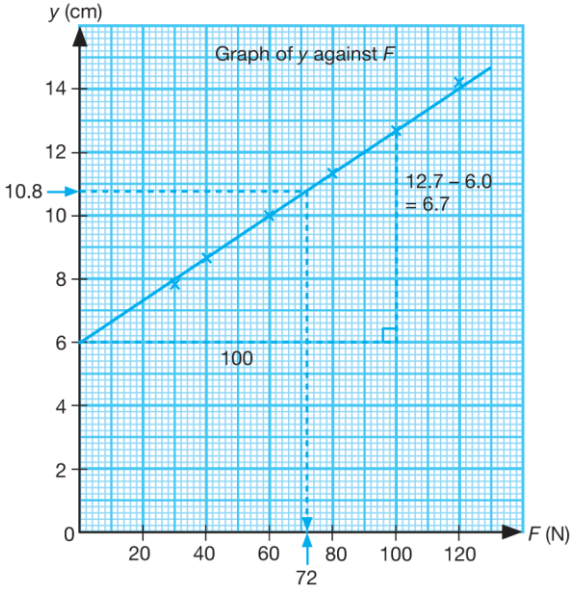
Form 4 Chapter 6
Linear Law
Fully-Worked Solutions

UPSKILL 6.1

1 (a)



(b) (i) $y = hF + k$

h = Gradient

$h = \dfrac{6.7}{100}$

$h = 0.067$

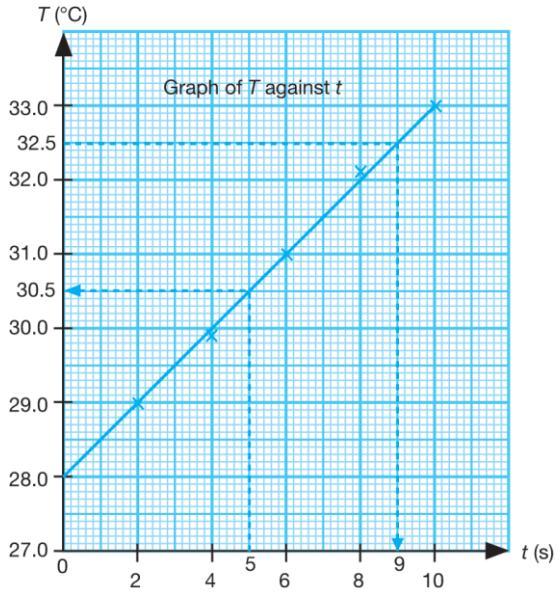
k = y-intercept

$k = 6$

(ii) When $F = 0$, $y = 6$ cm

(iii) When $y = 10.8$, $F = 72$ N

2 (a)



(b) (i) When $t = 0$, $T = 28^{\circ}$C

(ii) When $t = 5$, $T = 30.5^{\circ}$C

(iii) When $T = 32.5$, $t = 9$ s

UPSKILL 6.2

1 (a) $y = hx^3 + kx^2$

$$\frac{y}{x^2} = hx + k$$

$$Y = \frac{y}{x^2},\ X = x$$

Gradient = h
Y-intercept = k

(b) $y = hx + \frac{k}{x}$

$$xy = hx^2 + k$$

$$Y = xy,\ X = x^2$$

Gradient = h
Y-intercept = k

(c) $y = hx^k$

$$\log_{10} y = \log_{10} h + k \log_{10} x$$

$$\log_{10} y = k \log_{10} x + \log_{10} h$$

$$Y = \log_{10} y,\ X = \log_{10} x$$

Gradient = k
Y-intercept = $\log_{10} h$

(d) $\frac{h}{y} = \frac{k}{x} + 1$

$$\frac{1}{y} = \frac{k}{h}\left(\frac{1}{x}\right) + \frac{1}{h}$$

$$Y = \frac{1}{y},\ X = \frac{1}{x}$$

Gradient = $\frac{k}{h}$

Y-intercept = $\frac{1}{h}$

2 (a) $16p^2x = (y - q)^2$

$$4p\sqrt{x} = y - q$$

$$y = 4p\sqrt{x} + q$$

Graph of y against $\sqrt{x}$ is a straight line.
Gradient = $4p$
Y-intercept = q

(b) $yx^q = p$

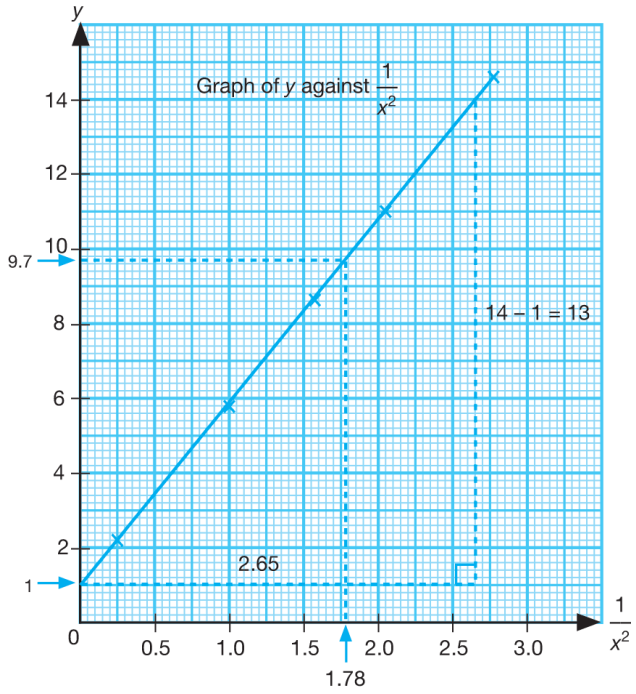
$$\log_{10} y + q \log_{10} x = \log_{10} p$$

$$\log_{10} y = -q \log_{10} x + \log_{10} p$$

Graph of $\log_{10} y$ against $\log_{10} x$ is a straight line.
Gradient = $-q$
Y-intercept = $\log_{10} p$

3 (a)

x	0.6	0.7	0.8	1.0	2.0
y	14.6	10.8	8.6	5.8	2.2
$\frac{1}{x^2}$	2.78	2.04	1.56	1.00	0.25



(b) (i) $y = \frac{h}{x^2} + k$

h = Gradient = $\frac{13}{2.65} = 4.9$

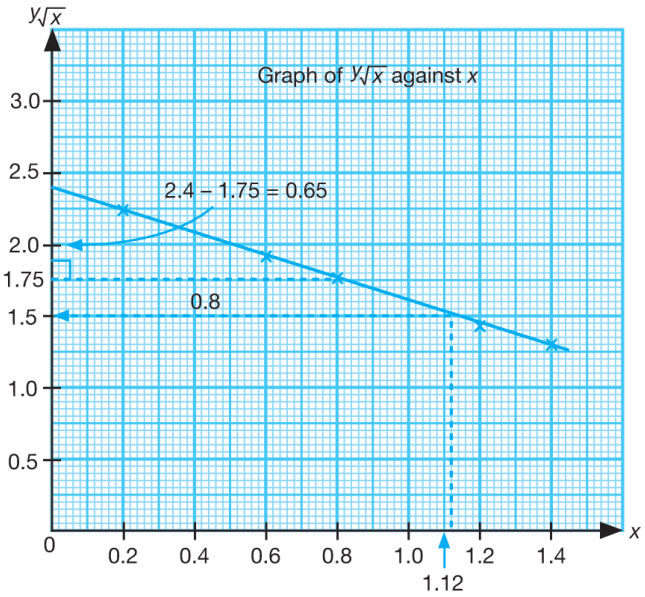
k = Y-intercept = 1

(ii) When $x = 0.75$, $\frac{1}{x^2} = 1.78$

From the graph, $y = 9.7$

4 (a)

x	0.2	0.6	0.8	1.2	1.4
y	5.03	2.45	1.96	1.32	1.10
$y\sqrt{x}$	2.25	1.90	1.75	1.45	1.30



(b) (i) $y = \dfrac{px}{\sqrt{x}} + \dfrac{q}{\sqrt{x}}$

$y\sqrt{x} = px + q$

q = Y-intercept
= 2.4

p = Gradient
$= -\dfrac{0.65}{0.8}$
= –0.8

(ii) When x = 1.12,

$y\sqrt{x} = 1.5$

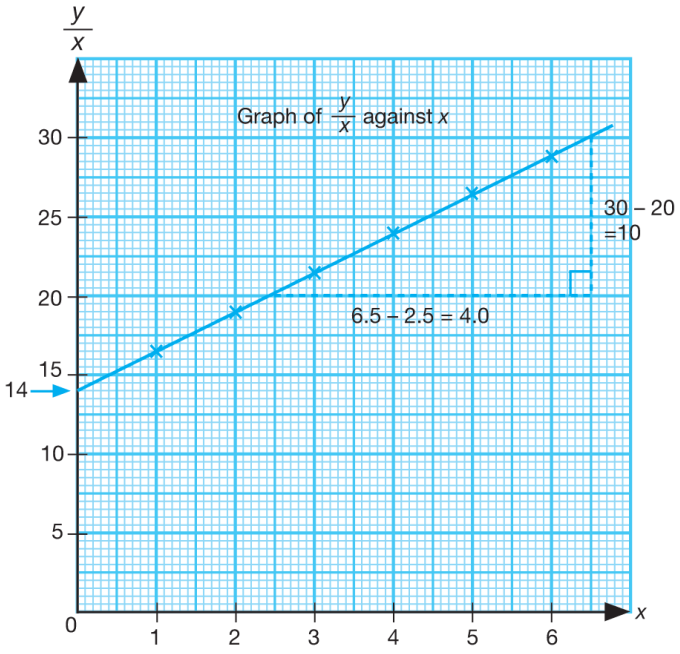
$y\sqrt{1.12} = 1.5$

$y = \dfrac{1.5}{\sqrt{1.12}}$

$y = 1.42$

5 (a)

x	1	2	3	4	5	6
y	16.3	37.5	63.7	95.0	131.3	172.5
$\frac{y}{x}$	16.3	18.75	21.23	23.75	26.26	28.75



(b) $y = px(x + q)$

$y = px^2 + pqx$

$\frac{y}{x} = px + pq$

p = Gradient

$p = \frac{10}{4} = 2.5$

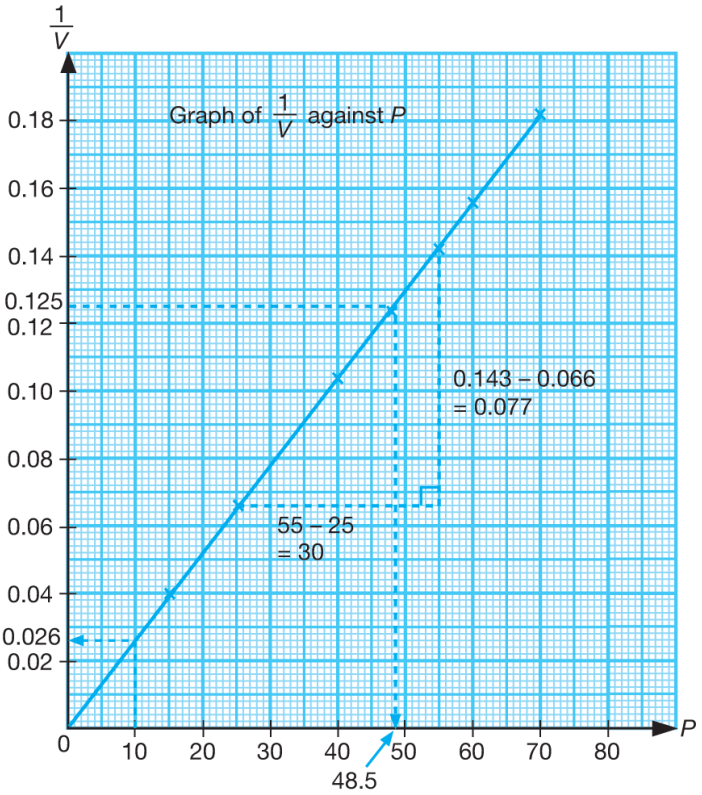
pq = Y-intercept

$2.5q = 14$

$q = 5.6$

6 (a)

P	15	25	40	48	55	60	70
V	25.0	15.1	9.6	8.1	7.0	6.4	5.5
$\frac{1}{V}$	0.040	0.066	0.104	0.123	0.143	0.156	0.182



(b) (i) $PV = d$

$$\frac{1}{PV} = \frac{1}{d}$$

$$\frac{1}{V} = \frac{1}{d}P$$

$$\frac{1}{d} = \text{ Gradient}$$

$$\frac{1}{d} = \frac{0.077}{30}$$

$$d = \frac{30}{0.077} = 390$$

(ii) When $P = 10$, $\frac{1}{V} = 0.026$

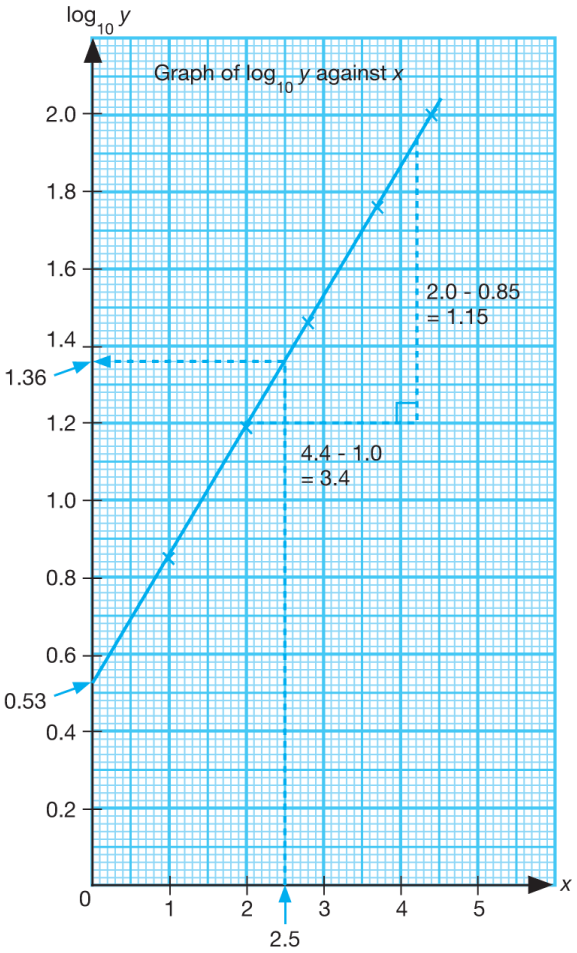
$V = 38.5 \text{ cm}^3$

(iii) When $V = 8$, $\frac{1}{V} = 0.125$

From the graph, $P = 48.5$ cm Hg

7 (a)

x	1.0	2.0	2.8	3.7	4.4
y	7.2	15.8	28.8	57.5	100
$\frac{y}{x}$	0.86	1.20	1.46	1.76	2.00



(b) (i) $y = hk^x$

$\log_{10} y = x \log_{10} k + \log_{10} h$

Y-intercept = 0.53

$\log_{10} h = 0.53$

$h = 3.4$

$\log_{10} k = \frac{1.15}{3.4} = 0.3382$

$k = 2.2$

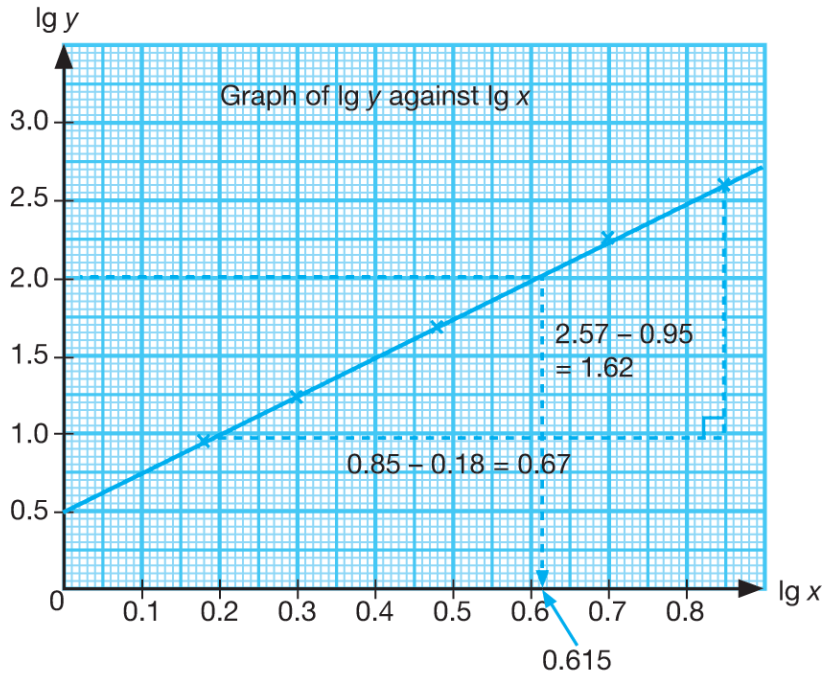
(ii) When $x = 2.5$, from the graph

$\log_{10} y = 1.36$

$y = 22.9$

8 (a)

x	1.5	2.0	3.0	5.0	7.0
y	9	17	47	170	370
$\log_{10} x$	0.18	0.30	0.48	0.70	0.85
$\log_{10} y$	0.95	1.23	1.67	2.23	2.57



(b) (i) $y = px^n$

$\log_{10} y = n \log_{10} x + \log_{10} p$

$\log_{10} p = Y$-intercept

$\log_{10} p = 0.5$

$p = 3.16$

n = Gradient

$n = \dfrac{1.62}{0.67} = 2.42$

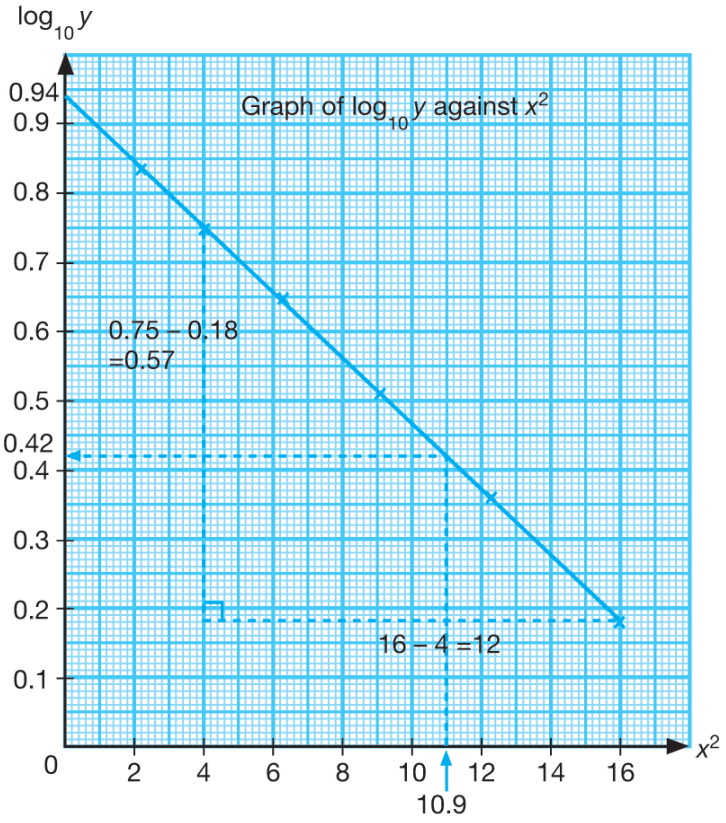
(ii) When $y = 100$, $\log_{10} y = 2$

From the graph, $\log_{10} x = 0.615$

$x = 4.12$

9 (a)

x	1.5	2.0	2.5	3.0	3.5	4.0
y	6.8	5.6	4.4	3.2	2.3	1.5
x^2	2.25	4.00	5.06	9.00	12.25	16.00
$\log_{10} y$	0.83	0.75	0.64	0.51	0.36	0.18



(b) (i) $y = \dfrac{t}{k^{x^2}}$

$\log_{10} y = \log_{10} t - x^2 \log_{10} k$

$\log_{10} y = -x^2 \log_{10} k + \log_{10} t$

$-\log_{10} k =$ Gradient

$-\log_{10} k = -\dfrac{0.57}{12}$

$\log_{10} k = \dfrac{0.57}{12} = 0.0475$

$k = 1.12$

$\log_{10} t =$ Y-intercept

$\log_{10} t = 0.94$

$t = 8.71$

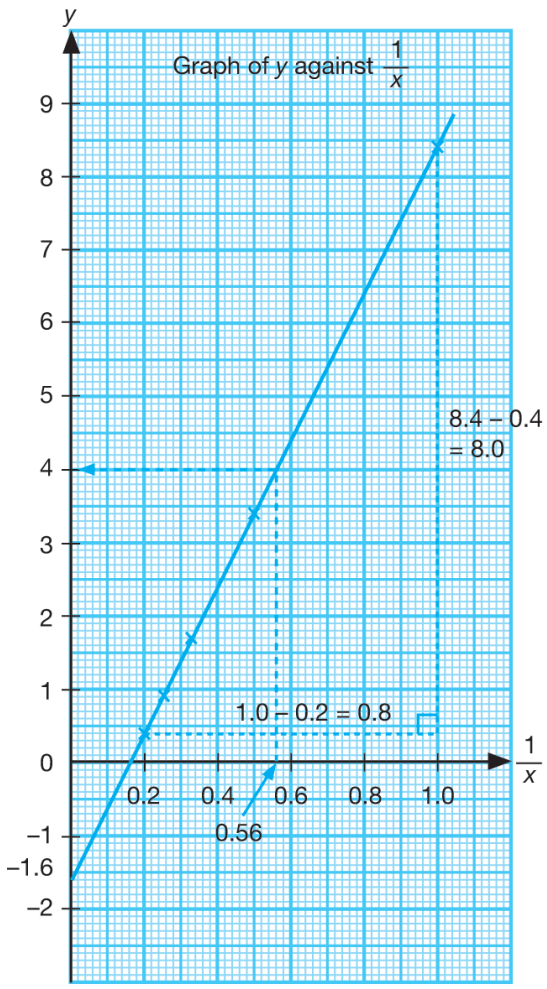
(ii) When $x = 3.3$, $x^2 = 10.89$

From the graph, $\log_{10} y = 0.42$

$y = 2.63$

10 (a)

x	1	2	3	4	5
y	8.4	3.4	1.7	0.9	0.4
$\frac{1}{x}$	1.00	0.50	0.33	0.25	0.20



(b) (i) $hy = t + \frac{2}{x}$

$y = \frac{t}{h} + \frac{2}{h}\left(\frac{1}{x}\right)$

$y = \frac{2}{h}\left(\frac{1}{x}\right) + \frac{t}{h}$

$\frac{2}{h}$ = Gradient

$\frac{2}{h} = \frac{8}{0.8}$

$h = 2 \times \frac{0.8}{8}$

$h = 0.2$

$\frac{t}{h}$ = Y-intercept

$\frac{t}{h} = -1.6$

$\frac{t}{0.2} = -1.6$

$t = -0.32$

(ii) When $x = 1.8$, $\frac{1}{x} = 0.56$

From the graph, $y = 4$

11 $\lg y = m\lg x + c$

$\lg y = m\lg x + 3$

$m = \dfrac{11-3}{4} = 2$

$\lg y = 2\lg x + 3$

$\lg y - 2\lg x = 3$

$\lg y - \lg x^2 = 3$

$\log_{10}\left(\dfrac{y}{x^2}\right) = 3$

$\dfrac{y}{x^2} = 10^3$

$y = 1\,000x^2$

12 $x - y = m(xy) + c$

For the point (2, 2), $xy = 2$ and $x - y = 2$,

$2 = m(2) + c$ … (1)

For the point (8, 5), $xy = 8$ and $x - y = 5$,

$5 = m(8) + c$ … (2)

(2) – (1) : $3 = 6m$

$m = \dfrac{1}{2}$

From (1) : $2 = \dfrac{1}{2}(2) + c$

$c = 1$

$x - y = \dfrac{1}{2}(xy) + 1$

$2x - 2y = xy + 2$

$xy + 2y = 2x - 2$

$(x+2)y = 2x - 2$

$y = \dfrac{2x-2}{x+2}$

13 $\dfrac{1}{y} = m\left(\dfrac{1}{x}\right) + c$

For the point (2, 7), $\dfrac{1}{x} = 2$ and $\dfrac{1}{y} = 7$

$7 = 2m + c$ … (1)

For the point (5, 1), $\dfrac{1}{x} = 5$ and $\dfrac{1}{y} = 1$

$1 = 5m + c$ … (2)

(2) – (1) : $-6 = 3m \Rightarrow m = -2$

$7 = 2(-2) + c$ … (1)

$c = 11$

$\dfrac{1}{y} = -2\left(\dfrac{1}{x}\right) + 11$

$\dfrac{1}{y} = \left(\dfrac{-2+11x}{x}\right)$

$y = \dfrac{x}{11x-2}$

14 $y = ab^{\sqrt{x}}$

$\log_{10} y = \log_{10} ab^{\sqrt{x}}$

$\log_{10} y = \log_{10} a + \log_{10} b^{\sqrt{x}}$

$\log_{10} y = \log_{10} a + \sqrt{x}\log_{10} b$

$\log_{10} y = \sqrt{x}\log_{10} b + \log_{10} a$

For the point (1, 5), $\sqrt{x} = 1$ and $\log_{10} y = 5$,

$5 = \log_{10} b + \log_{10} a$ … (1)

For the point (3, 1), $\sqrt{x} = 3$ and $\log_{10} y = 1$,

$1 = 3\log_{10} b + \log_{10} a$ … (2)

(1) – (2) :

$4 = -2\log_{10} b$

$\log_{10} b = -2$

$b = \dfrac{1}{100}$

From (2) :

$1 = 3\log_{10} b + \log_{10} a$

$1 = 3(-2) + \log_{10} a$

$\log_{10} a = 7$

$a = 10^7 = 10\,000\,000$

15 $y = \dfrac{a}{b^x}$

$\log_{10} y = \log_{10} a - x\log_{10} b$

$\log_{10} y = -x\log_{10} b + \log_{10} a$

For the point (–1, 8), $x = -1$ and $\log_{10} y = 8$,

$8 = -(-1)\log_{10} b + \log_{10} a$

$8 = \log_{10} b + \log_{10} a$ … (1)

For the point (3, 0), $x = 3$ and $\log_{10} y = 0$,

$0 = -3\log_{10} b + \log_{10} a$ … (2)

(1) – (2) : $8 = 4\log_{10} b$

$2 = \log_{10} b$

$b = 10^2 = 100$

From (1) :

$8 = 2 + \log_{10} a$

$\log_{10} a = 6$

$a = 10^4 = 1\ 000\ 000$

16 $\frac{1}{2}y = x + \frac{q}{x}$

$\frac{1}{2}xy = x^2 + q$

$xy = 2x^2 + 2q$

For the point (7, 2), $x^2 = 7$ and $xy = 7$,

$2 = 2(7) + 2q$

$2q = -12$

$q = -6$

For the point (0, h), $x^2 = 0$ and $xy = h$,

$xy = 2x^2 + 2q$

$h = 2(0) + 2(-6)$

$h = -12$

17 $y = \frac{10^4}{x^2}$

$\log_{10} y = \log_{10} 10^4 - \log_{10} x^2$

$\log_{10} y = 4 - 2\log_{10} x$

For the point (-1, p), $\log_{10} x = -1$ and

$\log_{10} y = p$,

$p = 4 - 2(-1) = 6$

For the point (r, -2), $\log_{10} x = r$ and

$\log_{10} y = -2$,

$-2 = 4 - 2r$

$r = 3$

18 $yx^n = d$

$\log_{10} y + n\log_{10} x = \log_{10} d$

$\log_{10} y = -n\log_{10} x + \log_{10} d$

Gradient = $-\frac{2}{5}$

$-n = -\frac{2}{5}$

$n = \frac{2}{5}$

For the point (5, 0), $\log_{10} x = 5$ and

$\log_{10} y = 0$

$\log_{10} y = -n\log_{10} x + \log_{10} d$

$0 = -\frac{2}{5}(5) + \log_{10} d$

$\log_{10} d = 2$

$d = 100$

19 $y = hk^{x-2}$

$\log_{10} y = \log_{10}\left(hk^{x-2}\right)$

$\log_{10} y = \log_{10} h + (x-2)\log_{10} k$

$\log_{10} y = (x-2)\log_{10} k + \log_{10} h$

Gradient = 2

$\log_{10} k = 2$

$k = 10^2 = 100$

For the point (3, 0), $(x-2) = 3$ and

$\log_{10} y = 0$

$\log_{10} y = (x-2)\log_{10} k + \log_{10} h$

$0 = 3\,(2) + \log_{10} h$

$\log_{10} h = -6$

$h = 10^{-6} = \frac{1}{1\ 000\ 000}$

20 $y = \frac{h}{x+k}$

$xy + ky = h$

$ky = -xy + h$

$y = -\frac{1}{k}xy + \frac{h}{k}$

Gradient = $-\frac{1}{2}$

$-\frac{1}{k} = -\frac{1}{2}$

$k = 2$

For the point (4, 4), $xy = 4$ and $y = 4$,

$y = -\frac{1}{k}xy + \frac{h}{k}$

$4 = -\frac{1}{2}(4) + \frac{h}{2}$

$8 = -4 + h$

$h = 12$

UPSKILL 6.3

1 (a) $10^M = a(T+1)^b$

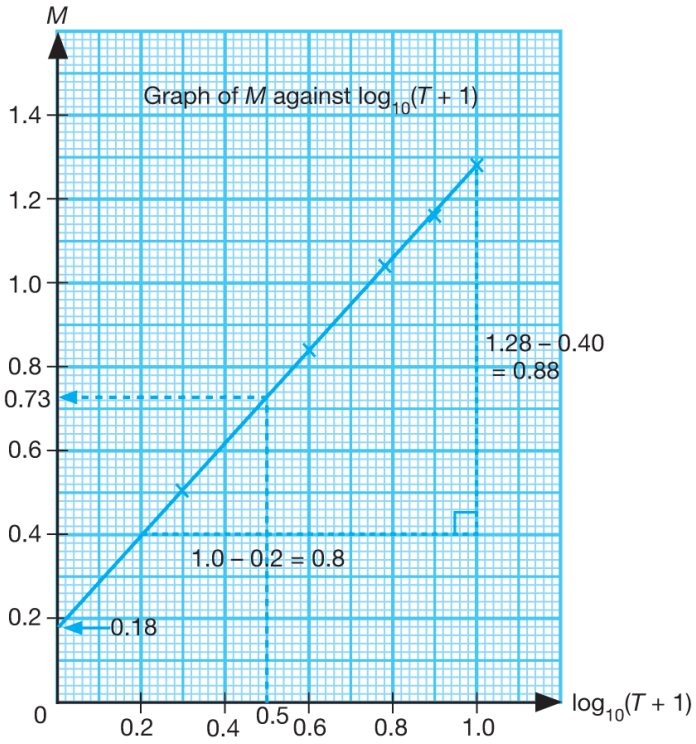
$M = \log_{10} a\,(T+1)^b$

$M = \log_{10} a + b\log_{10}(T+1)$

$M = b\log_{10}(T+1) + \log_{10} a$

(b)

T	1	3	5	7	9
M	0.51	0.84	1.04	1.18	1.28
$\log_{10}(T+1)$	0.300	0.600	0.780	0.90	1.00



(c) (i) $M = b\log_{10}(T+1) + \log_{10} a$

b = Gradient

$b = \dfrac{0.88}{0.8} = 1.1$

$\log_{10} a$ = Y-intercept

$\log_{10} a = 0.18$

$a = 1.51$

(ii) When $T = 2.16$, $\log_{10}(T+1) = 0.5$

From the graph, $M = 0.73$ mg

Summative Practice 6

1 $x^2y = a + bx$

$$xy = \frac{1}{x}a + b$$

For the point (3, 1), $\frac{1}{x} = 3$ and $xy = 1$,

$1 = 3a + b$ … (1)

For the point (7, 9), $\frac{1}{x} = 7$ and $xy = 9$,

$9 = 7a + b$ … (2)

(2) – (1) : $4a = 8$

$a = 2$

From (1) :

$1 = 3(2) + b$

$b = -5$

2 $y = \frac{x}{p + nx}$

$$\frac{1}{y} = \frac{p + nx}{x}$$

$$\frac{1}{y} = p\left(\frac{1}{x}\right) + n$$

$p =$ Gradient

$= \frac{0.2 - 0}{0 - 0.4}$

$= -0.5$

n = Y-intercept

$= 0.2$

3 $ay = x + \frac{b}{x}$

$$axy = x^2 + b$$

$$xy = \left(\frac{1}{a}\right)x^2 + \frac{b}{a}$$

For the point (3, 0), $x^2 = 3$ and $xy = 0$,

$0 = \frac{3}{a} + \frac{b}{a}$

$3 + b = 0$

$b = -3$

For the point (7, 2), $x^2 = 7$ and $xy = 2$,

$2 = \left(\frac{1}{a}\right)(7) + \frac{b}{a}$

$2a = 7 + b$

$2a = 7 - 3$

$a = 2$

4 $y = px^{n-1}$

$\log_{10} y = (n-1)\log_{10} x + \log_{10} p$,

$n - 1 =$ Gradient

$n - 1 = -2$

$n = -1$

For the point (2, 4), $\log_{10} x = 2$ and

$\log_{10} y = 4$,

$\log_{10} y = -2\log_{10} x + \log_{10} p$

$4 = -2(2) + \log_{10} p$

$\log_{10} p = 8$

$p = 10^8 = 100\,000\,000$

5 $(x + y) = mx^2 + c$

For the point (4, 1), $x^2 = 4$ and $(x + y) = 1$,

$1 = 4m + c$ … (1)

For the point (7, 2), $x^2 = 7$ and $(x + y) = 2$,

$2 = 7m + c$ … (2)

(2) – (1) : $3m = 1$

$m = \frac{1}{3}$

From (1) : $1 = \frac{4}{3} + c$

$c = -\frac{1}{3}$

Hence, $(x + y) = \frac{1}{3}x^2 - \frac{1}{3}$

$y = \frac{1}{3}x^2 - x - \frac{1}{3}$

6 $m = \frac{9}{3} = 3$

$\log_3 y = 3x$

$y = 3^{3x}$

$y = 27^x$

7 $\log_2 y = m\log_2 x + c$

For the point (–3, 4) : $\log_2 x = -3$ and

$\log_2 y = 4$,

$4 = -3m + c$ … (1)

For the point (–2, 1) : $\log_2 x = -2$ and $\log_2 y = 1$,

$1 = -2m + c$ … (2)

(1) – (2) : $-m = 3$

$m = -3$

From (1) : $4 = -3(-3) + c$

$c = -5$

Hence, $\log_2 y = -3\log_2 x - 5$

$\log_2 y + 3\log_2 x = -5$

$\log_2 y + \log_2 x^3 = -5$

$\log_2 \left(\frac{y}{x^3} \right) = -5$

$\frac{y}{x^3} = 2^{-5}$

$\frac{y}{x^3} = \frac{1}{32}$

$y = \frac{x^3}{32}$

8 $y = \frac{2}{x} + qx$

$\frac{y}{x} = 2\left(\frac{1}{x^2} \right) + q$

For the point (2, 7), $\frac{1}{x^2} = 2$ and $\frac{y}{x} = 7$,

$7 = 2(2) + q$

$q = 3$

For the point (4, p), $\frac{1}{x^2} = 4$ and $\frac{y}{x} = p$,

$p = 2(4) + q$

$p = 2(4) + 3 = 11$

9 $y = px + qx^{\frac{3}{2}}$

$\frac{y}{x} = p + q\left[\frac{x^{\left(\frac{3}{2}\right)}}{x} \right]$

$\frac{y}{x} = p + qx^{\frac{1}{2}}$

$\frac{y}{x} = p + q\sqrt{x}$

$\frac{y}{x} = q\sqrt{x} + p$

For the point (2, 10), $\sqrt{x} = 2$ and $\frac{y}{x} = 10$.

$10 = 2q + p$ … (1)

For the point (5, 4), $\sqrt{x} = 5$ and $\frac{y}{x} = 4$.

$4 = 5q + p$ … (2)

(1) – (2): $-3q = 6$

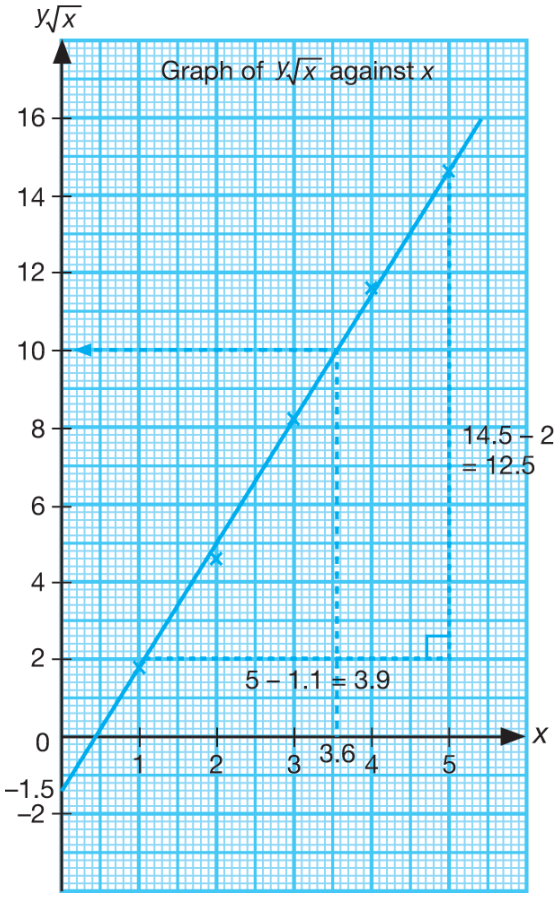
$q = -2$

From (1): $10 = 2(-2) + p$

$p = 14$

10 (a)

x	1	2	3	4	5
y	1.7	3.5	4.7	5.6	6.5
$y\sqrt{x}$	1.7	4.9	8.1	11.2	14.5



(b) (i) $y = a\sqrt{x} + \dfrac{d}{\sqrt{x}}$

$y\sqrt{x} = ax + d$

$a =$ Gradient

$= \dfrac{9.5}{3}$

$= 3.2$

d = Y-intercept

$= -1.5$

(ii) When $x = 3.6$, from the graph,

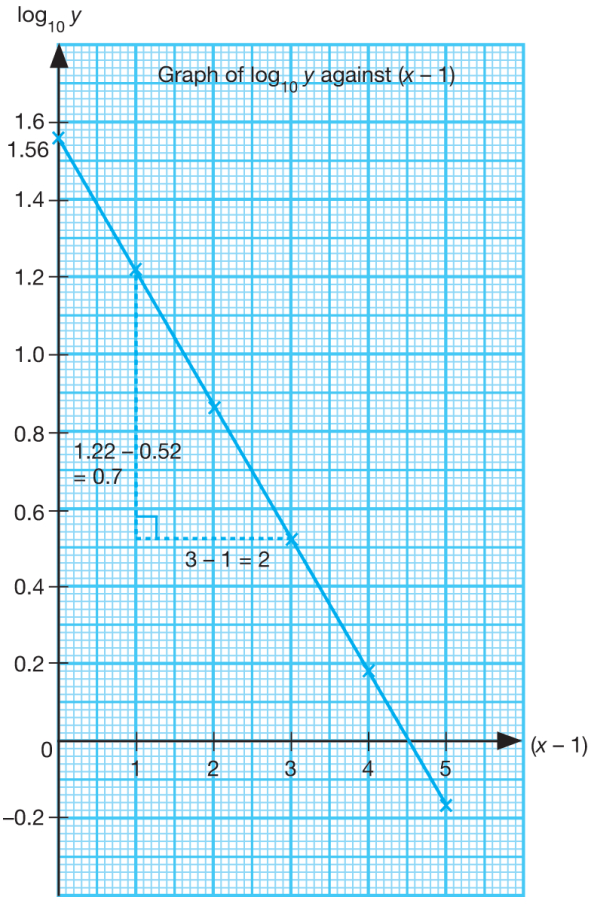
$y\sqrt{x} = 10$

$y\sqrt{3.6} = 10$

$y = \dfrac{10}{\sqrt{3.6}} = 5.27$

11 (a)

x	1	2	3	4	5	6
y	36.0	16.7	7.29	3.28	1.51	0.67
$x-1$	0	1	2	3	4	5
$\log_{10} y$	1.56	1.22	0.86	0.52	0.18	–0.17



(b) (i) $y = db^{x-1}$

$\log_{10} y = \log_{10} d + (x-1)\log_{10} b$

$\log_{10} y = (x-1)\log_{10} b + \log_{10} d$

Gradient $= -\dfrac{0.7}{2}$

$\log_{10} b = -0.35$

$b = 0.45$

Y-intercept = 1.56

$\log_{10} d = 1.56$

$d = 36.31$

(ii) When $x = 5.5$, $x - 1 = 4.5$.

From the graph $\log_{10} y = 0$

$y = 1$

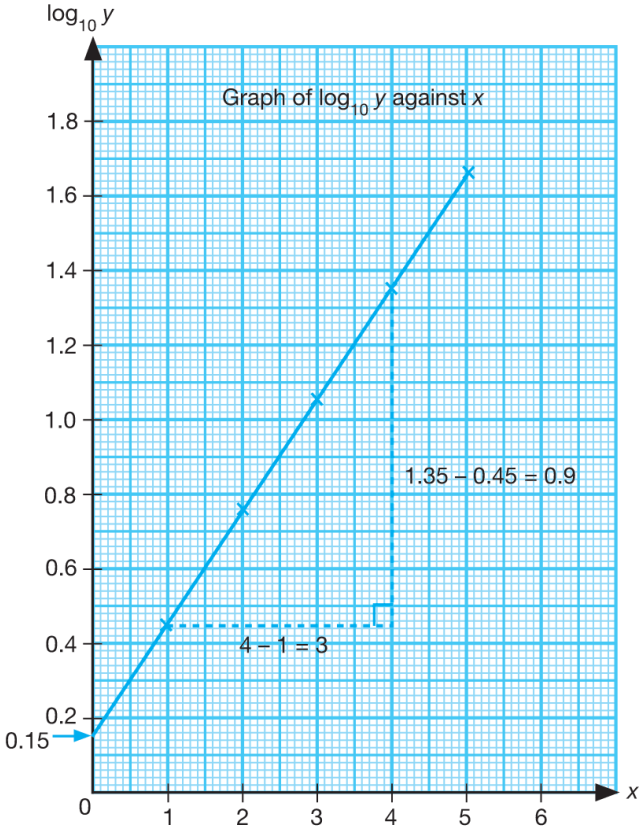
12 (a) $y = p^{b+x}$

$\log_{10} y = (b+x)\log_{10} p$

$\log_{10} y = x\log_{10} p + b\log_{10} p$

(b)

x	1	2	3	4	5
y	2.83	5.66	11.31	22.63	45.25
$\log_{10} y$	0.45	0.75	1.05	1.35	1.66



(c) $\log_{10} p =$ Gradient

$\log_{10} p = \dfrac{0.9}{3}$

$\log_{10} p = 0.3$

$p = 2$

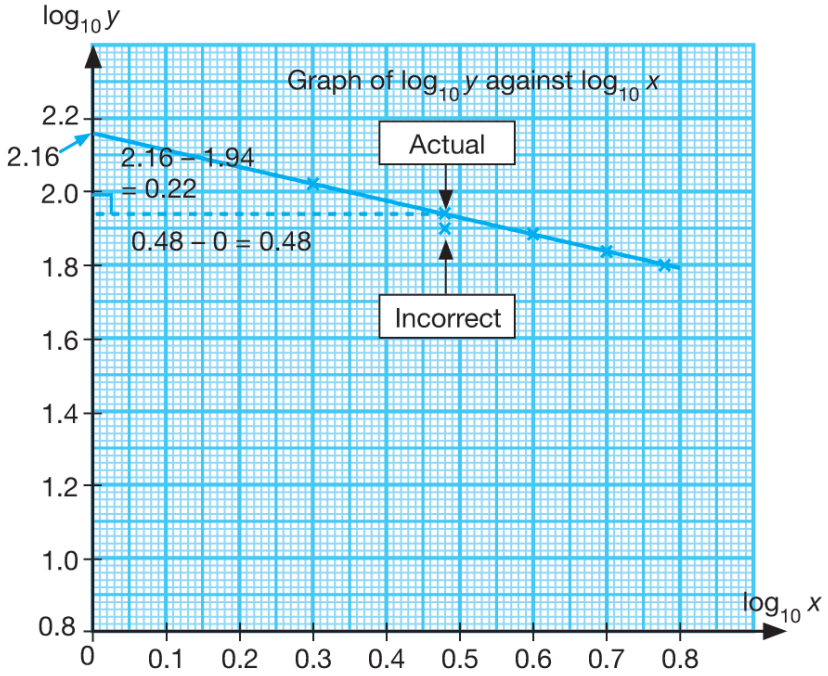
$b\log_{10} p = 0.15$

$b(0.3) = 0.15$

$b = \dfrac{0.15}{0.3} = 0.5$

13 (a)

x	2	3	4	5	6
y	106.0	78.5	75.0	67.0	61.0
$\log_{10} y$	2.03	1.89	1.88	1.83	1.79
$\log_{10} x$	0.30	0.48	0.60	0.70	0.78



(b) (i) The value of y which is incorrectly recorded is 78.5.

Actual:
$\log_{10} y = 1.94$
$y = 87.1$

(ii) $yx^n = k$

$\log_{10} y + n\log_{10} x = \log_{10} k$

$\log_{10} y = -n\log_{10} x + \log_{10} k$

$-n =$ Gradient

$-n = -\dfrac{0.22}{0.48}$

$n = 0.46$

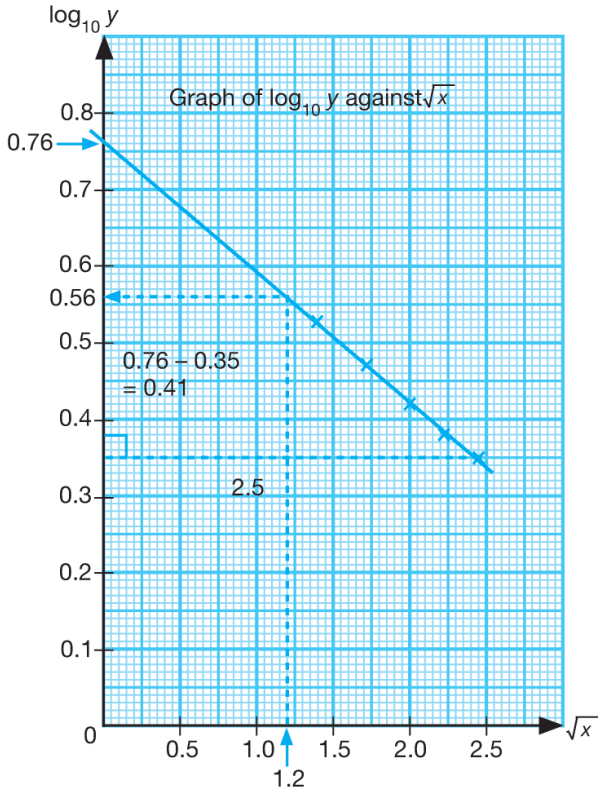
$\log_{10} k = Y$-intercept

$\log_{10} k = 2.16$

$k = 145$

14 (a)

x	2	3	4	5	6
y	3.39	2.92	2.63	2.39	2.23
$\sqrt{x}$	1.41	1.73	2.00	2.24	2.45
$\log_{10} y$	0.53	0.47	0.42	0.38	0.35



(b) (i) $y = qp^{\sqrt{x}}$

$\log_{10} y = \log_{10}\left(qp^{\sqrt{x}}\right)$

$\log_{10} y = \log_{10} q + \sqrt{x}\log_{10} p$

$\log_{10} y = \sqrt{x}\log_{10} p + \log_{10} q$

$\log_{10} q =$ Y-intercept

$\log_{10} q = 0.76$

$q = 5.75$

$\log_{10} p =$ Gradient

$\log_{10} p = -\dfrac{0.41}{2.5}$

$\log_{10} p = -0.164$

$p = 0.69$

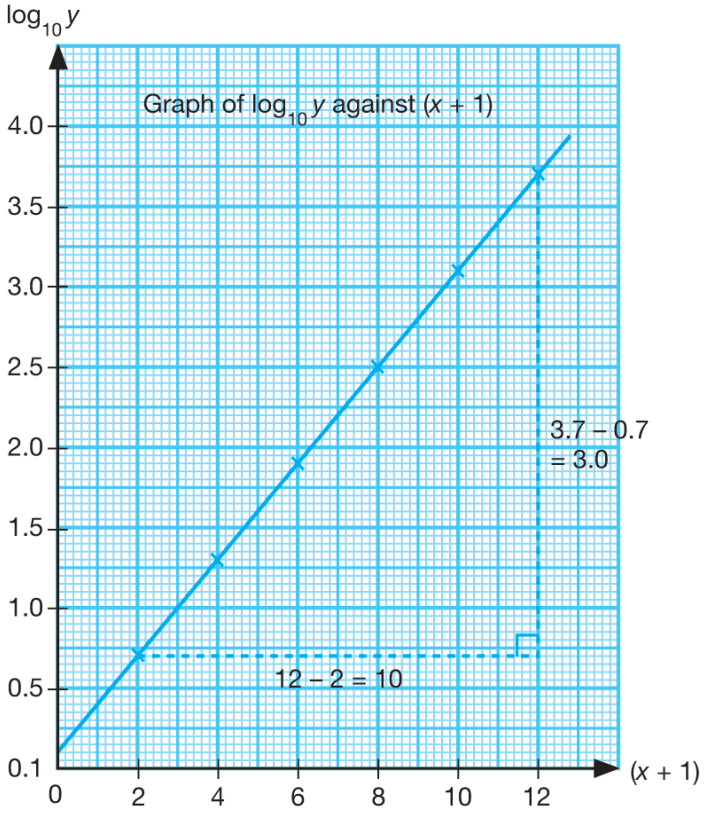
(ii) When $x = 1.44$, $\sqrt{x} = 1.2$.

From the graph, $\log_{10} y = 0.56$

$y = 3.63$

15 (a)

x	1	3	5	7	9	11
y	5	20	80	318	1 270	5 050
$\log_{10} y$	0.70	1.30	1.90	2.50	3.10	3.70
$(x + 1)$	2	4	6	8	10	12



(b) $y = hk^{x+1}$

$\log_{10} y = \log_{10} h + (x+1)\log_{10} k$

$\log_{10} y = (x+1)\log_{10} k + \log_{10} h$

$\log_{10} k$ = Gradient

$\log_{10} k = \dfrac{3}{10} = 0.3$

$k = 2$

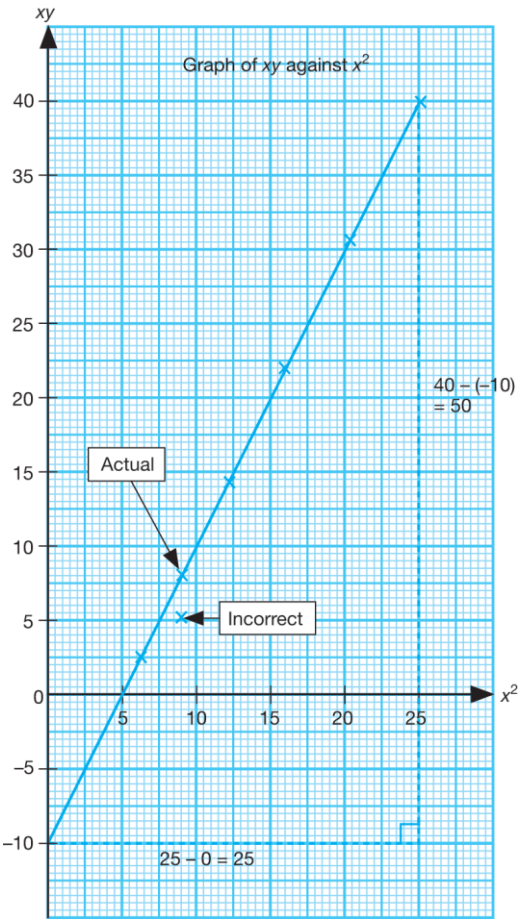
$\log_{10} h$ = Y-intercept

$\log_{10} h = 0.1$

$h = 1.26$

16 (a)

x	2.5	3.0	3.5	4.0	4.5	5.0
y	1.0	1.7	4.1	5.5	6.8	8.0
xy	2.5	5.1	14.4	22.0	30.5	40.0
x^2	6.25	9.00	12.25	16.00	20.25	25.00



(b) (i) The value of y which is incorrectly recorded is 1.7.

***Actual*:**

$xy = 8.1$

$3y = 8.1$

$y = 2.7$

(ii) $y = qx + \frac{p}{qx}$

$xy = qx^2 + \frac{p}{q}$

q = Gradient

$q = \frac{50}{25} = 2$

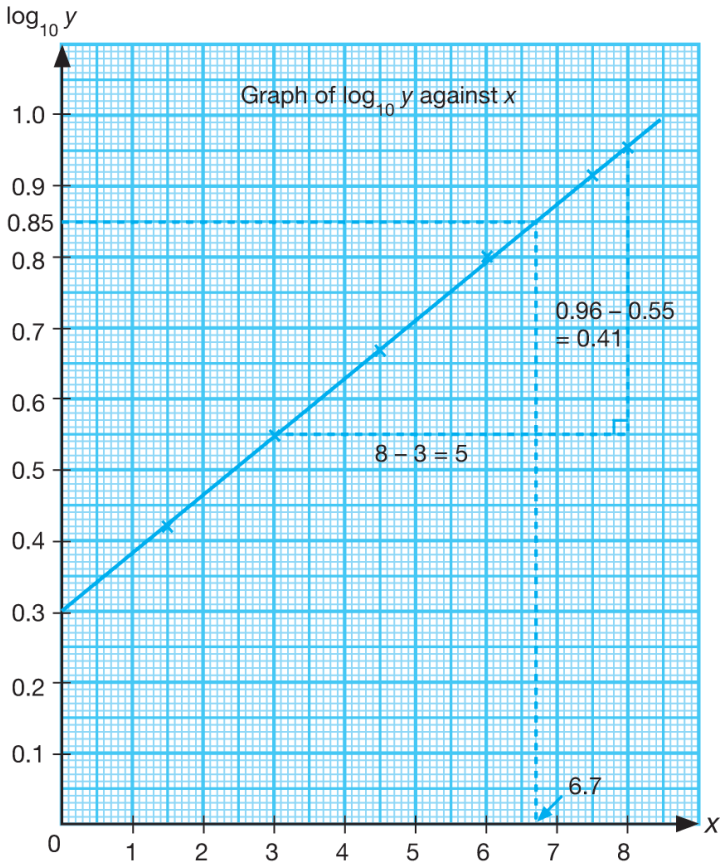
$\frac{p}{q} =$ Y-intercept

$\frac{p}{2} = -10$

$p = -20$

17 (a)

x	1.5	3.0	4.5	6.0	7.5	8.0
y	2.66	3.54	4.72	6.28	8.35	9.19
$\log_{10} y$	0.42	0.55	0.67	0.80	0.92	0.96



(b) (i) When $y = 7.1$, $\log_{10} y = 0.85$
From the graph, $x = 6.7$

(ii) $y = bd^{2x}$

$\log_{10} y = \log_{10} bd^{2x}$

$\log_{10} y = \log_{10} b + \log_{10} d^{2x}$

$\log_{10} y = \log_{10} b + 2x \log_{10} d$

$\log_{10} y = 2x \log_{10} d + \log_{10} b$

$\log_{10} b$ = Y-intercept

$\log_{10} b = 0.3$

$b = 2.0$

(iii) $2 \log_{10} d$ = Gradient

$2 \log_{10} d = \dfrac{0.41}{5}$

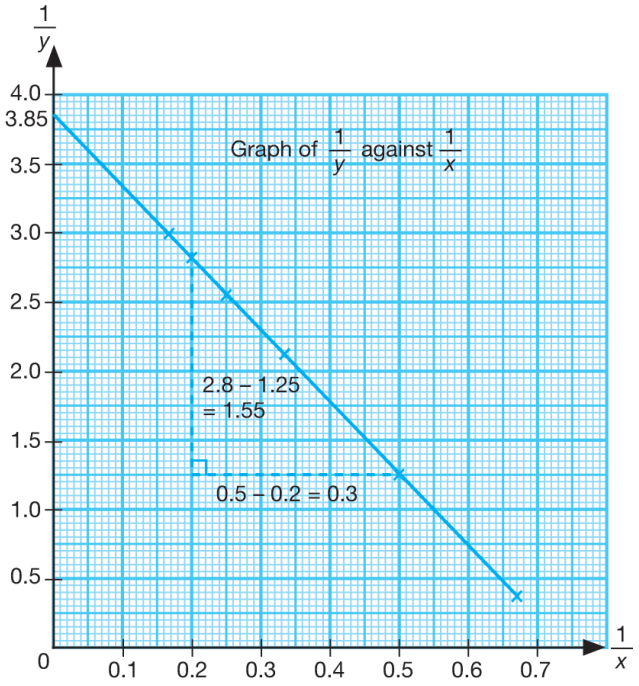
$\log_{10} d = 0.041$

$d = 1.1$

18 (a)

x	1.5	2.0	3.0	4.0	5.0	6.0
y	2.564	0.797	0.472	0.392	0.356	0.335
$\frac{1}{x}$	0.67	0.50	0.33	0.25	0.20	0.17
$\frac{1}{y}$	0.39	1.25	2.12	2.55	2.81	2.99

(b)



(c) $\frac{c}{y} = \frac{d}{x} + 1$

$\frac{c}{y} = \frac{d+x}{x}$

$\frac{1}{y} = \frac{d+x}{cx}$

$\frac{1}{y} = \frac{d}{c}\left(\frac{1}{x}\right) + \frac{1}{c}$

$\frac{1}{c} =$ Y-intercept

$\frac{1}{c} = 3.85$

$c = 0.26$

$\frac{d}{c} =$ Gradient

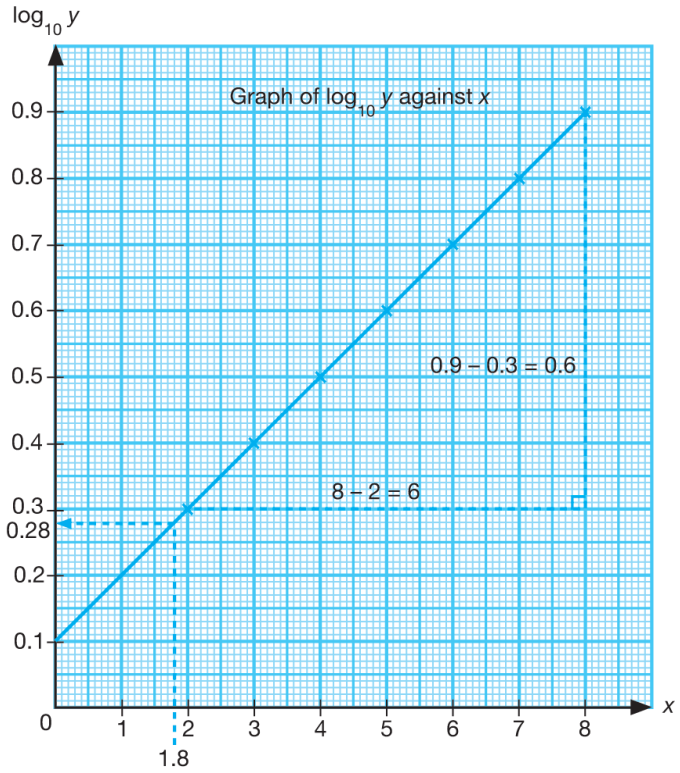
$\frac{d}{0.26} = -\frac{1.55}{0.3}$

$d = -\frac{1.55}{0.3} \times 0.26$

$d = -1.34$

19 (a)

x	3	4	5	6	7	8
y	2.49	3.15	3.96	5.01	6.29	7.95
$\log_{10} y$	0.4	0.5	0.6	0.7	0.8	0.9



(b) (i) $y = \dfrac{p^x}{b}$

$\log_{10} y = x \log_{10} p - \log_{10} b$

$\log_{10} p$ = Gradient

$\log_{10} p = \dfrac{0.6}{6} = 0.1$

$p = 1.26$

(ii) $-\log_{10} b$ = Y-intercept

$-\log_{10} b = 0.1$

$\log_{10} b = -0.1$

$b = 0.79$

(iii) When $x = 1.8$

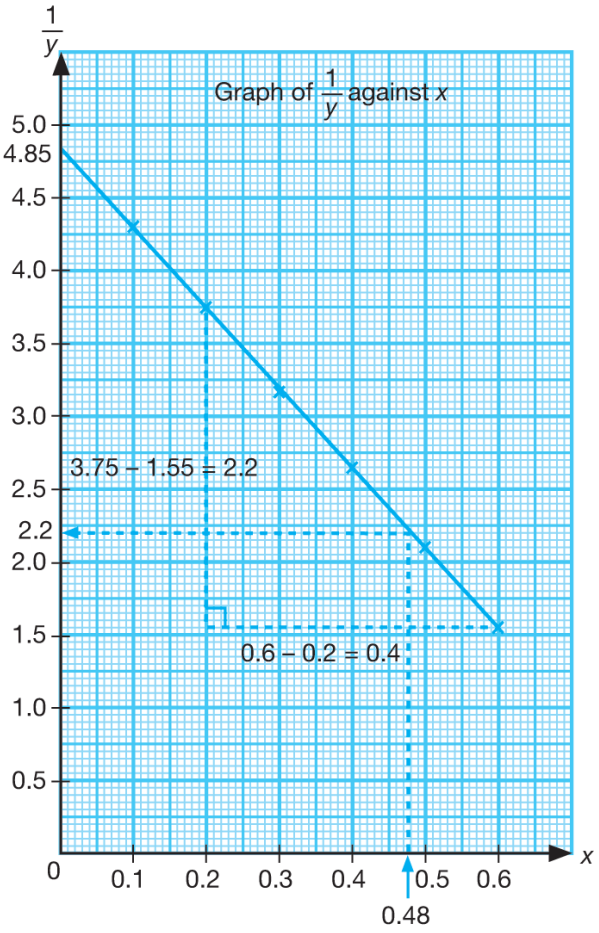
$\log_{10} y = 0.28$

$y = 1.91$

20 (a)

x	0.1	0.2	0.3	0.4	0.5	0.6
y	0.233	0.267	0.317	0.377	0.476	0.645
$\frac{1}{y}$	4.29	3.75	3.15	2.65	2.10	1.55

(b)



(c) $\frac{q}{y} = px + 1$

$\frac{1}{y} = \frac{p}{q}x + \frac{1}{q}$

(i) $\frac{1}{q} =$ Y-intercept

$\frac{1}{q} = 4.85$

$q = 0.206$

(ii) $\frac{p}{q} =$ Gradient

$\frac{p}{0.206} = -\frac{2.2}{0.4}$

$p = -1.13$

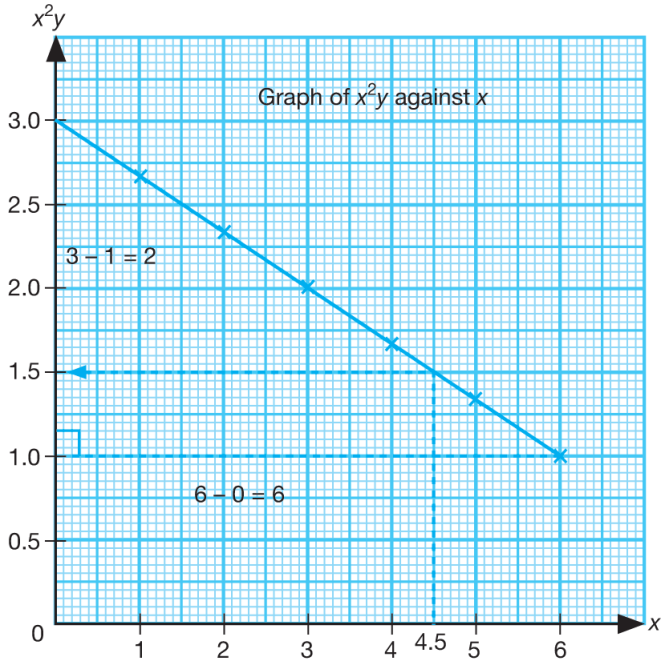
(iii) When $x = 0.48$, from the graph,

$\frac{1}{y} = 2.2 \Rightarrow y = 0.45$

21 (a)

x	1	2	3	4	5	6
y	2.667	0.583	0.222	0.104	0.053	0.028
x^2y	2.67	2.33	2.00	1.66	1.33	1.01

(b)



(c) (i) $y = \frac{a}{kx} + \frac{1}{kx^2}$

$x^2y = \left(\frac{a}{k}\right)x + \frac{1}{k}$

$\frac{1}{k}$ = Y-intercept

$\frac{1}{k} = 3.0$

$k = \frac{1}{3}$

(ii) $\frac{a}{k}$ = Gradient

$\frac{a}{\frac{1}{3}} = -\frac{2}{6}$

$a = -\frac{2}{6} \times \frac{1}{3}$

$a = -\frac{1}{9}$

(iii) When $x = 4.5$, from the graph,

$x^2y = 1.5$

$(4.5)^2 y = 1.5$

$y = \frac{1.5}{(4.5)^2}$

$y = \frac{1.5}{(4.5)^2}$

$y = 0.074$

22 (a) $R = k\left(2.718^{-\frac{a}{T}}\right)$

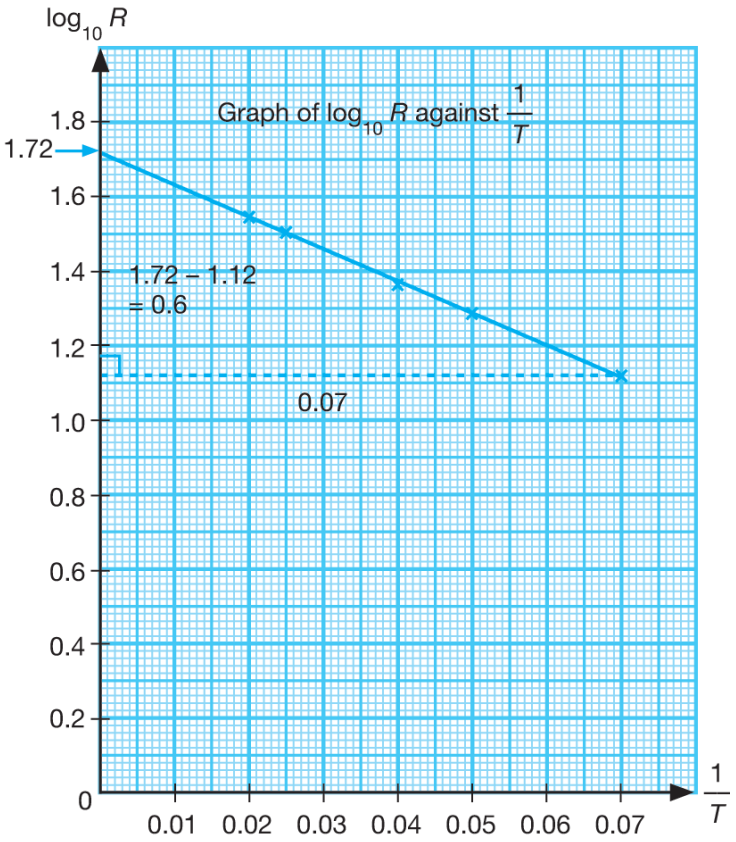
$$\log_{10} R = \log_{10} k - \frac{a}{T}\log_{10} 2.718$$

$$\log_{10} R = -\frac{a}{T}\log_{10} 2.718 + \log_{10} k$$

$$\log_{10} R = -0.4342\frac{a}{T} + \log_{10} k$$

(b)

T	15	20	25	40	50
R	13.2	19.0	22.9	31.6	34.6
$\log_{10} R$	1.12	1.28	1.40	1.50	1.54
$\frac{1}{T}$	0.07	0.05	0.04	0.03	0.02



(c) $\log_{10} k =$ Y-intercept

$$\log_{10} k = 1.72$$

$$k = 52.5$$

$$-0.4342a = \text{Gradient}$$

$$-0.4342a = -\frac{0.6}{0.07}$$

$$a = \frac{0.6}{0.07} \times \frac{1}{0.4342}$$

$$a = 19.7$$

SPM Spot

1 $y = \dfrac{h^x}{\sqrt{k}}$

$\log_{10} y = \log_{10}\left(\dfrac{h^x}{k^{\frac{1}{2}}}\right)$

$\log_{10} y = \log_{10} h^x - \log_{10} k^{\frac{1}{2}}$

$\log_{10} y = x\log_{10} h - \dfrac{1}{2}\log_{10} k$

At the point $\left(2, \dfrac{9}{2}\right)$, $x = 2$ and $\log_{10} y = \dfrac{9}{2}$.

$\dfrac{9}{2} = 2\log_{10} h - \dfrac{1}{2}\log_{10} k$ … (1)

At the point $\left(4, \dfrac{17}{2}\right)$, $x = 4$ and $\log_{10} y = \dfrac{17}{2}$.

$\dfrac{17}{2} = 4\log_{10} h - \dfrac{1}{2}\log_{10} k$ … (2)

(2) – (1): $4 = 2\log_{10} h$

$\log_{10} h = 2$

$h = 10^2$

$h = 100$

From (1):

$\dfrac{9}{2} = 2(2) - \dfrac{1}{2}\log_{10} k$

$\dfrac{1}{2}\log_{10} k = -\dfrac{1}{2}$

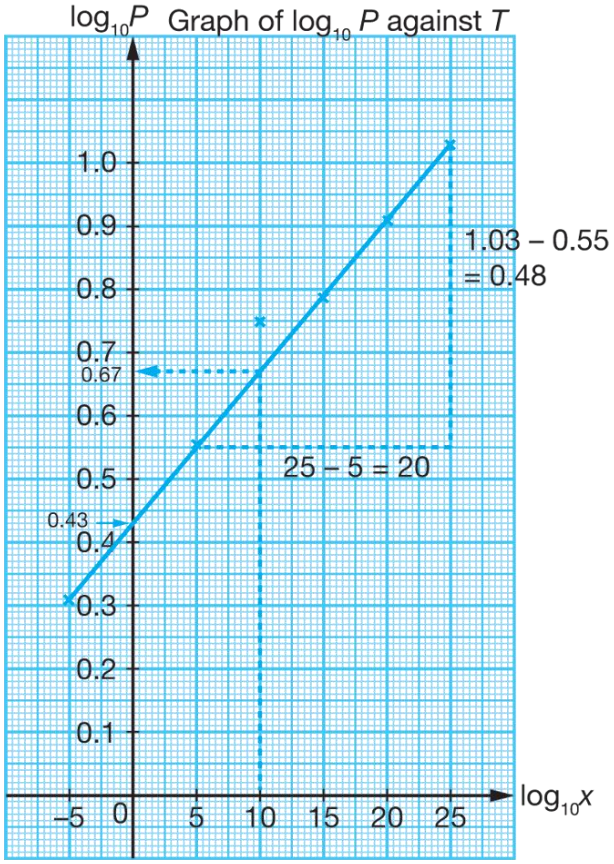
$\log_{10} k = -1$

$k = 10^{-1}$

$k = \dfrac{1}{10}$

2 (a)

T	–5	5	10	15	20	25
P	2.05	3.55	5.62	6.18	8.13	10.71
$\log_{10} P$	0.31	0.55	0.75	0.79	0.91	1.03



(b) (i) $P^2 = qr^T$

$2\log_{10} P = \log_{10} q + T\log_{10} r$

$2\log_{10} P = T\log_{10} r + \log_{10} q$

$\log_{10} P = \frac{1}{2}T\log_{10} r + \frac{1}{2}\log_{10} q$

$\frac{1}{2}\log_{10} r =$ Gradient

$\frac{1}{2}\log_{10} r = \frac{0.48}{20}$

$\log_{10} r = 0.048$

$r = 1.12$

$\frac{1}{2}\log_{10} q = Y$-intercept

$\frac{1}{2}\log_{10} q = 0.43$

$\log_{10} q = 0.86$

$q = 7.24$

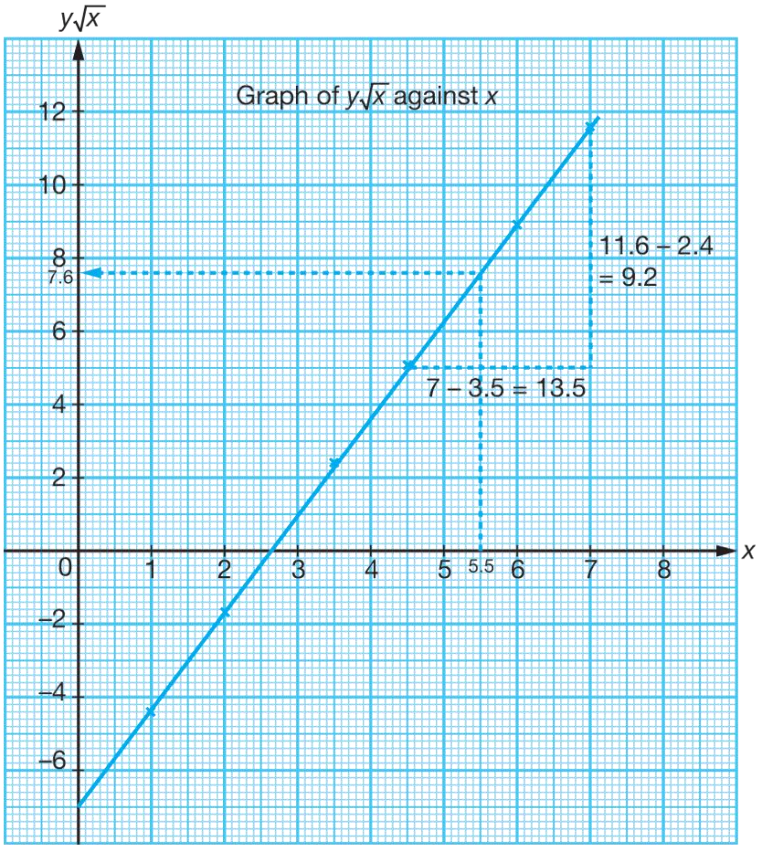
(ii) When $T = 10$,

$\log_{10} P = 0.67$

$P = 4.68$

3 (a)

x	1.0	2.0	3.5	4.5	6.0	7.0
y	–4.40	–1.13	1.29	2.36	3.67	4.38
$y\sqrt{x}$	–4.40	–1.60	2.41	5.01	8.99	11.60



(b) (i) When $x = 5.5$, $y\sqrt{x} = 7.6$

$$y\sqrt{5.5} = 7.6$$
$$y = 3.24$$

(ii) $$ay = b\sqrt{x} + \frac{1}{\sqrt{x}}$$

$$ay\sqrt{x} = bx + 1$$
$$y\sqrt{x} = \frac{b}{a}x + \frac{1}{a}$$

$\frac{1}{a}$ = Y-intercept

$$\frac{1}{a} = -7$$
$$a = -0.14$$

$\frac{b}{a}$ = Gradient

$$\frac{b}{-0.14} = \frac{9.2}{3.5}$$
$$\frac{b}{-0.14} = 2.6286$$
$$b = -0.37$$