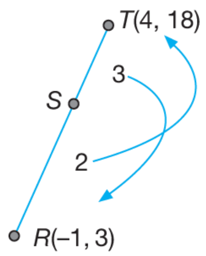
Form 4 Chapter 7
Coordinate Geometry
Fully-Worked Solutions

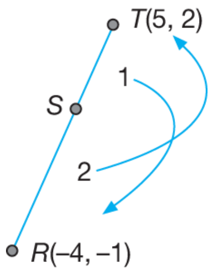
UPSKILL 7.1

1 (a)



$$S = \left(\frac{3(-1)+2(4)}{2+3}, \frac{3(3)+2(18)}{2+3}\right)$$
$$S = \left(\frac{5}{5}, \frac{45}{5}\right)$$
$$S = (1, 9)$$

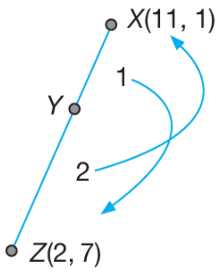
(b)



$$S = \left(\frac{1(-4)+2(5)}{2+1}, \frac{1(-1)+2(2)}{2+1}\right)$$
$$S = \left(\frac{6}{3}, \frac{3}{3}\right)$$
$$S = (2, 1)$$

2 (a) $2XY = YZ$

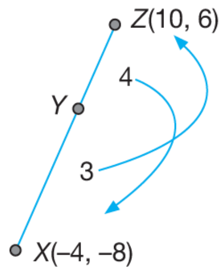
$$\frac{XY}{YZ} = \frac{1}{2}$$
$XY : YZ = 1 : 2$



$$Y = \left(\frac{2(11)+1(2)}{1+2}, \frac{2(1)+1(7)}{2+1}\right)$$
$$Y = \left(\frac{24}{3}, \frac{9}{3}\right)$$
$$Y = (8, 3)$$

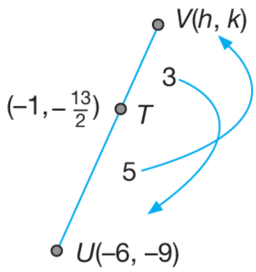
(b) $4XY = 3YZ$

$$\frac{XY}{YZ} = \frac{3}{4}$$
$XY : YZ = 3 : 4$



$$Y = \left(\frac{4(-4)+3(10)}{3+4}, \frac{4(-8)+3(6)}{3+4}\right)$$
$$Y = \left(\frac{14}{7}, \frac{-14}{7}\right)$$
$$Y = (2, -2)$$

3



Let $V = (h, k)$

$$T = \left(\frac{3(-6)+5h}{5+3}, \frac{3(-9)+5k}{5+3}\right)$$
$$T = \left(\frac{5h-18}{8}, \frac{5k-27}{8}\right)$$

But it is given that $T = \left(-1, -\frac{13}{2}\right)$.

Equating the x-coordinates:

$$\frac{5h-18}{8} = -1$$
$$5h - 18 = -8$$
$$5h = 10$$
$$h = 2$$

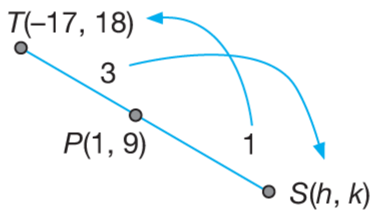
Equating the y-coordinates:

$$\frac{5k-27}{8} = -\frac{13}{2}$$
$$5k - 27 = -\frac{13}{2} \times 8$$

$5k = -52 + 27$
$5k = -25$
$k = -5$

Hence, the coordinates of point V are (2, –5).

4



$$P = \left(\frac{3h + 1(-17)}{1+3}, \frac{3k + 1(18)}{1+3}\right)$$
$$P = \left(\frac{3h - 17}{4}, \frac{3k + 18}{4}\right)$$

But it is given that the coordinates of point P are (1, 9).

Equating the x-coordinates:

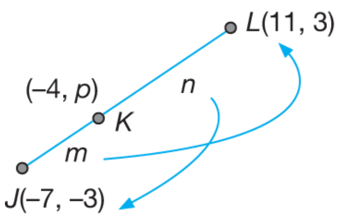
$$\frac{3h-17}{4} = 1$$
$$3h - 17 = 4$$
$$3h = 21$$
$$h = 7$$

Equating the y-coordinates:

$$\frac{3k+18}{4} = 9$$
$$3k = 18$$
$$k = 6$$

Hence, the coordinates of point S are (7, 6).

5



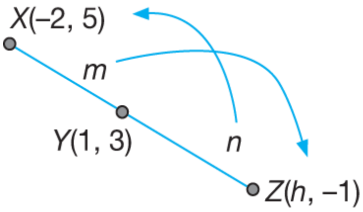
(a) Equating the x-coordinates:

$$\frac{n(-7) + m(11)}{m+n} = -4$$
$$-7n + 11m = -4m - 4n$$
$$15m = 3n$$
$$\frac{m}{n} = \frac{3}{15}$$
$$\frac{m}{n} = \frac{1}{5}$$
$$m : n = 1 : 5$$
$$JK : KL = 1 : 5$$

(b) Equating the y-coordinates:

$$p = \frac{5(-3) + 1(3)}{1+5} = -\frac{12}{6} = -2$$

6



(a) Equating the y-coordinates:

$$\frac{n(5) + m(-1)}{m+n} = 3$$
$$5n - m = 3m + 3n$$
$$4m = 2n$$
$$\frac{m}{n} = \frac{1}{2}$$
$$m : n = 1 : 2$$
$$XY : YZ = 1 : 2$$

(b) Equating the x-coordinates:

$$\frac{2(-2) + 1(h)}{1+2} = 1$$
$$-4 + h = 3$$
$$h = 7$$

UPSKILL 7.2a

1 (a) $y - y_1 = m(x - x_1)$

$$y - 2 = -2(x - 5)$$
$$y - 2 = -2x + 10$$
$$y = -2x + 12$$

(b) $y - 3 = \frac{3}{4}(x + 8)$

$$4(y - 3) = 3(x + 8)$$
$$4y - 12 = 3x + 24$$
$$4y = 3x + 36$$

2 (a) $\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$

$$\frac{y+1}{x-2} = \frac{0+1}{3-2}$$
$$\frac{y+1}{x-2} = 1$$
$$y + 1 = x - 2$$
$$y = x - 2 - 1$$
$$y = x - 3$$

(b) $\frac{y-3}{x+4}=\frac{-5-3}{2+4}$

$\frac{y-3}{x+4}=\frac{-5-3}{2+4}$

$\frac{y-3}{x+4}=\frac{-4}{3}$

$3(y-3)=-4(x+4)$

$3y-9=-4x-16$

$3y=-4x-7$

3 (a) $\frac{x}{3}+\frac{y}{(-5)}=1$

(b) $\frac{x}{(-8)}+\frac{y}{(-6)}=1$

(c) $\frac{x}{\left(-\frac{1}{2}\right)}+\frac{y}{\left(\frac{3}{4}\right)}=1$

4 (a) $\frac{y-3}{x+1}=\frac{15-3}{5+1}$

$\frac{y-3}{x+1}=2$

$y-3=2x+2$

$y=2x+5$

(b) $2x-y+5=0$

(c) $-\frac{2x}{5}+\frac{y}{5}=\frac{5}{5}$

$\frac{x}{-\frac{5}{2}}+\frac{y}{5}=1$

x-intercept = $-\frac{5}{2}$

y-intercept = 5

Gradient = 2

5 The equation of PQ is

$y-8=3(x-2)$

$y-8=3x-6$

$y=3x+2$ … (1)

The equation of RS is

$\frac{y+2}{x+6}=\frac{6+2}{2+6}$

$\frac{y+2}{x+6}=1$

$y+2=x+6$

$y=x+4$ … (2)

$y=3x+2$ … (1)

$y=x+4$ … (2)

Substitute (2) into (1) :

$x+4=3x+2$

$2x=2$

$x=1$

From (2) :

$y=1+4$

$y=5$

Hence, the coordinates of the point of intersection are (1, 5).

6 Point *P*

Equation of PQ: $3y=x+7$ … (1)

Equation of PR: $7y=-3x-5$ …(2)

(1) × 3: $9y=3x+21$ (3)

(2) + (3) : $16y=16$

$y=1$

From (1) : $3(1)=x+7$

$x=-4$

The coordinates of point P are (–4, 1).

Point *Q*

Equation of PQ: $3y=x+7$ …(1)

Equation of QR: $y=-5x+13$.(2)

(2) × 3 : $3y=-15x+39$ ..(3)

(1) – (3) : $16x-32=0$

$x=2$

From (2) : $y=-5(2)+13=3$

The coordinates of point Q are (2, 3).

Point *R*

Equation of PR: $7y=-3x-5$ …(1)

Equation of QR: $y=-5x+13$ …(2)

(2) × 7 : $7y=-35x+91$ …(3)

(1) – (3) : $0=32x-96$

$x=3$

From (2) : $y=-5(3)+13=-2$

The coordinates of point R are (3, –2).

7 The equation of the straight line CD is

$\frac{y-2}{x-4}=\frac{-4-2}{-5-4}$

$\frac{y-2}{x-4}=\frac{2}{3}$

$3y-6=2x-8$

$3y=2x-2$

Equation of CD: $3y=2x-2$ … (1)

Equation of ST: $2y=7x+10$ … (2)

$(1)\times 2: 6y=4x-4 \quad \dots (3)$
$(2)\times 3: 6y=21x+30 \quad \dots (4)$

$(3)-(4): 0=-17x-34$
$x=-2$
From (1): $3y=2(-2)-2$
$y=-2$
The coordinates of point P are $(-2,-2)$.

The equation of the straight line HK is
$y+2=-2(x+2)$
$y=-2x-6$

UPSKILL 7.2b

1 Gradient $PQ = \frac{8-18}{5-1}=-\frac{5}{2}$

Gradient $TU = \frac{8+2}{-5+1}=-\frac{5}{2}$

Hence, PQ is parallel to TU.

2 $3x+2y-1=0$
$2y=-3x+1$
$y=-\frac{3}{2}x+\frac{1}{2}$
$m_1=-\frac{3}{2}$

$-\frac{x}{2}-\frac{y}{3}=1$

$m_2=-\frac{y-\text{intercept}}{x-\text{intercept}}$
$=-\frac{-3}{-2}$
$=-\frac{3}{2}$

Hence, both straight lines are parallel.

3 $5x-2y+4=0$
$-2y=-5x-4$
$y=\frac{5}{2}x+2$
$m=\frac{5}{2}$

$y-4=\frac{5}{2}(x-5)$
$2y-8=5x-25$
$2y=5x-17$

UPSKILL 7.2c

1 m_1 = Gradient of PQ = $\frac{4-3}{8-4}=\frac{1}{4}$

m_2 = Gradient of TU = $\frac{5-1}{6-7}=-4$

$m_1m_2=\frac{1}{4}\times(-4)=-1$

Hence, PQ is perpendicular to TU.

2 m_1 = Gradient of KL = $\frac{5-4}{9-1}=\frac{1}{8}$

$2x+\frac{y}{4}=3$
$8x+y=12$
$y=-8x+12$
m_2 = Gradient of MN = -8
$m_1m_2=\frac{1}{8}\times(-8)=-1$

Hence, MN is perpendicular to TU.

3 $4x-3y+8=0$
$3y=4x+8$
$y=\frac{4}{3}x+\frac{8}{3}$
$m_1=\frac{4}{3}$

$\frac{x}{8}+\frac{y}{6}=1$
$m_2=-\frac{6}{8}=-\frac{3}{4}$
$m_1m_2=\frac{4}{3}\times\left(-\frac{3}{4}\right)=-1$

Hence, the straight line $4x-3y+8=0$ is perpendicular to the straight line $\frac{x}{8}+\frac{y}{6}=1$.

4 $4x+3y+2=0$
$3y=-4x-2$
$y=-\frac{4}{3}x-\frac{2}{3}$
$m=-\frac{4}{3}$

Gradient of the perpendicular line
$=\frac{3}{4}$

The equation of the perpendicular line is
$y-3=\frac{3}{4}(x+2)$
$4y-12=3x+6$
$4y=3x+18$

5 $N = \text{Midpoint} = \left(\frac{3-3}{2}, \frac{-1+6}{2}\right) = \left(0, \frac{5}{2}\right)$

$m_{PS} = \frac{-4-2}{1+5} = -1$

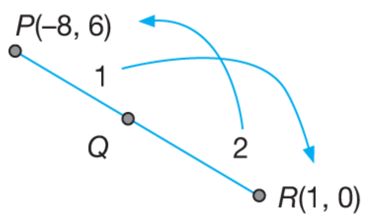
Thus, the gradient the perpendicular line is 1.
The equation of the perpendicular line is

$y - \frac{5}{2} = (1)(x - 0)$

$2y - 5 = 2x$

$2y = 2x + 5$

6 (a)



$Q = \left(\frac{2(-8)+1(1)}{1+2}, \frac{1(0)+2(6)}{1+2}\right)$

$Q = (-5, 4)$

$m_{PQR} = \frac{0-6}{1+8} = -\frac{2}{3}$

Gradient of the perpendicular line is $\frac{3}{2}$.

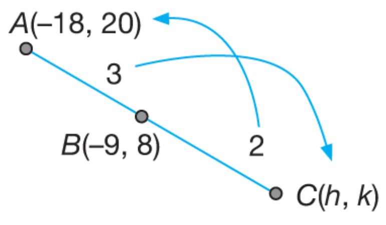
The equation of the perpendicular line is

$y - 4 = \frac{3}{2}(x+5)$

$2y - 8 = 3x + 15$

$2y = 3x + 23$

(b) Let C be point (h, k).



$\left(\frac{2(-18)+3h}{3+2}, \frac{2(20)+3k}{3+2}\right) = (-9, 8)$

$\left(\frac{3h-36}{5}, \frac{40+3k}{5}\right) = (-9, 8)$

Equating the x-coordinates,

$\frac{3h-36}{5} = -9$

$3h - 36 = -45$

$3h = -9$

$h = -3$

Equating the y-coordinates,

$\frac{40+3k}{5} = 8$

$40 + 3k = 40$

$3k = 0$

$k = 0$

Hence, the coordinates of point C are $(-3, 0)$.

$m_{ABC} = \frac{0-20}{-3+18} = -\frac{4}{3}$

Gradient of the perpendicular line is $\frac{3}{4}$.

The equation of perpendicular line is

$y - 0 = \frac{3}{4}(x+3)$

$4y = 3(x+3)$

$4y = 3x + 9$

7 Midpoint of KL is

$\left(\frac{-2+3}{2}, \frac{3+6}{2}\right) = \left(\frac{1}{2}, \frac{9}{2}\right)$.

$m_{KL} = \frac{6-3}{3+2} = \frac{3}{5}$

Gradient of perpendicular line = $-\frac{5}{3}$

The equation of the perpendicular bisector is

$y - \frac{9}{2} = -\frac{5}{3}\left(x - \frac{1}{2}\right)$

$6y - 27 = -10\left(x - \frac{1}{2}\right)$

$6y - 27 = -10x + 5$

$6y = -10x + 32$

$3y = -5x + 16$

8 (a) Midpoint of BC

$= \left(\frac{5-1}{2}, \frac{3+5}{2}\right)$

$= (2, 4)$

$m_{BC} = \frac{5-3}{-1-5} = -\frac{1}{3}$

Gradient of the perpendicular line $= 3$

The equation of the perpendicular bisector is

$y - 4 = 3(x - 2)$

$y - 4 = 3x - 6$

$y = 3x - 2 \quad \ldots (1)$

(b) The equation of the straight line AB is

$$\frac{y+4}{x+2}=\frac{3+4}{5+2}$$
$$\frac{y+4}{x+2}=1$$
$$y+4=x+2$$
$$y=x-2 \quad \ldots (2)$$

Substitute (1) into (2) :
$$3x-2=x-2$$
$$2x=0$$
$$x=0$$

From (2) :
$$y=x-2=0-2=-2$$

Hence, the required point of intersection is $(0,-2)$.

9 (a) $x+y=8$
$$y=-x+8$$
$$m_1=-1$$
$$m_{PR}=1$$

The equation of PR is
$$y-1=1(x-2)$$
$$y=x-1$$

(b) $m_{SQ}=-1$

The equation of SQ is
$$y-2=-(x-5)$$
$$y=-x+5+2$$
$$y=-x+7$$

(c) Equation of PTR: $y=x-1 \quad \ldots (1)$

Equation of STQ: $y=-x+7 \quad \ldots (2)$

Substitute (1) into (2) :
$$x-1=-x+7$$
$$2x=8$$
$$x=4$$

Substitute $x=4$ into (1) :
$$y=4-1=3$$

Hence, T is point (4, 3).

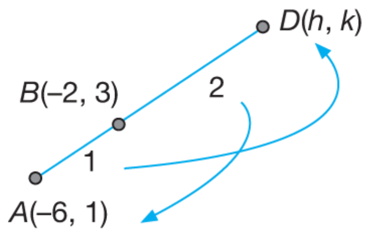
Let R be point (h, k).
T is the midpoint of PR.
$$\frac{2+h}{2}=4 \qquad \frac{1+k}{2}=3$$
$$h=6 \qquad k=5$$

Hence, R is point (6, 5).

Let S be point (a, b).
T is the midpoint of SQ.
$$\frac{5+a}{2}=4 \qquad \frac{2+b}{2}=3$$
$$a=3 \qquad b=4$$

Hence, S is point (3, 4).

10 (a) $AB=\frac{1}{2}BD$
$$\frac{AB}{BD}=\frac{1}{2}$$
$$AB : BD = 1 : 2$$



$$\left(\frac{2(-6)+h}{1+2}, \frac{2(1)+k}{1+2}\right)=(-2,3)$$
$$\left(\frac{h-12}{3}, \frac{k+2}{3}\right)=(-2,3)$$

Equating the x-coordinates:
$$\frac{h-12}{3}=-2$$
$$h=6$$

Equating the y-coordinates:
$$\frac{k+2}{3}=3$$
$$k+2=9$$
$$k=7$$

Hence, D is point (6, 7).

(b) $m_{ABD}=\frac{3-1}{-2+6}=\frac{1}{2}$

Thus, $m_{CE}=-2$

Hence, the equation of CE is
$$y-0=-2(x-7)$$
$$y=-2x+14 \quad \ldots (1)$$

(c) The equation of $ABCD$:
$$y-1=\frac{1}{2}(x+6)$$
$$2y-2=x+6$$
$$2y=x+8 \quad \ldots (2)$$

Substitute (1) into (2) :
$$2(-2x+14)=x+8$$
$$-4x+28=x+8$$
$$5x=20$$
$$x=4$$

Substitute $x = 4$ into (1) :

$y = -2(4) + 14$

$y = 6$

Hence, C is point (4, 6).

UPSKILL 7.3

1 (a) Area of triangle *PQR*

$$= \frac{1}{2}\begin{vmatrix} -4 & 1 & 3 & -4 \\ 1 & 2 & 5 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\left|-8+5+3-(1+6-20)\right|$$

$$= \frac{1}{2}\left|0-(-13)\right|$$

$$= \frac{13}{2} \text{ units}^2$$

(b) Area of triangle *KLM*

$$= \frac{1}{2}\begin{vmatrix} 2 & 9 & 5 & 2 \\ 3 & 4 & 1 & 3 \end{vmatrix}$$

$$= \frac{1}{2}\left|8+9+15-(27+20+2)\right|$$

$$= \frac{1}{2}\left|32-49\right|$$

$$= \frac{1}{2}\left|-17\right|$$

$$= \frac{17}{2} \text{ units}^2$$

(c) Area of triangle *ABC*

$$= \frac{1}{2}\begin{vmatrix} -2 & 4 & 1 & -2 \\ 1 & 2 & -3 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\left|-4-12+1-(4+2+6)\right|$$

$$= \frac{1}{2}\left|-15-12\right|$$

$$= \frac{1}{2}\left|-27\right|$$

$$= \frac{27}{2} \text{ units}^2$$

2 (a) Area of the quadrilateral *EFGH*

$$= \frac{1}{2}\begin{vmatrix} 2 & -3 & -2 & 4 & 2 \\ -2 & -1 & 5 & 0 & -2 \end{vmatrix}$$

$$= \frac{1}{2}\left|-2-15-8-(6+2+20)\right|$$

$$= \frac{1}{2}\left|-25-28\right|$$

$$= \frac{1}{2}\left|-53\right|$$

$$= \frac{53}{2} \text{ units}^2$$

(b) Area of the quadrilateral *ABCD*

$$= \frac{1}{2}\begin{vmatrix} -5 & 0 & 6 & 0 & -5 \\ 0 & 10 & 4 & 0 & 0 \end{vmatrix}$$

$$= \frac{1}{2}\left|-50-(60)\right|$$

$$= \frac{1}{2}\left|-110\right|$$

$$= \frac{1}{2}(110)$$

$$= 55 \text{ units}^2$$

3 Area of $\Delta ABC = 12$ units^2

$$\frac{1}{2}\begin{vmatrix} 1 & 7 & h & 1 \\ 2 & 8 & -10 & 2 \end{vmatrix} = 12$$

$$\begin{vmatrix} 1 & 7 & h & 1 \\ 2 & 8 & -10 & 2 \end{vmatrix} = 24$$

$$\left|8-70+2h-(14+8h-10)\right| = 24$$

$$\left|-66-6h\right| = 24$$

$$\left|-11-h\right| = 4$$

$$-11-h = \pm 4$$

$-11-h = 4$ or $-11-h = -4$

$-h = 15$ $h = -7$

$h = -15$

4 Area of the quadrilateral *TUVW* = 18 units^2

$$\frac{1}{2}\begin{vmatrix} -1 & p & 3 & 4 & -1 \\ 3 & -1 & 1 & 5 & 3 \end{vmatrix} = 18$$

$$\left|1+p+15+12-(3p-3+4-5\right| = 18$$

$$\left|32-2p\right| = 36$$

$$32-2p = \pm 36$$

$32-2p = 36$ or $32-2p = -36$

$-2p = 4$ $-2p = -68$

$p = -2$ $p = 34$

5 Area of $\Delta LMN = 0$

$$\frac{1}{2}\begin{vmatrix} -2 & 1 & 4 & -2 \\ 4 & h & 4h & 4 \end{vmatrix} = 0$$

$$-2h+4h+16-(4+4h-8h) = 0$$

$$6h+12 = 0$$

$$h = -2$$

6 (a)(i) Area of ΔABC

$$= \frac{1}{2}\begin{vmatrix} -8 & 6 & -1 & -8 \\ -4 & -6 & 2 & -4 \end{vmatrix}$$

$$= \frac{1}{2}\left| 48+12+4-(-24+6-16) \right|$$

$$= \frac{1}{2}|98|$$

= 49 units^2

(ii) $AC = \sqrt{(6+8)^2+(-6+4)^2}$

$AC=\sqrt{196+4}$

$AC = \sqrt{200}$

$AC=14.1421$ units

(b) Let the perpendicular distance from point B to the straight line AC be h units.

Area of ΔABC =49

$\frac{1}{2}\times\text{Base}\times\text{Height} = 49$

$\frac{1}{2}\times 14.1421\times h=49$

$h=6.930$

Hence, the shortest distance from B to AC is 6.930 units.

UPSKILL 7.4

1

$PA=5$

$\sqrt{(x-4)^2+(y-6)^2}=5$

$(x-4)^2+(y-6)^2=5^2$

$x^2-8x+16+y^2-12y+36-25=0$

$x^2-8x+y^2-12y+27=0$

2 $BQ = CQ$

$BQ^2 = CQ^2$

$(x-5)^2+(y-1)^2=(x+4)^2+(y-2)$

$x^2-10x+25+y^2-2y+1=$

$x^2+8x+16+y^2-4y+4$

$-18x+2y+6=0$

$-9x+y+3=0$

$y=9x-3$

3 $\frac{HJ}{HK}=\frac{2}{1}$

$HJ=2HK$

$HJ^2=2^2HK^2$

$(x-3)^2+(y-0)^2=4\left[(x-0)^2+(y+2)^2\right]$

$x^2-6x+9+y^2=4\left(x^2+y^2+4y+4\right)$

$x^2-6x+9+y^2=4x^2+4y^2+16y+16$

$3x^2+3y^2+6x+16y+7=0$

4 $\frac{KP}{KQ}=\frac{3}{2}$

$2KP=3KQ$

$4KP^2=9KQ^2$

$4\left[(x-2)^2+(y-1)^2\right]=9\left[(x+1)^2+(y+2)^2\right]$

$4\left(x^2-4x+4+y^2-2y+1\right)=$

$9\left(x^2+2x+1+y^2+4y+4\right)$

$4x^2-16x+4y^2-8y+20=$

$9x^2+18x+9y^2+36y+45$

$5x^2+5y^2+34x+44y+25=0$

5 $JM=2JN$

$JM^2=2^2JN^2$

$(x-0)^2+(y+1)^2=4\left[(x-2)^2+(y-0)^2\right]$

$x^2+y^2+2y+1=4\left(x^2-4x+4+y^2\right)$

$x^2+y^2+2y+1=4x^2-16x+16+4y^2$

$3x^2+3y^2-16x-2y+15=0$

(a) At the y-axis, $x = 0$

$3y^2-2y+15=0$

$b^2-4ac=(-2)^2-4(3)(15)=-176$

Since $b^2-4ac<0$, the locus of J does not intersect the y-axis.

(b) At the x-axis, $y = 0$

$3x^2-16x+15=0$

$b^2-4ac=(-16)^2-4(3)(15)=76$

Since $b^2-4ac>0$, the locus of J will intersect the x-axis.

6 Since $\angle APB = 90^\circ$

$m_{AP}\times m_{PB}=-1$

$\left(\frac{y+2}{x-3}\right)\left(\frac{y-0}{x-6}\right)=-1$

$\frac{y^2+2y}{x^2-9x+18}=-1$

$y^2+2y=-x^2+9x-18$

$x^2+y^2-9x+2y+18=0$

7 Since $\angle APB = 90^\circ$

$$m_{AP} \times m_{PB} = -1$$

$$\left(\frac{y-2}{x-0}\right)\left(\frac{y+2}{x-2}\right) = -1$$

$$\frac{y^2-4}{x^2-2x} = -1$$

$$y^2 - 4 = -x^2 + 2x$$

$$x^2 + y^2 - 2x - 4 = 0$$

8 Since $\angle MQN = 90^\circ$

$$m_{MQ} \times m_{QN} = -1$$

$$\left(\frac{y-4}{x-1}\right)\left(\frac{y-0}{x-3}\right) = -1$$

$$\frac{y^2-4y}{x^2-4x+3} = -1$$

$$y^2 - 4y = -x^2 + 4x - 3$$

$$x^2 + y^2 - 4x - 4y + 3 = 0$$

Summative Practice 7

1 (a) $m_{AB} = m_{DC} = 2$

The equation of AB is

$$y - 7 = 2(x-2)$$

$$y - 7 = 2x - 4$$

$$y = 2x + 3$$

(b) $m_{AD} = -\frac{1}{m_{DC}} = -\frac{1}{2}$

The equation of AD is

$$y - (-1) = -\frac{1}{2}(x-3)$$

$$2y + 2 = -x + 3$$

$$2y = -x + 1$$

(c) $y = 2x + 3 \ \dots (1)$

$2y = -x + 1 \ \dots (2)$

Substitute (1) into (2) :

$$2(2x+3) = -x + 1$$

$$4x + 6 = -x + 1$$

$$5x = -5$$

$$x = -1$$

From (1) :

$$y = 2(-1) + 3 = 1$$

Hence, the coordinates of point A are $(-1, 1)$.

(d) Area of ΔBAD

$$= \frac{1}{2}\begin{vmatrix} 2 & -1 & 3 & 2 \\ 7 & 1 & -1 & 7 \end{vmatrix}$$

$$= \frac{1}{2}\left| 2 + 1 + 21 - (-7 + 3 - 2) \right|$$

$$= 15 \text{ units}^2$$

Hence, the area of the rectangle $ABCD$

$$= 2 \times 15 = 30 \text{ units}^2$$

2 (a) $m_{PQ} = \frac{9-7}{6-2} = \frac{1}{2}$

$m_{PS} = -2$

The equation of PS is

$$y - 7 = -2(x-2)$$

$$y - 7 = -2x + 4$$

$$y = -2x + 11$$

(b) Substitute $y = -2x + 11$ into $7x - 2y = 44$,

$$7x - 2(-2x + 11) = 44$$

$$7x + 4x - 22 = 44$$

$$11x = 66$$

$$x = 6$$

When $x = 6$, $y = -2(6) + 11 = -1$

Hence, S is point $(6, -1)$.

(c) $T = \left(\frac{3(2)+1(6)}{1+3}, \frac{3(7)+1(-1)}{1+3}\right)$

$= (3, 5)$

(d) Area of $PQRS$ = 30 units^2

$$\frac{1}{2}\begin{vmatrix} 2 & 6 & k & 6 & 2 \\ 7 & -1 & \frac{7k-44}{2} & 9 & 7 \end{vmatrix} = 30$$

When $x = k$,

$7x - 2y = 44$

$7k - 2y = 44$

$y = \frac{7k-44}{2}$

$$-2 + 3(7k-44) + 9k + 42 - \left[42 - k + 3(7k-44) + 18\right] = 60$$

$$-2 + 3(7k-44) + 9k + 42 - 42 + k - 3(7k-44) - 18 = 60$$

$$-20 + 10k = 60$$

$$k = 8$$

When $k = 8$, $y = \frac{7(8)-44}{2} = 6$

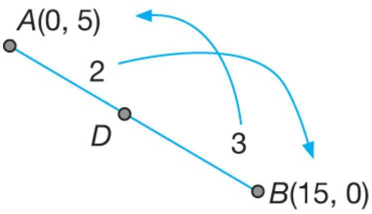
Hence, R is point (8, 6).

3 (a) The equation of AB is $\frac{x}{15}+\frac{y}{5}=1$.

(b) $3AD=2DB$

$\frac{AD}{DB}=\frac{2}{3}$

$AD:DB=2:3$



$D=\left(\frac{3(0)+2(15)}{2+3},\frac{3(5)+2(0)}{2+3}\right)$

$D=(6,\ 3)$

(c) $m_{AB}=\frac{0-5}{15-0}=-\frac{1}{3}$

$m_{CD}=3$

The equation of CD is

$y-3=3(x-6)$

$y-3=3x-18$

$y=3x-15$

Hence, the y-intercept is –15.

4 (a) Equation of AB : $y=6x-8$ … (1)

Equation of AN: $5y=2x+16$ .. (2)

Substitute (1) into (2) :

$5(6x-8)=2x+16$

$30x-40=2x+16$

$28x=56$

$x=2$

When x = 2,

$y=6(2)-8=4$

Hence, the coordinates of point A are (2, 4).

(b) M is the midpoint of AC.

Let C is point $(h,\ k)$.

$\left(\frac{2+h}{2},\frac{4+k}{2}\right)=(5,\ 8)$

Equating the x-coordinates:

$\frac{2+h}{2}=5$

$h=8$

Equating the y-coordinates:

$\frac{4+k}{2}=8$

$k=12$

Hence, the coordinates of point C are (8, 12).

(c) The equation of AN is

$5y=2x+16\Rightarrow y=\frac{2}{5}x+\frac{16}{5}$.

$m_{AN}=\frac{2}{5}$

Thus, $m_{CN}=-\frac{5}{2}$

The equation of CN is

$y-12=-\frac{5}{2}(x-8)$

$2y-24=-5x+40$

$2y=-5x+64$

(d) $m_{CD}=m_{AB}=6$

The equation of CD is

$y-12=6(x-8)$

$y-12=6x-48$

$y=6x-36$

(e) Equation of CD: $y=6x-36$ … (1)

Equation of AD: $5y=2x+16$ …(2)

Substitute (1) into (2) :

$5(6x-36)=2x+16$

$30x-180=2x+16$

$28x=196$

$x=7$

From (1):

$y=6x-36$

$y=6(7)-36=6$

Hence, the coordinates of point D are (7, 6).

(f) Area of triangle ADC

$=\frac{1}{2}\left|\begin{matrix}2 & 7 & 8 & 2\\ 4 & 6 & 12 & 4\end{matrix}\right|$

$=\frac{1}{2}\left|12+84+32-(28+48+24)\right|$

$=\frac{1}{2}(28)$

$=14$ units^2

Hence, the area of the parallelogram $ABCD$

$=14\times 2$

$=28$ units^2

5 (a)
$$OT = 2OV$$
$$OT^2 = 4OV^2$$
$$(10-0)^2 + (p-0)^2 = 4\left[(-5-0)^2 + (-10-0)^2\right]$$
$$100 + p^2 = 4(25+100)$$
$$p^2 = 100 + 400 - 100$$
$$p^2 = 400$$
$$p = 20$$

(b) $m_{UT} = m_{VO} = \dfrac{0+10}{0+5} = 2$

The equation of UT is
$$y - 10 = 2(x-20)$$
$$y - 10 = 2x - 40$$
$$y = 2x - 30$$

(c) $m_{UV} = -\dfrac{1}{m_{VO}} = -\dfrac{1}{2}$

The equation of UV is
$$y + 10 = -\frac{1}{2}(x+5)$$
$$2y + 20 = -x - 5$$
$$2y = -x - 5 - 20$$
$$2y = -x - 25$$

(d) Equation of UT: $y = 2x - 30$ … (1)

Equation of UV: $2y = -x - 25$ … (2)

Substitute (1) into (2) :
$$2y = -x - 25$$
$$2(2x-30) = -x - 25$$
$$4x - 60 = -x - 25$$
$$5x = 35$$
$$x = 7$$

From (1) :
$$y = 2x - 30$$
$$y = 2(7) - 30$$
$$y = -16$$

Hence, U is point $(7, -16)$.

(e) $O(0, 0)$, $T(20, 10)$, $U(7, -16)$, $V(-5, -10)$

Area of $OTUV$
$$= \frac{1}{2}\left|\begin{matrix} 0 & 20 & 7 & -5 & 0 \\ 0 & 10 & -16 & -10 & 0 \end{matrix}\right|$$
$$= \frac{1}{2}\left| 20(-16) + 7(-10) - (70+80) \right|$$
$$= \frac{1}{2}\left| -320 - 70 - 150 \right|$$
$$= \frac{1}{2}\left| -540 \right|$$
$$= 270 \text{ units}^2$$

6 (a) The equation of SQ is
$$x + 2y - 4 = 0$$
$$2y = -x + 4$$
$$y = -\frac{1}{2}x + 2$$
$$m_{SQ} = -\frac{1}{2}$$
$$m_{PR} = 2$$

The equation of PR is
$$y - 5 = 2(x+1)$$
$$y = 2x + 2 + 5$$
$$y = 2x + 7$$

(b) At point Q, (x-axis), $y = 0$:
$$y = -\frac{1}{2}x + 2$$
$$0 = -\frac{1}{2}x + 2$$
$$0 = -x + 4$$
$$x = 4$$

Q is point (4, 0).

Equation of PTR: $y = 2x + 7$ … (1)

Equation of STQ: $y = -\dfrac{1}{2}x + 2$ … (2)

Substitute (1) into (2) :
$$2x + 7 = -\frac{1}{2}x + 2$$
$$4x + 14 = -x + 4$$
$$5x = -10$$
$$x = -2$$

From (1):
$$y = 2x + 7$$
$$y = 2(-2) + 7$$
$$y = 3$$

Hence, T is point $(-2, 3)$.

Let R is point (h, k).
$$\left(\frac{h-1}{2}, \frac{k+5}{2}\right) = (-2, 3)$$
$$\frac{h-1}{2} = -2 \qquad \frac{k+5}{2} = 3$$
$$h = -3 \qquad k = 1$$

Hence, R is point $(-3, 1)$.

Let S is point (a, b).
$$\left(\frac{a+4}{2}, \frac{b+0}{2}\right) = (-2, 3)$$
$$\frac{a+4}{2} = -2 \qquad \frac{b}{2} = 3$$
$$a = -8 \qquad b = 6$$

Hence, S is point $(-8, 6)$.

(c) $P(-1, 5)$, $Q(4, 0)$, $R(-3, 1)$, $S(-8, 6)$
Area of $PQRS$

$$= \frac{1}{2}\left| \begin{matrix} -1 & 4 & -3 & -8 & -1 \\ 5 & 0 & 1 & 6 & 5 \end{matrix} \right|$$

$$= \frac{1}{2}\left| 4 - 18 - 40 - (20 - 8 - 6) \right|$$

$$= \frac{1}{2}\left| -60 \right|$$

$= 30$ units^2

7 (a) Midpoint of PR

$$= \left(\frac{-1+5}{2}, \frac{5+1}{2} \right)$$

$$= (2, 3)$$

(b) $5x + y - 2 = 0$

$$y = -5x + 2$$

$$\therefore m_{QS} = -5$$

The equation of QS is

$$y - 3 = -5(x - 2)$$
$$y = -5x + 10 + 3$$
$$y = -5x + 13$$

(c) $m_{PR} = \dfrac{1-5}{5-(-1)} = -\dfrac{2}{3}$

$$m_{QR} = \frac{3}{2}$$

The equation of QR is

$$y - 1 = \frac{3}{2}(x - 5)$$
$$2y - 2 = 3x - 15$$
$$2y = 3x - 13$$

(d) (i) Equation of QS: $y = -5x + 13$ … (1)
Equation of QR: $2y = 3x - 13$ … (2)
Substitute (1) into (2) :

$$2y = 3x - 13$$
$$2(-5x + 13) = 3x - 13$$
$$-10x + 26 = 3x - 13$$
$$13x = 39$$
$$x = 3$$

From (1) :

$$y = -5x + 13$$
$$y = -5(3) + 13 = -2$$

Hence, Q is point $(3, -2)$.

Let S is point (h, k).

$$\left(\frac{h+3}{2}, \frac{k-2}{2} \right) = (2, 3)$$

$$\frac{h+3}{2} = 2 \qquad \frac{k-2}{2} = 3$$
$$h + 3 = 4 \qquad k - 2 = 6$$
$$h = 1 \qquad k = 8$$

Hence, S is point $(1, 8)$.

(ii) $P(-1, 5)$, $Q(3, -2)$, $R(5, 1)$, $S(1, 8)$
Area of the parallelogram $PQRS$

$$= \frac{1}{2}\left| \begin{matrix} -1 & 3 & 5 & 1 & -1 \\ 5 & -2 & 1 & 8 & 5 \end{matrix} \right|$$

$$= \frac{1}{2}\left| 2 + 3 + 40 + 5 - (15 - 10 + 1 - 8) \right|$$

$$= \frac{1}{2}\left| 52 \right|$$

$= 26$ units^2

8 (a) $m_{QP} = \dfrac{2-8}{4-2} = -3$

$$m_{QR} = \frac{1}{3}$$

The equation of QR is

$$y - 8 = \frac{1}{3}(x - 2)$$
$$3y - 24 = x - 2$$
$$3y = x + 22$$

(b) Equation of QR: $3y = x + 22$ … (1)
Equation of PR: $y = x - 2$ … (2)
Substitute (2) into (1) :

$$3y = x + 22$$
$$3(x - 2) = x + 22$$
$$3x - 6 = x + 22$$
$$2x = 28$$
$$x = 14$$

From (2) :

$$y = x - 2$$
$$y = 14 - 2 = 12$$

Hence, R is point $(14, 12)$.

(c) Let S is point (h, k).
Midpoint of QS = Midpoint of PR

$$\left(\frac{h+2}{2}, \frac{k+8}{2} \right) = \left(\frac{4+14}{2}, \frac{2+12}{2} \right)$$

Equating the x-coordinates,

$$h + 2 = 18$$
$$h = 16$$

Equating the y-coordinates,

$$k + 8 = 14$$
$$k = 6$$

Hence, S is point $(16, 6)$.

(d) $P(4, 2)$, $Q(2, 8)$, $R(14, 12)$, $S(16, 6)$

Area of $PQRS$

$$= \frac{1}{2}\begin{vmatrix} 4 & 2 & 14 & 16 & 4 \\ 2 & 8 & 12 & 6 & 2 \end{vmatrix}$$

$$= \frac{1}{2}\left| 32+24+84+32-(4+112+192+24) \right|$$

$$= \frac{1}{2}\left| 172-332 \right|$$

$$= \frac{1}{2}\left| -160 \right|$$

= 80 units^2

9 (a) Equation of PQ: $3y = x+12$ … (1)

Equation of RQ: $3y = 5x-12$ … (2)

Substitute (2) into (1):

$$5x-12 = x+12$$
$$4x = 24$$
$$x = 6$$

From (1): $3y = 6+12$

$$y = 6$$

Hence, Q is point (6, 6).

(b) $m_{OR} = m_{PQ} = \frac{1}{3}$

The equation of OR is

$y = \frac{1}{3}x$ … (1)

The equation of QR is $3y = 5x-12$ (2)

Substitute (1) into (2):

$$3\left(\frac{1}{3}x\right) = 5x-12$$
$$x = 5x-12$$
$$4x = 12$$
$$x = 3$$

From (1):

$$y = \frac{1}{3}(3) = 1$$

Hence, R is point (3, 1).

(c) $m_{PQ} = \frac{1}{3}$

Thus, the gradient of the perpendicular line is -3.

The equation of the straight line that passes through the point R and is perpendicular to PQ is

$$y-1 = -3(x-3)$$
$$y = -3x+10$$

10 (a) $(m_{BC})(m_{BA}) = -1$ ← The straight line CB is perpendicular to the straight line BA.

$$\left(\frac{k-8}{h-(-6)}\right)\left(\frac{8-4}{-6-0}\right) = -1$$

$$\left(\frac{k-8}{h+6}\right)\left(\frac{2}{-3}\right) = -1$$

$$2(k-8) = 3(h+6)$$
$$2k-16 = 3h+18$$
$$3h-2k = -34$$

(b) Area of triangle ABC

$$= \frac{1}{2}\begin{vmatrix} 0 & -6 & h & 0 \\ 4 & 8 & k & 4 \end{vmatrix}$$

$$= \frac{1}{2}\left[-6k+4h-(-24+8h)\right]$$

$$= \frac{1}{2}(-6k-4h+24)$$

$= -3k-2h+12$ [Shown]

(c) If the area of the rectangle $ABCD$ is 104 units^2, hence the area of $\Delta ABC = 52$ units^2.

$$-3k-2h+12 = 52$$
$$-2h-3k = 40 \text{ … (1)}$$

From (a):

$3h-2k = -34$ … (2)

(1) × 2: $-4h-6k = 80$ … (3)

(2) × 3: $9h-6k = -102$ … (4)

(4) – (3): $13h = -182$

$$h = -14$$

Substitute $h = -14$ into (1):

$$-2(-14)-3k = 40$$
$$-3k = 40-28$$
$$k = -4$$

Hence, the coordinates point C are (–14, –4).

(d) $m_{AD} = m_{BC}$

$$m_{AD} = \frac{-4-8}{-14-(-6)} = \frac{3}{2}$$

The equation of AD is

$$y = mx+c$$
$$y = \frac{3}{2}x+4$$
$$2y = 3x+8$$
$$3x-2y = -8$$

$$\frac{3x}{-8}-\frac{2y}{-8}=\frac{-8}{-8}$$

$$\frac{x}{\left(-\frac{8}{3}\right)}+\frac{y}{4}=1$$

11 (a) Let R is point (x, y).

$$AR = BR$$

$$AR^2 = BR^2$$

$$[x-(-1)]^2+(y-4)^2=(x-1)^2+[y-(-2)]^2$$

$$(x+1)^2+(y-4)^2=(x-1)^2+(y+2)^2$$

$$x^2+2x+1+y^2-8y+16=x^2-2x+1+y^2+4y+4$$

$$4x-12y+12=0$$

$$x-3y+3=0$$

Hence, the equation of PQ is $x-3y+3=0$.

(b) (i) $x-3y+3=0 \quad \dots (1)$

$x+2y-7=0 \quad \dots (2)$

$(1)-(2):\ -5y+10=0$

$y=2$

From $(1):\ x-3(2)+3=0$

$x=3$

Hence, the coordinates of the traffic light are (3, 2).

(ii) When $C\left(1, \frac{4}{3}\right)$ is substituted into $x-3y+3=0$, then

$$1-3\left(\frac{4}{3}\right)+3$$

$$=0$$

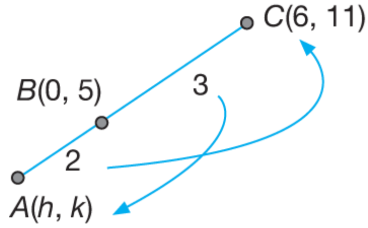
When $C\left(1, \frac{4}{3}\right)$ is substituted into $x+2y-7=0$, then

$$1+2\left(\frac{4}{3}\right)-7$$

$$=-\frac{10}{3}\ (\neq 0)$$

Hence, the road PQ passes through $C\left(1, \frac{4}{3}\right)$ and the road ST does not pass through the point $C\left(1, \frac{4}{3}\right)$.

12 (a) (i)



Let A be point (h, k).

$$\left(\frac{3h+2(6)}{2+3}, \frac{3k+2(11)}{2+3}\right)=(0, 5)$$

$$\left(\frac{3h+12}{5}, \frac{3k+22}{5}\right)=(0, 5)$$

Equating the x-coordinates,

$$\frac{3h+12}{5}=0$$

$$3h+12=0$$

$$3h=-12$$

$$h=-4$$

Equating the y-coordinates,

$$\frac{3k+22}{5}=5$$

$$3h+22=25$$

$$3h=3$$

$$h=1$$

Hence, the coordinates of point A are $(-4, 1)$.

(ii) The equation of the straight line AD is

$$\frac{y-1}{x-(-4)}=\frac{-7-1}{2-(-4)}$$

$$\frac{y-1}{x+4}=-\frac{4}{3}$$

$$3y-3=-4x-16$$

$$3y=-4x-13$$

(iii) Area of ΔACD

$$=\frac{1}{2}\left|\begin{matrix}-4 & 6 & 2 & -4\\ 1 & 11 & -7 & 1\end{matrix}\right|$$

$$=\frac{1}{2}\left|-44-42+2-(6+22+28)\right|$$

$$=\frac{1}{2}\left|-140\right|$$

$$=70 \text{ units}^2$$

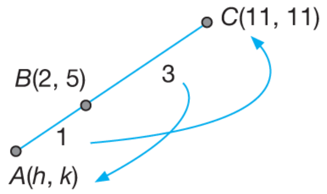
(b) Let P be point (x, y).

$$PC = 2PD$$
$$\sqrt{(x-6)^2+(y-11)^2} = 2\sqrt{(x-2)^2+[y-(-7)]^2}$$
$$(x-6)^2+(y-11)^2 = 2^2\left[(x-2)^2+[y-(-7)]^2\right]$$
$$x^2-12x+36+y^2-22y+121 = 4\left[x^2-4x+4+y^2+14y+49\right]$$
$$x^2-12x+y^2-22y+157 = 4x^2-16x+4y^2+56y+212$$
$$3x^2+3y^2-4x+78y+55=0$$

13 (a) (i) Area of $PQRS$

$$= \frac{1}{2}\begin{vmatrix} -5 & 14 & 15 & 0 & -5 \\ 4 & 19 & 2 & 1 & 4 \end{vmatrix}$$
$$= \frac{1}{2}\left|-95+28+15-(56+285-5)\right|$$
$$= \frac{1}{2}\left|-388\right|$$
$$= 194 \text{ units}^2$$

(ii)



Let A be point (h, k)

$CB : BA = 3 : 1$

$$\left(\frac{1(11)+3h}{3+1}, \frac{1(11)+3k}{3+1}\right) = (2, 5)$$
$$\left(\frac{11+3h}{4}, \frac{11+3k}{4}\right) = (2, 5)$$

Equating the x-coordinates,

$$\frac{11+3h}{4} = 2$$
$$11+3h = 8$$
$$3h = -3$$
$$h = -1$$

Equating the y-coordinates,

$$\frac{11+3k}{4} = 5$$
$$11+3k = 20$$
$$3k = 9$$
$$k = 3$$

Hence, A is point $(-1, 3)$.

(b) Let T be point (x, y).

$$TC = 3$$
$$\sqrt{(x-11)^2+(y-11)^2} = 3$$
$$(x-11)^2+(y-11)^2 = 9$$
$$x^2-22x+121+y^2-22y+121 = 9$$
$$x^2+y^2-22x-22y+233 = 0$$

14 (a) (i) The equation of PS is

$$2y = 5x-23$$
$$y = \frac{5}{2}x - \frac{23}{2}$$
$$m_{PS} = \frac{5}{2}$$
$$m_{PQ} = -\frac{1}{m_{PS}} = -\frac{1}{\left(\frac{5}{2}\right)} = -\frac{2}{5}$$

The equation of PQ is

$$y-(-2) = -\frac{2}{5}[x-(-2)]$$
$$5(y+2) = -2(x+2)$$
$$5y+10 = -2x-4$$
$$5y = -2x-14$$

(ii) Equation of PS: $2y = 5x-23 \;\ldots (1)$

Equation of PQ: $5y = -2x-14 \;\ldots (2)$

$$10y = 25x-115 \quad \ldots (1)\times 5$$
$$(-)\; 10y = -4x-28 \quad \ldots (2)\times 2$$
$$0 = 29x-87$$
$$x = 3$$

When $x = 3$,

$$2y = 5(3)-23$$
$$y = -4$$

Hence, P is point $(3, -4)$.

(b) The equation of PS is $2y = 5x-23$.

When $y = 1$, $2(1) = 5x-23$

$$5x = 25$$
$$x = 5$$

Hence, S is point $(5, 1)$.

Let T be point (x, y).

$$TS = 5$$
$$\sqrt{(x-5)^2+(y-1)^2} = 5$$
$$(x-5)^2+(y-1)^2 = 5^2$$
$$x^2-10x+25+y^2-2y+1 = 25$$
$$x^2-10x+y^2-2y+1 = 0$$

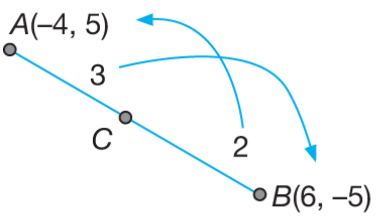
15 (a) Area of ΔAOB

$$= \frac{1}{2}\begin{vmatrix} -4 & 0 & 6 & -4 \\ 5 & 0 & -5 & 5 \end{vmatrix}$$

$$= \frac{1}{2}|30-20|$$

$= 5$ units^2

(b)



$$C = \left(\frac{2(-4)+3(6)}{3+2}, \frac{2(5)+3(-5)}{3+2}\right)$$

$$= (2, -1)$$

(c) Let Q be point (x, y).

$$QB = 2QA$$

$$\sqrt{(x-6)^2+[y-(-5)]^2} = 2\sqrt{[x-(-4)]^2+(y-5)^2}$$

$$(x-6)^2+(y+5)^2 = 2^2\left[(x+4)^2+(y-5)^2\right]$$

$$x^2-12x+36+y^2+10y+25 = 4\left(x^2+8x+16+y^2-10y+25\right)$$

$$x^2-12x+36+y^2+10y+25 = 4x^2+32x+64+4y^2-40y+100$$

$$3x^2+3y^2+44x-50y+103=0$$

16 (a) (i) $y = -\frac{4}{3}x$

$$m_{OA} = -\frac{4}{3}$$

$$m_{AB} = -\frac{1}{\left(-\frac{4}{3}\right)} = \frac{3}{4}$$

$$4y = kx+25$$

$$y = \frac{k}{4}x+\frac{25}{4}$$

$$m_{AB} = \frac{k}{4}$$

$$\therefore \frac{k}{4} = \frac{3}{4} \Rightarrow k = 3$$

(ii) $4y = 3x+25 \ \dots (1)$

$$y = -\frac{4}{3}x \ \dots (2)$$

Substitute (2) into (1) :

$$4\left(-\frac{4}{3}x\right) = 3x+25$$

$$-16x = 9x+75$$

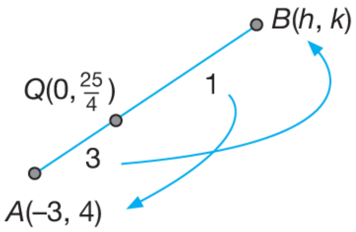
$$25x = -75$$

$$x = -3$$

From (2) : $y = -\frac{4}{3}(-3) = 4$

Hence, A is point $(-3, 4)$.

(b) (i)



Equating the x-coordinates,

$$\frac{1(-3)+3h}{3+1} = 0$$

$$h = 1$$

Equating the y-coordinates,

$$\frac{1(4)+3k}{3+1} = \frac{25}{4}$$

$$4+3k = 25$$

$$k = 7$$

Hence, B is point (1, 7)

(ii) $m_{CB} = m_{OA} = -\frac{4}{3}$

The equation of BC is

$$y-7 = -\frac{4}{3}(x-1)$$

$$3y-21 = -4x+4$$

$$3y = -4x+25$$

(c) $PA = 2PB$

$$PA^2 = 2^2 PB^2$$

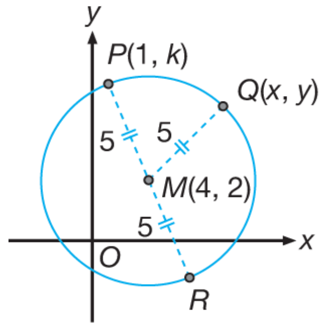
$$(x+3)^2+(y-4)^2 = 2^2\left[(x-1)^2+(y-7)^2\right]$$

$$x^2+6x+9+y^2-8y+16 = 4\left(x^2-2x+1+y^2-14y+49\right)$$

$$x^2+6x+y^2-8y+25 = 4x^2-8x+4y^2-56y+200$$

$$3x^2-14x+3y^2-48y+175=0$$

17



(a) Let Q be point (x, y).

$$QM = 5$$
$$\sqrt{(x-4)^2 + (y-2)^2} = 5$$
$$(x-4)^2 + (y-2)^2 = 5^2$$
$$x^2 - 8x + 16 + y^2 - 4y + 4 = 25$$
$$x^2 - 8x + y^2 - 4y - 5 = 0$$

(b) (i) Since $P(1, k)$ lies on the locus of Q,

$$1^2 - 8(1) + k^2 - 4k - 5 = 0$$
$$k^2 - 4k - 12 = 0$$
$$(k+2)(k-6) = 0$$

$k = -2$ or 6

$k = -2$ is not accepted.

$\therefore k = 6$

(ii) Since the points P and R lie on the locus of Q, the distances of the points from point M are equal. This means that M is the midpoint of PR.

Let R be point (a, b).

$$\left(\frac{1+a}{2}, \frac{6+b}{2}\right) = (4, 2)$$

By comparison,

$\frac{1+a}{2} = 4$ and $\frac{6+b}{2} = 2$

$a = 7$

$b = -2$

Hence, R is point $(7, -2)$.

(c) $O(0, 0)$, $P(1, 6)$, $R(7, -2)$

Area of ΔOPR

$$= \frac{1}{2}\left|\begin{matrix} 0 & 1 & 7 & 0 \\ 0 & 6 & -2 & 0 \end{matrix}\right|$$
$$= \frac{1}{2}\left|-2-42\right|$$
$$= \frac{1}{2}\left|-44\right|$$
$$= \left|-22\right|$$
$$= 22 \text{ units}^2$$

18 (a) (i) Area of ΔOAB

$$= \frac{1}{2}\left|\begin{matrix} 0 & -3 & 6 & 0 \\ 0 & -5 & 1 & 0 \end{matrix}\right|$$
$$= \frac{1}{2}\left|-3-(-30)\right|$$
$$= \frac{1}{2}\left|27\right|$$
$$= 13.5 \text{ units}^2$$

(ii) $AB = \sqrt{(6-(-3))^2 + (1-(-5))^2}$

$$= \sqrt{9^2 + 6^2}$$
$$= \sqrt{117}$$
$$= 10.82 \text{ units}$$

(b) (i) Let $P = (x, y)$

$$PA = 2PB$$
$$(PA)^2 = (2PB)^2$$
$$PA^2 = 4PB^2$$
$$(x-(-3))^2 + (y-(-5))^2 = 4\left((x-6)^2 + (y-1)^2\right)$$
$$(x+3)^2 + (y+5)^2 = 4\left((x-6)^2 + (y-1)^2\right)$$
$$x^2 + 6x + 9 + y^2 + 10y + 25 = 4\left(x^2 - 12x + 36 + y^2 - 2y + 1\right)$$

$$x^2 + 6x + 9 + y^2 + 10y + 25 = 4x^2 - 48x + 144 + 4y^2 - 8y + 4$$
$$0 = 3x^2 - 54x + 3y^2 - 18y + 114$$
$$x^2 - 18x + y^2 - 6y + 38 = 0$$

(ii) At the y-axis, $x = 0$.

$$\therefore 0^2 - 18(0) + y^2 - 6y + 38 = 0$$
$$y^2 - 6y + 38 = 0$$

$$b^2 - 4ac$$
$$= (-6)^2 - 4(1)(38)$$
$$= 36 - 152$$
$$= -116$$

Since $b^2 - 4ac < 0$, the quadratic equation does not have real roots. Hence, the locus of P will not intersect the y-axis.

19 (a) (i) Gradient of the straight line $2x - y - 5 = 0 \Rightarrow y = 2x - 5$ is 2.

$\therefore m_{BC} = 2$

$\therefore m_{AB} = -\frac{1}{m_{BC}} = -\frac{1}{2}$

Hence, the equation of the straight line AB is

$y - y_1 = m(x - x_1)$

$y - (-5) = -\frac{1}{2}[x - (-10)]$

At point A (–10, –5), $x_1 = -10$, $y_1 = -5$.

$2(y + 5) = -(x + 10)$

$2y + 10 = -x - 10$

$2y = -x - 20$

(ii) Equation of BC: $2x - y - 5 = 0$ ... (1)

Equation of AB: $x + 2y + 20 = 0$ ... (2)

$$\begin{array}{rl} 4x - 2y - 10 = 0 & \text{... (1)} \times 2 \\ (+) \quad x + 2y + 20 = 0 & \text{... (2)} \\ \hline 5x \qquad + 10 = 0 & \\ \therefore x = -2 & \end{array}$$

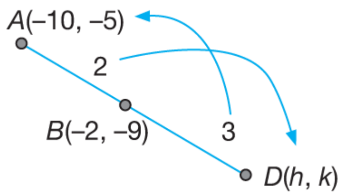
From (1):

$2(-2) - y - 5 = 0$

$\therefore y = -9$

Hence, B is point (–2, –9).

(b)



$B = (-2, -9)$

$\left(\frac{3(-10) + 2h}{2 + 3}, \frac{3(-5) + 2k}{2 + 3}\right) = (-2, -9)$

$\left(\frac{-30 + 2h}{5}, \frac{-15 + 2k}{5}\right) = (-2, -9)$

Equating the x-coordinates:

$\frac{-30 + 2h}{5} = -2$

$2h = 20$

$h = 10$

Equating the y-coordinates:

$\frac{-15 + 2k}{5} = -9$

$2k = -30$

$k = -15$

Hence, D is point $(10, -15)$.

Area of ΔADO

$= \frac{1}{2}\begin{vmatrix} -10 & 10 & 0 & -10 \\ -5 & -15 & 0 & -5 \end{vmatrix}$

$= \frac{1}{2}|150 - (-50)|$

$= \frac{1}{2}|200|$

$= 100$ units^2

(c) Let P be point (x, y).

Since $\angle APB = 90^\circ$, AP is perpendicular to PB.

Hence, $(m_{AP})(m_{PB}) = -1$

$\left(\frac{y - (-5)}{x - (-10)}\right)\left(\frac{y - (-9)}{x - (-2)}\right) = -1$

$\frac{(y + 5)(y + 9)}{(x + 10)(x + 2)} = -1$

$(y + 5)(y + 9) = -(x + 10)(x + 2)$

$y^2 + 14y + 45 = -(x^2 + 12x + 20)$

$x^2 + y^2 + 12x + 14y + 65 = 0$

20 (a) (i) Radius of circle,

$MA = \sqrt{(1 + 3)^2 + (3 - 0)^2}$

$MA = \sqrt{25}$

$MA = 5$

$MR = 5$

$\sqrt{(x - 1)^2 + (y - 3)^2} = 5$

$x^2 - 2x + 1 + y^2 - 6y + 9 = 5^2$

$x^2 + y^2 - 2x - 6y - 15 = 0$

(ii) $4^2 + k^2 - 2(4) - 6k - 15 = 0$

$16 + k^2 - 8 - 6k - 15 = 0$

$k^2 - 6k - 7 = 0$

$(k + 1)(k - 7) = 0$

$k = -1$ or $k = 7$

$k = -1$ is not accepted because the question states that $k > 0$.

$\therefore k = 7$

(b) Gradient $MA = \dfrac{3-0}{1+3} = \dfrac{3}{4}$

Hence, gradient $MC = -\dfrac{4}{3}$

The equation of MC is

$$y - 0 = -\frac{4}{3}(x+3)$$
$$3y = -4x - 12$$

At C, $x = 0$

$$3y = -4(0) - 12$$
$$y = -4$$

Hence, C is point $(0, -4)$.

Area of OAC

$$= \frac{1}{2}\begin{vmatrix} 0 & -3 & 0 & 0 \\ 0 & 0 & -4 & 0 \end{vmatrix}$$
$$= \frac{1}{2}|12|$$
$$= 6 \text{ units}^2$$

SPM Spot

1 (a) Gradient of the line $AMC = \dfrac{3-9}{-5-(-9)}$

$$= -\frac{3}{2}$$

Gradient of the perpendicular bisector

$$= -\left(-\frac{2}{3}\right) = \frac{2}{3}$$

Equation of the perpendicular bisector is

$$y - 3 = \frac{2}{3}[x - (-5)]$$
$$3y - 9 = 2x + 10$$
$$3y = 2x + 19$$

(b) For point $B(-8, k)$,

$$3k = 2(-8) + 19$$
$$3k = 3$$
$$k = 1$$

Thus, B is point $(-8, 1)$.

Let C be point (p, q).
Equating the x-coordinates,

$$\frac{-9+p}{2} = -5$$
$$-9 + p = -10$$
$$p = -1$$

Equating the y-coordinates,

$$\frac{9+q}{2} = 3$$
$$9 + q = 6$$
$$q = -3$$

Thus, C is point $(-1, -3)$.
Area of $ABCD$

$$= \frac{1}{2}\begin{vmatrix} -9 & -8 & -1 & 1 & -9 \\ 9 & 1 & -3 & 9 & 9 \end{vmatrix}$$
$$= \frac{1}{2}\left|-9 + 24 - 9 + 9 - (-72 - 1 - 3 - 81)\right|$$
$$= \frac{1}{2}(172)$$
$$= 86 \text{ units}^2$$

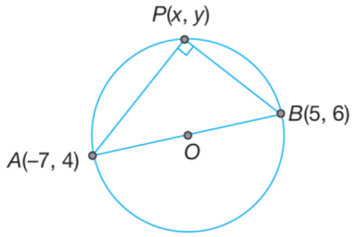
(c) $AD = 9 + 1 = 10$ units

$$\sqrt{[x-(-9)]^2 + (y-9)^2} = 10$$
$$x^2 + 18x + 81 + y^2 - 18y + 81 = 100$$

Hence, the equation of the locus of P is

$$x^2 + y^2 + 18x - 18y + 62 = 0$$

2 (a) (i)

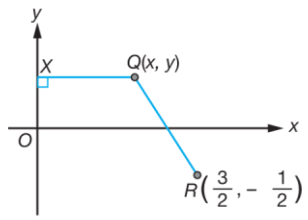


$$m_{AP} = \frac{y-4}{x+7}$$
$$m_{BP} = \frac{y-6}{x-5}$$

The angle at the circumference of a semicircle is always 90°.

$$\left(\frac{y-4}{x+7}\right)\left(\frac{y-6}{x-5}\right) = -1$$
$$(y-4)(y-6) = -(x+7)(x-5)$$
$$y^2 - 10y + 24 = -\left(x^2 + 2x - 35\right)$$
$$y^2 - 10y + x^2 + 2x - 11 = 0$$

(ii)



$$\sqrt{\left(x-\frac{3}{2}\right)^2+\left(y+\frac{1}{2}\right)^2}=2.5x$$

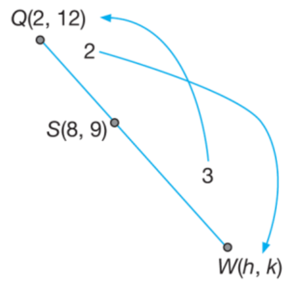
$$\left(x-\frac{3}{2}\right)^2+\left(y+\frac{1}{2}\right)^2=6.25x^2$$

$$x^2-3x+\frac{9}{4}+y^2+y+\frac{1}{4}=6.25x^2$$

$$4x^2-12x+9+4y^2+4y+1=25x^2$$

$$21x^2+12x-4y^2-4y-10=0$$

(b)



(i) The coordinates of point S are

$$\left(\frac{3(2)+2h}{2+3}, \frac{3(12)+2k}{2+3}\right), \text{ i.e.}$$

$$\left(\frac{6+2h}{5}, \frac{36+2k}{5}\right).$$

But it is given that the coordinates of point S are (8, 9).

By comparison,

$$\frac{6+2h}{5}=8$$

$$6+2h=40$$

$$2h=34$$

$$h=17$$

$$\frac{36+2k}{5}=9$$

$$36+2k=45$$

$$2k=9$$

$$k=4.5$$

Hence, the coordinates of the point W are (17, 4.5).

(ii) Area of the quadrilateral $PSRQ$ = 45 units^2

$$\frac{1}{2}\begin{vmatrix} 0 & 8 & 8 & 2 & 0 \\ y & 9 & 16 & 12 & y \end{vmatrix}=45$$

$$128+96+2y-(8y+72+32)=90$$

$$120-6y=90$$

$$6y=30$$

$$y=5$$

Hence, the y-intercept of the straight line PQ is 5.