatrix}1\\-3\end{pmatrix} + 3\begin{pmatrix}-2\\4\end{pmatrix}$

$= \begin{pmatrix}-3\\29\end{pmatrix}$

3
$$t\underline{b} - k\underline{a} = \underline{c}$$
$$t\left(\underline{i} + 2\underline{j}\right) - k\left(3\underline{i} + 4\underline{j}\right) = 3\underline{i} - 2\underline{j}$$
$$(t - 3k)\underline{i} + (2t - 4k)\underline{j} = 3\underline{i} - 2\underline{j}$$

Equating the coefficients of $\underline{i}$,

$t - 3k = 3$

$2t - 6k = 6 \ \ldots (1)$

Equating the coefficients of $\underline{j}$,

$2t - 4k = -2 \ \ldots (2)$

$(1) - (2): \quad -2k = 8$

$k = -4$

From (1) :

$t - 3(-4) = 3$

$t = -9$

4
$$p\underline{a} + k\underline{b} = \underline{c}$$
$$p\begin{pmatrix}2\\1\end{pmatrix} + k\begin{pmatrix}3\\-2\end{pmatrix} = \begin{pmatrix}13\\-4\end{pmatrix}$$

$2p + 3k = 13 \ \ldots (1)$

$p - 2k = -4 \ \ldots (2)$

$p = 2k - 4 \ \ldots (3)$

Substitute (3) into (1) :

$2(2k - 4) + 3k = 13$

$4k - 8 + 3k = 13$

$7k = 21$

$k = 3$

From (3) :

$p = 2(3) - 4 = 2$

5 (a) $\overrightarrow{OP} = h\underline{a} + k\underline{b}$

$$\begin{pmatrix}5\\-3\end{pmatrix} = h\begin{pmatrix}3\\0\end{pmatrix} + k\begin{pmatrix}1\\3\end{pmatrix}$$

$3h + k = 5 \ \ldots (1)$

$3k = -3 \ \ldots (2)$

From (2) : $k = -1$

From (1) : $3h - 1 = 5$

$h = 2$

$\therefore \overrightarrow{OP} = 2\underline{a} - \underline{b}$

(b) $\overrightarrow{PQ} = p\underset{\sim}{a} + q\underset{\sim}{b}$

$$\begin{pmatrix}-1\\6\end{pmatrix} = p\begin{pmatrix}3\\0\end{pmatrix} + q\begin{pmatrix}1\\3\end{pmatrix}$$

$3p + q = -1 \quad \ldots (1)$

$3q = 6 \quad \ldots (2)$

From (2) : $q = 2$

From (1) : $3p + 2 = -1$

$p = -1$

$\therefore \overrightarrow{PQ} = -\underset{\sim}{a} + 2\underset{\sim}{b}$

6 Since $\underset{\sim}{u}$ and $\underset{\sim}{v}$ are parallel, thus

$\underset{\sim}{u} = m\underset{\sim}{v}$ [m is a constant.]

$3i + 6j = m(k\underset{\sim}{i} - 2\underset{\sim}{j})$

$3i + 6j = mk\underset{\sim}{i} - 2m\underset{\sim}{j}$

Equating the coefficients of $\underset{\sim}{j}$,

$-2m = 6$

$m = -3$

Equating the coefficients of $\underset{\sim}{i}$,

$mk = 3$

$-3k = 3$

$k = -1$

7 $\underset{\sim}{r} = k\underset{\sim}{s}$

$2\underset{\sim}{i} + (p+3)\underset{\sim}{j} = k[(p-5)\underset{\sim}{i} - 8\underset{\sim}{j}]$

$2\underset{\sim}{i} + (p+3)\underset{\sim}{j} = k(p-5)\underset{\sim}{i} - 8k\underset{\sim}{j}$

Equating the coefficients of $\underset{\sim}{i}$,

$2 = k(p-5)$

$2 = kp - 5k \quad \ldots (1)$

Equating the coefficients of $\underset{\sim}{j}$,

$p + 3 = -8k$

$p = -8k - 3 \quad \ldots (2)$

Substitute (2) into (1) :

$2 = k(-8k-3) - 5k$

$8k^2 + 8k + 2 = 0$

$4k^2 + 4k + 1 = 0$

$(2k+1)(2k+1) = 0$

$k = -\frac{1}{2}$

From (2) :

When $k = -\frac{1}{2}$,

$p = -8\left(-\frac{1}{2}\right) - 3 = 1$

8 $\overrightarrow{PQ} = m\overrightarrow{PR}$

$\overrightarrow{OQ} - \overrightarrow{OP} = m\left(\overrightarrow{OR} - \overrightarrow{OP}\right)$

$5i - 2j - (3\underset{\sim}{i} + \underset{\sim}{j}) = m\left[k\underset{\sim}{i} - 6\underset{\sim}{j} - (3\underset{\sim}{i} + \underset{\sim}{j})\right]$

$2\underset{\sim}{i} - 3\underset{\sim}{j} = m\left[(k-3)\underset{\sim}{i} - 7\underset{\sim}{j}\right]$

$2\underset{\sim}{i} - 3\underset{\sim}{j} = m(k-3)\underset{\sim}{i} - 7m\underset{\sim}{j}$

Equating the coefficients of- $\underset{\sim}{j}$,

$-7m = -3$

$m = \frac{3}{7}$

Equating the coefficients of- $\underset{\sim}{i}$,

$m(k-3) = 2$

$\frac{3}{7}(k-3) = 2$

$3(k-3) = 14$

$3k - 9 = 14$

$3k = 23$

$k = \frac{23}{3}$

9 $|\underset{\sim}{u}| = |\underset{\sim}{v}|$

$\sqrt{(k-2)^2 + 4^2} = \sqrt{(k-1)^2 + 3^2}$

$(k-2)^2 + 16 = (k-1)^2 + 9$

$k^2 - 4k + 4 + 16 = k^2 - 2k + 1 + 9$

$20 - 4k = 10 - 2k$

$2k = 10$

$k = 5$

10 $\underset{\sim}{x} - \underset{\sim}{y} = 3\underset{\sim}{i} + k\underset{\sim}{j} - (4\underset{\sim}{i} - 3\underset{\sim}{j})$

$= -\underset{\sim}{i} + (k+3)\underset{\sim}{j}$

$|x - y| = \sqrt{5}$

$\sqrt{(-1)^2 + (k+3)^2} = \sqrt{5}$

$1 + k^2 + 6k + 9 = 5$

$k^2 + 6k + 5 = 0$

$(k+1)(k+5) = 0$

$k = -1$ or $k = -5$

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1 (a) Resultant vector

$= (16\underline{i} + 12\underline{j}) + (6\underline{i} - 8\underline{j})$

$= 22\underline{i} + 4\underline{j}$

(b) Magnitude

$= \sqrt{22^2 + 4^2}$

$= \sqrt{500}$

$= 22.36 \text{ km h}^{-1}$

2 (a) Resultant vector

$= 500\underline{i} - 300\underline{j} + (-60\underline{i} - 80\underline{j})$

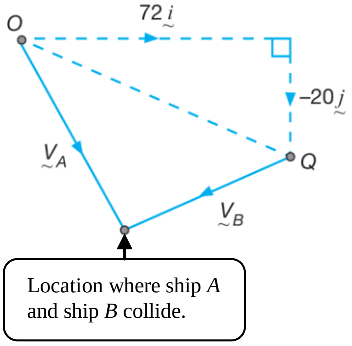
$= 440\underline{i} - 380\underline{j}$

(b) Magnitude

$= \sqrt{440^2 + (-380)^2}$

$= 581.38 \text{ km h}^{-1}$

3



Position vector of ship *A* after *t* hours is

$t(6\underset{\sim}{i} - 8\underset{\sim}{j}) = \begin{pmatrix} 6t\,\underset{\sim}{i} - 8t\,\underset{\sim}{j} \end{pmatrix}$ … (1)

Position vector of ship *B* after *t* hours is

$\begin{pmatrix} 72\underset{\sim}{i} - 20\underset{\sim}{j} \end{pmatrix}$ km $+\ t \begin{pmatrix} -12\underset{\sim}{i} - 3\underset{\sim}{j} \end{pmatrix}$

$= \left[(72 - 12t)\underset{\sim}{i} + (-20 - 3t)\underset{\sim}{j} \right]$ … (2)

Equating (1) and (2):

$\begin{pmatrix} 6t\,\underset{\sim}{i} - 8t\,\underset{\sim}{j} \end{pmatrix} = \left[(72 - 12t)\underset{\sim}{i} + (-20 - 3t)\underset{\sim}{j} \right]$

Equating the coefficient of $\underset{\sim}{i}$:

$6t = 72 - 12t$

$18t = 72$

$t = 4$

Equating the coefficient of $\underset{\sim}{j}$:

$-8t = -20 - 3t$

$8t = 20 + 3t$

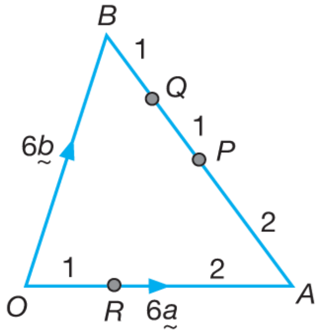
$5t = 20$

$t = 4$

Hence, ship *A* and ship *B* will collide after 4 hours.

Summative Practice 8

1



(a) $\overrightarrow{OQ} = \overrightarrow{OB} + \overrightarrow{BQ}$

$= 6\underline{b} + \frac{1}{4}\overrightarrow{BA}$

$= 6\underline{b} + \frac{1}{4}(-6\underline{b} + 6\underline{a})$

$= 6\underline{b} - \frac{3}{2}\underline{b} + \frac{3}{2}\underline{a}$

$= \frac{9}{2}\underline{b} + \frac{3}{2}\underline{a}$

$= \frac{3}{2}(3\underline{b} + \underline{a})$ … (1)

(b) $\overrightarrow{RP} = \overrightarrow{RA} + \overrightarrow{AP}$

$= \frac{2}{3}\overrightarrow{OA} + \frac{1}{2}\overrightarrow{AB}$

$= \frac{2}{3}(6\underline{a}) + \frac{1}{2}\left(\overrightarrow{AO} + OB\right)$

$= 4\underline{a} + \frac{1}{2}(-6\underline{a} + 6\underline{b})$

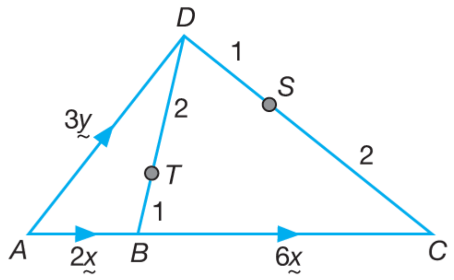
$= \underline{a} + 3\underline{b}$

Since $\overrightarrow{OQ} = \frac{3}{2}\overrightarrow{RP}$, $\overrightarrow{OQ}$ can be

expressed as a scalar multiple of $\overrightarrow{RP}$.

Thus, $\overrightarrow{OQ}$ is parallel to $\overrightarrow{RP}$.

2



(a) (i) $\overrightarrow{AS} = \overrightarrow{AD} + \overrightarrow{DS}$

$= 3\underline{y} + \frac{1}{3}\overrightarrow{DC}$

$= 3\underline{y} + \frac{1}{3}\left(-3\underline{y} + 8\underline{x}\right)$

$= 2\underline{y} + \frac{8}{3}\underline{x}$

(ii) $\overrightarrow{TC} = \overrightarrow{TD} + \overrightarrow{DC}$

$= \frac{2}{3}\left(\overrightarrow{BD}\right) + \overrightarrow{DA} + \overrightarrow{AC}$

$= \frac{2}{3}\left(-2\underline{x} + 3\underline{y}\right) + \left(-3\underline{y} + 8\underline{x}\right)$

$= -\frac{4}{3}\underline{x} + 2\underline{y} - 3\underline{y} + 8\underline{x}$

$= \frac{20}{3}\underline{x} - \underline{y}$

(b) $\overrightarrow{AT} = \overrightarrow{AB} + \overrightarrow{BT}$

$= 2\underline{x} + \frac{1}{3}\overrightarrow{BD}$

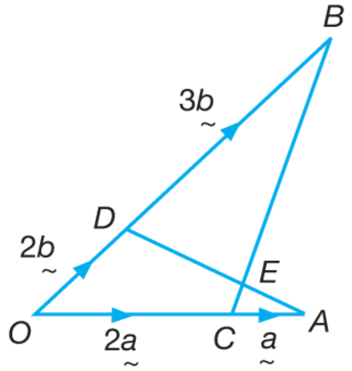
$= 2\underline{x} + \frac{1}{3}\left(-2\underline{x} + 3\underline{y}\right)$

$= \frac{4}{3}\underline{x} + \underline{y}$

$= \frac{1}{2}\overrightarrow{AS}$

Hence, $\overrightarrow{AT}$ can be expressed as a scalar multiple of $\overrightarrow{AS}$ and *A* is a common point. Thus, the points *A*, *T* and *S* are collinear.

3



(a) (i) $\overrightarrow{OE} = \overrightarrow{OA} + \overrightarrow{AE}$

$= 3\underline{a} + h\,\overrightarrow{AD}$

$= 3\underline{a} + h\left(-3\underline{a} + 2\underline{b}\right)$

$= \left(3 - 3h\right)\underline{a} + 2h\underline{b}$ ... (1)

(ii) $\overrightarrow{OE} = \overrightarrow{OB} + \overrightarrow{BE}$

$= 5\underline{b} + k\,\overrightarrow{BC}$

$= 5\underline{b} + k\left(-5\underline{b} + 2\underline{a}\right)$

$= \left(5 - 5k\right)\underline{b} + 2k\underline{a}$ ... (2)

(b) $\left(3 - 3h\right)\underline{a} + 2h\underline{b} = \left(5 - 5k\right)\underline{b} + 2k\underline{a}$

Equating the coefficients of $\underline{a}$,

$3 - 3h = 2k$ ... (1)

Equating the coefficients of $\underline{b}$,

$2h = 5 - 5k$

$h = \frac{5 - 5k}{2}$ ... (2)

Substitute (2) into (1) :

$3 - 3\left(\frac{5 - 5k}{2}\right) = 2k$

$6 - 3(5 - 5k) = 4k$

$6 - 15 + 15k = 4k$

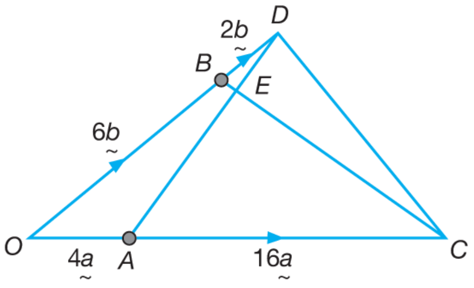
$11k = 9$

$k = \frac{9}{11}$

From (2) :

$h = \frac{5 - 5\left(\frac{9}{11}\right)}{2} = \frac{5}{11}$

4



(a) (i) $\overrightarrow{AD} = -4\underset{\sim}{a} + 8\underset{\sim}{b}$

(ii) $\overrightarrow{BC} = -6\underset{\sim}{b} + 20\underset{\sim}{a}$

(b) $\overrightarrow{AE} = \overrightarrow{AB} + \overrightarrow{BE}$

$h\overrightarrow{AD} = -4\underset{\sim}{a} + 6\underset{\sim}{b} + k\overrightarrow{BC}$

$h(-4\underset{\sim}{a} + 8\underset{\sim}{b}) = -4\underset{\sim}{a} + 6\underset{\sim}{b} + k(-6\underset{\sim}{b} + 20\underset{\sim}{a})$

$-4h\underset{\sim}{a} + 8h\underset{\sim}{b} = -4\underset{\sim}{a} + 6\underset{\sim}{b} - 6k\underset{\sim}{b} + 20k\underset{\sim}{a}$

$-4h\underset{\sim}{a} + 8h\underset{\sim}{b} = (-4 + 20k)\underset{\sim}{a} + (6\underset{\sim}{b} - 6k)\underset{\sim}{b}$

Equating the coefficients of $\underset{\sim}{a}$,

$-4h = -4 + 20k \ \dots (1)$

Equating the coefficients of $\underset{\sim}{b}$,

$8h = 6 - 6k$

$h = \dfrac{6-6k}{8} \ \dots (2)$

Substitute (2) into (1) :

$-4\left(\dfrac{6-6k}{8}\right) = -4 + 20k$

$-\left(\dfrac{6-6k}{2}\right) = -4 + 20k$

$-(3-3k) = -4 + 20k$

$-3 + 3k = -4 + 20k$

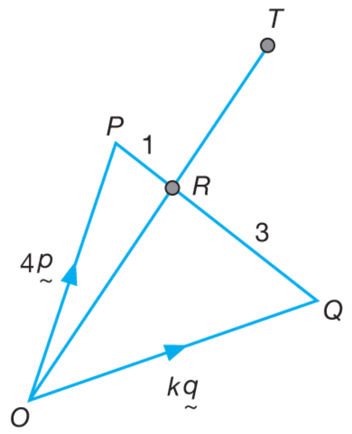
$17k = 1$

$k = \dfrac{1}{17}$

From (2) :

$h = \dfrac{6 - 6\left(\dfrac{1}{17}\right)}{8} = \dfrac{12}{17}$

5



(a) $\overrightarrow{OR} = \overrightarrow{OP} + \overrightarrow{PR}$

$= 4\underset{\sim}{p} + \dfrac{1}{4}\overrightarrow{PQ}$

$= 4\underset{\sim}{p} + \dfrac{1}{4}\left(\overrightarrow{PO} + \overrightarrow{OQ}\right)$

$= 4\underset{\sim}{p} + \dfrac{1}{4}(-4\underset{\sim}{p} + k\underset{\sim}{q})$

$= 4\underset{\sim}{p} - \underset{\sim}{p} + \dfrac{k}{4}\underset{\sim}{q}$

$= 3\underset{\sim}{p} + \dfrac{k}{4}\underset{\sim}{q}$

(b) $\overrightarrow{OR} = m\overrightarrow{RT}$

$3\underset{\sim}{p} + \dfrac{k}{4}\underset{\sim}{q} = m\left(2\underset{\sim}{p} + \dfrac{5}{3}\underset{\sim}{q}\right)$

$3\underset{\sim}{p} + \dfrac{k}{4}\underset{\sim}{q} = 2m\underset{\sim}{p} + \dfrac{5}{3}m\underset{\sim}{q}$

Equating the coefficients of $\underset{\sim}{p}$,

$2m = 3$

$m = \dfrac{3}{2}$

Equating the coefficients of $\underset{\sim}{q}$,

$\dfrac{k}{4} = \dfrac{5}{3}m$

$\dfrac{k}{4} = \dfrac{5}{3}\left(\dfrac{3}{2}\right)$

$k = 10$

6 (a) $\overrightarrow{PR} = \overrightarrow{PQ} + \overrightarrow{QR}$

$= 4\underline{u} + \frac{3}{2}\overrightarrow{PS}$

$= 4\underline{u} + \frac{3}{2}(12\underline{v})$

$= 4\underline{u} + 18\underline{v}$

Given $\overrightarrow{PS} = \frac{2}{3}\overrightarrow{QR}$, thus $\overrightarrow{QR} = \frac{3}{2}\overrightarrow{PS}$.

(b) (i) $\overrightarrow{TX} = m\,\overrightarrow{PQ}$

$= m(4\underline{u})$

$= 4m\underline{u}$

(ii) P, X and R are collinear.

$\overrightarrow{PX} = k\,\overrightarrow{PR}$

$\overrightarrow{PT} + \overrightarrow{TX} = k(4\underline{u} + 18\underline{v})$

$\frac{4}{3}\overrightarrow{PS} + 4m\underline{u} = 4k\underline{u} + 18k\underline{v}$

Given | From (b) (i)

$\frac{3}{4}(12\underline{v}) + 4m\underline{u} = 4k\underline{u} + 18k\underline{v}$

$9\underline{v} + 4m\underline{u} = 4k\underline{u} + 18k\underline{v}$

Equating the coefficients of $\underline{v}$,

$9 = 18k$

$k = \frac{1}{2}$

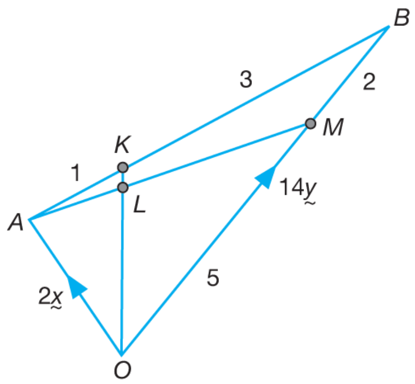
Equating the coefficients of $\underline{u}$,

$4m = 4k$

$4m = 4\left(\frac{1}{2}\right)$

$m = \frac{1}{2}$

7



(a) (i) $\overrightarrow{OM} = \frac{5}{7}\overrightarrow{OB}$

$= \frac{5}{7}(14\underline{y})$

$= 10\underline{y}$

(ii) $\overrightarrow{AK} = \frac{1}{4}\overrightarrow{AB}$

$= \frac{1}{4}(-2\underline{x} + 10\underline{y})$

$= -\frac{1}{2}\underline{x} + \frac{7}{2}\underline{y}$

(b) (i) $\overrightarrow{AL} = p\,\overrightarrow{AM}$

$= p(-2\underline{x} + 10\underline{y})$

$= -2p\underline{x} + 10p\underline{y}$

(ii) $\overrightarrow{KL} = q\,\overrightarrow{KO}$

$= q\left(\overrightarrow{KA} + \overrightarrow{AO}\right)$

$= q\left(\frac{1}{4}\overrightarrow{BA} - 2\underline{x}\right)$

$= q\left[\frac{1}{4}(-14y + 2x) - 2\underline{x}\right]$

$= q\left(-\frac{7}{2}y + \frac{1}{2}x - 2x\right)$

$= q\left(-\frac{7}{2}y - \frac{3}{2}x\right)$

$= -\frac{7}{2}q\underline{y} - \frac{3}{2}q\underline{x}$

(c) $\overrightarrow{AK} = \overrightarrow{AL} + \overrightarrow{LK}$

$$\frac{7}{2}y - \frac{1}{2}x = -2px + 10py + \frac{7}{2}qy + \frac{3}{2}qx$$

$$\frac{7}{2}\underset{\sim}{y} - \frac{1}{2}\underset{\sim}{x} = \left(10p + \frac{7}{2}q\right)\underset{\sim}{y} + \left(-2p + \frac{3}{2}q\right)\underset{\sim}{x}$$

Equating the coefficients of $\underset{\sim}{x}$:

$$-\frac{1}{2} = -2p + \frac{3}{2}q$$

$$-4p + 3q = -1 \ \ldots (1)$$

Equating the coefficients of $\underset{\sim}{y}$:

$$10p + \frac{7}{2}q = \frac{7}{2}$$

$$20p + 7q = 7 \ \ldots (2)$$

$$-20p + 15q = -5 \ \ldots (1) \times 5$$

$$(+) \quad 20p + 7q = 7 \ \ldots (2)$$

$$22q = 2$$

$$q = \frac{1}{11}$$

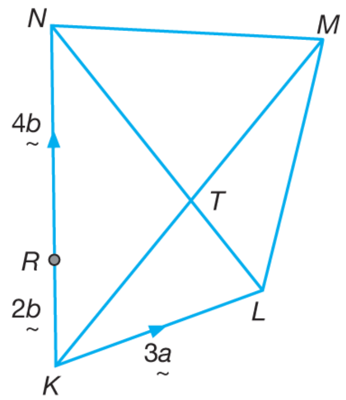
From (1):

$$-4p + 3\left(\frac{1}{11}\right) = -1$$

$$-4p = -\frac{14}{11}$$

$$p = \frac{7}{22}$$

8



(a) (i) $\overrightarrow{KR} = \frac{1}{3}\overrightarrow{KN}$

$$\overrightarrow{KN} = 3\overrightarrow{KR}$$

$$\overrightarrow{KN} = 3(2\underset{\sim}{b})$$

$$\overrightarrow{KN} = 6\underset{\sim}{b}$$

$$\overrightarrow{NL} = \overrightarrow{NK} + \overrightarrow{KL}$$

$$= -6\underset{\sim}{b} + 3\underset{\sim}{a}$$

(ii) $\overrightarrow{KT} = \overrightarrow{KL} + \overrightarrow{LT}$

$$= \overrightarrow{KL} + \frac{1}{3}\overrightarrow{LN}$$

$$= 3\underset{\sim}{a} + \frac{1}{3}(6\underset{\sim}{b} - 3\underset{\sim}{a})$$

$$= 3\underset{\sim}{a} + 2\underset{\sim}{b} - \underset{\sim}{a}$$

$$= 2\underset{\sim}{a} + 2\underset{\sim}{b}$$

(b) (i) $\overrightarrow{NM} = \overrightarrow{NK} + \overrightarrow{KM}$

$$= \overrightarrow{NK} + \frac{1}{q}\overrightarrow{KT}$$

$$= -6\underset{\sim}{b} + \frac{1}{q}(2\underset{\sim}{a} + 2\underset{\sim}{b})$$

$$= -6\underset{\sim}{b} + \frac{2}{q}\underset{\sim}{a} + \frac{2}{q}\underset{\sim}{b}$$

$$= \left(\frac{2}{q} - 6\right)\underset{\sim}{b} + \frac{2}{q}\underset{\sim}{a}$$

(ii) But it is given that $\overrightarrow{NM} = 3p\underset{\sim}{a} - 2\underset{\sim}{b}$.

By comparison,

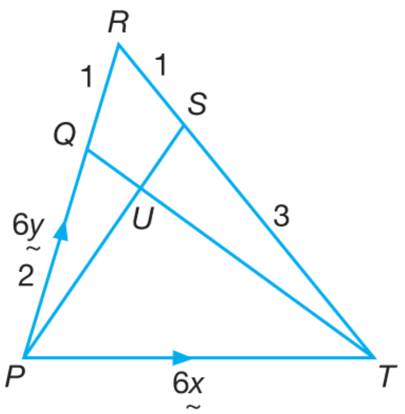
$$\frac{2}{q} - 6 = -2 \qquad 3p = \frac{2}{q}$$

$$\frac{2}{q} = 4 \qquad p = \frac{2}{3q}$$

$$q = \frac{1}{2} \qquad p = \frac{2}{3\left(\frac{1}{2}\right)}$$

$$p = \frac{4}{3}$$

9



(a) (i) $\overrightarrow{TR} = \overrightarrow{TP} + \overrightarrow{PR} = -6\underset{\sim}{x} + 6\underset{\sim}{y}$

(ii) $\overrightarrow{PS} = \overrightarrow{PT} + \overrightarrow{TS}$

$$= \overrightarrow{PT} + \frac{3}{4}\overrightarrow{TR}$$

$$= 6\underset{\sim}{x} + \frac{3}{4}\left(-6\underset{\sim}{x} + 6\underset{\sim}{y}\right)$$

$$= 6\underset{\sim}{x} - \frac{9}{2}\underset{\sim}{x} + \frac{9}{2}\underset{\sim}{y}$$

$$= \frac{3}{2}\underset{\sim}{x} + \frac{9}{2}\underset{\sim}{y}$$

(b) $\overrightarrow{PU} = h\,\overrightarrow{PS}$

$$= h\left(\frac{3}{2}\underset{\sim}{x} + \frac{9}{2}\underset{\sim}{y}\right)$$

$$= \frac{3}{2}h\underset{\sim}{x} + \frac{9}{2}h\underset{\sim}{y}$$

$\overrightarrow{PU} = \overrightarrow{PT} + k\overrightarrow{TQ}$

$$= 6\underset{\sim}{x} + k(-6\underset{\sim}{x} + 4\underset{\sim}{y})$$

$$= (6 - 6k)\underset{\sim}{x} + 4k\underset{\sim}{y}$$

Equating the coefficients of $\underset{\sim}{x}$,

$$\frac{3}{2}h = 6 - 6k$$

$$3h = 12 - 12k$$

$$h = 4 - 4k \quad \ldots (1)$$

Equating the coefficients of $\underset{\sim}{y}$,

$$4k = \frac{9}{2}h$$

$$8k = 9h \quad \ldots (2)$$

Substitute (1) into (2) :

$$8k = 9(4 - 4k)$$

$$8k = 36 - 36k$$

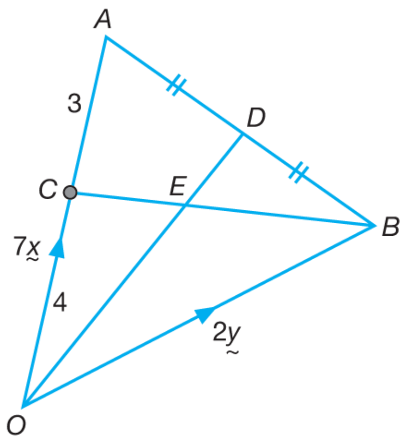
$$44k = 36$$

$$k = \frac{9}{11}$$

From (1) :

$$h = 4 - 4\left(\frac{9}{11}\right) = \frac{8}{11}$$

10



(a) (i) $\overrightarrow{BC} = \overrightarrow{BO} + \overrightarrow{OC}$

$$= -2\underset{\sim}{y} + \frac{4}{7}\overrightarrow{OA}$$

$$= -2\underset{\sim}{y} + \frac{4}{7}(7\underset{\sim}{x})$$

$$= -2\underset{\sim}{y} + 4\underset{\sim}{x}$$

(ii) $\overrightarrow{OD} = \overrightarrow{OB} + \overrightarrow{BD}$

$= 2\underline{y} + \frac{1}{2}\overrightarrow{BA}$

$= 2\underline{y} + \frac{1}{2}(-2\underline{y} + 7\underline{x})$

$= 2\underline{y} - \underline{y} + \frac{7}{2}\underline{x}$

$= \underline{y} + \frac{7}{2}\underline{x}$

(b) (i) $\overrightarrow{OE} = p\,\overrightarrow{OD}$

$= p\left(\underline{y} + \frac{7}{2}\underline{x}\right)$

$= p\underline{y} + \frac{7}{2}p\underline{x} \quad \ldots (1)$

(ii) $\overrightarrow{OE} = \overrightarrow{OB} + \overrightarrow{BE}$

$= \overrightarrow{OB} + q\,\overrightarrow{BC}$

$= 2\underline{y} + q(-2\underline{y} + 4\underline{x})$

$= (2 - 2q)\underline{y} + 4q\underline{x} \quad \ldots (2)$

(c) Equating (1) and (2),

$p\underline{y} + \frac{7}{2}p\underline{x} = (2 - 2q)\underline{y} + 4q\underline{x}$

Equating the coefficients of $\underline{y}$,

$p = 2 - 2q \quad \ldots (3)$

Equating the coefficients of $\underline{x}$,

$\frac{7}{2}p = 4q$

$7p = 8q$

$7(2 - 2q) = 8q$

$14 - 14q = 8q$

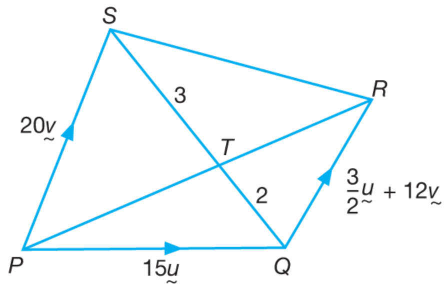
$14 = 22q$

$q = \frac{7}{11}$

From (3),

$p = 2 - 2\left(\frac{7}{11}\right) = \frac{8}{11}$

11



(a) (i) $\overrightarrow{QS} = \overrightarrow{QP} + \overrightarrow{PS}$

$= -15\underline{u} + 20\underline{v}$

(ii) $\overrightarrow{PT} = \overrightarrow{PQ} + \overrightarrow{QT}$

$= 15\underline{u} + \frac{2}{5}\overrightarrow{QS}$

$= 15\underline{u} + \frac{2}{5}(-15\underline{u} + 20\underline{v})$

$= 15\underline{u} - 6\underline{u} + 8\underline{v}$

$= 9\underline{u} + 8\underline{v}$

(b) $\overrightarrow{PT} = k\,\overrightarrow{TR}$

$9\underline{u} + 8\underline{v} = k\left(\overrightarrow{TQ} + \overrightarrow{QR}\right)$

$= k\left(6\underline{u} - 8\underline{v} - \frac{3}{2}\underline{u} + 12\underline{v}\right)$

$= k\left(\frac{9}{2}\underline{u} + 4\underline{v}\right)$

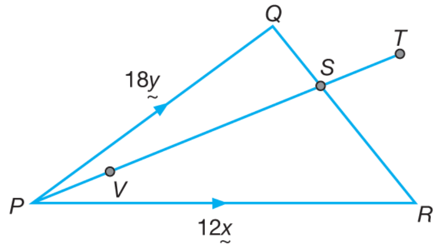
$= \frac{9}{2}k\underline{u} + 4k\underline{v}$

Equating the coefficients of $\underline{u}$,

$9 = \frac{9}{2}k$

$k = 2 \Rightarrow PT : TR = 2 : 1$

12



(a) (i) $\overrightarrow{QR} = \overrightarrow{QP} + \overrightarrow{PR}$

$= -18\underline{y} + 12\underline{x}$

(ii) $\overrightarrow{PS} = \overrightarrow{PQ} + \overrightarrow{QS}$

$= \overrightarrow{PQ} + \frac{1}{3}\overrightarrow{QR}$

$= 18\underline{y} + \frac{1}{3}\left(-18\underline{y} + 12\underline{x}\right)$

$= 18\underline{y} - 6\underline{y} + 4\underline{x}$

$= 12\underline{y} + 4\underline{x}$

(b) Using the triangle addition rule,

$\overrightarrow{PV} + \overrightarrow{VQ} = \overrightarrow{PQ}$

$m\,\overrightarrow{PS} - n(2\underline{x} - 18\underline{y}) = 18\underline{y}$

$\overrightarrow{VQ} = -\overrightarrow{QV}$

$m\left(12\underline{y} + 4\underline{x}\right) - n(2\underline{x} - 18\underline{y}) = 18\underline{y}$

$12m\underline{y} + 4m\underline{x} - 2n\underline{x} + 18n\underline{y} = 18\underline{y}$

$(12m + 18n)\underline{y} + (4m - 2n)\underline{x} = 18\underline{y}$

Equating the coefficients of $\underline{y}$,

$12m + 18n = 18$

$2m + 3n = 3 \;\ldots (1)$

Equating the coefficients of $\underline{x}$,

$4m - 2n = 0$

$2m - n = 0 \;\ldots (2)$

$(1) - (2):\quad 4n = 3$

$n = \frac{3}{4}$

From (2), $2m - \frac{3}{4} = 0$

$m = \frac{3}{8}$

(c) Since the points P, S and T are collinear,

$\overrightarrow{PS} = k\,\overrightarrow{PT}$ (k is a constant.)

$12\underline{y} + 4\underline{x} = k\left(h\underline{x} + 18\underline{y}\right)$

$12\underline{y} + 4\underline{x} = hk\underline{x} + 18k\underline{y}$

Equating the coefficients of $\underline{y}$,

$18k = 12$

$k = \frac{2}{3}$

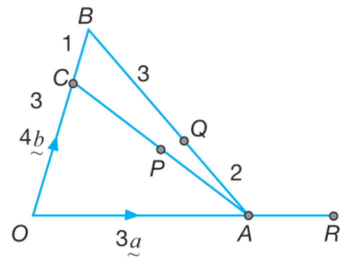
Equating the coefficients of $\underline{x}$,

$hk = 4$

$\frac{2}{3}h = 4$

$h = 6$

13



(a) (i) $\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$

$= -3\underset{\sim}{a} + 4\underset{\sim}{b}$

(ii) $\overrightarrow{OQ} = \overrightarrow{OA} + \overrightarrow{AQ}$

$= 3\underset{\sim}{a} + \frac{2}{5}\overrightarrow{AB}$

$= 3\underset{\sim}{a} + \frac{2}{5}\left(-3\underset{\sim}{a} + 4\underset{\sim}{b}\right)$

$= \frac{9}{5}\underset{\sim}{a} + \frac{8}{5}\underset{\sim}{b}$

(b) (i) Since CQR is a straight line, then

$\overrightarrow{CQ} = k\,\overrightarrow{CR}$

> **HOT TIPS**
>
> For any vector $\overrightarrow{AB}$,
> $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$ is always true such that O is the origin.

$\overrightarrow{CQ} = k\,\overrightarrow{CR}$

$\overrightarrow{OQ} - \overrightarrow{OC} = k\left(\overrightarrow{OR} - \overrightarrow{OC}\right)$

$\frac{9}{5}\underset{\sim}{a} + \frac{8}{5}\underset{\sim}{b} - 3\underset{\sim}{b} = k\left[n\,\overrightarrow{OA} - 3\underset{\sim}{b}\right]$

$$\frac{9}{5}\underset{\sim}{a}+\frac{8}{5}\underset{\sim}{b}-3\underset{\sim}{b}=k\left[n\left(3\underset{\sim}{a}\right)-3\underset{\sim}{b}\right]$$

$$\frac{9}{5}\underset{\sim}{a}-\frac{7}{5}\underset{\sim}{b}=3nk\,\underset{\sim}{a}-3k\underset{\sim}{b}$$

Equating the coefficients of $\underset{\sim}{b}$,

$$-\frac{7}{5}=-3k$$

$$k=\frac{7}{15}$$

Equating the coefficients of $\underset{\sim}{a}$,

$$\frac{9}{5}=3kn$$

$$\frac{9}{5}=3\left(\frac{7}{15}\right)n$$

$$3=\left(\frac{7}{3}\right)n$$

$$n=\frac{9}{7}$$

(ii) $\overrightarrow{CQ}=k\overrightarrow{CR}$

$$\overrightarrow{CQ}=\frac{7}{15}\overrightarrow{CR}$$

$$\frac{CQ}{CR}=\frac{7}{15}$$

$$CQ:CR=7:15$$

CAUTION!

Writing $\dfrac{\overrightarrow{CQ}}{\overrightarrow{CR}}$ is incorrect because a vector cannot be divided by another vector. You can state them in the modulus form $\dfrac{\left|\overrightarrow{CQ}\right|}{\left|\overrightarrow{CR}\right|}$ or $\dfrac{CQ}{CR}$.

(c) $\overrightarrow{OP}=\overrightarrow{OA}+\overrightarrow{AP}$

$$=3\underset{\sim}{a}+\frac{1}{2}\overrightarrow{AC}$$

$$=3\underset{\sim}{a}+\frac{1}{2}\left(-3\underset{\sim}{a}+3\underset{\sim}{b}\right)$$

$$=\frac{3}{2}\underset{\sim}{a}+\frac{3}{2}\underset{\sim}{b}$$

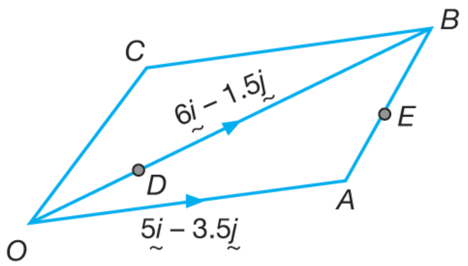
$$=\frac{3}{2}\left(\underset{\sim}{a}+\underset{\sim}{b}\right)$$

$$\overrightarrow{OQ}=\frac{9}{5}\underset{\sim}{a}+\frac{8}{5}\underset{\sim}{b}$$

$$=\frac{1}{5}\left(9\underset{\sim}{a}+8\underset{\sim}{b}\right)$$

Since $\overrightarrow{OP}$ cannot be expressed as a scalar multiple of $\overrightarrow{OQ}$, then the points O, P and Q are not collinear.
Hence, player A can be seen by player C without blocked by player B.

14 (a)



(i) $\overrightarrow{OD}=\frac{1}{3}\overrightarrow{OB}$

$$=\frac{1}{3}\left(6\underset{\sim}{i}-1.5\underset{\sim}{j}\right)$$

$$=2\underset{\sim}{i}-0.5\underset{\sim}{j}$$

(ii) $\overrightarrow{OE}=\overrightarrow{OA}+\overrightarrow{AE}$

$$=\overrightarrow{OA}+\frac{1}{4}\overrightarrow{AB}$$

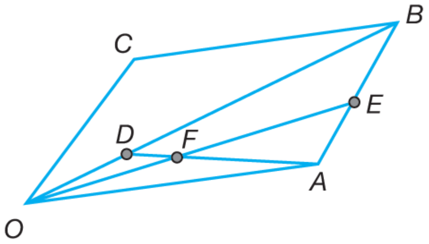
$$=5\underset{\sim}{i}+3.5\underset{\sim}{j}+\frac{1}{4}\left(\overrightarrow{AO}+\overrightarrow{OB}\right)$$

$$=5\underset{\sim}{i}+3.5\underset{\sim}{j}+\frac{1}{4}\left(-5\underset{\sim}{i}-3.5\underset{\sim}{j}+6\underset{\sim}{i}-1.5\underset{\sim}{j}\right)$$

$$=5\underset{\sim}{i}+3.5\underset{\sim}{j}+\frac{1}{4}\left(\underset{\sim}{i}-5\underset{\sim}{j}\right)$$

$$=\frac{21}{4}\underset{\sim}{i}+\frac{9}{4}\underset{\sim}{j}$$

(b)



(i) $\overrightarrow{OF} = k\,\overrightarrow{OE}$

$$= k\left(\frac{21}{4}\underline{i} + \frac{9}{4}\underline{j}\right)$$

$$= \frac{21}{4}k\underline{i} + \frac{9}{4}k\underline{j} \quad \ldots (1)$$

(ii) $\overrightarrow{OF} = \overrightarrow{OD} + \overrightarrow{DF}$

$$= 2\underline{i} - 0.5\underline{j} + t\,\overrightarrow{DA}$$

$$= 2\underline{i} - 0.5\underline{j} + t\left(\overrightarrow{DO} + \overrightarrow{OA}\right)$$

$$= 2\underline{i} - 0.5\underline{j} + t\left(-2\underline{i} + 0.5\underline{j} + 5\underline{i} + 3.5\underline{j}\right)$$

$$= 2\underline{i} - 0.5\underline{j} + t\left(3\underline{i} + 4\underline{j}\right)$$

$$= (2 + 3t)\underline{i} + (4t - 0.5)\underline{j} \quad \ldots (2)$$

(c) Equating (1) and (2) :

$$\frac{21}{4}k\underline{i} + \frac{9}{4}k\underline{j} = (2 + 3t)\underline{i} + (4t - 0.5)\underline{j}$$

Equating the coefficients of $\underline{i}$,

$$\frac{21}{4}k = 2 + 3t$$

$$21k = 8 + 12t \quad \ldots (3)$$

Equating the coefficients of $\underline{j}$,

$$\frac{9}{4}k = 4t - 0.5$$

$$9k = 16t - 2 \quad \ldots (4)$$

$(3) \times 9 : 189k = 72 + 108t \quad \ldots (5)$

$(4) \times 21 : 189k = -42 + 336t \quad \ldots (6)$

$(5) - (6) : 0 = 114 - 228t$

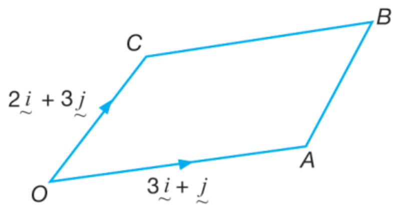
$$t = \frac{1}{2}$$

From (5) :

$$189k = 72 + 108 \times \frac{1}{2}$$

$$k = \frac{126}{189} = \frac{2}{3}$$

15



(a) (i) $\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB}$

$$= 3\underline{i} + \underline{j} + 2\underline{i} + 3\underline{j}$$

$$= 5\underline{i} + 4\underline{j}$$

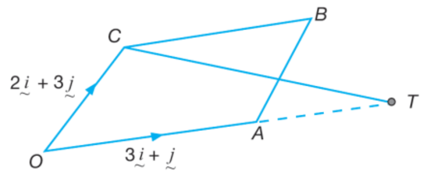
(ii) $\left|\overrightarrow{OB}\right| = \sqrt{5^2 + 4^2} = \sqrt{41}$

Unit vector in the direction of $\overrightarrow{OB}$

$$= \frac{1}{\sqrt{41}}\begin{pmatrix}5\\4\end{pmatrix}$$

$$= \begin{pmatrix}\frac{5}{\sqrt{41}}\\ \frac{4}{\sqrt{41}}\end{pmatrix}$$

(b)



(i) $\overrightarrow{AT} = \overrightarrow{AC} + \overrightarrow{CT}$

$$= \overrightarrow{AO} + \overrightarrow{OC} + \overrightarrow{CT}$$

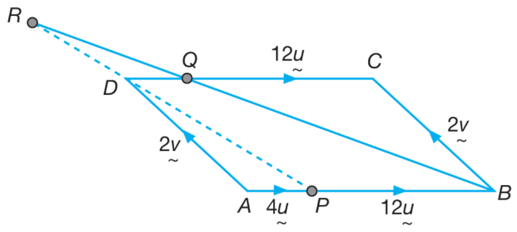
$$= -3\underline{i} - \underline{j} + 2\underline{i} + 3\underline{j} + 16\underline{i} + 3\underline{j}$$

$$= 15\underline{i} + 5\underline{j}$$

(ii) $\overrightarrow{AT} = 5\left(3\underline{i} + \underline{j}\right) = 5\overrightarrow{OA}$

Since $\overrightarrow{AT}$ can be expressed as a scalar multiple of $\overrightarrow{OA}$ and A is a common point, thus the points O, A dan T are collinear.

16



(a) (i) $\overrightarrow{BQ} = \overrightarrow{BC} + \overrightarrow{CQ}$

$= 2\underline{v} - 12\underline{u}$

(ii) $\overrightarrow{PD} = \overrightarrow{PA} + \overrightarrow{AD}$

$= -4\underline{u} + 2\underline{v} \qquad \ldots (1)$

$\overrightarrow{PR} = \overrightarrow{PB} + \overrightarrow{BR}$

$= \overrightarrow{PB} + \frac{3}{2}\overrightarrow{BQ}$

$= 12\underline{u} + \frac{3}{2}(2\underline{v} - 12\underline{u})$

$= -6\underline{u} + 3\underline{v}$

$= 3(-2\underline{u} + \underline{v}) \qquad \ldots (2)$

From (1) : $\overrightarrow{PD} = -4\underline{u} + 2\underline{v}$,

$= 2(-2\underline{u} + \underline{v})$

$-2\underline{u} + \underline{v} = \frac{1}{2}\overrightarrow{PD}$

Hence, from (2) :

$\overrightarrow{PR} = 3(-2\underline{u} + \underline{v}) = 3\left(\frac{1}{2}\overrightarrow{PD}\right)$

$\overrightarrow{PR} = \frac{3}{2}\overrightarrow{PD}$

Since $\overrightarrow{PR}$ can be expressed as a scalar multiple of $\overrightarrow{PD}$ and P is a common point, thus the points P, D dan R are collinear.

(b) (i) $\overrightarrow{PD} = -4\underline{u} + 2\underline{v}$

$= -4(3\underline{i}) + 2(-\underline{i} + 6\underline{j})$

$= -14\underline{i} + 12\underline{j}$

(ii) $\left|\overrightarrow{PD}\right| = \sqrt{(-14)^2 + 12^2} = \sqrt{340}$

Unit vector in the direction of $\overrightarrow{PD}$

$= \frac{1}{\sqrt{340}}(-14\underline{i} + 12\underline{j})$

$= \frac{2}{\sqrt{4 \times 85}}(-7\underline{i} + 6\underline{j})$

$= \frac{2}{2\sqrt{85}}(-7\underline{i} + 6\underline{j})$

$= -\frac{7}{\sqrt{85}}\underline{i} + \frac{6}{\sqrt{85}}\underline{j}$

17 (a) (i) $\overrightarrow{BC} = \overrightarrow{BA} + \overrightarrow{AC}$

$= 2\underline{i} - 3\underline{j} - 6\underline{i} + 6\underline{j}$

$= -4\underline{i} + 3\underline{j}$

(ii) $\left|\overrightarrow{BC}\right| = \sqrt{(-4)^2 + 3^2} = 5$

Unit vector in the direction of $\overrightarrow{BC}$

$= \frac{1}{5}\left(-4\underline{i} + 3\underline{j}\right)$

$= -\frac{4}{5}\underline{i} + \frac{3}{5}\underline{j}$

(b) $\overrightarrow{AD} = k\,\overrightarrow{BC}$

$p\underline{i} - 12\underline{j} = k\left(-4\underline{i} + 3\underline{j}\right)$

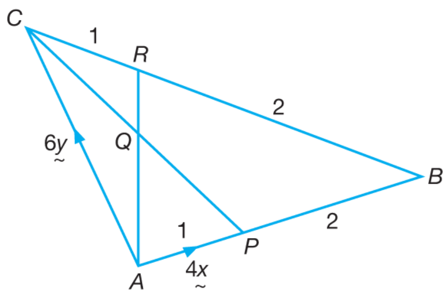
$p\underline{i} - 12\underline{j} = -4k\underline{i} + 3k\underline{j}$

$3k = -12 \qquad p = -4k$

$k = -4 \qquad\quad = -4(-4)$

$\qquad\qquad\quad = 16$

18



(a) (i) $\overrightarrow{CP} = \overrightarrow{CA} + \overrightarrow{AP}$

$= -6\underline{y} + 4\underline{x}$

(ii) $\overrightarrow{CR} = \frac{1}{3}\overrightarrow{CB}$.

$= \frac{1}{3}\left(\overrightarrow{CA} + \overrightarrow{AB}\right)$

$= \frac{1}{3}\left(-6\underline{y} + 12\underline{x}\right)$

$= -2\underline{y} + 4\underline{x}$

(b) $\overrightarrow{CR} = -2\underline{y} + 4\underline{x}$

$\overrightarrow{CR} = -2\left(-3\underline{i} + 4\underline{j}\right) + 4\left(2\underline{i} + \underline{j}\right)$

$\overrightarrow{CR} = 14\underline{i} - 4\underline{j}$

$\left|\overrightarrow{CR}\right| = \sqrt{14^2 + (-4)^2} = \sqrt{212} = 14.56$

(c) Using the triangle addition rule,

$\overrightarrow{CQ} + \overrightarrow{QR} = \overrightarrow{CR}$

$m\overrightarrow{CP} + n\overrightarrow{AR} = \overrightarrow{CR}$

$m(-6\underline{y} + 4\underline{x}) + n\left(\overrightarrow{AC} + \overrightarrow{CR}\right) = \overrightarrow{CR}$

$m(-6\underline{y} + 4\underline{x}) + n\left[6\underline{y} + (-2\underline{y} + 4\underline{x})\right] = -2\underline{y} + 4\underline{x}$

$\left(4m + 4n\right)\underline{x} + \left(-6m + 4n\right)\underline{y} = -2\underline{y} + 4\underline{x}$

Equating the coefficients of $\underline{x}$,

$4m + 4n = 4$

$2m + 2n = 2$ … (1)

Equating the coefficients of $\underline{y}$,

$-6m + 4n = -2$

$-3m + 2n = -1$ … (2)

(1) – (2) : $5m = 3$

$m = \frac{3}{5}$

From (2) :

$-3\left(\frac{3}{5}\right) + 2n = -1$

$2n = \frac{4}{5}$

$n = \frac{2}{5}$

SPM Spot

1 (a) If the points F, G and H are collinear,

then $\overrightarrow{FG} = k\overrightarrow{GH}$, where k is a constant.

$\overrightarrow{FG} = k\overrightarrow{GH}$

$6\underset{\sim}{p} - 4\underset{\sim}{q} = k\left[4\underset{\sim}{p} - (2u-1)\underset{\sim}{q}\right]$

$6\underset{\sim}{p} - 4\underset{\sim}{q} = 4k\underset{\sim}{p} - k(2u-1)\underset{\sim}{q}$

Equating the coefficients of $\underset{\sim}{p}$,

$4k = 6$

$k = \frac{3}{2}$

Equating the coefficients of $\underset{\sim}{q}$,

$-4 = -k(2u-1)$

$-4 = -\frac{3}{2}(2u-1)$

$-8 = -3(2u-1)$

$-8 = -6u + 3$

$6u = 3 + 8$

$u = \frac{11}{6}$

(b) $\overrightarrow{FG} = \frac{3}{2}\overrightarrow{GH}$

$\frac{\left|\overrightarrow{FG}\right|}{\left|\overrightarrow{GH}\right|} = \frac{3}{2} = 3 : 2$

2 (a) $\underset{\sim}{v}_A = \left(4\underset{\sim}{i} + p\underset{\sim}{j}\right)$ m per min

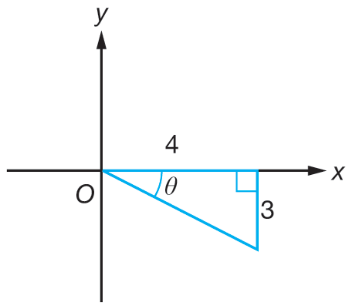
$\left|\underset{\sim}{v}_A\right| = \sqrt{16 + p^2} = 5$

$16 + p^2 = 25$

$p^2 = 9$

$p = -3$

(b) $\underset{\sim}{v}_A = 4\underset{\sim}{i} - 3\underset{\sim}{j}$



$\tan\theta = \frac{3}{4}$

$\theta = 36.9^o$

The direction (bearing) of boat *A*

$= (90^o + 36.9^o)$

$= 126.9^o$

(c) The position vector of boat *A*

$= \left(-2\underset{\sim}{i} + 8\underset{\sim}{j}\right) + t\left(4\underset{\sim}{i} - 3\underset{\sim}{j}\right)$

$= (4t - 2)\underset{\sim}{i} + (-3t + 8)\underset{\sim}{j}$

The position vector of boat *B*

$= \left(q\underset{\sim}{i} - 2\underset{\sim}{j}\right) + t\left(3\underset{\sim}{i} - \underset{\sim}{j}\right)$

$= (3t + q)\underset{\sim}{i} + (-t - 2)\underset{\sim}{j}$

Equating the coefficients of $\underset{\sim}{j}$,

$-3t + 8 = -t - 2$

$2t = 10$

$t = 5$

Hence, boats *A* and *B* meet after 5 minutes.

(i) Equating the coefficients of $\underset{\sim}{i}$,

$4t - 2 = 3t + q$

$4(5) - 2 = 3(5) + q$

$18 = 15 + q$

$q = 3$

(ii) The position vector where the two boats meet

$= (4\times5 - 2)\underset{\sim}{i} + (-3\times5 + 8)\underset{\sim}{j}$

$= 18\underset{\sim}{i} - 7\underset{\sim}{j}$

3 (a) (i) $\overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{AP}$

$= \overrightarrow{OA} + \frac{1}{3}\overrightarrow{AB}$

$= 2\underset{\sim}{a} + \frac{1}{3}\left(-2\underset{\sim}{a} + 2\underset{\sim}{b}\right)$

$= \frac{4}{3}\underset{\sim}{a} + \frac{2}{3}\underset{\sim}{b}$

(ii) $\overrightarrow{BQ} = \overrightarrow{BO} + \overrightarrow{OQ}$

$= \overrightarrow{BO} + k\overrightarrow{OP}$

$= -2\underset{\sim}{b} + k\left(\frac{4}{3}\underset{\sim}{a} + \frac{2}{3}\underset{\sim}{b}\right)$

$= -2\underset{\sim}{b} + \frac{4}{3}k\underset{\sim}{a} + \frac{2}{3}k\underset{\sim}{b}$

$= \frac{4}{3}k\underset{\sim}{a} + \left(\frac{2}{3}k - 2\right)\underset{\sim}{b}$

(b) $\overrightarrow{BQ} = h\overrightarrow{BC}$

$\frac{4}{3}k\underset{\sim}{a} + \left(\frac{2}{3}k - 2\right)\underset{\sim}{b} = h\left(\overrightarrow{BO} + \overrightarrow{OC}\right)$

$\frac{4}{3}k\underset{\sim}{a} + \left(\frac{2}{3}k - 2\right)\underset{\sim}{b} = h\left(-2\underset{\sim}{b} + 3\overrightarrow{OA}\right)$

$\frac{4}{3}k\underset{\sim}{a} + \left(\frac{2}{3}k - 2\right)\underset{\sim}{b} = -2h\underset{\sim}{b} + 6h\underset{\sim}{a}$

Equating the coefficients of $\underset{\sim}{a}$,

$\frac{4}{3}k = 6h$

$\frac{2}{3}k = 3h \quad \ldots (1)$

Equating the coefficients of $\underset{\sim}{b}$,

$\frac{2}{3}k - 2 = -2h$

$h = 1 - \frac{1}{3}k \quad \ldots (2)$

Substitute (2) into (1) :

$\frac{2}{3}k = 3\left(1 - \frac{1}{3}k\right)$

$2k = 9\left(1 - \frac{1}{3}k\right)$

$2k = 9 - 3k$

$5k = 9$

$k = \frac{9}{5}$

From (2) :

$h = 1 - \frac{1}{3}\left(\frac{9}{5}\right) = \frac{2}{5}$

(c) $\overrightarrow{BQ} = h\,\overrightarrow{BC}$

$\overrightarrow{BQ} = \frac{2}{5}\,\overrightarrow{BC}$

$BQ : QC = 2 : 3$