

Form 4 Chapter 5
Progressions
Fully-Worked Solutions

UPSKILL 5.1a

1 (a) $T_2 - T_1 = -13 - (-17) = 4$

$$T_3 - T_2 = -9 - (-13) = 4$$

Since $T_2 - T_1 = T_3 - T_2 = 4$ (a constant), then the number sequence is an arithmetic progression.

(b) $T_2 - T_1 = \frac{3}{4} - 1 = -\frac{1}{4}$

$$T_3 - T_2 = \frac{1}{2} - \frac{3}{4} = -\frac{1}{4}$$

$$T_4 - T_3 = \frac{1}{4} - \frac{1}{2} = -\frac{1}{4}$$

Since $T_2 - T_1 = T_3 - T_2 = T_4 - T_3$ (a constant), then the number sequence is an arithmetic progression.

(c) $T_2 - T_1 = \frac{7}{5} - \frac{3}{5} = \frac{2}{5}$

$$T_3 - T_2 = 2 - \frac{7}{5} = \frac{3}{5}$$

Since $T_2 - T_1 \neq T_3 - T_2$, then the number sequence is not an arithmetic progression.

2 (a) $T_2 - T_1 = 50 - 42 = 8$

$$T_3 - T_2 = 58 - 50 = 8$$

Since $T_2 - T_1 = T_3 - T_2 = 8$ (a constant), then the number sequence is an arithmetic progression. Common difference = 8.

(b) $T_2 - T_1 = \frac{17}{12} - \frac{7}{4} = -\frac{1}{3}$

$$T_3 - T_2 = \frac{13}{12} - \frac{17}{12} = -\frac{1}{3}$$

Since $T_2 - T_1 = T_3 - T_2 = -\frac{1}{3}$ (a

constant), then the number sequence is an arithmetic progression. Common

difference = $-\frac{1}{3}$

3 $T_1 = p(p+1) = p^2 + p$

$$T_2 = p(p+3) = p^2 + 3p$$

$$T_3 = p(p+5) = p^2 + 5p$$

$$T_2 - T_1 = p^2 + 3p - (p^2 + p) = 2p$$

$$T_3 - T_2 = p^2 + 5p - (p^2 + 3p) = 2p$$

Since $T_2 - T_1 = T_3 - T_2 = 2p$ (a constant), then the number sequence is an arithmetic progression.

Common difference = $2p$

UPSKILL 5.1b

1 (a) 4, 12, 20, ...

$$a = 4$$

$$d = 12 - 4 = 8$$

$$T_6 = a + 5d = 4 + 5(8) = 44$$

(b) 2, $3\frac{1}{2}$, 5, ...

$$a = 2$$

$$d = 3\frac{1}{2} - 2 = 1\frac{1}{2}$$

$$T_{11} = a + 10d = 2 + 10\left(\frac{3}{2}\right) = 17$$

(c) -9, -3, 3, ...

$$a = -9$$

$$d = -3 - (-9) = 6$$

$$T_n = a + (n-1)(d)$$

$$= -9 + (n-1)(6)$$

$$= -9 + 6n - 6$$

$$= 6n - 15$$

2 $T_n = 23$

$$a + (n-1)d = 23$$

$$2 + (n-1)(3) = 23$$

$$2 + 3n - 3 = 23$$

$$3n = 24$$

$$n = 8$$

Hence, T_8 is 23.

3 (a) 9, 3, -3, ..., -45

$$T_n = -45$$

$$a + (n-1)d = -45$$

$$9 + (n-1)(-6) = -45$$

$$n-1 = \frac{-45-9}{-6}$$

$$n-1 = 9$$

$$n = 10$$

Number of terms is 10.

(b) $5, 6\frac{1}{2}, 8, \dots, 26$

$$T_n = 26$$

$$a + (n-1)d = 26$$

$$5 + (n-1)\left(\frac{3}{2}\right) = 26$$

$$\frac{3}{2}(n-1) = 21$$

$$n-1 = 21 \times \frac{2}{3}$$

$$n-1 = 14$$

$$n = 15$$

Number of terms is 15.

4 $154, 161, \dots, 294$

$$T_n = 294$$

$$154 + (n-1)(7) = 294$$

$$7(n-1) = 294 - 154$$

$$7(n-1) = 140$$

$$n-1 = 20$$

$$n = 21$$

Number of multiples of 7 is 21.

5 $x^2 - 2, 3x + 2, 11, \dots$

$$3x + 2 - (x^2 - 2) = 11 - (3x + 2)$$

$$-x^2 + 3x + 4 = -3x + 9$$

$$x^2 - 6x + 5 = 0$$

$$(x-5)(x-1) = 0$$

$$x = 5 \text{ or } 1$$

6 $4, p, q, \frac{35}{2}, \dots$

$$a = 4$$

$$T_4 = \frac{35}{2}$$

$$a + 3d = \frac{35}{2}$$

$$4 + 3d = \frac{35}{2}$$

$$8 + 6d = 35$$

$$6d = 27$$

$$d = 4.5$$

$$p = 4 + 4.5 = 8.5$$

$$q = 8.5 + 4.5 = 13$$

7 (a) $T_4 = 13$

$$a + 3d = 13 \dots (1)$$

$$T_{10} = 31$$

$$a + 9d = 31 \dots (2)$$

$$(2) - (1) : 6d = 18$$

$$d = 3$$

$$\text{From (1) : } a + 3(3) = 13$$

$$a = 13 - 9$$

$$a = 4$$

(b) $T_{25} = 4 + 24(3)$

$$= 76$$

8 $T_5 = 14$

$$a + 4d = 14 \dots (1)$$

$$T_{13} = -18$$

$$a + 12d = -18 \dots (2)$$

$$(2) - (1) : 8d = -32$$

$$d = -4$$

$$\text{From (1) : } a + 4d = 14$$

$$a + 4(-4) = 14$$

$$a = 30$$

$$T_9 = a + 8d = 30 + 8(-4) = -2$$

9 (a) $T_3 = 9$

$$a + 2d = 9 \dots (1)$$

$$T_7 = 49$$

$$a + 6d = 49 \dots (2)$$

$$(2) - (1) : 4d = 40$$

$$d = 10$$

$$\text{From (1) :}$$

$$a + 2(10) = 9$$

$$a = -11$$

$$T_{13} = a + 12d = -11 + 12(10) = 109$$

(b) $T_n = 79$

$$a + (n-1)d = 79$$

$$-11 + (n-1)(10) = 79$$

$$10(n-1) = 90$$

$$n-1 = 9$$

$$n = 10$$

The value of the 10th term is 79.

$$10 \quad T_4 = 36$$

$$a + 3d = 36 \dots (1)$$

$$T_{10} = 78$$

$$a + 9d = 78 \dots (2)$$

$$(2) - (1) : 6d = 42$$

$$d = 7$$

From (1) :

$$a + 3(7) = 36$$

$$a = 15$$

$$T_n > 1\,000$$

$$a + (n-1)d > 1\,000$$

$$15 + (n-1)(7) > 1\,000$$

$$7(n-1) > 985$$

$$n-1 > 140.7$$

$$n > 141.7$$

The smallest integer value of n is 142.

$$T_{142} = a + 141d = 15 + 141(7) = 1\,002$$

11 If x , y and z are three consecutive term of an arithmetic progression, hence

$$y - x = z - y$$

or $x - y = y - z \dots (1)$

$$y + z, z + x, x + y.$$

$$\begin{array}{ccc} \uparrow & \uparrow & \uparrow \\ T_1 & T_2 & T_3 \end{array}$$

$$T_2 - T_1 = z + x - (y + z) = x - y$$

$$T_3 - T_2 = x + y - (z + x) = y - z$$

From (1), $x - y = y - z$.

Hence, $T_2 - T_1 = T_3 - T_2$.

Hence, $y + z$, $z + x$ and $x + y$ are three consecutive terms of an arithmetic progression.

12 (a) $a = 3\,000$, $d = 250$

$$T_7 = a + 6d = 3\,000 + 6(250) = 4\,500$$

Encik Sulaiman's monthly salary in the 7th year is RM4 500.

(b)

$$T_n = 5\,250$$

$$a + (n-1)d = 5\,250$$

$$3\,000 + (n-1)(250) = 5\,250$$

$$250(n-1) = 2\,250$$

$$n-1 = 9$$

$$n = 10$$

Hence, Encik Sulaiman's monthly salary is RM2 250 in the 10th year of his service.

13 (a) 60, 57, 54, ...

$$a = 60, d = -3$$

$$T_8 = a + 7d = 60 + 7(-3) = 39$$

The distance of walk at the 8th minute is 39 m.

(b)

$$T_n = 30$$

$$a + (n-1)d = 30$$

$$60 + (n-1)(-3) = 30$$

$$(n-1)(-3) = -30$$

$$n = 11$$

The distance of walk is 30 m at the 11th minute.

UPSKILL 5.1c

1 (a) $a = -11.5$, $d = -9 - (-11.5) = 2.5$

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$S_{10} = \frac{10}{2} [2(-11.5) + (9)(2.5)]$$

$$S_{10} = -2.5$$

(b) $a = -1$, $d = 1 - (-1) = 2$

$$S_n = \frac{n}{2} [2(-1) + (n-1)(2)]$$

$$S_n = \frac{n}{2} (-2 + 2n - 2)$$

$$S_n = \frac{n}{2} (2n - 4)$$

$$S_n = n^2 - 2n$$

(c) $a = -1$, $d = 3 - (-1) = 4$

$$T_n = 43$$

$$-1 + (n-1)(4) = 43$$

$$(n-1)(4) = 44$$

$$(n-1) = 11$$

$$n = 12$$

$$S_n = \frac{n}{2}(a+l)$$

$$S_{12} = \frac{12}{2}(-1+43)$$

$$S_{12} = \frac{12}{2}(-1+43)$$

$$S_{12} = 252$$

$$(d) a = \frac{3}{2}, d = \frac{5}{4} - \frac{3}{2} = -\frac{1}{4}$$

$$T_n = -1$$

$$\frac{3}{2} + (n-1)\left(-\frac{1}{4}\right) = -1$$

$$(n-1)\left(-\frac{1}{4}\right) = -1 - \frac{3}{2}$$

$$(n-1)\left(-\frac{1}{4}\right) = -\frac{5}{2}$$

$$n-1 = \frac{-5}{-\frac{1}{4}}$$

$$n-1 = 10$$

$$n = 11$$

$$S_n = \frac{n}{2}(a+l)$$

$$S_{11} = \frac{11}{2}\left(\frac{3}{2} + (-1)\right)$$

$$S_{11} = 2\frac{3}{4}$$

2 301, 308, ..., 994

$$a = 301, d = 7$$

$$T_n = 994$$

$$301 + (n-1)(7) = 994$$

$$(n-1)(7) = 693$$

$$n-1 = 99$$

$$n = 100$$

$$S_n = \frac{n}{2}(a+l)$$

$$S_{100} = \frac{100}{2}(301+994) = 64\ 750$$

3 -100, -95, ..., 155

$$a = -100, d = 5$$

$$T_n = 155$$

$$a + (n-1)d = 155$$

$$-100 + (n-1)(5) = 155$$

$$5(n-1) = 255$$

$$n-1 = 51$$

$$n = 52$$

$$T_n = \frac{n}{2}(a+l)$$

$$T_{52} = \frac{52}{2}(-100+155)$$

$$T_{52} = 1\ 430$$

4 27, 24, 21, ...

$$a = 27, d = -3$$

$$S_n = 0$$

$$\frac{n}{2}[2a + (n-1)d] = 0$$

$$\frac{n}{2}[2(27) - 3(n-1)] = 0$$

$$n(54 - 3n + 3) = 0$$

$$n(57 - 3n) = 0$$

$$3n - 57 = 0$$

$$n = 19$$

5 10, 7, 4, ...

$$a = 10, d = -3$$

$$S_n = -35$$

$$\frac{n}{2}[2a + (n-1)d] = -35$$

$$\frac{n}{2}[2(10) + (n-1)(-3)] = -35$$

$$n(20 - 3n + 3) = -70$$

$$n(23 - 3n) = -70$$

$$23n - 3n^2 = -70$$

$$3n^2 - 23n - 70 = 0$$

$$(n-10)(3n+7) = 0$$

$$n = 10 \text{ or } n = -\frac{7}{3}$$

$$n = -\frac{7}{3} \text{ is not accepted.}$$

$$\therefore n = 10$$

6 2, 5, 8, ...

$$a = 2, d = 3$$

$$S_n = 301$$

$$\frac{n}{2}[2a + (n-1)d] = 301$$

$$\frac{n}{2}[2(2) + (n-1)(3)] = 301$$

$$n(4 + 3n - 3) = 602$$

$$n(3n+1) = 602$$

$$3n^2 + n - 602 = 0$$

$$(n-14)(3n-43) = 0$$

$$n = 14 \text{ or } n = \frac{43}{3}$$

$$n = \frac{43}{3} \text{ is not accepted.}$$

$$\therefore n = 14$$

7 $T_4 = 13$
 $a + 3d = 13 \dots (1)$

$$T_{10} = 31$$

$$a + 9d = 31 \dots (2)$$

$$(2) - (1) : 6d = 18$$

$$d = 3$$

From (1) :
 $a + 3(3) = 13$
 $a = 4$

$$S_{14} = \frac{14}{2} [2(4) + 13(3)] = 329$$

8 $S_8 = 8$
 $\frac{8}{2}(2a + 7d) = 8$
 $2a + 7d = 2 \dots (1)$

$$S_{16} = 144$$

$$\frac{16}{2}(2a + 15d) = 144$$

$$2a + 15d = 18 \dots (2)$$

$$(2) - (1) : 8d = 16$$

$$d = 2$$

From (1) : $2a + 7(2) = 2$
 $a = -6$

$$T_{16} = a + 15d = -6 + 15(2) = 24$$

9 $T_4 = 7$
 $a + 3d = 7 \dots (1)$
 $2a + 6d = 14 \dots (1) \times 2$

$$S_{10} = 145$$

$$\frac{10}{2}(2a + 9d) = 145$$

$$2a + 9d = 29 \dots (2)$$

$$(2) - (1) : 3d = 15$$

$$d = 5$$

From (1) : $a + 3(5) = 7$
 $a = -8$

$$S_{11} = \frac{11}{2} [2(-8) + 10(5)] = 187$$

10 $4, 7, 10, \dots$
 $a = 4, d = 3$

$$S_{12} - S_3$$

$$= \frac{12}{2} [2(4) + 11(3)] - \frac{3}{2} [2(4) + 2(3)]$$

$$= 246 - 21$$

$$= 225$$

11 (a) $T_2 = 15$
 $a + d = 15 \dots (1)$

$$T_4 = 23$$

$$a + 3d = 23 \dots (2)$$

$$2d = 8$$

$$d = 4$$

From (1) :
 $a + 4 = 15$
 $a = 11$

(b) $S_{10} - S_4$
 $= \frac{10}{2} [2(11) + 9(4)] - \frac{4}{2} [2(11) + 3(4)]$
 $= 290 - 68$
 $= 222$

12 $S_n = \frac{n}{2}(a + l) = 360$

$$\frac{n}{2}(10 + 80) = 360$$

$$45n = 360$$

$$n = 8$$

$$\frac{n}{2} [2(10) + (n-1)d] = 360$$

$$\frac{8}{2} [20 + (8-1)d] = 360$$

$$4(20 + 7d) = 360$$

$$20 + 7d = 90$$

$$7d = 70$$

$$d = 10$$

Alternative Method:

$$T_n = 10 + (n-1)d = 80$$

$$10 + (8-1)d = 80$$

$$d = 10$$

$$13 \text{ (a) } T_1 = \frac{1}{3}\pi h^2 k$$

$$T_2 = \frac{1}{3}\pi h^2(k+2) = \frac{1}{3}\pi h^2 k + \frac{2}{3}\pi h^2$$

$$T_3 = \frac{1}{3}\pi h^2(k+4) = \frac{1}{3}\pi h^2 k + \frac{4}{3}\pi h^2$$

$$T_2 - T_1 = \frac{2}{3}\pi h^2$$

$$T_3 - T_2 = \frac{2}{3}\pi h^2$$

$$\text{Since } T_2 - T_1 = T_3 - T_2 =$$

$\frac{2}{3}\pi h^2$ (constant), hence the volumes of cones form an arithmetic progression.

$$\text{Common difference} = \frac{2}{3}\pi h^2$$

$$(b) \quad T_4 = 30\pi$$

$$a + 3d = 30\pi$$

$$\frac{1}{3}\pi h^2 k + 3\left(\frac{2}{3}\pi h^2\right) = 30\pi$$

$$h^2 k + 6h^2 = 90 \dots (1)$$

$$S_5 = 120\pi$$

$$\frac{5}{2}[2a + 4d] = 120\pi$$

$$5a + 10d = 120\pi$$

$$a + 2d = 24\pi$$

$$\frac{1}{3}\pi h^2 k + 2\left(\frac{2}{3}\pi h^2\right) = 24\pi$$

$$h^2 k + 4h^2 = 72 \dots (2)$$

$$2h^2 = 18$$

$$h^2 = 9$$

$$h = 3$$

Radius of base = 3 cm

$$3^2 k + 6(3)^2 = 90$$

$$9k + 54 = 90$$

$$k = 4$$

Height = 4 cm

$$14 \quad S_n = 2n^2 + 3n$$

$$(a) \quad T_3 = S_3 - S_2$$

$$= 2(3)^2 + 3(3) - [2(2)^2 + 3(2)]$$

$$= 27 - 14$$

$$= 13$$

$$(b) \quad T_{10} = S_{10} - S_9$$

$$= 2(10)^2 + 3(10) - [2(9)^2 + 3(9)]$$

$$= 230 - 189$$

$$= 41$$

$$15 \quad S_n = 6n - 3n^2$$

$$(a) \quad T_1 = S_1$$

$$= 6 - 3$$

$$= 3$$

$$(b) \quad T_2 = S_2 - S_1$$

$$= 6(2) - 3(2)^2 - (6 - 3)$$

$$= -3$$

$$d = T_2 - T_1$$

$$d = -3 - 3$$

$$d = -6$$

$$16 \text{ (a) } a, a+15, a+30, \dots$$

$$d = 15$$

$$S_n = 3\ 850$$

$$\frac{n}{2}[2(50) + (n-1)(15)] = 3\ 850$$

$$n(85 + 15n) = 3\ 850$$

$$15n^2 + 85n - 3\ 850 = 0$$

$$3n^2 + 17n - 770 = 0$$

$$n = \frac{-17 \pm \sqrt{17^2 - 4(3)(-770)}}{2(3)}$$

$$n = \frac{17 \pm \sqrt{9\ 529}}{6}$$

$$n = -13.44 \text{ or } 19.10$$

$$n = -13.44 \text{ is not accepted.}$$

$$n = 19.10$$

The integer value of n is 20.

Hence, the number of months taken by Gurmit to pay all her loans is 20.

$$(b) \text{ The amount paid in the first 19 months}$$

$$= \frac{19}{2}[2(50) + (18)(15)] = \text{RM}3\ 515$$

Hence, the payment in the last month

$$= \text{RM}(3\ 850 - 3\ 515)$$

$$= \text{RM}335$$

UPSKILL 5.2a

1 (a) $\frac{1}{4}, 1, 4, \dots$

$$\frac{T_2}{T_1} = \frac{1}{\frac{1}{4}} = 4$$

$$\frac{T_3}{T_2} = \frac{4}{1} = 4$$

Since $\frac{T_2}{T_1} = \frac{T_3}{T_2} = 4$ (a constant), hence

The number sequence is a geometric progression.

(b) $1, -\frac{1}{2}, \frac{1}{4}, -\frac{1}{8}, \dots$

$$\frac{T_2}{T_1} = \frac{-\frac{1}{2}}{1} = -\frac{1}{2}$$

$$\frac{T_3}{T_2} = \frac{\frac{1}{4}}{-\frac{1}{2}} = -\frac{1}{2}$$

$$\frac{T_4}{T_3} = \frac{-\frac{1}{8}}{\frac{1}{4}} = -\frac{1}{2}$$

Since $\frac{T_2}{T_1} = \frac{T_3}{T_2} = \frac{T_4}{T_3} = -\frac{1}{2}$ (constant),

hence the number sequence is a geometric progression.

(c) $\frac{2}{5}, \frac{3}{5}, \frac{4}{5}, \dots$

$$\frac{T_2}{T_1} = \frac{\frac{3}{5}}{\frac{2}{5}} = \frac{3}{2}$$

$$\frac{T_3}{T_2} = \frac{\frac{4}{5}}{\frac{3}{5}} = \frac{4}{3}$$

Since $\frac{T_2}{T_1} \neq \frac{T_3}{T_2}$, hence the number

sequence is not a geometric progression.

2 (a) $\frac{3}{16}, \frac{3}{4}, 3, \dots$

$$\frac{T_2}{T_1} = \frac{\frac{3}{4}}{\frac{3}{16}} = 4$$

$$\frac{T_3}{T_2} = \frac{3}{\frac{3}{4}} = 4$$

Since $\frac{T_2}{T_1} = \frac{T_3}{T_2} = 4$ (a constant), hence

the number sequence is a geometric progression.

Common ratio = 4

(b) $\frac{1}{3}, -\frac{1}{30}, \frac{1}{300}, -\frac{1}{3000}, \dots$

$$\frac{T_2}{T_1} = \frac{-\frac{1}{30}}{\frac{1}{3}} = -\frac{1}{10}$$

$$\frac{T_3}{T_2} = \frac{\frac{1}{300}}{-\frac{1}{30}} = -\frac{1}{10}$$

$$\frac{T_4}{T_3} = \frac{-\frac{1}{3000}}{\frac{1}{300}} = -\frac{1}{10}$$

Since $\frac{T_2}{T_1} = \frac{T_3}{T_2} = \frac{T_4}{T_3} = -\frac{1}{10}$ (a constant),

hence the number sequence is a geometric

progression. Common ratio = $-\frac{1}{10}$.

(c) $\frac{p}{p+2}, p, p^2 + 2p, \dots$

$$\frac{T_2}{T_1} = \frac{p}{\frac{p}{p+2}} = p+2$$

$$\frac{T_3}{T_2} = \frac{p(p+2)}{p} = p+2$$

Since $\frac{T_2}{T_1} = \frac{T_3}{T_2} = p+2$ (a constant), hence

the number sequence is a geometric progression. Common ratio = $p+2$.

$$3 \quad T_1 = \text{Area of } ABCD = bh$$

$$T_2 = \text{Area of } ABPQ = b\left(\frac{h}{2}\right) = \frac{bh}{2}$$

$$T_3 = \text{Area of } ABKL = b\left(\frac{h}{4}\right) = \frac{bh}{4}$$

$$T_4 = \text{Area of } ABTU = b\left(\frac{h}{8}\right) = \frac{bh}{8}$$

$$\frac{T_2}{T_1} = \frac{\frac{bh}{2}}{bh} = \frac{1}{2}$$

$$\frac{T_3}{T_2} = \frac{\frac{bh}{4}}{\frac{bh}{2}} = \frac{1}{2}$$

$$\frac{T_4}{T_3} = \frac{\frac{bh}{8}}{\frac{bh}{4}} = \frac{1}{2}$$

Since $\frac{T_2}{T_1} = \frac{T_3}{T_2} = \frac{T_4}{T_3} = \frac{1}{2}$, hence the areas of

$ABCD$, $ABPQ$, $ABKL$ and $ABTU$ form a geometric progression.

$$\text{Common ratio} = \frac{1}{2}.$$

UPSKILL 5.2b

$$1 \text{ (a) } 8, 4, 2, \dots$$

$$a = 8, r = \frac{1}{2}$$

$$T_{11} = ar^{n-1} = (8)\left(\frac{1}{2}\right)^{10} = \frac{1}{128}$$

$$(b) \frac{16}{27}, \frac{8}{9}, \frac{4}{3}, \dots$$

$$a = \frac{16}{27}, r = \frac{\frac{8}{9}}{\frac{16}{27}} = \frac{3}{2}$$

$$T_5 = ar^4 = \left(\frac{16}{27}\right)\left(\frac{3}{2}\right)^4 = 3$$

$$(c) 3, 1, \frac{1}{3}, \dots$$

$$a = 3, r = \frac{1}{3}$$

$$T_n = 3\left(\frac{1}{3}\right)^{n-1} = \frac{3}{3^{n-1}} = 3^{1-(n-1)} = 3^{2-n}$$

$$2 \text{ (a) } \frac{1}{4}, \frac{1}{6}, \frac{1}{9}$$

$$a = \frac{1}{4}, r = \frac{\frac{1}{6}}{\frac{1}{4}} = \frac{2}{3}$$

$$T_n = \frac{4}{81}$$

$$\left(\frac{1}{4}\right)\left(\frac{2}{3}\right)^{n-1} = \frac{4}{81}$$

$$\left(\frac{2}{3}\right)^{n-1} = \frac{16}{81}$$

$$\left(\frac{2}{3}\right)^{n-1} = \left(\frac{2}{3}\right)^4$$

$$n-1 = 4$$

$$n = 5$$

Number of terms = 5

$$(b) 2, \frac{2}{3}, \frac{2}{9}, \dots$$

$$a = 2, r = \frac{\frac{2}{3}}{2} = \frac{1}{3}$$

$$T_n = \frac{2}{729}$$

$$2\left(\frac{1}{3}\right)^{n-1} = \frac{2}{729}$$

$$\left(\frac{1}{3}\right)^{n-1} = \frac{1}{729}$$

$$\left(\frac{1}{3}\right)^{n-1} = \left(\frac{1}{3}\right)^6$$

$$n-1 = 6$$

$$n = 7$$

Number of terms = 7

$$(c) 1\,000, -200, 40, \dots$$

$$a = 1\,000, r = \frac{-200}{1\,000} = -\frac{1}{5}$$

$$T_n = -\frac{8}{625}$$

$$ar^{n-1} = -\frac{8}{625}$$

$$(1\,000)\left(-\frac{1}{5}\right)^{n-1} = -\frac{8}{625}$$

$$\left(-\frac{1}{5}\right)^{n-1} = -\frac{8}{625\,000}$$

$$\left(-\frac{1}{5}\right)^{n-1} = -\frac{1}{78\,125}$$

$$\left(-\frac{1}{5}\right)^{n-1} = \left(-\frac{1}{5}\right)^7$$

$$n-1=7$$

$$n=8$$

Number of terms = 8

3 (a) $x-2, x+4, 4x+7, \dots$

$$\frac{x+4}{x-2} = \frac{4x+7}{x+4}$$

$$(x+4)^2 = (4x+7)(x-2)$$

$$x^2 + 8x + 16 = 4x^2 - x - 14$$

$$3x^2 - 9x - 30 = 0$$

$$x^2 - 3x - 10 = 0$$

$$(x-5)(x+2) = 0$$

$$x=5 \text{ or } x=-2$$

$$\text{When } x=5, r = \frac{x+4}{x-2} = \frac{9}{3} = 3$$

$$\text{When } x=-2, r = \frac{-2+4}{-2-2} = -\frac{1}{2}$$

4 (a) $x, x+4, 2x+2, \dots$

$$\frac{x+4}{x} = \frac{2x+2}{x+4}$$

$$(x+4)^2 = x(2x+2)$$

$$x^2 + 8x + 16 = 2x^2 + 2x$$

$$x^2 - 6x - 16 = 0$$

$$(x-8)(x+2) = 0$$

$$x=8 \text{ or } x=-2$$

$x=-2$ is not accepted because the question states that all the terms are positive.

$$\therefore x=8$$

(b) 8, 12, 18, ...

$$T_2 = x$$

$$ar = 8$$

$$a\left(\frac{12}{8}\right) = 8$$

$$a = \frac{16}{3}$$

$$T_6 = ar^5 = \left(\frac{16}{3}\right)\left(\frac{12}{8}\right)^5 = \frac{81}{2}$$

$$5 (a) T_2 = \frac{1}{2}$$

$$ar = \frac{1}{2} \dots (1)$$

$$T_4 = \frac{1}{128}$$

$$ar^3 = \frac{1}{128} \dots (2)$$

$$\frac{(2)}{(1)} : \frac{ar^3}{ar} = \frac{\frac{1}{128}}{\frac{1}{2}} = \frac{1}{64}$$

$$r^2 = \frac{1}{64}$$

$$r = \frac{1}{8}$$

$$\text{From (1)} : a\left(\frac{1}{8}\right) = \frac{1}{2}$$

$$a = 4$$

$$(b) T_n = ar^{n-1}$$

$$T_n = 4\left(\frac{1}{8}\right)^{n-1}$$

$$T_n = 2^2\left(\frac{1}{2^{3(n-1)}}\right)$$

$$T_n = 2^{2-3(n-1)}$$

$$T_n = 2^{5-3n}$$

$$6 T_2 + T_3 = 12$$

$$ar + ar^2 = 12$$

$$ar(1+r) = 12 \dots (1)$$

$$T_3 + T_4 = 4$$

$$ar^2 + ar^3 = 4$$

$$ar^2(1+r) = 4 \dots (2)$$

$$\frac{(2)}{(1)} : \frac{ar^2(1+r)}{ar(1+r)} = \frac{4}{12}$$

$$r = \frac{1}{3}$$

From (1) :

$$ar(1+r) = 12$$

$$\frac{1}{3}\left(1+\frac{1}{3}\right)a = 12$$

$$\frac{4}{9}a = 12$$

$$a = 27$$

7 $3, 5, 8\frac{1}{3}, \dots$

$$a = 3, r = \frac{5}{3}$$

$$T_n > 60$$

$$3\left(\frac{5}{3}\right)^{n-1} > 60$$

$$\left(\frac{5}{3}\right)^{n-1} > 20$$

$$(n-1)\lg\left(\frac{5}{3}\right) > \lg 20$$

$$n-1 > \frac{\lg 20}{\lg\left(\frac{5}{3}\right)}$$

$$n-1 > 5.86$$

$$n > 6.86$$

The smallest term which exceeds 60 is the 7th term.

8 $27, 18, 12, \dots$

$$a = 27, r = \frac{18}{27} = \frac{2}{3}$$

$$T_n < 1$$

$$27\left(\frac{2}{3}\right)^{n-1} < 1$$

$$\left(\frac{2}{3}\right)^{n-1} < \frac{1}{27}$$

$$(n-1)\lg\left(\frac{2}{3}\right) < \lg\frac{1}{27}$$

$$-0.1761(n-1) < -1.4314$$

$$n-1 > \frac{-1.4314}{-0.1761}$$

$$n-1 > 8.03$$

$$n > 9.03$$

The smallest term which is less than 1 is the 10th term.

9 $a = 40\,000, r = 1.05$

$$T_{11} = ar^t = 40\,000(1.05)^{10}$$

$$= \text{RM}65\,155.79$$

10 $a = 80\,000$

$$r = 0.9$$

The value of the car after 5 years:

$$T_6 = ar^5 = 80\,000(0.9)^5 = \text{RM}47\,239.20$$

11 $T_3 = 52\,920$

$$ar^2 = 52\,920 \dots (1)$$

$$T_5 = 58\,344.30$$

$$ar^4 = 58\,344.30 \dots (2)$$

$$\frac{(2)}{(1)} : \frac{ar^4}{ar^2} = \frac{58\,344.30}{52\,920}$$

$$r^2 = 1.1025$$

$$r = 1.05$$

Hence, the percentage of the annual increment is 5%.

$$1.05 = \frac{100+5}{100}$$

UPSKILL 5.2c

1 (a) 1, 2, 4,

$$a = 1, r = 2$$

$$S_{10} = \frac{a(r^n - 1)}{r - 1} = \frac{1(2^{10} - 1)}{2 - 1} = 1\ 023$$

(b) 4, -6, 9, ...

$$a = 4, r = -\frac{3}{2}$$

$$S_5 = \frac{4 \left[\left(-\frac{3}{2} \right)^5 - 1 \right]}{-\frac{3}{2} - 1}$$

$$S_5 = \frac{4 \left(-\frac{275}{32} \right)}{\left(-\frac{5}{2} \right)} = \frac{-\frac{275}{8}}{-\frac{5}{2}} = 13\frac{3}{4}$$

(c) 48, 12, 3, ..., $\frac{3}{16}$

$$a = 48, r = \left(\frac{1}{4} \right)$$

$$T_n = \frac{3}{16}$$

$$ar^{n-1} = \frac{3}{16}$$

$$48 \left(\frac{1}{4} \right)^{n-1} = \frac{3}{16}$$

$$\left(\frac{1}{4} \right)^{n-1} = \frac{1}{256}$$

$$\left(\frac{1}{4} \right)^{n-1} = \left(\frac{1}{4} \right)^4$$

$$n - 1 = 4$$

$$n = 5$$

$$S_5 = \frac{48 \left[1 - \left(\frac{1}{4} \right)^5 \right]}{1 - \frac{1}{4}}$$

$$S_5 = \frac{48 \left[1 - \left(\frac{1}{1\ 024} \right) \right]}{\frac{3}{4}}$$

$$S_5 = 48 \left(\frac{1\ 023}{1\ 024} \right) \times \frac{4}{3}$$

$$S_5 = 64 \left(\frac{1\ 023}{1\ 024} \right)$$

$$S_5 = \frac{1\ 023}{16} = 63\frac{15}{16}$$

2 $r = \frac{1}{2}$

$$S_4 = 11\frac{1}{4}$$

$$\frac{a \left[1 - \left(\frac{1}{2} \right)^4 \right]}{1 - \frac{1}{2}} = \frac{45}{4}$$

$$\frac{15}{16}a = \frac{45}{4} \times \frac{1}{2}$$

$$a = \frac{\frac{45}{8}}{\frac{15}{16}} = 6$$

3 $a = 8$

$$T_2 = 4$$

$$r = \frac{4}{8} = \frac{1}{2}$$

$$S_n = 15\frac{1}{2}$$

$$\frac{8 \left[1 - \left(\frac{1}{2} \right)^n \right]}{1 - \frac{1}{2}} = \frac{31}{2}$$

$$16 \left[1 - \left(\frac{1}{2} \right)^n \right] = \frac{31}{2}$$

$$1 - \left(\frac{1}{2} \right)^n = \frac{31}{2(16)}$$

$$\left(\frac{1}{2} \right)^n = \frac{1}{32}$$

$$n = 5$$

4 $a = 5, r = 2$

$$\begin{aligned} S_{11} - S_5 &= \frac{5(2^{11} - 1)}{2 - 1} - \frac{5(2^5 - 1)}{2 - 1} \\ &= 10\ 235 - 155 \\ &= 10\ 080 \end{aligned}$$

5 $T_4 = 54$

$$ar^3 = 54 \dots (1)$$

$$T_7 = 1\ 458$$

$$ar^6 = 1\ 458 \dots (2)$$

$$\frac{(2)}{(1)} : \frac{ar^6}{ar^3} = \frac{1\ 458}{54}$$

$$r^3 = 27$$

$$r = 3$$

From (1) :

$$a(3)^3 = 54$$

$$a = \frac{54}{27}$$

$$a = 2$$

$$S_8 - S_3$$

$$= \frac{2(3^8 - 1)}{3 - 1} - \frac{2(3^3 - 1)}{3 - 1}$$

$$= 6\ 560 - 26$$

$$= 6\ 534$$

6 $S_2 = 25$

$$\frac{a(r^2 - 1)}{r - 1} = 25$$

$$\frac{a(r+1)(r-1)}{r-1} = 25$$

$$a(r+1) = 25 \dots (1)$$

$$T_3 = 80$$

$$ar^2 = 80 \dots (2)$$

$$\frac{(2)}{(1)} : \frac{ar^2}{a(r+1)} = \frac{80}{25}$$

$$\frac{r^2}{r+1} = \frac{16}{5}$$

$$5r^2 = 16r + 16$$

$$5r^2 - 16r - 16 = 0$$

$$(r-4)(5r+4) = 0$$

$$r = 4 \text{ or } r = -\frac{4}{5}$$

From (1) : when $r = 4$,

$$a(4+1) = 25$$

$$a = 5$$

From (1) : when $r = -\frac{4}{5}$,

$$a\left(-\frac{4}{5} + 1\right) = 25$$

$$\frac{1}{5}a = 25$$

$$a = 125$$

7 $2, 3, \frac{9}{2}, \dots$

$$S_n > 60$$

$$\frac{2\left[\left(\frac{3}{2}\right)^n - 1\right]}{\frac{3}{2} - 1} > 60$$

$$4\left[\left(\frac{3}{2}\right)^n - 1\right] > 60$$

$$\left(\frac{3}{2}\right)^n - 1 > 15$$

$$\left(\frac{3}{2}\right)^n > 16$$

$$n \lg 1.5 > \lg 16$$

$$n > \frac{\lg 16}{\lg 1.5}$$

$$n > 6.84$$

Hence, the 7th term exceeds 60.

8 $a = 1, r = 2$

$$S_{18} = \frac{1(2^{18} - 1)}{2 - 1} = \text{RM}262\ 143$$

9 $a = 4, r = 3$

$$S_9 = \frac{4(3^9 - 1)}{3 - 1} = 39\ 364$$

10 (a) $a = 600, r = 0.95$

$$T_7 = 600(0.95)^{7-1} = \text{RM}441 \text{ (correct to the nearest RM)}$$

(b) $T_n < 370$

$$ar^{n-1} < 370$$

$$600(0.95)^{n-1} < 370$$

$$0.95^{n-1} < \frac{37}{60}$$

$$(n-1)\lg 0.95 < \lg \frac{37}{60}$$

$$-0.0223(n-1) < -0.2099$$

$$n-1 > \frac{-0.2099}{-0.0223}$$

$$n-1 > 9.41$$

$$n > 10.41$$

Minimum value of $n = 11$

(c) $S_8 = \frac{600(1-0.95^8)}{1-0.95} = \text{RM}4\ 039$ (correct to the nearest RM)

11 $S_n = 3(2^n - 1)$

(a) $T_3 = S_3 - S_2$
 $= 3(2^3 - 1) - 3(2^2 - 1)$
 $= 21 - 9$
 $= 12$

(b) $T_6 = S_6 - S_5$
 $T_6 = 3(2^6 - 1) - 3(2^5 - 1)$
 $= 189 - 93$
 $= 96$

12 $S_n = 200(1 - 0.5^n)$

(a) $T_1 = S_1 = 200(1 - 0.5^1) = 100$

(b) $T_2 = S_2 - S_1$
 $= 200(1 - 0.5^2) - 100$
 $= 50$
 $r = \frac{T_2}{T_1} = \frac{50}{100} = \frac{1}{2}$

13 (a) $r = \frac{100+6}{100} = 1.06$

$$S_n = \frac{a(1.06^n - 1)}{1.06 - 1} = \frac{a(1.06^n - 1)}{0.06}$$

(b) $\frac{a(1.06^n - 1)}{0.06} > 10a$

$$\frac{1.06^n - 1}{0.06} > 10$$

$$1.06^n - 1 > 0.6$$

$$1.06^n > 1.6$$

$$\log_{10} 1.06^n > \log_{10} 1.6$$

$$n \log_{10} 1.06 > \log_{10} 1.6$$

$$0.0253n > 0.2041$$

$$n > \frac{0.2041}{0.0253}$$

$$n > 8.0672$$

Hence, the minimum value of n is 9.

1 (a) 4, 2, 1, ...

$$S_\infty = \frac{4}{1 - \frac{1}{2}} = 8$$

(b) 6, 2, $\frac{2}{3}$, ...

$$S_\infty = \frac{6}{1 - \frac{1}{3}} = 9$$

(c) $-\frac{2}{3}, \frac{4}{9}, -\frac{8}{27}, \dots$

$$r = \frac{\frac{4}{9}}{-\frac{2}{3}} = -\frac{2}{3}$$

$$S_\infty = \frac{-\frac{2}{3}}{1 - \left(-\frac{2}{3}\right)} = -\frac{2}{5}$$

2 $S_\infty = 16\frac{2}{3}$

$$\frac{10}{1-r} = \frac{50}{3}$$

$$50(1-r) = 30$$

$$1-r = \frac{3}{5}$$

$$r = \frac{2}{5}$$

3 (a) $k+26, k+2, k-6, \dots$

$$\frac{k+2}{k+26} = \frac{k-6}{k+2}$$

$$(k+2)^2 = (k+26)(k-6)$$

$$k^2 + 4k + 4 = k^2 - 6k + 26k - 156$$

$$16k - 160 = 0$$

$$k = \frac{160}{16} = 10$$

(b) 10+26, 10+2, 10-6, ...

$$36, 12, 4, \dots$$

$$r = \frac{12}{36} = \frac{1}{3}$$

(c) $S_\infty = \frac{36}{1 - \left(\frac{1}{3}\right)} = 54$

$$4 \quad T_3 = 2$$

$$ar^2 = 2 \dots (1)$$

$$T_5 = \frac{1}{8}$$

$$ar^4 = \frac{1}{8} \dots (2)$$

$$\frac{(2)}{(1)} : r^2 = \frac{1}{16}$$

$$r = \frac{1}{4}$$

$$\text{From (1)} : a\left(\frac{1}{4}\right)^2 = 2$$

$$a = 32$$

$$S_\infty = \frac{32}{1 - \left(\frac{1}{4}\right)} = 42\frac{2}{3}$$

$$5 \quad T_2 = \frac{3}{2}$$

$$ar = \frac{3}{2} \dots (1)$$

$$S_\infty = 6$$

$$\frac{a}{1-r} = 6 \dots (2)$$

$$\frac{(2)}{(1)} : \frac{a}{ar} = \frac{6}{\frac{3}{2}}$$

$$\frac{a}{1-r} \times \frac{1}{ar} = 4$$

$$\frac{1}{r(1-r)} = 4$$

$$1 = 4r(1-r)$$

$$1 = 4r - 4r^2$$

$$4r^2 - 4r + 1 = 0$$

$$(2r-1)^2 = 0$$

$$r = \frac{1}{2}$$

From (1) :

$$a\left(\frac{1}{2}\right) = \frac{3}{2}$$

$$a = 3$$

$$6 \quad S_\infty = \frac{27}{8}$$

$$\frac{a}{1-r} = \frac{27}{8} \dots (1)$$

$$S_2 = \frac{15}{8}$$

$$T_1 + T_2 = \frac{15}{8}$$

$$a + ar = \frac{15}{8}$$

$$a(1+r) = \frac{15}{8} \dots (2)$$

$$\frac{(2)}{(1)} : \frac{a(1+r)}{a} = \frac{\frac{15}{8}}{\frac{27}{8}}$$

$$(1+r)(1-r) = \frac{5}{9}$$

$$1-r^2 = \frac{5}{9}$$

$$r^2 = 1 - \frac{5}{9}$$

$$r^2 = \frac{4}{9}$$

$$r = \pm \frac{2}{3}$$

From (2) :

$$\text{When } r = \frac{2}{3}, a\left(1 + \frac{2}{3}\right) = \frac{15}{8}$$

$$a = \frac{\frac{15}{8}}{\frac{5}{3}}$$

$$a = \frac{9}{8}$$

$$\text{When } r = -\frac{2}{3}, a\left(1 - \frac{2}{3}\right) = \frac{15}{8}$$

$$a = \frac{\frac{15}{8}}{\frac{1}{3}}$$

$$a = \frac{45}{8}$$

$$\begin{aligned}
7 \text{ (a) } & 0.888\dots \\
& = 0.8 + 0.08 + 0.0008 + \dots \\
& = \frac{0.8}{1-0.1} \\
& = \frac{0.8}{1-0.1} \\
& = \frac{0.8}{0.9} \\
& = \frac{8}{9}
\end{aligned}$$

$$\begin{aligned}
\text{(b) } & 0.454545\dots \\
& = 0.45 + 0.0045 + 0.00000045 \\
& = \frac{0.45}{1-0.01} \\
& = \frac{0.45}{0.99} \\
& = \frac{45}{99} \\
& = \frac{5}{11}
\end{aligned}$$

$$\begin{aligned}
\text{(c) } & 0.228228228\dots \\
& = 0.228 + 0.000000228 + \\
& \quad 0.000000000228 \\
& = \frac{0.228}{1-0.999} \\
& = \frac{228}{999} \\
& = \frac{76}{333}
\end{aligned}$$

$$\begin{aligned}
8 \text{ (a) } T_1 & = \text{Area of } PQR = \frac{1}{2}x^2 \sin \theta \\
T_2 & = \text{Area of } PQ_1R_1 = \frac{1}{2}\left(\frac{x}{2}\right)^2 \sin \theta \\
& = \frac{1}{8}x^2 \sin \theta \\
T_3 & = \text{Area of } PQ_2R_2 = \frac{1}{2}\left(\frac{x}{4}\right)^2 \sin \theta \\
& = \frac{1}{32}x^2 \sin \theta \\
T_4 & = \text{Area of } PQ_3R_3 = \frac{1}{2}\left(\frac{x}{8}\right)^2 \sin \theta \\
& = \frac{1}{128}x^2 \sin \theta
\end{aligned}$$

$$\frac{T_2}{T_1} = \frac{\frac{1}{8}x^2 \sin \theta}{\frac{1}{2}x^2 \sin \theta} = \frac{1}{4}$$

$$\frac{T_3}{T_2} = \frac{\frac{1}{32}x^2 \sin \theta}{\frac{1}{8}x^2 \sin \theta} = \frac{1}{4}$$

$$\frac{T_4}{T_3} = \frac{\frac{1}{128}x^2 \sin \theta}{\frac{1}{32}x^2 \sin \theta} = \frac{1}{4}$$

Since $\frac{T_2}{T_1} = \frac{T_3}{T_2} = \frac{T_4}{T_3} = \frac{1}{4}$ (a constant),

hence the areas of the triangles form a geometric progression.

$$\text{Common ratio} = \frac{1}{4}$$

(b) (i) $a = T_1 = \text{Area of } PQR$

$$\begin{aligned}
& = \frac{1}{2}(24)^2 \sin 30^\circ \\
& = 144
\end{aligned}$$

$$T_5 = ar^4 = 144\left(\frac{1}{4}\right)^4 = \frac{9}{16} \text{ cm}^2$$

$$\begin{aligned}
\text{(ii) } T_\infty & = \frac{a}{1-r} = \frac{\frac{1}{2}(24)^2 \sin 30^\circ}{1-\frac{1}{4}} \\
& = \frac{144}{\frac{3}{4}} \\
& = 192 \text{ cm}^2
\end{aligned}$$

Summative Practice 5

1 (a) $k-7, k-1, 2k-2, \dots$

$$k-1-(k-7) = 2k-2-(k-1)$$

$$6 = k-1$$

$$k = 7$$

(b) 0, 6, 12, ...

$$S_9 - S_3$$

$$= \frac{9}{2}(2a+8d) - \frac{3}{2}(2a+2d)$$

$$= \frac{9}{2}[2(0)+8(6)] - \frac{3}{2}[2(0)+2(6)]$$

$$= 216 - 18$$

$$= 198$$

2 4, 7, 10, ...

$$a = 4, d = 3$$

$$S_{23} - S_3$$

$$= \frac{23}{2}(2a+22d) - \frac{3}{2}(2a+2d)$$

$$= \frac{23}{2}[2(4)+22(3)] - \frac{3}{2}[2(4)+2(3)]$$

$$= 851 - 21$$

$$= 830$$

3 $S_3 = 117$

$$\frac{3}{2}[2a+(3-1)(7)] = 117$$

$$2a+14 = 117 \times \frac{2}{3}$$

$$2a+14 = 78$$

$$a = 32$$

The required three consecutive terms are 32, 39 and 46.

4 50, 46, 42, ...

$$T_n < 0$$

$$50 + (n-1)(-4) < 0$$

$$-4(n-1) < -50$$

$$n-1 > \frac{-50}{-4}$$

$$n-1 > 12.5$$

$$n > 13.5$$

Hence, the smallest value of n is 14.

5 $a = 1\ 200, d = 80$

$$T_{15} = a + 14d = 1\ 200 + 14(80) = \text{RM}2\ 320$$

6 $a = 150, d = 5$

$$S_{24} = \frac{24}{2}(2a+23d)$$

$$= 12[2(150)+23(5)]$$

$$= \text{RM}4\ 980$$

7 1, 2, 3, ...

$$S_{20} = \frac{20}{2}[2(1)+19(1)] = 210 \text{ bricks}$$

8 **Condensed milk**

$$T_n = a + (n-1)d$$

$$T_n = 65 + (n-1)(-5)$$

$$T_n = 65 - 5n + 5$$

$$T_n = 70 - 5n \dots (1)$$

Evaporated milk

$$T_n = a + (n-1)d$$

$$T_n = 45 + (n-1)(-3)$$

$$T_n = 45 - 3n + 3$$

$$T_n = 48 - 3n \dots (2)$$

Substitute (1) into (2):

$$70 - 5n = 48 - 3n$$

$$22 = 2n$$

$$n = 11$$

Hence, the balance of the number of cans of the condensed milk and evaporated milk are the same after 10 days.

At the 0th day (T_1), the stall has 65 cans of condensed milk and 45 cans of evaporated milk. Hence, T_{11} is the 10 day.

9 $4, 4\frac{1}{7}, 4\frac{2}{7}, \dots$

$$S_{15} = \frac{15}{2}(2a+14d)$$

$$= \frac{15}{2}\left[2(4)+14\left(\frac{1}{7}\right)\right]$$

$$= 75 \text{ minutes}$$

$$= 1 \text{ hour } 15 \text{ minutes [less than 1.5 hours]}$$

Hence, Nathan is qualified for the state level run.

$$10 \quad T_1 = \frac{1}{3}\pi r^2 h$$

$$T_2 = \frac{1}{3}\pi r^2 (h+1)$$

$$T_3 = \frac{1}{3}\pi r^2 (h+2)$$

$$T_2 - T_1 = \frac{1}{3}\pi r^2 (h+1) - \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi r^2$$

$$\begin{aligned} T_3 - T_2 &= \frac{1}{3}\pi r^2 (h+2) - \frac{1}{3}\pi r^2 (h+1) \\ &= \frac{1}{3}\pi r^2 \end{aligned}$$

$$\text{Since } T_2 - T_1 = T_3 - T_2 = \frac{1}{3}\pi r^2$$

(a constant), hence the volumes of cones form an arithmetic progression.

$$\text{Common difference} = \frac{1}{3}\pi r^2$$

- 11 (a) Let the length of the side of the smallest equilateral triangle = x cm

$$3x, 3(x+2), 3(x+4), \dots$$

$$a = 3x, d = 6$$

$$S_5 = 90$$

$$\frac{5}{2}[2(3x) + 4(6)] = 90$$

$$6x + 24 = 90 \times \frac{2}{5}$$

$$6x = 12$$

$$x = 2$$

Hence, the length of each side of the smallest equilateral triangle is 2 cm.

- (b) 6, 12, 18, ...

$$S_n = 350$$

$$\frac{n}{2}[2(6) + (n-1)(6)] = 350$$

$$n(12 + 6n - 6) = 700$$

$$n(6n - 4) = 700$$

$$6n^2 - 4n - 700 = 0$$

$$3n^2 - 2n - 350 = 0$$

$$n = \frac{2 \pm \sqrt{(-2)^2 - 4(3)(-350)}}{2(3)}$$

$$n = \frac{2 \pm \sqrt{4 + 4200}}{6}$$

$$n = 11.14 \text{ or } n = -10.47$$

$$n = 10.47 \text{ is not accepted.}$$

Hence, the number of complete triangles that can be formed is 10.

$$12 \text{ (a)} \quad S_n = 360^\circ$$

$$\frac{n}{2}(a+l) = 360$$

$$\frac{n}{2}(20+100) = 360$$

$$60n = 360$$

$$n = 6$$

$$\text{(b)} \quad T_n = 100$$

$$a + (n-1)d = 100$$

$$20 + (6-1)d = 100$$

$$5d = 80$$

$$d = 16^\circ$$

$$\text{(c)} \quad \pi r^2 = 25\pi$$

$$r = 5$$

Angle of the 2nd sector

$$= 20^\circ + 16^\circ$$

$$= 36^\circ$$

Area of the 2nd sector

$$= \frac{36}{360} \times \pi(5)^2$$

$$= \frac{5}{2}\pi \text{ cm}^2$$

- 13 **Particle A**

$$120, 116, 112, \dots$$

$$a = 120, d = -4$$

Particle B

$$x, x-5, x-10, \dots$$

$$a = x, d = -5$$

$$\text{(a)} \quad T_n = 0$$

$$120 + (n-1)(-4) = 0$$

$$-4(n-1) = -120$$

$$n-1 = 30$$

$$n = 31$$

$$\text{(b)} \quad T_{31} = 0$$

$$x + (n-1)(-5) = 0$$

$$x + (31-1)(-5) = 0$$

$$x = 150$$

$$\text{(c)} \quad S_{31} \text{ (particle A)}$$

$$= \frac{31}{2}[2(120) + 30(-4)]$$

$$= 1\,860 \text{ cm}$$

$$\begin{aligned}
S_{31} \text{ (particle B)} \\
&= \frac{31}{2}[2(150) + 30(-5)] \\
&= 2\,325 \text{ cm}
\end{aligned}$$

$$\begin{aligned}
\text{Difference in distance travelled} \\
&= 2\,325 - 1\,860 \\
&= 465 \text{ cm}
\end{aligned}$$

$$14 \quad T_1 = \pi(3)^2(5) = 45\pi$$

$$T_2 = \pi(3)^2(7) = 63\pi$$

$$T_3 = \pi(3)^2(9) = 81\pi$$

$$a = 45\pi, \quad r = 63\pi - 45\pi = 18\pi$$

$$\begin{aligned}
\text{(a) } T_{15} &= a + 14d \\
&= 45\pi + 14(18\pi) \\
&= 297\pi \text{ cm}^3
\end{aligned}$$

$$\begin{aligned}
\text{(b) } S_n &= 1\,260\pi \\
\frac{n}{2}[2(45\pi) + (n-1)(18\pi)] &= 1\,260\pi
\end{aligned}$$

$$\frac{n}{2}(90 + 18n - 18) = 1\,260$$

$$\frac{n}{2}(72 + 18n) = 1\,260$$

$$n(36 + 9n) = 1\,260$$

$$36n + 9n^2 = 1\,260$$

$$9n^2 + 36n - 1\,260 = 0$$

$$n^2 + 4n - 140 = 0$$

$$(n-10)(n+14) = 0$$

$$n = 10 \text{ or } n = -14$$

$n = -14$ is not accepted.

Hence, $n = 10$.

$$15 \quad T_n = 500$$

$$a + (n-1)d = 500$$

$$4000 + (n-1)(-250) = 500$$

$$(n-1)(-250) = -3500$$

$$n-1 = 14$$

$$n = 15$$

$$S_{15} = \frac{n}{2}[2a + (n-1)d]$$

$$S_{15} = \frac{15}{2}[2(4000) + (15-1)(-250)]$$

$$S_{15} = 33\,750$$

$$\begin{aligned}
\text{Total cost} \\
&= 33\,750 \times 0.50 = \text{RM}16\,875
\end{aligned}$$

$$16 \quad \frac{1}{3}, 1, 3, \dots$$

$$a = \frac{1}{3}, \quad r = 3$$

$$S_n = 364\frac{1}{3}$$

$$\frac{\frac{1}{3}(3^n - 1)}{3-1} = \frac{1\,093}{3}$$

$$\frac{3^n - 1}{6} = \frac{1\,093}{3}$$

$$3^n - 1 = 2\,186$$

$$3^n = 2\,187$$

$$3^n = 3^7$$

$$n = 7$$

$$17 \quad T_3 - T_2 = 20a$$

$$ar^2 - ar = 20a$$

$$r^2 - r - 20 = 0$$

$$(r-5)(r+4) = 0$$

$$r = 5 \text{ or } -4$$

$$18 \quad S_n = \frac{5}{2}(3^n - 1)$$

$$T_3 = S_3 - S_2$$

$$= \frac{5}{2}(3^3 - 1) - \frac{5}{2}(3^2 - 1)$$

$$= 65 - 20$$

$$= 45$$

$$19 \quad a = 3, \quad r = 3$$

$$T_7 = ar^6 = (3)(3)^6 = 2\,187 \text{ cells}$$

$$20 \quad a = 2\,000, \quad r = 1.05$$

$$S_8 = \frac{2\,000(1.05^8 - 1)}{1.05 - 1} = \text{RM}19\,098$$

$$21 \quad A_1 = \frac{1}{2}k^2$$

$$A_2 = \frac{1}{2}\left(\frac{k}{2}\right)^2 = \frac{1}{8}k^2$$

$$A_3 = \frac{1}{2}\left(\frac{k}{4}\right)^2 = \frac{1}{32}k^2$$

$$\frac{A_2}{A_1} = \frac{\frac{1}{8}k^2}{\frac{1}{2}k^2} = \frac{1}{4}$$

$$\frac{A_3}{A_2} = \frac{\frac{1}{32}k^2}{\frac{1}{8}k^2} = \frac{1}{4}$$

Since $\frac{A_2}{A_1} = \frac{A_3}{A_2} = \frac{1}{4}$ (a constant), hence

the areas of the right-angled triangles form a geometric progression.

$$\text{Common ratio} = \frac{1}{4}$$

22 (a) 50, 40, 32, ...

$$a = 50, r = \frac{4}{5}$$

$$T_8 = ar^7 = 50\left(\frac{4}{5}\right)^7 = 10.49 \text{ cm}$$

$$(b) \quad T_n < 5$$

$$50\left(\frac{4}{5}\right)^{n-1} < 5$$

$$\left(\frac{4}{5}\right)^{n-1} < \frac{5}{50}$$

$$(n-1) \lg\left(\frac{4}{5}\right) < \lg\frac{1}{10}$$

$$-0.097(n-1) < -1$$

$$n-1 > \frac{-1}{-0.097}$$

$$n-1 > 10.31$$

$$n > 11.31$$

Hence, height of the 12th bounce is less than 5 cm.

23 (a) 3, 6, 12, ...

$$T_6 = 3(2)^5 = 96 \text{ members}$$

$$(b) \quad S_9 = \frac{3[2^9 - 1]}{2 - 1} = 1533 \text{ members}$$

$$24 (a) \quad T_1 = \frac{1}{2}bt$$

$$T_2 = \frac{1}{2}(2b)(2t) = 2bt$$

$$T_3 = \frac{1}{2}(4b)(4t) = 8bt$$

$$\frac{T_2}{T_1} = \frac{2bt}{\frac{1}{2}bt} = 4$$

$$\frac{T_3}{T_2} = \frac{8bt}{2bt} = 4$$

Since $\frac{T_2}{T_1} = \frac{T_3}{T_2} = 4$ (a constant), the

areas of the right-angled triangles form a geometric progression.

Common ratio = 4

(b) $b = 6, t = 3$

$$a = T_1 = \frac{1}{2}(6)(3) = 9$$

$$(i) \quad T_8 = ar^7 = 9(4)^7 = 147\,456 \text{ cm}^2$$

$$(ii) \quad S_4 = \frac{9(4^4 - 1)}{4 - 1} = 765 \text{ cm}^2$$

25 (a)

$$\begin{array}{ccc} & \times 1.05 & \times 1.05 \\ & \curvearrowright & \curvearrowright \\ 24\,000, & 25\,200, & 26\,460, \dots \rightarrow \text{J.G.} \end{array}$$

$$T_6 = 24\,000(1.05)^{6-1} = \text{RM}30\,631$$

(b)

$$T_n > 35\,000$$

$$24\,000(1.05^{n-1}) > 35\,000$$

$$1.05^{n-1} > \frac{35}{24}$$

$$(n-1) \lg 1.05 > \lg\left(\frac{35}{24}\right)$$

$$n-1 > \frac{\lg\left(\frac{35}{24}\right)}{\lg 1.05}$$

$$n-1 > 7.73$$

$$n > 8.73$$

Minimum value of $n = 9$

$$(c) \quad S_6 = \frac{24\,000(1.05^6 - 1)}{1.05 - 1} = \text{RM}163\,246$$

26 (a) $T_7 = 8T_4$

$$ar^6 = 8ar^3$$

$$\frac{r^6}{r^3} = 8$$

$$r^3 = 8$$

$$r = 2$$

(b) (i) $S_n = 3069$

$$\frac{3(2^n - 1)}{2 - 1} = 3069$$

$$2^n - 1 = 1023$$

$$2^n = 1024$$

$$2^n = 2^{10}$$

$$n = 10$$

(ii) $T_{10} = ar^9 = 3(2)^9 = 1\ 536\ \text{cm}$

27 $T_2 = 48$

$$ar = 48 \dots (1)$$

$$T_4 = 27$$

$$ar^3 = 27 \dots (2)$$

$$\frac{(2)}{(1)} : \frac{ar^3}{ar} = \frac{27}{48}$$

$$r^2 = \frac{9}{16}$$

$$r = \frac{3}{4}$$

From (1) :

$$a\left(\frac{3}{4}\right) = 48$$

$$a = 48 \times \frac{4}{3}$$

$$a = 64$$

$$S_\infty = \frac{64}{1 - \frac{3}{4}} = 256$$

28 (a) $T_3 = 10$

$$ar^2 = 10 \dots (1)$$

$$T_3 + T_4 = 15$$

$$10 + ar^3 = 15$$

$$ar^3 = 5 \dots (2)$$

$$\frac{(2)}{(1)} : \frac{ar^3}{ar^2} = \frac{5}{10} = \frac{1}{2}$$

$$r = \frac{1}{2}$$

From (1) :

$$a\left(\frac{1}{2}\right)^2 = 10$$

$$a = 10(4)$$

$$a = 40$$

(b) $S_\infty = \frac{40}{1 - \frac{1}{2}} = 80$

29 (a) Let $AB = r$

$$T_1 = \frac{1}{4} \pi r^2$$

$$T_2 = \frac{1}{4} \pi \left(\frac{r}{2}\right)^2 = \frac{1}{16} \pi r^2$$

$$T_3 = \frac{1}{4} \pi \left(\frac{r}{4}\right)^2 = \frac{1}{64} \pi r^2$$

$$\frac{T_2}{T_1} = \frac{\frac{1}{16} \pi r^2}{\frac{1}{4} \pi r^2} = \frac{1}{4}$$

$$\frac{T_3}{T_2} = \frac{\frac{1}{64} \pi r^2}{\frac{1}{16} \pi r^2} = \frac{1}{4}$$

Since $\frac{T_2}{T_1} = \frac{T_3}{T_2} = \frac{1}{4}$ (a constant), hence

the areas of the quadrants form a geometric progression.

$$\text{Common ratio} = \frac{1}{4}$$

(b) $AB = 6\ \text{cm}$

$$a = \frac{1}{4} \pi (6)^2 = 9\pi$$

$$S_\infty = \frac{9\pi}{1 - \frac{1}{4}} = 12\pi\ \text{cm}^2$$

$$30 \text{ (a) } T_1 = \frac{1}{2}(50+40)(2) = 90$$

$$T_2 = \frac{1}{2}(40+32)(2) = 72$$

$$a = 90, r = \frac{72}{90} = \frac{4}{5}$$

$$T_5 = ar^4 = 90\left(\frac{4}{5}\right)^4 = 36.864 \text{ cm}^2$$

$$(b) T_\infty = \frac{90}{1 - \frac{4}{5}} = 450 \text{ cm}^2$$

SPM Spot

$$1 \text{ (a) } S_{20} = 176$$

$$\frac{20}{2}(2a+19d) = 176$$

$$20a+190d = 176$$

$$10a+95d = 88 \quad \dots (1)$$

$$S_5 = 14$$

$$\frac{5}{2}(2a+4d) = 14$$

$$5a+10d = 14 \quad \dots (2)$$

$$(2) \times 2 : 10a+20d = 28 \quad \dots (3)$$

$$(3) - (1) : -75d = -60$$

$$d = 0.8$$

$$\text{From (2) : } 5a+10(0.8) = 14$$

$$5a = 6$$

$$a = 1.2$$

Hence, the length of the longest part,
 $T_{20} = 1.2+19(0.8) = 16.4 \text{ cm}$

$$(b) \text{ (i) } a = 30, r = \frac{115}{100} = 1.15$$

$$S_{31} = \frac{30(1.15^{31}-1)}{1.15-1} = 15\,029$$

Total case = 15 029

$$(ii) n = 31, a = 2\,284$$

$$T_{31} = 45\,464$$

$$(2\,284)r^{31-1} = 45\,464$$

$$r^{30} = \frac{11\,366}{571}$$

$$r = 1.105$$

$$\frac{100+x}{100} = 1.105$$

$$x = 10.5\%$$

Rate of increase = 10.5%

$$2 \text{ (a) } S_4 = 16(S_8 - S_4)$$

$$S_4 = 16S_8 - 16S_4$$

$$17S_4 = 16S_8$$

$$17 \left[\frac{a(1-r^4)}{1-r} \right] = 16 \left[\frac{a(1-r^8)}{1-r} \right]$$

$$17(1-r^4) = 16(1-r^8)$$

$$17 \left[\frac{(1+r^2)(1-r^2)}{(1-r^2)} \right] = 16 \left[\frac{(1+r^4)(1-r^4)}{(1-r^2)} \right]$$

$$17 \left[\frac{(1+r^2)(1-r^2)}{(1-r^2)} \right] = 16 \left[\frac{(1+r^4)(1-r^4)}{(1-r^2)} \right]$$

$$17 = 16 + 16r^4$$

$$16r^4 = 1$$

$$r^4 = \frac{1}{16}$$

$$r = \frac{1}{2}$$

$$T_4 - T_8 = 237 \frac{3}{16}$$

$$ar^3 - ar^7 = \frac{3\,795}{16}$$

$$a\left(\frac{1}{2}\right)^3 - a\left(\frac{1}{2}\right)^7 = \frac{3\,795}{16}$$

$$\frac{1}{8}a - \frac{1}{128}a = \frac{3\,795}{16}$$

$$\frac{15}{128}a = \frac{3\,795}{16}$$

$$a = \frac{3\,795}{16} \times \frac{128}{15}$$

$$a = 2\,024$$

$$(b) \text{ (i) } a = x, d = 20$$

$$S_{36} = 110\,880$$

$$\frac{36}{2}[2x+(36-1)20] = 110\,880$$

$$18[2x+700] = 110\,880$$

$$2x+700 = 6\,160$$

$$x+350 = 3\,080$$

$$x = 2\,730$$

(ii) Shukri has cut $\frac{1}{3}$ of the roll of paper,

which is $\frac{1}{3} \times 36 = 12$ parts.

$$T_{12} = 2\,730 + (12-1)(20) = 2\,950$$

$$\begin{aligned}\text{For Shukor, } T_{13} &= 2\,950 + 20 \\ &= 2\,970\end{aligned}$$

$$a = 2\,970, d = 20, n = \frac{2}{3} \times 36 = 24$$

$$\begin{aligned}S_{24} &= \frac{24}{2} [2(2\,970) + (24-1)(20)] \\ &= 76\,800\end{aligned}$$

Hence, the total area of the paper cut
by Shukor

$$\begin{aligned}&= 76\,800 \times 500 \\ &= 38\,400\,000 \\ &= 3.84 \times 10^7 \text{ mm}^2\end{aligned}$$