Form 4 Chapter 3
Systems of Equations
Fully-Worked Solutions

UPSKILL 3.1a

1 (a) $p+q+r=4 \ldots (1)$
$2p-q+6r=8 \ldots (2)$
$p+2q-3r=0 \ldots (3)$

$$\begin{array}{rl} & p+q+r=4 \quad \ldots (1) \\ (+) & 2p-q+6r=8 \quad \ldots (2) \\ \hline & 3p+7r=12 \ldots (4) \end{array}$$

$$\begin{array}{rl} & 2p+2q+2r=8 \ldots (1)\times 2 \\ (-) & p+2q-3r=0 \ldots (3) \\ \hline & p+5r=8 \ldots (5) \end{array}$$

$$\begin{array}{rl} & 3p+7r=12 \ldots (4) \\ (-) & 3p+15r=24 \ldots (5)\times 3 \\ \hline & -8r=-12 \end{array}$$

$$r=\frac{-12}{-8}$$

$$r=\frac{3}{2}$$

From (4) :

$$3p+7\left(\frac{3}{2}\right)=12$$

$$6p+21=24$$

$$6p=3$$

$$p=\frac{1}{2}$$

From (1) :

$$\frac{1}{2}+q+\frac{3}{2}=4$$

$$q+2=4$$

$$q=2$$

(b) $r+s+t=0 \ldots (1)$
$5r-3s+5t=12 \ldots (2)$
$10r+s-5t=6 \ldots (3)$

$$\begin{array}{rl} & r+s+t=0 \ldots (1) \\ (-) & 10r+s-5t=6 \ldots (2) \\ \hline & -9r+6t=-6 \end{array}$$

$$-3r+2t=-2 \ldots (4)$$

$$\begin{array}{rl} & 3r+3s+3t=0 \ldots (1)\times 3 \\ (+) & 5r-3s+5t=12 \ldots (2) \\ \hline & 8r+8t=12 \end{array}$$

$$2r+2t=3 \ldots (5)$$

$$\begin{array}{rl} & -3r+2t=-2 \ldots (4) \\ (-) & 2r+2t=3 \ldots (5) \\ \hline & -5r=-5 \end{array}$$

$$r=1$$

From (4) : $-3(1)+2t=-2$

$$2t=3-2$$

$$t=\frac{1}{2}$$

From (1) : $1+s+\frac{1}{2}=0$

$$s=-1.5$$

(c) $2b+3d+m=-1$ … (1)
$b-d-m=-8$ … (2)
$3b+d+4m=11$ … (3)

$b-d-m=-8$ … (2)
(+) $3b+d+4m=11$ … (3)

$4b+3m=3$ … (4)

$2b+3d+m=-1$ … (1)
(+) $3b-3d-3m=-24$ … (2) × 3

$5b-2m=-25$ … (5)

$8b+6m=6$ … (4) × 2
(+) $15b-6m=-75$ … (5) × 3

$23b=-69$
$b=-3$

From (4) : $4(-3)+3m=3$
$3m=15$
$m=5$

From (1) :
$2(-3)+3d+5=-1$
$3d=0$
$d=0$

2 (a) $h+k=2-m$
$h+k+m=2$ … (1)

$2h+3m=k+9$
$2h-k+3m=9$ … (2)

$k+2=-3h+2m$
$3h+k-2m=-2$ … (3)

$h+k+m=2$ … (1)
(+) $2h-k+3m=9$ … (2)

$3h+4m=11$ … (4)

$h+k+m=2$ … (1)
(–) $3h+k-2m=-2$ … (3)

$-2h+3m=4$ … (5)

$6h+8m=22$ … (4) × 2
(+) $-6h+9m=12$ … (5) × 3

$17m=34$
$m=2$

From (4) : $3h+4m=11$
$3h+4(2)=11$
$3h=3$
$h=1$

From (1) :
$1+k+2=2$
$k=-1$

(b) $2h + k = m - 2$

$2h + k - m = -2 \quad \ldots (1)$

$h - m = k - 6$

$h - k - m = -6 \quad \ldots (2)$

$h - 7 = -2k - 3m$

$h + 2k + 3m = 7 \quad \ldots (3)$

$2h + k - m = -2 \quad \ldots (1)$

(+) $h - k - m = -6 \quad \ldots (2)$

$3h - 2m = -8 \quad \ldots (4)$

$4h + 2k - 2m = -4 \quad \ldots (1) \times 2$

(–) $h + 2k + 3m = 7 \quad \ldots (3)$

$3h - 5m = -11 \quad \ldots (5)$

$3h - 2m = -8 \quad \ldots (4)$

(–) $3h - 5m = -11 \quad \ldots (5)$

$3m = 3$

$m = 1$

From (4) : $3h - 2(1) = -8$

$3h = -6$

$h = -2$

From (1) : $2(-2) + k - 1 = -2$

$k = -2 + 4 + 1$

$k = 3$

(c) $2h + 3k = 9 - m$

$2h + 3k + m = 9 \ \ldots (1)$

$h - 11 = k - 4m$

$h - k + 4m = 11 \ \ldots (2)$

$k - 5m = -3h - 23$

$3h + k - 5m = -23 \ \ldots (3)$

$h - k + 4m = 11 \ \ldots (2)$

(+) $3h + k - 5m = -23 \ \ldots (3)$

$4h - m = -12 \ \ldots (4)$

$2h + 3k + m = 9 \quad \ldots (1)$

(+) $3h - 3k + 12m = 33 \ \ldots (2)$

$5h + 13m = 42 \ \ldots (5)$

$52h - 13m = -156 \ \ldots (4) \times 13$

(+) $5h + 13m = 42 \ \ldots (5)$

$57h = -114$

$h = -2$

From (4) : $4(-2) - m = -12$

$m = 4$

From (1) : $2h + 3k + m = 9$

$2(-2) + 3k + 4 = 9$

$3k = 9 + 4 - 4$

$k = 3$

UPSKILL 3.1b

1 The total amount invested in ASB, ASW and ASM is RM40 000.

$x + y + z = 40\,000$ … (1)

The amount invested in ASM is RM10 000 more than that of ASW.

$z - y = 10\,000$ … (2)

Dividend = RM3 400

$\frac{7}{100}x + \frac{9}{100}y + \frac{9}{100} = 3\,400$

$7x + 9y + 9z = 340\,000$ … (3)

$7x + 7y + 7z = 280\,000$ … (1) × 7

(–) $7x + 9y + 9z = 340\,000$ … (3)

$-2y - 2z = -60\,000$

$-y - z = -30\,000$ … (4)

$z - y = 10\,000$ … (2)

(–) $-y - z = -30\,000$ … (4)

$2z = 40\,000$

$z = 20\,000$

From (2) : $20\,000 - y = 10\,000$

$y = 10\,000$

From (1) : $x + y + z = 40\,000$

$x + 10\,000 + 20\,000 = 40\,000$

$x = 10\,000$

2 $2x + 2y + z = 46\,000$ … (1)

$3x + 4y + 2z = 82\,000$ … (2)

$4x + 4y + 3z = 100\,000$ … (3)

$3x + 4y + 2z = 82\,000$ … (2)

(–) $4x + 4y + 3z = 100\,000$ … (3)

$-x - z = -18\,000$ … (4)

$4x + 4y + 2z = 92\,000$ … (1) × 2

(–) $3x + 4y + 2z = 82\,000$ … (2)

$x = 10\,000$

From (4) : $-x - z = -18\,000$

$-10\,000 - z = -18\,000$

$z = 8\,000$

From (1) :

$2x + 2y + z = 46\,000$

$2(10\,000) + 2y + 8\,000 = 46\,000$

$2y = 18\,000$

$y = 9\,000$

3 $60x+80y+70z=1\,660 \ldots (1)$
$80x+30y+50z=1\,100 \ldots (2)$
$30x+20y+10z=430 \ \ldots (3)$

$60x+80y+70z=1\,660 \ldots (1)$
(–) $60x+40y+20z=860 \ \ldots (3) \times 2$
$40y+50z=800 \ \ldots (4)$

$480x+640y+560z=13\,280 \ldots (1) \times 8$
(–) $480x+180y+300z=6\,600 \ldots (2) \times 6$
$460y+260z=6\,680 \ \ldots (5)$

$4y+5z=80 \quad \ldots (4) \times \frac{1}{10}$
$23y+13z=334 \ \ldots (5) \times \frac{1}{20}$

$4y+5z=80 \ \ldots (6)$
$23y+13z=334 \ldots (7)$

$52y+65z=1\,040 \quad \ldots (6) \times 13$
(–) $115y+65z=1\,670 \quad \ldots (7) \times 5$
$-63y=-630$
$y=10$

From (6) : $4(10)+5z=80$
$z=8$

From (1) :
$60x+80(10)+70(8)=1\,660$
$x=5$

UPSKILL 3.2a

1 (a) $x+y=1 \qquad \ldots (1)$
$2x^2+2y^2=17 \qquad \ldots (2)$

From (1) :
$y=1-x \qquad (3)$

Substitute (3) into (2) :
$2x^2+2(1-x)^2=17$
$2x^2+2\left(1-2x+x^2\right)-17=0$
$2x^2+2-4x+2x^2-17=0$
$4x^2-4x-15=0$
$(2x-5)(2x+3)=0$
$x=\frac{5}{2}$ or $x=-\frac{3}{2}$

From (3) :
When $x=\frac{5}{2}$, $y=1-\frac{5}{2}=-\frac{3}{2}$
When $x=-\frac{3}{2}$, $y=1-\left(-\frac{3}{2}\right)=\frac{5}{2}$

(b) $x+1=2y \qquad \ldots (1)$
$x^2+xy-26=0 \qquad \ldots (2)$

From (1) :
$x=2y-1 \qquad (3)$

Substitute (3) into (2) :
$(2y-1)^2+(2y-1)y-26=0$
$4y^2-4y+1+2y^2-y-26=0$
$6y^2-5y-25=0$
$(2y-5)(3y+5)=0$
$y=\frac{5}{2}$ or $y=-\frac{5}{3}$

When $y=\frac{5}{2}$, $x=2\left(\frac{5}{2}\right)-1=4$
When $y=-\frac{5}{3}$, $x=2\left(-\frac{5}{3}\right)-1=-\frac{13}{3}$

(c) $2x+y+2=0$ … (1)

$x^2+y^2+2x+6y+1=0$ … (2)

From (1) :

$y=-2x-2$ … (3)

Substitute (3) into (2) :

$x^2+(-2x-2)^2+2x+6(-2x-2)+1=0$

$x^2+4x^2+8x+4+2x-12x-12+1=0$

$5x^2-2x-7=0$

$(5x-7)(x+1)=0$

$x=\frac{7}{5}$ or $x=-1$

When $x=\frac{7}{5}$, $y=-2\left(\frac{7}{5}\right)-2=-\frac{24}{5}$

When $x=-1$, $y=-2(-1)-2=0$

(d) $x+y=2$ … (1)

$x^2+xy+y^2=12$ … (2)

From (1) :

$y=2-x$ … (3)

Substitute (3) into (2) :

$x^2+x(2-x)+(2-x)^2=12$

$x^2+2x-x^2+4-4x+x^2-12=0$

$x^2-2x-8=0$

$(x-4)(x+2)=0$

$x=4$ or $x=-2$

From (3) :

When $x=4$, $y=2-4=-2$

When $x=-2$, $y=2-(-2)=4$

2 (a) $2x+3y=2$ … (1)

$12x^2+18y^2=5$ … (2)

From (1) :

$2x+3y=2$

$y=\frac{2-2x}{3}$ … (3)

Substitute (3) into (2) :

$12x^2+18\left(\frac{2-2x}{3}\right)^2=5$

$12x^2+18\left(\frac{4-8x+4x^2}{9}\right)-5=0$

$12x^2+2\left(4-8x+4x^2\right)-5=0$

$12x^2+8-16x+8x^2-5=0$

$20x^2-16x+3=0$

$(2x-1)(10x-3)=0$

$x=\frac{1}{2}$ or $x=\frac{3}{10}$

From (3) :

When $x=\frac{1}{2}$, $y=\frac{2-2\left(\frac{1}{2}\right)}{3}=\frac{1}{3}$

When $x=\frac{3}{10}$, $y=\frac{2-2\left(\frac{3}{10}\right)}{3}=\frac{7}{15}$

(b) $3x+2y=1$ … (1)

$6xy+12y^2=1$ … (2)

From (1) :

$x=\frac{1-2y}{3}$ … (3)

Substitute (3) into (2) :

$6y\left(\frac{1-2y}{3}\right)+12y^2=1$

$2y(1-2y)+12y^2-1=0$

$2y-4y^2+12y^2-1=0$

$8y^2+2y-1=0$

$(4y-1)(2y+1)=0$

$y=\frac{1}{4}$ or $y=-\frac{1}{2}$

From (3) :

When $y=\frac{1}{4}$, $x=\frac{1-2\left(\frac{1}{4}\right)}{3}=\frac{1}{6}$

When $y=-\frac{1}{2}$, $x=\frac{1-2\left(-\frac{1}{2}\right)}{3}=\frac{2}{3}$

(c) $2x+3y=4$ … (1)

$x^2+xy+y^2=3$ … (2)

From (1) :

$y=\frac{4-2x}{3}$ … (3)

Substitute (3) into (2) :

$x^2+x\left(\frac{4-2x}{3}\right)+\left(\frac{4-2x}{3}\right)^2-3=0$

$9x^2+3x(4-2x)+(4-2x)^2-27=0$

$9x^2+12x-6x^2+16-16x+4x^2-27=0$

$7x^2-4x-11=0$

$(7x-11)(x+1)=0$

$x=\frac{11}{7}$ or $x=-1$

From (3) :

When $x=\frac{11}{7}$, $y=\frac{4-2\left(\frac{11}{7}\right)}{3}=\frac{2}{7}$

When $x=-1$, $y=\frac{4-2(-1)}{3}=2$

3 (a) $x-\frac{1}{2}y=\frac{5}{2}$

$2x-y=5$

$y=2x-5$ … (1)

$2x+y=-\frac{2}{y}$

$2xy+y^2=-2$

$2xy+y^2+2=0$ … (2)

Substitute (1) into (2) :

$2x(2x-5)+(2x-5)^2+2=0$

$4x^2-10x+4x^2-20x+25+2=0$

$8x^2-30x+27=0$

$(4x-9)(2x-3)=0$

$x=\frac{9}{4}$ or $x=\frac{3}{2}$

From (1) :

When $x=\frac{9}{4}$, $y=2\left(\frac{9}{4}\right)-5=-\frac{1}{2}$

When $x=\frac{3}{2}$, $y=2\left(\frac{3}{2}\right)-5=-2$

(b) $\frac{x}{4}+\frac{y}{3}=1$

$\frac{3x+4y}{12}=1$

$3x+4y=12$

$y=\frac{12-3x}{4}$ … (1)

$\frac{3}{y}-\frac{2}{x}=\frac{7}{12}$

$\frac{3x-2y}{xy}=\frac{7}{12}$

$36x-24y=7xy$

$36x-7xy-24y=0$ … (2)

Substitute (1) into (2) :

$36x-7x\left(\frac{12-3x}{4}\right)-24\left(\frac{12-3x}{4}\right)=0$

$144x-7x(12-3x)-24(12-3x)=0$

$144x-84x+21x^2-288+72x=0$

$21x^2+132x-288=0$

$7x^2+44x-96=0$

$(7x-12)(x+8)=0$

$x=\frac{12}{7}$ or $x=-8$

From (1) :

When $x=\frac{12}{7}$, $y=\frac{12-3\left(\frac{12}{7}\right)}{4}=\frac{12}{7}$

When $x=-8$, $y=\frac{12-3(-8)}{4}=9$

4 (a) $12y^2-5x^2=2y-5x=7$

$2y-5x=7$

$y=\frac{7+5x}{2}$ … (1)

$12y^2-5x^2=7$ … (2)

Substitute (1) into (2) :

$12\left(\frac{5x+7}{2}\right)^2-5x^2-7=0$

$12\left(\frac{25x^2+70x+49}{4}\right)-5x^2-7=0$

$3(25x^2+70x+49)-5x^2-7=0$

$75x^2+210x+147-5x^2-7=0$

$70x^2+210x+140=0$

$x^2+3x+2=0$

$(x+1)(x+2)=0$

$x=-1$ or $x=-2$

From (1) :

When $x=-1$, $y=\frac{7+5(-1)}{2}=1$

When $x=-2$, $y=\frac{7+5(-2)}{2}=-\frac{3}{2}$

(b) $x^2-xy+y^2=2x+2y=12$

$2x+2y=12$

$x+y=6$

$y=-x+6$ … (1)

$x^2-xy+y^2=12$ … (2)

Substitute (1) into (2) :

$x^2-x(-x+6)+(-x+6)^2=12$

$x^2+x^2-6x+x^2-12x+36-12=0$

$3x^2-18x+24=0$

$x^2-6x+8=0$

$(x-4)(x-2)=0$

$x=4$ or $x=2$

From (1) :
When $x = 4$, $y = -4 + 6 = 2$
When $x = 2$, $y = -2 + 6 = 4$

5 $x + 3y + 1 = 0$

$x = -3y - 1$ … (1)

$x^2 + xy + y^2 = 4$ … (2)

Substitute (1) into (2) :

$(-3y - 1)^2 + y(-3y - 1) + y^2 - 4 = 0$

$9y^2 + 6y + 1 - 3y^2 - y + y^2 - 4 = 0$

$7y^2 + 5y - 3 = 0$

$$y = \frac{-5 \pm \sqrt{5^2 - 4(7)(-3)}}{2(7)}$$

$$y = \frac{-5 \pm \sqrt{109}}{14}$$

$y = 0.3886$ or $y = -1.103$

From (1) :
When $y = 0.3886$,
$x = -3(0.3886) - 1 = -2.166$
When $y = -1.103$,
$x = -3(-1.103) - 1 = 2.309$

6 $3x + y + 6 = x^2 + x - y = 2$

$3x + y + 6 = 2$

$3x + y = -4$

$y = -3x - 4$ … (1)

$x^2 + x - y = 2$

$y = x^2 + x - 2$ … (2)

Substitute (2) into (1) :

$3x + (x^2 + x - 2) = -4$

$x^2 + 4x + 2 = 0$

$$x = \frac{-4 \pm \sqrt{4^2 - 4(1)(2)}}{2}$$

$$x = \frac{-4 \pm \sqrt{8}}{2}$$

$x = -0.586$ or $x = -3.414$

From (1) :
When $x = -0.586$, $y = -3(-0.586) - 4 = -2.242$
When $x = -3.441$, $y = -3(-3.414) - 4 = 6.242$

7 $my + kx = 8$

$2m + 3k = 8$

$$m = \frac{8 - 3k}{2} \quad \text{… (1)}$$

$m^2 y - k^2 x + 10 = 0$

$2m^2 - 3k^2 + 10 = 0$ … (2)

Substitute (1) into (2) :

$$2\left(\frac{8 - 3k}{2}\right)^2 - 3k^2 + 10 = 0$$

$$2\left(\frac{64 - 48k + 9k^2}{4}\right) - 3k^2 + 10 = 0$$

$64 - 48k + 9k^2 - 6k^2 + 20 = 0$

$3k^2 - 48k + 84 = 0$

$k^2 - 16k + 28 = 0$

$(k - 2)(k - 14) = 0$

$k = 2$ or $k = 14$

From (1) :

When $k = 2$, $m = \frac{8 - 3(2)}{2} = 1$

When $k = 14$, $m = \frac{8 - 3(14)}{2} = -17$

UPSKILL 3.2b

1 Let the two numbers be x and y.

$y - x = 2$

$y = x + 2$ … (1)

$x^2 + y^2 = 20$ … (2)

From (1) : $y = x + 2$ … (3)
Substitute (3) into (2) :

$x^2 + (x + 2)^2 = 20$

$x^2 + x^2 + 4x + 4 - 20 = 0$

$2x^2 + 4x - 16 = 0$

$x^2 + 2x - 8 = 0$

$(x - 2)(x + 4) = 0$

$x = 2$ or $x = -4$

$x = -4$ is not accepted.
Hence, $x = 2$.
When $x = 2$, from (3),
$y = 2 + 2 = 4$
Hence, the two required numbers are 2 and 4.

2 Perimeter = 48

$4(10x)+2y=48$

$20x+y=24$

$y=24-20x$ … (1)

Area = 144 cm^2

$2\left(\frac{1}{2}\times 12x\times 8x\right)+12xy=144$

$96x^2+12xy=144$

$8x^2+xy-12=0$ … (2)

Substitute (1) into (2) :

$8x^2+x(24-20x)=12$

$8x^2+24x-20x^2=12$

$-12x^2+24x-12=0$

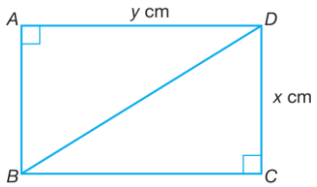
$x^2-2x+1=0$

$(x-1)^2=0$

$x=1$

From (1) : $y=24-20x=24-20(1)=4$

3



Perimeter = 18

$2x+2y=18$

$x+y=9$

$y=9-x$ … (1)

$BD^2=x^2+y^2=45$ … (2)

Substitute (1) into (2) :

$x^2+(9-x)^2=45$ … (2)

$x^2+81-18x+x^2-45=0$

$2x^2-18x+36=0$

$x^2-9x+18=0$

$x = 6$ or $x = 3$

$x = 6$ is not accepted.

Hence, $x = 3$

From (1), when $x = 3$, $y = 9 – 3 = 6$

Hence, length = 6 cm and width = 3 cm

4 Perimeter = 56 cm

$4(x+1)+4(y-2)+4(x+2)=56$

$(x+1)+(y-2)+(x+2)=14$

$2x+y=13$

$y=13-2x$

Base area = 15

$(x+1)(y-2)=15$

$xy-2x+y-2-15=0$

$x(13-2x)-2x+(13-2x)-17=0$

$-2x^2+13x-2x-2x+13-17=0$

$-2x^2+9x-4=0$

$2x^2-9x+4=0$

$(x-4)(2x-1)=0$

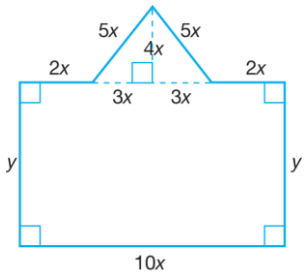
$x=4$ or $x=\frac{1}{2}$

$x=\frac{1}{2}$ is not accepted.

Hence, $x=4$.

$y=13-2x=13-2(4)=5$.

5



Perimeter = 36 m

$10x+4x+10x+2y=36$

$24x+2y=36$

$12x+y=18$

$y=18-12x$ … (1)

Area of the enclosed region = 72

$10xy+\frac{1}{2}(6x)(4x)=72$

$10x(18-12x)+\frac{1}{2}(6x)(4x)=72$

$10x(18-12x)+(3x)(4x)=72$

$10x(2)(9-6x)+12x^2-72=0$

$5x(9-6x)+3x^2-18=0$

$45x-30x^2+3x^2-18=0$

$-27x^2+45x-18=0$

$9x^2-15x+6=0$

$3x^2-5x+2=0$

$(x-1)(3x-2)=0$

$x=1$ or $x=\frac{2}{3}$

From (1) :
When $x = 1$, $y = 18 - 12(1) = 6$

When $x = \frac{2}{3}$, $y = 18 - 12\left(\frac{2}{3}\right) = 10$

6 Perimeter = 22 cm

$3x + 4x + 5x + 2x + 2y = 22$

$14x + 2y = 22$

$7x + y = 11$

$y = 11 - 7x$ … (1)

Area = 10 cm^2

$\frac{1}{2}(4x)(3x) + xy = 10$

$6x^2 + xy = 10$

$6x^2 + x(11 - 7x) = 10$

$6x^2 + 11x - 7x^2 - 10 = 0$

$-x^2 + 11x - 10 = 0$

$x^2 - 11x + 10 = 0$

$(x - 10)(x - 1) = 0$

$x = 10$ or $x = 1$

$x = 10$ is not accepted.

$x = 1$

From (1) : $y = 11 - 7(1) = 4$

Summative Practice 3

1 $2(x + y) - 3z = 9$

$2x + 2y - 3z = 9$ … (1)

$3x - 5(y + z) = -10$

$3x - 5y - 5z = -10$ … (2)

$x + 2y + 3z = 3$ … (3)

(1) + (3) : $3x + 4y = 12$ … (4)

(1) × 5 : $10x + 10y - 15z = 45$ … (5)

(2) × 3 : $9x - 15y - 15z = -30$ … (6)

(5) – (6) : $x + 25y = 75$ … (7)

(7) × 3 : $3x + 75y = 225$ … (8)

(8) – (4) : $71y = 213$

$y = 3$

Substitute $y = 3$ into (8) :

$3x + 75(3) = 225$

$x = 0$

Substitute $x = 0$ and $y = 3$ into (1) :

$2(0) + 2(3) - 3z = 9$

$z = -1$

2 $2x + 6y + 5z + 1 = 0$

$2x + 6y + 5z = -1$ … (1)

$30(x - y - z) + 11 = 0$

$30x - 30y - 30z = -11$ … (2)

$6(x + y) - 10z - 9 = 0$

$6x + 6y - 10z = 9$ … (3)

(1) × 2 : $4x + 12y + 10z = -2$ … (4)

(1) × 6 : $12x + 36y + 30z = -6$ … (5)

(3) + (4) : $10x + 18y = 7$ … (6)

(2) + (5) : $42x + 6y = -17$ … (7)

(7) × 3 : $126x + 18y = -51$ … (8)

(8) – (6) : $116x = -58$

$x = -\frac{1}{2}$

Substitute $x = -\frac{1}{2}$ into (6) :

$10\left(-\frac{1}{2}\right) + 18y = 7$

$18y = 12$

$y = \frac{2}{3}$

Substitute $x = -\frac{1}{2}$ and $y = \frac{2}{3}$ into (1) :

$2\left(-\frac{1}{2}\right) + 6\left(\frac{2}{3}\right) + 5z = -1$

$-1 + 4 + 5z = -1$

$5z = -4$

$z = -\frac{4}{5}$

3 $\frac{1}{2}y+\frac{1}{3}z=26$

$3y+2z=156$ … (1)

$\frac{1}{3}x+\frac{1}{4}z=23$

$4x+3z=276$ … (2)

$\frac{1}{2}x+\frac{1}{4}y=28$

$2x+y=112$ … (3)

$y=-2x+112$ … (4)

Substitute (4) into (1) :

$3(-2x+112)+2z=156$

$-6x+336+2z=156$

$-6x+2z=-180$

$-3x+z=-90$ … (5)

(5) × 3 : $-9x+3z=-270$ … (6)

(2) – (6) : $13x=546$

$x=42$

Substitute $x=42$ into (2) :

$4(42)+3z=276$

$3z=108$

$z=36$

Substitute $x=42$ into (4) :

$y=-2(42)+112=28$

4 $5x+2y+3z=23.60$ … (1)

$3x+y+2z=14.20$ … (2)

$x+3z=4y \Rightarrow x-4y+3z=0$ … (3)

(2) × 2 : $6x+2y+4z=28.4$ … (4)

(2) × 4 : $12x+4y+8z=56.8$ … (5)

(4) – (1) : $x+z=4.8$ … (6)

(3) + (5) : $13x+11z=56.8$ … (7)

(6) × 11 : $11x+11z=52.8$ … (8)

(7) – (8) : $2x=4$

$x=2$

Substitute $x=2$ into (7) :

$13(2)+11z=56.8$

$11z=30.8$

$z=2.80$

Substitute $x=2$ and $z=2.80$ into (1) :

$5(2)+2y+3(2.8)=23.60$

$2y=5.2$

$y=2.6$

Hence, the prices of a cup of coffee, a cup of Milo and a glass of orange juice are RM2.00, RM2.60 and RM2.80 respectively.

5 $x+y+2z=260$ … (1)

$x+y+4z=340$ … (2)

$8x+10y+15z=2\ 200$ … (3)

(2) – (1) : $2z=80$

$z=40$

Substitute $z=40$ into (1):

$x+y+2(40)=260$

$x+y=180$ … (4)

Substitute $z=40$ into (3) :

$8x+10y+15(40)=2\ 200$

$8x+10y=1\ 600$

$4x+5y=800$ … (5)

(4) × 4 : $4x+4y=720$ … (6)

(5) – (6) : $y=80$

Substitute y = 80 into (5) :

$4x+5(80)=800$

$4x$ = 400

x = 100

Hence, the numbers of voice recorded model *P*, *Q* and *R* are 100 , 80 and 40 units respectively.

6 $5x+3y+4z=360$ … (1)

$x+y+2z=120$ … (2)

$6x+4y+5z=450$ … (3)

(2) × 5 : $5x+5y+10z=600$ … (4)

(2) × 6 : $6x+6y+12z=720$ … (5)

(4) – (1) : $2y+6z=240$ … (6)

(5) – (3) : $2y+7z=270$ … (7)

(7) – (6) : $z=30$

Substitute $z=30$ into (7) :

$2y+7(30)=270$

$2y=60$

y = 30

Substitute $y = 30$ and $z = 30$ into (1) :

$5x + 3(30) + 4(30) = 360$

$5x = 150$

$x = 30$

Hence, the number of nitrile, vinyl and surgical gloves produced are 30, 30 and 30 respectively.

7 $x + y + z = 240$ … (1)

$x + y = 3z \Rightarrow x + y - 3z = 0$ … (2)

$3x + 2y + 4z = 700$ … (3)

(1) – (2) :

$4z = 240$

$z = 60$

Substitute $z = 60$ into (1) :

$x + y + 60 = 240$

$x + y = 180$ … (4)

Substitute $z = 60$ into (3) :

$3x + 2y + 4(60) = 700$

$3x + 2y = 460$ … (5)

(4) × 2 : $2x + 2y = 360$ …. (6)

(5) – (6) : $x = 100$

Substitute $x = 100$ into (4) :

$100 + y = 180$

$y = 80$

Hence, the number of oranges, apples and pineapples sold are 100, 80 and 60 respectively.

8 (a) $y - 2x = 7$ … (1)

$4x^2 + y^2 = 37$ … (2)

From (1) :

$y = 2x + 7$ … (3)

Substitute (3) into (2) :

$4x^2 + (2x + 7)^2 = 37$

$4x^2 + 4x^2 + 28x + 49 - 37 = 0$

$8x^2 + 28x + 12 = 0$

$4x^2 + 14x + 6 = 0$

$2x^2 + 7x + 3 = 0$

$(2x + 1)(x + 3) = 0$

$x = -\frac{1}{2}$ or $x = -3$

From (3) :

When $x = -\frac{1}{2}$, $y = 2\left(-\frac{1}{2}\right) + 7 = 6$

When $x = -3$, $y = 2(-3) + 7 = 1$

(b) $2x + y = 5$ … (1)

$x^2 - xy = 12$ … (2)

From (1) :

$y = 5 - 2x$ … (3)

Substitute (3) into (2) :

$x^2 - x(5 - 2x) - 12 = 0$

$x^2 - 5x + 2x^2 - 12 = 0$

$3x^2 - 5x - 12 = 0$

$(x - 3)(3x + 4) = 0$

$x = 3$ or $x = -\frac{4}{3}$

From (3) :

When $x = 3$, $y = 5 - 2(3) = -1$

When $x = -\frac{4}{3}$, $y = 5 - 2\left(-\frac{4}{3}\right) = \frac{23}{3}$

(c) $3x - y = 7$ … (1)

$x^2 - xy + y^2 = 7$ … (2)

From (1) :

$y = 3x - 7$ … (3)

Substitute (3) into (2) :

$x^2 - x(3x - 7) + (3x - 7)^2 - 7 = 0$

$x^2 - 3x^2 + 7x + 9x^2 - 42x + 49 - 7 = 0$

$7x^2 - 35x + 42 = 0$

$x^2 - 5x + 6 = 0$

$(x - 3)(x - 2) = 0$

$x = 3$ or $x = 2$

From (3) :

When $x = 3$, $y = 3(3) - 7 = 2$

When $x = 2$, $y = 3(2) - 7 = -1$

9 (a) $2x + 3y = 7$ … (1)

$x^2 + y^2 = 7 - xy$ … (2)

From (1) :

$y = \frac{7 - 2x}{3}$ … (3)

Substitute (3) into (2) :

$x^2 + \left(\frac{7 - 2x}{3}\right)^2 - 7 + x\left(\frac{7 - 2x}{3}\right) = 0$

$x^2 + \left(\frac{49 - 28x + 4x^2}{9}\right) - 7 + x\left(\frac{7 - 2x}{3}\right) = 0$

$9x^2 + 49 - 28x + 4x^2 - 63 + 3x(7 - 2x) = 0$

$9x^2 + 49 - 28x + 4x^2 - 63 + 21x - 6x^2 = 0$

$7x^2 - 7x - 14 = 0$

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$x = 2$ or $x = -1$

From (3) :

When $x = 2$, $y = \frac{7-2(2)}{3} = 1$

When $x = -1$, $y = \frac{7-2(-1)}{3} = 3$

(b) $2x + 3y = 1$ … (1)

$3x^2 + 4xy - y^2 - 6 = 0$ … (2)

From (1) : $y = \frac{1-2x}{3}$ … (3)

Substitute (3) into (2) :

$3x^2 + 4x\left(\frac{1-2x}{3}\right) - \left(\frac{1-2x}{3}\right)^2 - 6 = 0$

$3x^2 + 4x\left(\frac{1-2x}{3}\right) - \frac{1-4x+4x^2}{9} - 6 = 0$

$27x^2 + 12x(1-2x) - \left(1-4x+4x^2\right) - 54 = 0$

$27x^2 + 12x - 24x^2 - 1 + 4x - 4x^2 - 54 = 0$

$-x^2 + 16x - 55 = 0$

$x^2 - 16x + 55 = 0$

$(x-5)(x-11) = 0$

$x = 5$ or $x = 11$

From (3) :

When $x = 5$, $y = \frac{1-2(5)}{3} = -3$

When $x = 11$, $y = \frac{1-2(11)}{3} = -7$

10 (a) $\frac{x}{3} + \frac{y}{4} = \frac{9}{2}$

$4x + 3y = 54$

$y = \frac{54-4x}{3}$ … (1)

$\frac{3}{x} + \frac{4}{y} = 1$

$3y + 4x = xy$ … (2)

Substitute (1) into (2) :

$3\left(\frac{54-4x}{3}\right) + 4x - x\left(\frac{54-4x}{3}\right) = 0$

$54 - 4x + 4x - x\left(\frac{54-4x}{3}\right) = 0$

$162 - x(54-4x) = 0$

$162 - 54x + 4x^2 = 0$

$2x^2 - 27x + 81 = 0$

$(x-9)(2x-9) = 0$

$x = 9$ or $x = \frac{9}{2}$

From (1) :

When $x = 9$ $y = \frac{54-4(9)}{3} = 6$

When $x = \frac{9}{2}$, $y = \frac{54-4\left(\frac{9}{2}\right)}{3} = 12$

(b) $\frac{2y}{3} - \frac{x}{2} = \frac{2}{3}$

$4y - 3x = 4$

$y = \frac{4+3x}{4}$ … (1)

$\frac{2}{x} + \frac{3}{y} = 3$

$2y + 3x = 3xy$

$2y + 3x - 3xy = 0$ … (2)

Substitute (1) into (2) :

$2\left(\frac{4+3x}{4}\right) + 3x - 3x\left(\frac{4+3x}{4}\right) = 0$

$2(4+3x) + 12x - 3x(4+3x) = 0$

$8 + 6x + 12x - 12x - 9x^2 = 0$

$-9x^2 + 6x + 8 = 0$

$9x^2 - 6x - 8 = 0$

$(3x-4)(3x+2) = 0$

$x = \frac{4}{3}$ or $x = -\frac{2}{3}$

From (1) :

When $x = \frac{4}{3}$, $y = \frac{4+3\left(\frac{4}{3}\right)}{4} = 2$

When $x = -\frac{2}{3}$, $y = \frac{4+3\left(-\frac{2}{3}\right)}{4} = \frac{1}{2}$

11 (a) $4y - 4x = 24$

$y - x = 6$

$y = x + 6$ … (1)

$3x^2 + y^2 - 4y = 24$ … (2)

Substitute (1) into (2) :

$3x^2 + (x+6)^2 - 4(x+6) - 24 = 0$

$3x^2 + x^2 + 12x + 36 - 4x - 24 - 24 = 0$

$4x^2 + 8x - 12 = 0$

$x^2 + 2x - 3 = 0$

$(x-1)(x+3) = 0$

$x = 1$ or $x = -3$

From (1) :

When $x = 1$, $y = 1 + 6 = 7$

When $x = -3$, $y = -3 + 6 = 3$

(b) $3(4x - 3y) = 45$

$4x - 3y = 15$

$x = \frac{15 + 3y}{4}$ … (1)

$8x^2 - 27y^2 = 45$ … (2)

Substitute (1) into (2) :

$8\left(\frac{15 + 3y}{4}\right)^2 - 27y^2 = 45$

$8\left(\frac{225 + 90y + 9y^2}{16}\right) - 27y^2 - 45 = 0$

$\left(\frac{225 + 90y + 9y^2}{2}\right) - 27y^2 - 45 = 0$

$225 + 90y + 9y^2 - 54y^2 - 90 = 0$

$-45y^2 + 90y + 135 = 0$

$45y^2 - 90y - 135 = 0$

$9y^2 - 18y - 27 = 0$

$3y^2 - 6y - 9 = 0$

$y^2 - 2y - 3 = 0$

$(y - 3)(y + 1) = 0$

$y = 3$ or $y = -1$

From (1) :

When $y = 3$, $x = \frac{15 + 3(3)}{4} = 6$

When $y = -1$, $x = \frac{15 + 3(-1)}{4} = 3$

12 $Q = P$

$-2y = 3 - x$

$x = 3 + 2y$ … (1)

$Q = R$

$-2y = x(5 + 9y)$

$-2y = 5x + 9xy$

$-2y - 5x - 9xy = 0$ … (2)

Substitute (1) into (2) :

$-2y - 5(2y + 3) - 9y(2y + 3) = 0$

$-2y - 10y - 15 - 18y^2 - 27y = 0$

$-18y^2 - 39y - 15 = 0$

$18y^2 + 39y + 15 = 0$

$6y^2 + 13y + 5 = 0$

$(3y + 5)(2y + 1) = 0$

$y = -\frac{5}{3}$ or $-\frac{1}{2}$

From (1) :

When $y = -\frac{5}{3}$, $x = 3 + 2\left(-\frac{5}{3}\right) = -\frac{1}{3}$

When $y = -\frac{1}{2}$, $x = 3 + 2\left(-\frac{1}{2}\right) = 2$

13 $y + 2x = 4$

$3k + 2(2h) = 4$

$3k + 4h = 4$

$k = \frac{4 - 4h}{3}$ … (1)

$\frac{2}{y} - \frac{3}{2x} = 1$

$\frac{2}{3k} - \frac{3}{2(2h)} = 1$

$\frac{8h - 9k}{12hk} = 1$

$8h - 9k = 12hk$ … (2)

Substitute (1) into (2) :

$9h - 9\left(\frac{4 - 4h}{3}\right) = 12h\left(\frac{4 - 4h}{3}\right)$

$8h - 3(4 - 4h) = 4h(4 - 4h)$

$8h - 12 + 12h = 16h - 16h^2$

$16h^2 + 4h - 12 = 0$

$4h^2 + h - 3 = 0$

$(4h - 3)(h + 1) = 0$

$h = \frac{3}{4}$ or -1

From (1) :

When $h = \frac{3}{4}$, $k = \frac{4 - 4\left(\frac{3}{4}\right)}{3} = \frac{1}{3}$

When $h = -1$, $k = \frac{4 - 4(-1)}{3} = \frac{8}{3}$

14 $x - \frac{1}{2}y = \frac{5}{2}$

$2x - y = 5$

$y = 2x - 5$ … (1)

$2x + y = -\frac{2}{y}$

$2xy + y^2 = -2$

$2x(2x - 5) + (2x - 5)^2 + 2 = 0$

$4x^2 - 10x + 4x^2 - 20x + 25 + 2 = 0$

$8x^2 - 30x + 27 = 0$

$(4x - 9)(2x - 3) = 0$

$x = \frac{9}{4}$ or $x = \frac{3}{2}$

From (1) :

When $x=\frac{9}{4}$, $y=2\left(\frac{9}{4}\right)-5=-\frac{1}{2}$

When $x=\frac{3}{2}$, $y=2\left(\frac{3}{2}\right)-5=-2$

15 $x-y=1$

$x=1+y$ … (1)

$x^2+3y=6$ … (2)

Substitute (1) into (2) :

$(1+y)^2+3y=6$

$1+2y+y^2+3y-6=0$

$y^2+5y-5=0$

$y=\frac{-5\pm\sqrt{5^2-4(1)(-5)}}{2(1)}$

$y=\frac{-5\pm\sqrt{45}}{2(1)}$

$y=0.854$ or -5.854

When $y=0.854$, $x=1.854$

When $y=-5.854$, $x=-4.851$

16 $3x+y+4=0$

$y=-3x-4$ … (1)

$xy+40=y^2$

$x(-3x-4)+40-(-3x-4)^2=0$

$-3x^2-4x+40-\left(9x^2+24x+16\right)=0$

$-3x^2-4x+40-9x^2-24x-16=0$

$-12x^2-28x+24=0$

$(x+3)(3x-2)=0$

$x=-3$ or $x=\frac{2}{3}$

From (1) :

When $x=-3$, $y=-3(-3)-4=5$

When $x=\frac{2}{3}$, $y=-3\left(\frac{2}{3}\right)-4=-6$

17 $m-2n=-1$

$m=2n-1$ … (1)

$mn+n-3m=0$

$n(2n-1)+n-3(2n-1)=0$

$2n^2-n+n-6n+3=0$

$2n^2-6n+3=0$

$n=\frac{6\pm\sqrt{(-6)^2-4(2)(3)}}{2(2)}$

$n=\frac{6\pm\sqrt{12}}{4}$

n = 2.366 or n = 0.634

From (1) :

When n = 2.366, $m=2(2.366)-1$ = 3.732

When n = 0.634, $m=2(0.634)-1$ = 0.268

18 Perimeter = 42

$10+x+y+30-3y+x=42$

$2x-2y-2=0$

$x-y-1=0$

$x=y+1$ … (1)

Area = 86 cm^2

$10x+(10-y)^2=86$

$10x+100-20y+y^2-86=0$

$10x+14-20y+y^2=0$

Substitute $x=y+1$,

$10(y+1)+14-20y+y^2=0$

$10y+10+14-20y+y^2=0$

$y^2-10y+24=0$

$(y-6)(y-4)=0$

$y=6$ or $y=4$

From (1) :

$y=6$, $x=6+1=7$

$y=4$, $x=4+1=5$

19 Perimeter = 70 cm

$5x+y+12x+y+13x=70$

$30x+2y=70$

$15x+y=35$

$y=35-15x$ … (1)

Area = 240 cm^2

$\frac{1}{2}\times12x\times5x+12xy=240$

$30x^2+12xy=240$

$10x^2+4xy=80$

$5x^2+2xy=40$ … (2)

Substitute (1) into (2) :

$5x^2+2x(35-15x)=40$

$5x^2+70x-30x^2-40=0$

$-25x^2+70x-40=0$

$5x^2-14x+8=0$

$(x-2)(5x-4)=0$

$x=2$ or $x=\frac{4}{5}$

From (1) :

When $x=2$, $y=35-15(2)=5$

When $x=\frac{4}{5}$, $y=35-15\left(\frac{4}{5}\right)=23$

20 Area = 28 cm^2

$7xy = 28$

$xy = 4$ … (1)

Perimeter = 26 cm

$7x + y + y + \left(\frac{22}{7} \times \frac{7}{2}x\right) = 26$

$7x + 2y + 11x = 26$

$18x + 2y = 26$

$9x + y = 13$

$y = 13 - 9x$ … (2)

Substitute (2) into (1) :

$x(13 - 9x) = 4$

$13x - 9x^2 = 4$

$9x^2 - 13x + 4 = 0$

$(x - 1)(9x - 4) = 0$

$x = 1$ or $x = \frac{4}{9}$

$x = \frac{4}{9}$ is not accepted.

$\therefore x = 1$

From (2) : $y = 13 - 9(1) = 4$

21 Perimeter of fish pond = 48 m

$2(x - 10) + 2(30 - y) = 48$

$x - 10 + 30 - y = 24$

$x - y = 4$

$y = x - 4$ … (1)

Area of the region planted with papayas = 460 m^2

$(30 \times 10) + y(x - 10) = 460$

$-160 + xy - 10y = 0$ … (2)

Substitute (1) into (2) :

$-160 + x(x - 4) - 10(x - 4) = 0$

$-160 + x^2 - 4x - 10x + 40 = 0$

$x^2 - 14x - 120 = 0$

$(x - 20)(x + 6) = 0$

$x = 20$ or $x = -6$

$x = -6$ is not accepted.

Hence, $x = 20$

When $x = 20$, $y = 20 - 4 = 16$

22 Perimeter = 30

$x + (x + y) + (2x + 3) = 30$

$4x + y = 27$

$y = 27 - 4x$ … (1)

Using the Pythagoras' Theorem,

$x^2 + (x + y)^2 = (2x + 3)^2$

$x^2 + x^2 + 2xy + y^2 = 4x^2 + 12x + 9$

$2x^2 - 2xy - y^2 + 12x + 9 = 0$ … (2)

$2x^2 - 2x(27 - 4x) - (27 - 4x)^2 + 12x + 9 = 0$

$2x^2 - 54x + 8x^2 - (729 - 216x + 16x^2) + 12x + 9 = 0$

$-6x^2 + 174x - 720 = 0$

$6x^2 - 174x + 720 = 0$

$x^2 - 29x + 120 = 0$

$(x - 5)(x - 24) = 0$

$x = 5$ or $x = 24$

$x = 24$ is not accepted because it will make the value of y to be negative.

Hence, $x = 5$.

From (1) : $y = 27 - 4(5) = 7$

The lengths of the sides are x = 5 m, $(x + y)$ = 12 m and $(2x + 3)$ = 13 m respectively.

SPM Spot

1 $x + y + z = 0$ … (1)

$2x - 3y + 2z = 5$ … (2)

$x + 4y - 3z = -9$ … (3)

(1) – (3) : $-3y + 4z = 9$ … (4)

(1) × 2 : $2x + 2y + 2z = 0$ … (5)

(5) – (2) : $5y = -5$

$y = -1$

Substitute $y = -1$ into (4) :

$-3(-1) + 4z = 9$

$4z = 6$

$z = \frac{3}{2}$

Substitute $y = -1$ and $z = \frac{3}{2}$ into (1) :

$x - 1 + \frac{3}{2} = 0$

$x = -\frac{1}{2}$

2 The length of AB is twice the arc length PQ

$2y = \left(\frac{1}{4} \times 2\pi r\right) \times 2$

$2y = \pi r$

$2y = \pi \times 6x$

$y = 3\pi x$ … (1)

Area of the shaded region = $4(90x+27)\pi$

$$24xy-\frac{1}{4}\pi r^2 = 4(90x+27)\pi$$

$$24xy-\frac{1}{4}\pi(6x)^2 = (360x+108)\pi$$

$$24xy-9\pi x^2 = (360x+108)\pi \ldots(2)$$

Substitute (1) into (2) :

$$24x(3\pi x)-9\pi x^2 = (360x+108)\pi$$

$$72x^2-9x^2 = 360x+108$$

$$63x^2 = 360x+108$$

$$7x^2-40x-12 = 0$$

$$(x-6)(7x+2) = 0$$

$$x = 6 \text{ or } x = -\frac{2}{7}$$

$x = -\frac{2}{7}$ is not accepted.

$\therefore x = 6$

From (1) : $y = 3\pi(6)$

$$y = 18\pi$$

