Form 4 Chapter 2
Quadratic Functions
Fully-Worked Solutions

UPSKILL 2.1a

1 (a) $6x^2 - 7x - 3 = 0$

$$6x^2 - 7x = 3$$
$$x^2 - \frac{7}{6}x = \frac{3}{6}$$
$$x^2 - \frac{7}{6}x = \frac{1}{2}$$
$$x^2 - \frac{7}{6}x + \left(-\frac{7}{6} \times \frac{1}{2}\right)^2 = \frac{1}{2} + \left(-\frac{7}{6} \times \frac{1}{2}\right)^2$$
$$x^2 - \frac{7}{6}x + \frac{49}{144} = \frac{1}{2} + \frac{49}{144}$$
$$\left(x - \frac{7}{12}\right)^2 = \frac{121}{144}$$
$$x - \frac{7}{12} = \pm\frac{11}{12}$$
$$x = \frac{7}{12} + \frac{11}{12} \quad \text{or} \quad x = \frac{7}{12} - \frac{11}{12}$$
$$x = \frac{3}{2} \qquad\qquad x = -\frac{1}{3}$$

(b) $2p^2 - 10p + 3 = 0$

$$2p^2 - 10p = -3$$
$$p^2 - 5p = -\frac{3}{2}$$
$$p^2 - 5p + \left(-\frac{5}{2}\right)^2 = -\frac{3}{2} + \left(-\frac{5}{2}\right)^2$$
$$p^2 - 5p + \frac{25}{4} = -\frac{3}{2} + \frac{25}{4}$$
$$\left(p - \frac{5}{2}\right)^2 = \frac{19}{4}$$
$$p - \frac{5}{2} = \pm\sqrt{\frac{19}{4}}$$
$$p = \frac{5}{2} + \sqrt{\frac{19}{4}} \quad \text{or} \quad p = \frac{5}{2} - \sqrt{\frac{19}{4}}$$
$$p = 4.679 \qquad\qquad p = 0.321$$

2 (a) $s^2 + 1 = -\frac{10}{3}s$

$$3s^2 + 3 + 10s = 0$$
$$3s^2 + 10s + 3 = 0$$
$$s = \frac{-10 \pm \sqrt{10^2 - 4(3)(3)}}{2(3)}$$
$$s = \frac{-10 \pm \sqrt{64}}{6}$$
$$s = -\frac{1}{3} \text{ or } -3$$

(b) $\frac{11v - 2}{v + 3} = 2v$

$$11v - 2 = 2v^2 + 6v$$
$$2v^2 - 5v + 2 = 0$$
$$v = \frac{5 \pm \sqrt{(-5)^2 - (2)(2)}}{2(2)}$$
$$v = \frac{1}{2} \text{ or } 2$$

(c) $8 + x(2x + 35) = 10x(2x - 1)$

$$8 + 2x^2 + 35x = 20x^2 - 10x$$
$$18x^2 - 45x - 8 = 0$$
$$x = \frac{45 \pm \sqrt{(-45)^2 - 4(18)(-8)}}{2(18)}$$
$$x = \frac{45 \pm 51}{36}$$
$$x = \frac{8}{3} \text{ or } -\frac{1}{6}$$

3 (a) $(x - 1)(4x - 9) = 10x - 5$

$$4x^2 - 13x + 9 = 10x - 5$$
$$4x^2 - 13x - 10x + 9 + 5 = 0$$
$$4x^2 - 23x + 14 = 0$$
$$x = \frac{23 \pm \sqrt{(-23)^2 - 4(4)(14)}}{2(4)}$$
$$x = \frac{23 \pm \sqrt{305}}{8}$$
$$x = 5.058 \text{ or } 0.692$$

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(b) $\frac{z}{3}+4=z^2$

$z+12=3z^2$

$3z^2-z-12=0$

$z=\frac{1\pm\sqrt{(-1)^2-4(3)(-12)}}{2(3)}$

$z=\frac{1\pm\sqrt{145}}{6}$

$z=2.174$ or -1.840

(c) $\frac{y^2+3y-1}{y^2-y-1}=2$

$y^2+3y-1=2y^2-2y-2$

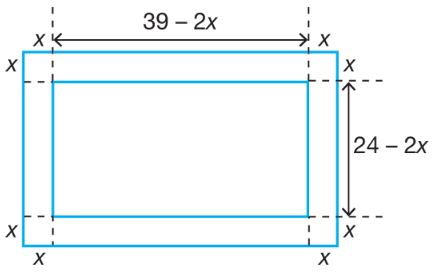
$y^2-5y-1=0$

$y=\frac{5\pm\sqrt{(-5)^2-4(-1)}}{2}$

$y=\frac{5\pm\sqrt{29}}{2}$

$y=-0.193$ or 5.193

4



Total area of the four pieces of wood

$= 39\times24 - (39-2x)(24-2x)$

$=936-(936-78x-48x+4x^2)$

$=126x-4x^2$

It is given that the total area of the four pieces of wood $=180$ cm^2

$-4x^2+126x=180$

$4x^2-126x+180=0$

$x=\frac{126\pm\sqrt{(-126)^2-4(4)(180)}}{2(4)}$

$x=\frac{126\pm\sqrt{12\,996}}{8}$

$x=1.5$ or 30

$x=30$ is not accepted.

$x=1.5$

UPSKILL 2.1b

1 (a) S.O.R. $=\frac{2}{3}+\frac{1}{4}=\frac{11}{12}$

P.O.R. $=\frac{2}{3}\times\frac{1}{4}=\frac{1}{6}$

The quadratic equation is

$x^2-\frac{11}{12}x+\frac{1}{6}=0$

$12x^2-11x+2=0$

(b) S.O.R. $=-5+4=-1$

P.O.R. $=-5\times4=-20$

The quadratic equation is

$x^2+x-20=0$

(c) S.O.R. $=-3-3=-6$

P.O.R. $=(-3)(-3)=9$

The quadratic equation is

$x^2+6x+9=0$

(d) S.O.R. $=\frac{2}{3}-\frac{2}{5}=\frac{4}{15}$

P.O.R. $=\frac{2}{3}\times\left(-\frac{2}{5}\right)=-\frac{4}{15}$

The quadratic equation is

$x^2-\frac{4}{15}x-\frac{4}{15}=0$

$15x^2-4x-4=0$

(e) S.O.R. $=-3-\frac{1}{2}=-\frac{7}{2}$

P.O.R. $=-3\left(-\frac{1}{2}\right)=\frac{3}{2}$

The quadratic equation is

$x^2+\frac{7}{2}x+\frac{3}{2}=0$

$2x^2+7x+3=0$

2 (a) $2x^2+4x-7=0$

S.O.R. $=-\frac{b}{a}=-\frac{4}{2}=-2$

P.O.R. $=\frac{c}{a}=-\frac{7}{2}$

(b) $3h^2-10h+5=0$

S.O.R. $=\frac{10}{3}$, P.O.R. $=\frac{5}{3}$

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3 (a) $2p^2+2p-3=0$

The roots are α and β.

Sum of roots = $-\frac{b}{a}$

$\alpha+\beta=-\frac{2}{2}=-1$

Product of roots = $\frac{c}{a}$

$\alpha\beta=-\frac{3}{2}$

The new roots are $\alpha+2$ and $\beta+2$.

S.O.R. = $(\alpha+2)+(\beta+2)$

$=\alpha+\beta+4$

$=-1+4$

$=3$

P.O.R. = $(\alpha+2)(\beta+2)$

$=\alpha\beta+2(\alpha+\beta)+4$

$=-\frac{3}{2}+2(-1)+4$

$=\frac{1}{2}$

The new quadratic equation is

$p^2-3p+\frac{1}{2}=0$

$2p^2-6p+1=0$

(b) The new roots are $\frac{2}{\alpha}$ and $\frac{2}{\beta}$.

S.O.R. = $\frac{2}{\alpha}+\frac{2}{\beta}$

$=\frac{2(\alpha+\beta)}{\alpha\beta}$

$=\frac{2(-1)}{-\frac{3}{2}}$

$=\frac{4}{3}$

P.O.R. = $\frac{2}{\alpha}\times\frac{2}{\beta}$

$=\frac{4}{\alpha\beta}$

$=\frac{4}{-\frac{3}{2}}$

$=-\frac{8}{3}$

The new quadratic equation is

$p^2-\frac{4}{3}p-\frac{8}{3}=0$

$3p^2-4p-8=0$

4 (a) $2t^2-5t+1=0$

The roots are α and β.

Sum of roots = $-\frac{b}{a}$

$\alpha+\beta=\frac{5}{2}$

Product of roots = $\frac{c}{a}$

$\alpha\beta=\frac{1}{2}$

The new roots are $\frac{\alpha}{3}$ and $\frac{\beta}{3}$.

S.O.R. = $\frac{\alpha}{3}+\frac{\beta}{3}$

$=\frac{\alpha+\beta}{3}$

$=\frac{\frac{5}{2}}{3}$

$=\frac{5}{6}$

P.O.R. = $\frac{\alpha}{3}\times\frac{\beta}{3}$

$=\frac{\alpha\beta}{9}$

$=\frac{\frac{1}{2}}{9}$

$=\frac{1}{18}$

The new quadratic equation is

$t^2-\frac{5}{6}t+\frac{1}{18}=0$

$18t^2-15t+1=0$

(b) The new roots are $3-\alpha$ and $3-\beta$.

S.O.R. = $3-\alpha+3-\beta$

$=6-(\alpha+\beta)$

$=6-\frac{5}{2}$

$=\frac{7}{2}$

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$$\text{P.O.R.} = (3-\alpha)(3-\beta)$$
$$= 9-(\alpha+\beta)+\alpha\beta$$
$$= 9-\frac{5}{2}+\frac{1}{2}$$
$$= 7$$

The new quadratic equation is

$$t^2-\frac{7}{2}t+7=0$$
$$2t^2-7t+14=0$$

5 $x^2+9x+q=0$

The roots are α and 2α.

$$\text{S.O.R.} = -\frac{b}{a}$$
$$\alpha+2\alpha=-9$$
$$3\alpha=-9$$
$$\alpha=-3$$

$$\text{P.O.R.} = \frac{c}{a}$$
$$\alpha\times2\alpha=q$$
$$q=2(-3)^2=18$$

6 $5x^2+px+1=0$

The roots are α and 5α.

$$\text{S.O.R.} = -\frac{b}{a}$$
$$\alpha+5\alpha=-\frac{p}{5}$$
$$6\alpha=-\frac{p}{5}$$
$$\alpha=-\frac{p}{30}$$

$$\text{P.O.R.} = \frac{c}{a}$$
$$\alpha\times5\alpha=\frac{1}{5}$$
$$5\alpha^2=\frac{1}{5}$$
$$5\left(-\frac{p}{30}\right)^2=\frac{1}{5}$$
$$\frac{p^2}{900}=\frac{1}{25}$$
$$p^2=\frac{900}{25}$$
$$p^2=\frac{900}{25}$$
$$p^2=36$$
$$p=\pm6$$

7 $2x^2-(d+3)x+d=0$

The roots are α and 4α.

$$\text{S.O.R.} = -\frac{b}{a}$$
$$\alpha+4\alpha=\frac{d+3}{2}$$
$$2(\alpha+4\alpha)=d+3$$
$$10\alpha=d+3 \quad \dots (1)$$
$$\alpha=\frac{d+3}{10}$$

$$\text{P.O.R.} = \frac{c}{a}$$
$$\alpha\times4\alpha=\frac{d}{2}$$
$$4\alpha^2=\frac{d}{2}$$
$$\alpha^2=\frac{d}{8}$$
$$\left(\frac{d+3}{10}\right)^2=\frac{d}{8}$$
$$\frac{(d+3)^2}{100}=\frac{d}{8}$$
$$\frac{(d+3)^2}{25}=\frac{d}{2}$$
$$2(d+3)^2=25d$$
$$2(d^2+6d+9)=25d$$
$$2d^2+12d+18-25d=0$$
$$2d^2-13d+18=0$$
$$(d-2)(2d-9)=0$$
$$d=2 \text{ or } \frac{9}{2}$$

8 $2x^2+hx-4=0$

The roots are 4 and k.

$$\text{S.O.R.} = -\frac{b}{a}$$
$$k+4=-\frac{h}{2}$$
$$2k+8=-h$$
$$h=-2k-8 \quad \dots (1)$$

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P.O.R. $= \dfrac{c}{a}$

$4k = -\dfrac{4}{2}$

$k = -\dfrac{1}{2}$

From (1) : $h = -2\left(-\dfrac{1}{2}\right) - 8 = -7$

9 $8x^2 + 26x + k = 0$

The roots are $-\dfrac{5}{2}$ and m.

S.O.R. $= -\dfrac{b}{a}$

$-\dfrac{5}{2} + m = -\dfrac{26}{8}$

$m = -\dfrac{26}{8} + \dfrac{5}{2}$

$m = -\dfrac{3}{4}$

P.O.R. $= \dfrac{c}{a}$

$-\dfrac{5}{2}m = \dfrac{k}{8}$

$-\dfrac{5}{2}\left(-\dfrac{3}{4}\right) = \dfrac{k}{8}$

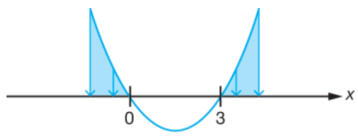
$\dfrac{15}{8} = \dfrac{k}{8}$

$k = 15$

UPSKILL 2.1c

1 (a) $x^2 - 3x \geq 0$

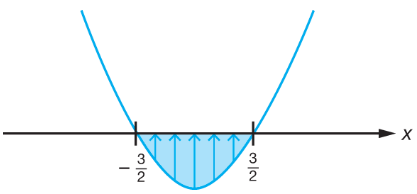
$x(x-3) \geq 0$



The required range of values of x is $x \leq 0$ or $x \geq 3$.

(b) $4x^2 - 9 \leq 0$

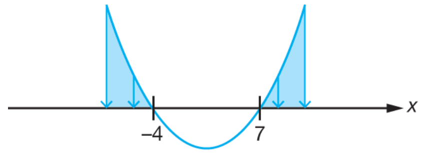
$(2x+3)(2x-3) \leq 0$



The required range of values of x is $-\dfrac{3}{2} \leq x \leq \dfrac{3}{2}$.

(c) $x^2 - 3x - 28 > 0$

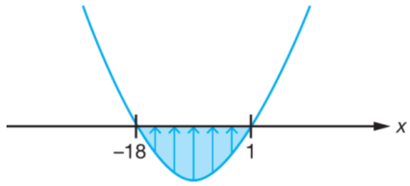
$(x+4)(x-7) > 0$



The required range of values of x is $x < -4$ or $x > 7$.

(d) $x^2 + 17x - 18 < 0$

$(x+18)(x-1) < 0$

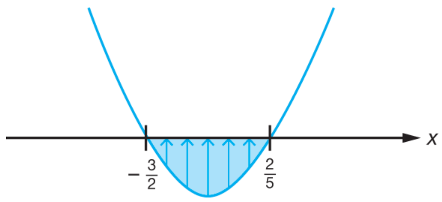


The required range of values of x is $-18 < x < 1$.

(e) $6 - 11x - 10x^2 \geq 0$

$10x^2 + 11x - 6 \leq 0$

$(2x+3)(5x-2) \leq 0$

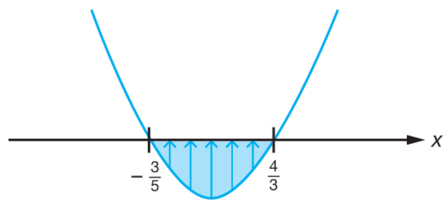


The required range of values of x is $-\dfrac{3}{2} \leq x \leq \dfrac{2}{5}$.

(f) $12 + 11x - 15x^2 \geq 0$

$15x^2 - 11x - 12 \leq 0$

$(5x+3)(3x-4) \leq 0$



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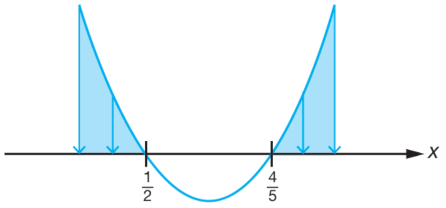
The range of values of x is $-\frac{3}{5} \le x \le \frac{4}{3}$.

(g) $10x^2 > 13x - 4$

$10x^2 - 13x + 4 > 0$

$(2x-1)(5x-4) > 0$



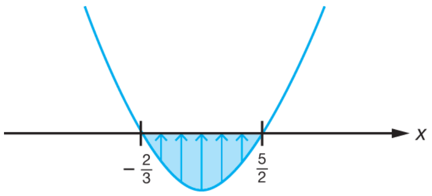
The required range of values of x is

$x < \frac{1}{2}$ or $x > \frac{4}{5}$.

(h) $11x + 10 \ge 6x^2$

$6x^2 - 11x - 10 \le 0$

$(3x+2)(2x-5) \le 0$



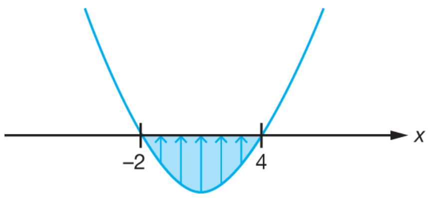
The required range of values of x is

$-\frac{2}{3} \le x \le \frac{5}{2}$.

(i) $x(x-2) \le 8$

$x^2 - 2x - 8 \le 0$

$(x+2)(x-4) \le 0$



The range of values of x is $-2 \le x \le 4$.

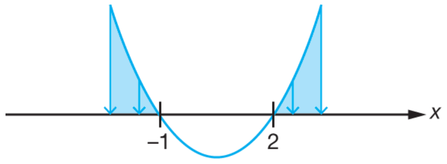
(j) $(2x-1)^2 > 9$

$4x^2 - 4x + 1 - 9 > 0$

$4x^2 - 4x - 8 > 0$

$x^2 - x - 2 > 0$

$(x+1)(x-2) > 0$



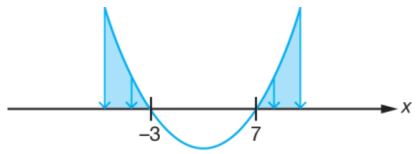
The required range of values of x is
$x < -1$ or $x > 2$.

(k) $(x+1)(x-5) \ge 16$

$x^2 - 4x - 5 - 16 \ge 0$

$x^2 - 4x - 21 \ge 0$

$(x+3)(x-7) \ge 0$



The required range of values of x is
$x \le -3$ or $x \ge 7$.

2 $-1 < x^2 + 3x + 1 \le 1$.

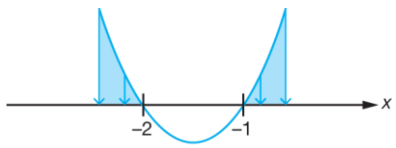
The first inequality is

$-1 < x^2 + 3x + 1$

$x^2 + 3x + 1 + 1 > 0$

$x^2 + 3x + 2 > 0$

$(x+1)(x+2) > 0$



The range of values of x is
$x < -2$ or $x > -1$ … (1)

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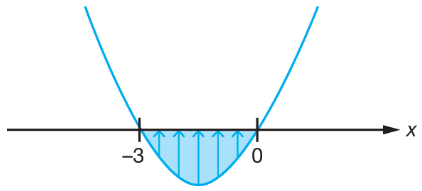
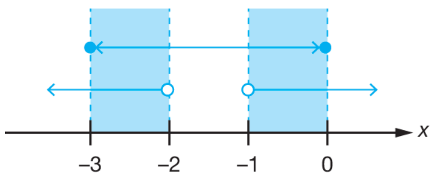
The second inequality is

$$x^2 + 3x + 1 \le 1$$
$$x^2 + 3x \le 0$$
$$x(x+3) \le 0$$



The range of values of x is
$-3 \le x \le 0$ … (2)

Combining (1) and (2) :



The required range of values of x is
$-3 \le x < -2$ or $-1 < x \le 0$.

UPSKILL 2.2a

1 (a) $2x^2 - 8x + 3 = 0$

$b^2 - 4ac = (-8)^2 - 4(2)(3) = 40$

Since $b^2 - 4ac > 0$, then the quadratic equation has real and distinct roots.

(b) $3x^2 - 2x + 9 = 0$

$b^2 - 4ac = (-2)^2 - 4(3)(9) = -104$

Since $b^2 - 4ac < 0$, then the quadratic equation does not have real roots.

(c) $x^2 + 10x + 25 = 0$

$b^2 - 4ac = 10^2 - 4(1)(25) = 0$

Since $b^2 - 4ac = 0$, then the quadratic equation has real and equal roots.

(d) $-2x^2 + 6x + 3 = 0$

$b^2 - 4ac = 6^2 - 4(-2)(3) = 60$

Since $b^2 - 4ac > 0$, then the quadratic equation has real and distinct roots.

(e) $3x^2 - 6x + 4 = 0$

$b^2 - 4ac = (-6)^2 - 4(3)(4) = -12$

Since $b^2 - 4ac < 0$, then the quadratic equation does not have real roots.

(f) $4x^2 - 12x + 9 = 0$

$b^2 - 4ac = (-12)^2 - 4(4)(9) = 0$

Since $b^2 - 4ac = 0$, then the quadratic equation has real and equal roots.

UPSKILL 2.2b

1 (a) $x^2 - 2hx + 3h + 4 = 0$

$a = 1,\ b = -2h,\ c = 3h + 4$

$$b^2 - 4ac = 0$$
$$(-2h)^2 - 4(1)(3h+4) = 0$$
$$4h^2 - 12h - 16 = 0$$
$$h^2 - 3h - 4 = 0$$
$$(h+1)(h-4) = 0$$
$$h = -1 \text{ or } 4$$

(b) $x^2 - 2(3+h)x - h - 1 = 0$

$a = 1,\ b = -2(3+h),\ c = -h - 1$

$$b^2 - 4ac = 0$$
$$[-2(3+h)]^2 - 4(1)(-h-1) = 0$$
$$4(3+h)^2 + 4h + 4 = 0$$
$$(3+h)^2 + h + 1 = 0$$
$$h^2 + 6h + 9 + h + 1 = 0$$
$$h^2 + 7h + 10 = 0$$
$$(h+2)(h+5) = 0$$
$$h = -2 \text{ or } -5$$

(c) $hx^2 + 8x = 8hx - 36$

$hx^2 + 8x - 8hx + 36 = 0$

$a = h,\ b = 8 - 8h,\ c = 36$

$$b^2 - 4ac = 0$$
$$(8 - 8h)^2 - 4h(36) = 0$$
$$64 - 128h + 64h^2 - 144 = 0$$
$$64h^2 - 128h - 80 = 0$$
$$4h^2 - 8h - 5 = 0$$
$$(2h+1)(2h-5) = 0$$
$$h = -\frac{1}{2} \text{ or } \frac{5}{2}$$

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(d) $(3-h)x^2+h+1=2(h+1)x$

$(3-h)x^2-2(h+1)x+h+1=0$

$a=3-h,\ b=-2(h+1),\ c=h+1$

$b^2-4ac=0$

$[-2(h+1)]^2-4(3-h)(h+1)=0$

$4(h+1)^2-4(3h+3-h^2-h)=0$

$(h+1)^2-(-h^2+2h+3)=0$

$h^2+2h+1+h^2-2h-3=0$

$2h^2-2=0$

$h^2-1=0$

$h=\pm 1$

2 $x^2+2kx=5p-1$

$x^2+2kx+1-5p=0$

$a=1,\ b=-2k,\ c=-5p+1$

$b^2-4ac=0$

$(-2k)^2-4(1)(-5p+1)=0$

$4k^2+20p-4=0$

$k^2+5p-1=0$

$5p=1-k^2$

$p=\dfrac{1-k^2}{5}$

3 $3mx+q=2-2x^2$

$2x^2+3mx+q-2=0$

$a=2,\ b=3m,\ c=q-2$

$b^2-4ac=0$

$(3m)^2-4(2)(q-2)=0$

$9m^2-8q+16=0$

$8q=9m^2+16$

$q=\dfrac{9m^2+16}{8}$

4 (a) $x^2+k=kx-3$

$x^2-kx+k+3=0$

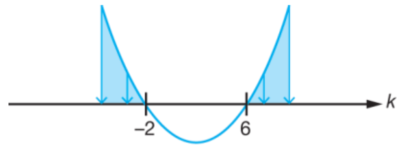
$a=1,\ b=-k,\ k+3$

$b^2-4ac>0$

$(-k)^2-4(1)(k+3)>0$

$k^2-4k-12>0$

$(k+2)(k-6)>0$



Hence, the required range of values of k is $k<-2$ or $k>6$.

(b) $kx^2+k=6-8x$

$kx^2+8x+k-6=0$

$a=k,\ b=8,\ c=k-6$

$b^2-4ac>0$

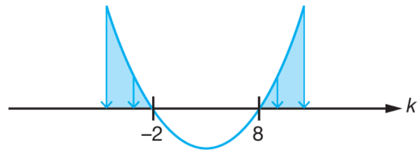
$8^2-4k(k-6)>0$

$16-k(k-6)>0$

$16-k^2+6k>0$

$k^2-6k-16<0$

$(k+2)(k-8)<0$



The required range of values of k is $-2<k<8$.

(c) $x^2+2k=(2k-3)x$

$x^2-(2k-3)x+2k=0$

$a=1,\ b=-(2k-3),\ c=2k$

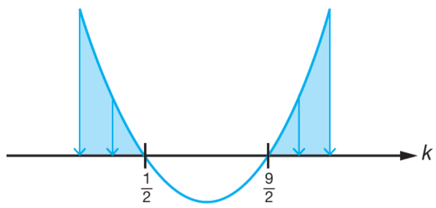
$b^2-4ac>0$

$[-(2k-3)]^2-4(1)(2k)>0$

$4k^2-12k+9-8k>0$

$4k^2-20k+9>0$

$(2k-9)(2k-1)>0$



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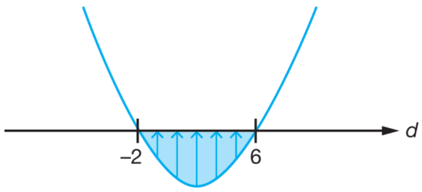
The required range of values of k is

$k < \frac{1}{2}$ or $k > \frac{9}{2}$.

5 (a) $x^2 - dx + d + 3 = 0$

$a = 1, b = -d, c = d + 3$

$$b^2 - 4ac < 0$$
$$(-d)^2 - 4(1)(d+3) < 0$$
$$d^2 - 4d - 12 < 0$$
$$(d+2)(d-6) < 0$$



The required range of values of d is

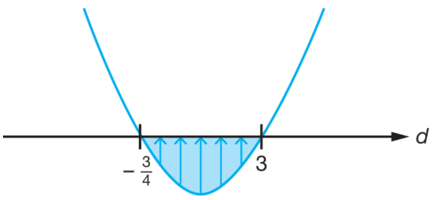
$-2 < d < 6$.

(b) $dx^2 + 4dx = -9 - x^2$

$$(d+1)x^2 - 4dx + 9 = 0$$

$a = d + 1, b = -4d, c = 9$

$$b^2 - 4ac < 0$$
$$(-4d)^2 - 4(d+1)(9) < 0$$
$$16d^2 - 36(d+1) < 0$$
$$4d^2 - 9(d+1) < 0$$
$$4d^2 - 9d - 9 < 0$$
$$(d-3)(4d+3) < 0$$



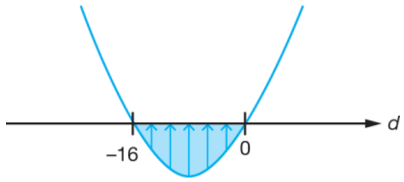
The required range of values of d is

$-\frac{3}{4} < d < 3$.

(c) $(2 - 3d)x^2 + 2 = (d - 4)x$

$a = (2 - 3d), b = -(d - 4), c = 2$

$$b^2 - 4ac < 0$$
$$[-(d-4)]^2 - 4(2-3d)(2) < 0$$
$$d^2 - 8d + 16 - 16 + 24d < 0$$
$$d^2 + 16d < 0$$
$$d(d+16) < 0$$



The required range of values of d is

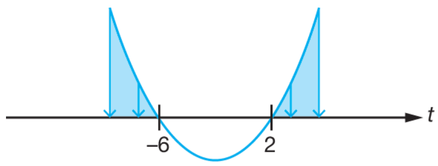
$-16 < d < 0$.

6 $2x^2 - tx + 1 = 2x - 1$

$$2x^2 - tx - 2x + 2 = 0$$

$a = 2, b = -t - 2, c = 2$

$$b^2 - 4ac \geq 0$$
$$(-t-2)^2 - 4(2)(2) \geq 0$$
$$t^2 + 4t + 4 - 16 \geq 0$$
$$t^2 + 4t - 12 \geq 0$$
$$(t+6)(t-2) \geq 0$$



The required range of values of k is

$t \leq -6$ or $t \geq 2$.

UPSKILL 2.3a

1 (a) $f(x) = 9x^2 - 12x + 8$

Since $a > 0$, then the graph has the shape of $\cup$.

(b) $g(x) = -2x^2 - 5x + 3$

Since $a < 0$, then the graph has the shape of $\cap$.

(c) $h(x) = (x+1)^2 - 4$

Since $a > 0$, then the graph has the shape of $\cup$.

(d) $m(x) = 1 - (2 - x)^2$

Since $a < 0$, then the graph has the shape of $\cap$.

UPSKILL 2.3b

1 (a) The shape of the graph is $\cup$ and it touches the x-axis at only a point.

Its function is $d(x) = 4x^2 - 20x + 25$

because $(-20)^2 - 4(4)(25) = 0$.

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(b) The shape of the graph is $\cup$ and it intersects the x-axis at two different points.

Its function is $p(x) = x^2 - 6x + 8$

because $(-6)^2 - 4(1)(8) = 4$ (> 0).

(c) The shape of the graph is $\cap$ and it does not intersect the x-axis.

Its function is $q(x) = -2x^2 - 3x - 4$

because $(-3)^2 - 4(-2)(-4) = -23$ (< 0).

(d) The shape of the graph is $\cap$ and it intersects the x-axis at two different points.

Its function is $h(x) = 5x - 6 - x^2 =$

$-x^2 + 5x - 6$ because

$5^2 - 4(-1)(-6) = 1$ (> 0).

(e) The shape of the graph is $\cup$ and it does not intersect the x-axis.

Its function is $k(x) = x^2 - 2x + 4$

because $(-2)^2 - 4(1)(4) = -12$.

(f) The shape of the graph is $\cap$ and it touches the x-axis at only a point.

Its function is

$m(x) = 8x - 16 - x^2 = -x^2 + 8x - 16$

because $(8)^2 - 4(-1)(-16) = 0$.

2 (a) No real roots

(b) Real and distinct roots

(c) Real and equal roots

(d) No real roots

(e) Real and equal roots

(f) Real and distinct roots

3 (a) $f(x) = -2x^2 + 3x - 4$

$b^2 - 4ac = 3^2 - 4(-2)(-4) = -23$ (< 0)

The graph of $f(x)$ will not intersect the x-axis.

(b) $g(x) = 4x^2 - 3x - 5$

$b^2 - 4ac = (-3)^2 - 4(4)(-5) = 89$ (> 0)

The graph of $f(x)$ will intersect the x-axis.

(c) $m(x) = (x-2)^2 + 3$

$= x^2 - 4x + 4 + 3$

$= x^2 - 4x + 7$

$b^2 - 4ac = (-4)^2 - 4(1)(7) = -12$ (< 0)

The graph of $f(x)$ will not intersect the x-axis.

(d) $n(x) = 5 - (2x+1)^2$

$= 5 - \left(4x^2 + 4x + 1\right)$

$= -4x^2 - 4x + 4$

$b^2 - 4ac = (-4)^2 - 4(-4)(4) = 80$ (> 0)

The graph of $f(x)$ will intersect the x-axis.

4 (a) $f(x) = x^2 - (w+4)x + 1$

$b^2 - 4ac = 0$

$[-(w+4)]^2 - 4(1)(1) = 0$

$w^2 + 8w + 16 - 4 = 0$

$w^2 + 8w + 12 = 0$

$(w+2)(w+6) = 0$

$w = -2$ or -6

(b) $g(x) = x^2 - wx + w + 3$

$b^2 - 4ac = 0$

$(-w)^2 - 4(1)(w+3) = 0$

$w^2 - 4w - 12 = 0$

$(w+2)(w-6) = 0$

$w = -2$ or 6

(c) $h(x) = (4-2w)x^2 + 3wx - 2w - 1$

$b^2 - 4ac = 0$

$(3w)^2 - 4(4-2w)(-2w-1) = 0$

$9w^2 - 4\left(-6w + 4w^2 - 4\right) = 0$

$9w^2 + 24w - 16w^2 + 16 = 0$

$-7w^2 + 24w + 16 = 0$

$7w^2 - 24w - 16 = 0$

$(w-4)(7w+4) = 0$

$w = 4$ or $-\frac{4}{7}$

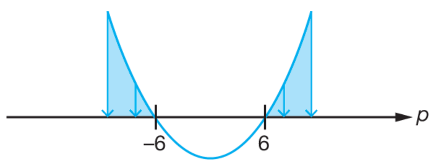
5 (a) $f(x) = 3x^2 + px + 3$

$b^2 - 4ac > 0$

$p^2 - 4(3)(3) > 0$

$p^2 - 36 > 0$

$(p+6)(p-6) > 0$



The required range of values of p is $p < -6$ or $p > 6$.

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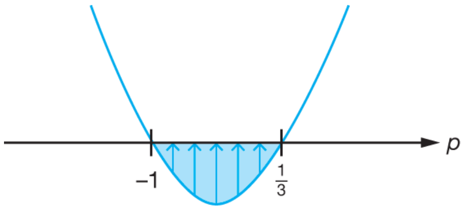
(b) $g(x) = px^2 + (p+1)x + p + 1$

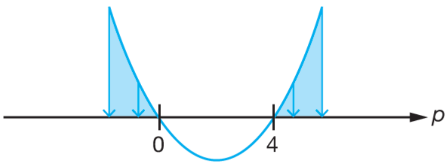
$$b^2 - 4ac > 0$$
$$(p+1)^2 - 4p(p+1) > 0$$
$$p^2 + 2p + 1 - 4p^2 - 4p > 0$$
$$-3p^2 - 2p + 1 > 0$$
$$3p^2 + 2p - 1 < 0$$
$$(p+1)(3p-1) < 0$$



The required range of values of p is
$-1 < p < \frac{1}{3}$.

(c) $m(x) = x^2 + (2-2p)x + 2p + 1$

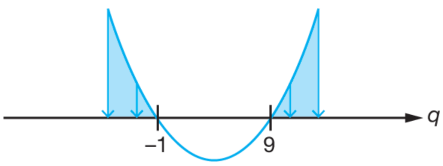
$$b^2 - 4ac > 0$$
$$(2-2p)^2 - 4(1)(2p+1) > 0$$
$$4 - 8p + 4p^2 - 8p - 4 > 0$$
$$4p^2 - 16p > 0$$
$$p^2 - 4p > 0$$
$$p(p-4) > 0$$



The required range of values of p is
$p < 0$ or $p > 4$.

6 (a) $f(x) = qx^2 + 6x + q - 8$

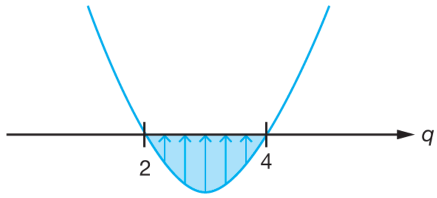
$$b^2 - 4ac < 0$$
$$6^2 - 4q(q-8) < 0$$
$$36 - 4q^2 + 32q < 0$$
$$-4q^2 + 32q + 36 < 0$$
$$q^2 - 8q - 9 > 0$$
$$(q-9)(q+1) > 0$$



The required range of values of q is
$q < -1$ or $q > 9$.

(b) $g(x) = 4x^2 + 4(3-q)x + 1$

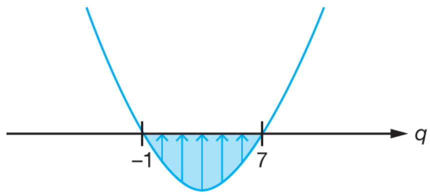
$$b^2 - 4ac < 0$$
$$[4(3-q)]^2 - 4(4)(1) < 0$$
$$16(3-q)^2 - 16 < 0$$
$$(3-q)^2 - 1 < 0$$
$$9 - 6q + q^2 - 1 < 0$$
$$q^2 - 6q + 8 < 0$$
$$(q-2)(q-4) < 0$$



The required range of values of q is
$2 < q < 4$.

(c) $m(x) = x^2 + (q-1)x + q + 2$

$$b^2 - 4ac < 0$$
$$(q-1)^2 - 4(1)(q+2) < 0$$
$$q^2 - 2q + 1 - 4q - 8 < 0$$
$$q^2 - 6q - 7 < 0$$
$$(q+1)(q-7) < 0$$



The required range of values of q is
$-1 < q < 7$.

UPSKILL 2.3c

1 (a) $f(x) = x^2 - 2x + 3$

$$= x^2 - 2x + \left[\frac{(-2)}{2}\right]^2 - \left[\frac{(-2)}{2}\right]^2 + 3$$
$$= x^2 - 2x + 1 - 1 + 3$$
$$= (x-1)^2 + 2$$

Minimum value = 2 when x = 1.

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(b) $g(x) = 2 + 6x - x^2$

$= -(x^2 - 6x - 2)$

$= -\left[x^2 - 6x + \left(\frac{-6}{2}\right)^2 - \left(\frac{-6}{2}\right)^2 - 2\right]$

$= -(x^2 - 6x + 9 - 9 - 2)$

$= -[(x-3)^2 - 11]$

$= -(x-3)^2 + 11$

Maximum value = 11 when x = 3.

(c) $q(x) = 2x^2 + 8x - 1$

$= 2\left(x^2 + 4x - \frac{1}{2}\right)$

$= 2\left[x^2 + 4x + \left(\frac{4}{2}\right)^2 - \left(\frac{4}{2}\right)^2 - \frac{1}{2}\right]$

$= 2\left(x^2 + 4x + 4 - 4 - \frac{1}{2}\right)$

$= 2\left[(x+2)^2 - \frac{9}{2}\right]$

$= 2(x+2)^2 - 9$

Minimum value = –9 when x = –2

(d) $m(x) = 5 - 4x - 2x^2$

$= -2\left(x^2 + 2x - \frac{5}{2}\right)$

$= -2\left[x^2 + 2x + \left(\frac{2}{2}\right)^2 - \left(\frac{2}{2}\right)^2 - \frac{5}{2}\right]$

$= -2\left(x^2 + 2x + 1 - 1 - \frac{5}{2}\right)$

$= -2\left[(x+1)^2 - \frac{7}{2}\right]$

$= -2(x+1)^2 + 7$

Maximum value = 7 when $x = -1$

(e) $n(x) = 5x^2 + 8x - 10$

$= 5\left(x^2 + \frac{8}{5}x - 2\right)$

$= 5\left[x^2 + \frac{8}{5}x + \left(\frac{1}{2} \times \frac{8}{5}\right)^2 - \left(\frac{1}{2} \times \frac{8}{5}\right)^2 - 2\right]$

$= 5\left(x^2 + \frac{8}{5}x + \frac{16}{25} - \frac{16}{25} - 2\right)$

$= 5\left[\left(x + \frac{4}{5}\right)^2 - \frac{66}{25}\right]$

$= 5\left[\left(x + \frac{4}{5}\right)^2 - \frac{66}{25}\right]$

$= 5\left(x + \frac{4}{5}\right)^2 - \frac{66}{5}$

Minimum value = $-\frac{66}{5}$ when $x = -\frac{4}{5}$.

(f) $p(x) = 6x - 9 - 4x^2$

$= -4\left(x^2 - \frac{6}{4}x + \frac{9}{4}\right)$

$= -4\left(x^2 - \frac{3}{2}x + \frac{9}{4}\right)$

$= -4\left[x^2 - \frac{3}{2}x + \left(-\frac{3}{4}\right)^2 - \left(-\frac{3}{4}\right)^2 + \frac{9}{4}\right]$

$= -4\left(x^2 - \frac{3}{2}x + \frac{9}{16} - \frac{9}{16} + \frac{9}{4}\right)$

$= -4\left[\left(x - \frac{3}{4}\right)^2 + \frac{27}{16}\right]$

$= -4\left(x - \frac{3}{4}\right)^2 - \frac{27}{4}$

Maximum value = $-\frac{27}{4}$ when $x = \frac{3}{4}$.

2 $f(x) = x^2 + 6x + k$

$= x^2 + 6x + \left(\frac{6}{2}\right)^2 - \left(\frac{6}{2}\right)^2 + k$

$= x^2 + 6x + 9 - 9 + k$

$= (x+3)^2 - 9 + k$

Minimum value = –2

$-9 + k = -2$

$k = 7$

3 $m(x) = 3x(2-x) + q$

$= 6x - 3x^2 + q$

$= -3\left(x^2 - 2x - \frac{q}{3}\right)$

$= -3\left[x^2 - 2x + \left(\frac{-2}{2}\right)^2 - \left(\frac{-2}{2}\right)^2 - \frac{q}{3}\right]$

$= -3\left(x^2 - 2x + 1 - 1 - \frac{q}{3}\right)$

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$$= -3\left[(x-1)^2 - 1 - \frac{q}{3}\right]$$
$$= -3(x-1)^2 + 3 + q$$

Maximum value = 5
$$3 + q = 5$$
$$q = 2$$

4 $g(x) = 4 - 3x - dx^2$
$$= -d\left(x^2 + \frac{3}{d} - \frac{4}{d}\right)$$
$$= -d\left[x^2 + \frac{3}{d} + \left(\frac{3}{2d}\right)^2 - \left(\frac{3}{2d}\right)^2 - \frac{4}{d}\right]$$
$$= -d\left[x^2 + \frac{3}{d} + \left(\frac{9}{4d^2}\right) - \left(\frac{9}{4d^2}\right) - \frac{4}{d}\right]$$
$$= -d\left(x + \frac{3}{2d}\right)^2 + \left(\frac{9}{4d}\right) + 4$$

Maximum value = $\frac{41}{8}$
$$\frac{9}{4d} + 4 = \frac{41}{8}$$
$$\frac{9}{4d} = \frac{9}{8}$$
$$d = 2$$

5 $f(x) = -3x^2 + px + 18$
$$= -3\left(x^2 - \frac{p}{3}x - 6\right)$$
$$= -3\left[x^2 - \frac{p}{3}x + \left(-\frac{p}{6}\right)^2 - \left(-\frac{p}{6}\right)^2 - 6\right]$$
$$= -3\left[x^2 - \frac{p}{3}x + \left(\frac{p^2}{36}\right) - \left(\frac{p^2}{36}\right) - 6\right]$$
$$= -3\left[\left(x - \frac{p}{6}\right)^2 - \left(\frac{p^2}{36}\right) - 6\right]$$
$$= -3\left(x - \frac{p}{6}\right)^2 + \left(\frac{p^2}{12}\right) + 18$$

Maximum value = q when $x = -2$

Maximum value = $\left(\frac{p^2}{12}\right) + 18$ when

$$x = \frac{p}{6}$$

By comparison, $\frac{p}{6} = -2$
$$p = -12$$

$$q = \left(\frac{p^2}{12}\right) + 18$$
$$q = \left(\frac{144}{12}\right) + 18 = 12 + 18 = 30$$

6 $f(x) = tx^2 - 12x + 20$
$$= t\left(x^2 - \frac{12}{t} + \frac{20}{t}\right)$$
$$= t\left[x^2 - \frac{12}{t} + \left(-\frac{12}{2t}\right)^2 - \left(-\frac{12}{2t}\right)^2 + \frac{20}{t}\right]$$
$$= t\left[x^2 - \frac{12}{t} + \left(\frac{36}{t^2}\right) - \left(\frac{36}{t^2}\right) + \frac{20}{t}\right]$$
$$= t\left[\left(x - \frac{6}{t}\right)^2 - \left(\frac{36}{t^2}\right) + \frac{20}{t}\right]$$
$$= t\left(x - \frac{6}{t}\right)^2 - \frac{36}{t} + 20$$

$f(x)$ has a maximum value when $x = \frac{6}{t}$.

But it is given that $f(x)$ has a maximum value when $x = -2$.

By comparison, $\frac{6}{t} = -2$
$$t = -3$$

Maximum value = $-\frac{36}{t} + 20$
$$= -\frac{36}{-3} + 20$$
= 12 + 20
= 32

UPSKILL 2.3d

1 (a) $g(x) = a(x-h)^2 + k$
From the graph,
$$g(x) = a(x-3)^2 - 2$$
By comparison, $h = 3$ and $k = -2$

(b) $g(x) = a(x-3)^2 - 2$
When $x = 0$, $y = -6$
$$g(0) = a(0-3)^2 - 2 = -6$$
$$9a = -4$$
$$a = -\frac{4}{9}$$

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(c) The equation of the axis of symmetry is $x = 3$.

2 (a) Equation of axis of symmetry is

$$x = \frac{-1+3}{2}$$
$$x = 1$$

(b) $f(x) = a(x-h)^2 + k$

But it is given that

$$f(x) = a(x-1)^2 - 5$$

Hence, $h = 1$ and $k = -5$

(c) When x = 3, y = 0.

$$f(3) = a(3-1)^2 - 5$$
$$0 = 4a - 5$$
$$a = \frac{5}{4}$$

3 (a) The equation of the axis of symmetry is

$$x = \frac{1+3}{2}$$
$$x = 2$$

(b) $f(x) = a(x-k)^2 - 4$

$$x = k$$

It is found in (a) that $x = 2$.
By comparison, k = 2.

(c) $f(x) = a(x-2)^2 - 4$

The coordinates of the minimum point are (2, –4).

(d) When x = 1, y = 0

$$f(x) = a(x-k)^2 - 4$$
$$0 = a(1-2)^2 - 4$$
$$a = 4$$

(e) When the curve is reflected in the x-axis, each term will change, i.e.

$$f(x) = -4(x-2)^2 + 4$$

(f) When the curve is reflected in the y-axis, the value of h will change, i.e.

$$f(x) = 4(x+2)^2 - 4$$

UPSKILL 2.3e

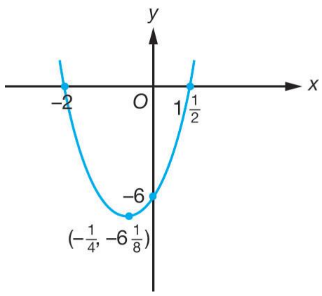
1 (a) $f(x) = (2x-3)(x+2)$

$$= 2x^2 + x - 6$$
$$= 2\left(x^2 + \frac{x}{2} - 3\right)$$
$$= 2\left[x^2 + \frac{x}{2} + \left(\frac{1}{2(2)}\right)^2 - \left(\frac{1}{2(2)}\right)^2 - 3\right]$$
$$= 2\left[x^2 + \frac{x}{2} + \left(\frac{1}{16}\right) - \left(\frac{1}{16}\right) - 3\right]$$
$$= 2\left[\left(x + \frac{1}{4}\right)^2 - \frac{49}{16}\right]$$
$$= 2\left(x + \frac{1}{4}\right)^2 - \frac{49}{8}$$

The minimum point is $\left(-\frac{1}{4}, -6\frac{1}{8}\right)$.

The y-intercept is –6.

The x-intercepts are $1\frac{1}{2}$ and –2.



The equation of the axis of symmetry is $x = -\frac{1}{4}$.

(b) $g(x) = (1+x)(3-2x)$

$$= -2x^2 + x + 3$$
$$= -2\left(x^2 - \frac{1}{2}x - \frac{3}{2}\right)$$
$$= -2\left[x^2 - \frac{1}{2}x + \left(\frac{-1}{2\times 2}\right)^2 - \left(\frac{-1}{2\times 2}\right)^2 - \frac{3}{2}\right]$$
$$= -2\left[x^2 - \frac{1}{2}x + \left(\frac{1}{16}\right) - \left(\frac{1}{16}\right) - \frac{3}{2}\right]$$
$$= -2\left[\left(x - \frac{1}{4}\right)^2 - \left(\frac{25}{16}\right)\right]$$

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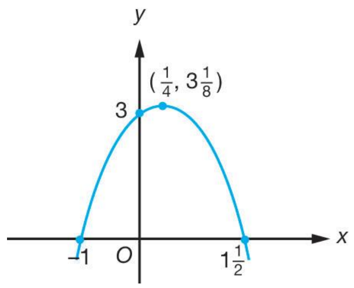
$$= -2\left(x - \frac{1}{4}\right)^2 + \frac{25}{8}$$

The maximum point is $\left(\frac{1}{4}, 3\frac{1}{8}\right)$.

The y-intercept is 3.

The x-intercepts are –1 and $1\frac{1}{2}$.



The equation of the axis of symmetry is $x = \frac{1}{4}$.

(c) $h(x) = 3x^2 + 12x - 4$

$$= 3\left(x^2 + 4x - \frac{4}{3}\right)$$

$$= 3\left(x^2 + 4x - \frac{4}{3}\right)$$

$$= 3\left[x^2 + 4x + \left(\frac{4}{2}\right)^2 - \left(\frac{4}{2}\right)^2 - \frac{4}{3}\right]$$

$$= 3\left(x^2 + 4x + 4 - 4 - \frac{4}{3}\right)$$

$$= 3\left[(x+2)^2 - 16\right]$$

$$= 3(x+2)^2 - 16$$

The minimum point is $(-2, -16)$.

y-intercept = –4

On the x-axis (y = 0),

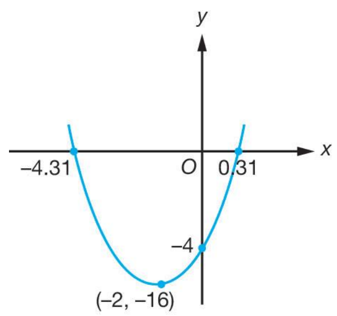
$$3x^2 + 12x - 4 = 0$$

$$x = \frac{-12 \pm \sqrt{12^2 - 4(3)(-4)}}{2(3)}$$

$$x = \frac{-12 \pm \sqrt{192}}{6}$$

$x = -4.31$ or 0.72

The curve intersects the x-axis at (–4.31, 0.72).



The equation of the axis of symmetry is $x = -2$.

(d) $m(x) = 2x^2 + 7x + 11$

$$= 2\left(x^2 + \frac{7}{2}x + \frac{11}{2}\right)$$

$$= 2\left(x^2 + \frac{7}{2}x + \left(\frac{1}{2} \times \frac{7}{2}\right)^2 - \left(\frac{1}{2} \times \frac{7}{2}\right)^2 + \frac{11}{2}\right)$$

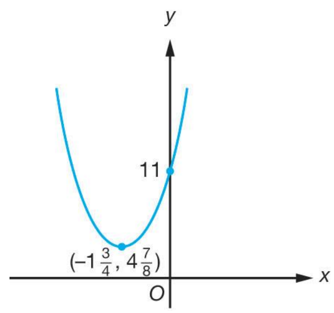
$$= 2\left(x^2 + \frac{7}{2}x + \frac{49}{16} - \left(\frac{49}{16}\right) + \frac{11}{2}\right)$$

$$= 2\left[\left(x + \frac{7}{4}\right)^2 + \frac{39}{16}\right]$$

$$= 2\left(x + \frac{7}{2}\right)^2 + \frac{39}{8}$$

The minimum point is $\left(-1\frac{3}{4}, 4\frac{7}{8}\right)$

y-intercept = 11



The equation of the axis of symmetry is $x = -\frac{7}{4}$.

(e) $n(x) = 1 - 2x - x^2$

$$= -x^2 - 2x + 1$$

$$= -(x^2 + 2x - 1)$$

$$= -\left[x^2 + 2x + \left(\frac{2}{2}\right)^2 - \left(\frac{2}{2}\right)^2 - 1\right]$$

$$= -(x^2 + 2x + 1 - 1 - 1)$$

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$= -\left[(x+1)^2 - 2\right]$

$= -(x+1)^2 + 2$

The maximum point is $(-1, 2)$.

y-intercept = 1

On the x-axis, $y = 0$

$-x^2 - 2x + 1 = 0$

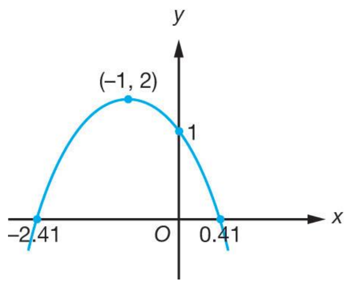
$x^2 + 2x - 1 = 0$

$$x = \frac{-2 \pm \sqrt{2^2 - 4(1)(-1)}}{2(1)}$$

$$x = \frac{-2 \pm \sqrt{8}}{2(1)}$$

$x = 0.21$ or 0.41

The curve intersects the x-axis at (–2.41, 0) and (0.41, 0).



The equation of the axis of symmetry is $x = -1$.

(f) $p(x) = 2x - 3 - 2x^2$

$= -2x^2 + 2x - 3$

$$= -2\left(x^2 - x + \frac{3}{2}\right)$$

$$= -2\left[x^2 - x + \left(-\frac{1}{2}\right)^2 - \left(-\frac{1}{2}\right)^2 + \frac{3}{2}\right]$$

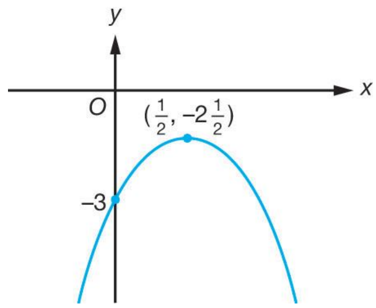
$$= -2\left(x^2 - x + \frac{1}{4} - \frac{1}{4} + \frac{3}{2}\right)$$

$$= -2\left[\left(x - \frac{1}{2}\right)^2 + \frac{5}{4}\right]$$

$$= -2\left(x - \frac{1}{2}\right)^2 - \frac{5}{2}$$

The maximum point is $\left(\frac{1}{2}, -\frac{5}{2}\right)$.

y-intercept = –3



The equation of the axis of symmetry is $x = \frac{1}{2}$.

2 (a) $f(x) = (x-2)^2 - (2x-3)^2$

$= x^2 - 4x + 4 - \left(4x^2 - 12x + 9\right)$

$= x^2 - 4x + 4 - 4x^2 + 12x - 9$

$= -3x^2 + 8x - 5$

$$= -3\left(x^2 - \frac{8}{3}x + + \frac{5}{3}\right)$$

$$= -3\left(x^2 - \frac{8}{3}x + 16 - 16 - 5\right)$$

$$= -3\left[x^2 - \frac{8}{3}x + \left(-\frac{8}{(2)(3)}\right)^2 - \left(-\frac{8}{(2)(3)}\right)^2 + \frac{5}{3}\right]$$

$$= -3\left[x^2 - \frac{8}{3}x + \frac{16}{9} - \frac{16}{9} + \frac{5}{3}\right]$$

$$= -3\left[\left(x - \frac{4}{3}\right)^2 - \frac{1}{9}\right]$$

$$= -3\left(x - \frac{4}{3}\right)^2 + \frac{1}{3}$$

The maximum point is $\left(\frac{4}{3}, \frac{1}{3}\right)$.

y-intercept = –5

At the x-axis ($y = 0$),

$-3x^2 + 8x - 5 = 0$

$3x^2 - 8x + 5 = 0$

$(3x - 5)(x - 1) = 0$

$x = \frac{5}{3}$ or 1

The curve will intersect the x-axis at (1, 0) and $\left(\frac{5}{3}, 0\right)$

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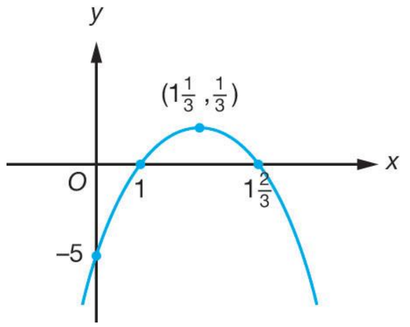
(b)



3 (a) $g(x)=\frac{1}{2}\left[(x+5)^2+(x-3)^2\right]$

$=\frac{1}{2}\left(x^2+10x+25+x^2-6x+9\right)$

$=\frac{1}{2}\left(2x^2+4x+34\right)$

$=x^2+2x+17$

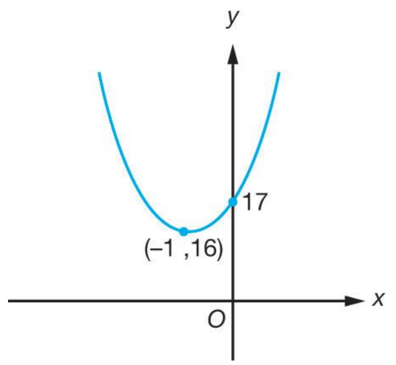
$=x^2+2x+\left(\frac{2}{2}\right)^2-\left(\frac{2}{2}\right)^2+17$

$=x^2+2x+1-1+17$

$=(x+1)^2+16$

The minimum point of the curve is (–1, 16).
y-intercept = 17

(b)



4 (a) $f(x)=x^2+px+5$

$= x^2+px+\left(\frac{p}{2}\right)^2-\left(\frac{p}{2}\right)^2+5$

$= x^2+px+\frac{p^2}{4}-\frac{p^2}{4}+5$

$=\left(x+\frac{p}{2}\right)^2-\frac{p^2}{4}+5$

Minimum point is $\left(-\frac{p}{2}, -\frac{p^2}{4}+5\right)$

Given minimum point is $\left(q, \frac{11}{4}\right)$

By comparison, $-\frac{p^2}{4}+5=\frac{11}{4}$

$-p^2+20=11$

$p^2=9$

$p=3$

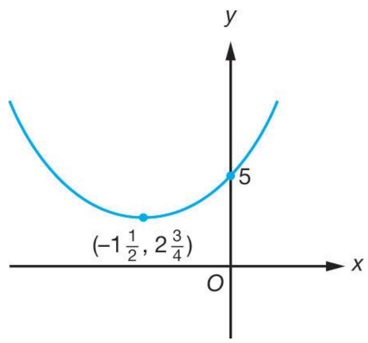
$q=-\frac{p}{2}=-\frac{3}{2}$

(b) $f(x)=\left(x+\frac{3}{2}\right)^2-\frac{3^2}{4}+5$

$=\left(x+\frac{3}{2}\right)^2+\frac{11}{4}$

Minimum point is $\left(-\frac{3}{2}, \frac{11}{4}\right)$ i.e. $\left(-1\frac{1}{2}, 2\frac{3}{4}\right)$.

y-intercept = 5



5 (a) $g(x)=-x^2+hx-4$

$=-\left(x^2-hx+4\right)$

$= -\left[x^2-hx+\left(\frac{h}{2}\right)^2-\left(\frac{h}{2}\right)^2+4\right]$

$=-\left(x^2-hx+\frac{h^2}{4}-\frac{h^2}{4}+4\right)$

$=-\left[\left(x-\frac{h}{2}\right)^2-\frac{h^2}{4}+4\right]$

$=-\left(x-\frac{h}{2}\right)^2+\frac{h^2}{4}-4$

Maximum point is $\left(\frac{h}{2}, \frac{h^2}{4}-4\right)$.

Given maximum point is $(k, -3)$.

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By comparison, $\frac{h^2}{4} - 4 = -3$

$$h^2 = 4$$
$$h = 2$$

$$k = \frac{h}{2} = \frac{2}{2} = 1$$

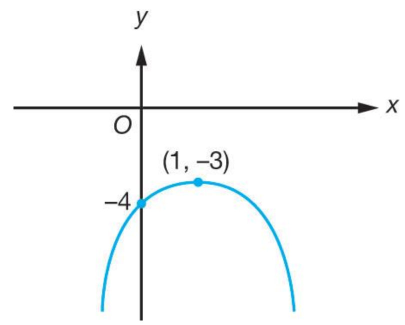
(b) $-\left(x - \frac{h}{2}\right)^2 + \frac{h^2}{4} - 4$

$$= -\left(x - \frac{2}{2}\right)^2 + \frac{2^2}{4} - 4$$

$$= -(x-1)^2 - 3$$

Maximum point is $(1, -3)$.

y-intercept = –4



6 (a) $f(x) = 2x^2 - 7x + 5$

$$= 2\left(x^2 - \frac{7}{2}x + \frac{5}{2}\right)$$

$$= 2\left[x^2 - \frac{7}{2}x + \left(-\frac{7}{2\times 2}\right)^2 - \left(-\frac{7}{2\times 2}\right)^2 + \frac{5}{2}\right]$$

$$= 2\left(x^2 - \frac{7}{2}x + \frac{49}{16} - \frac{49}{16} + \frac{5}{2}\right)$$

$$= 2\left[\left(x - \frac{7}{4}\right)^2 - \frac{9}{16}\right]$$

$$= 2\left(x - \frac{7}{4}\right)^2 - \frac{9}{8}$$

Minimum point is $\left(\frac{7}{4}, -\frac{9}{8}\right)$, i.e. $\left(1\frac{3}{4}, -1\frac{1}{8}\right)$.

y-intercept = 5

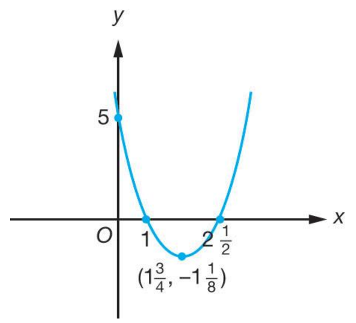
On the x-axis, $y = 0$

$$2x^2 - 7x + 5 = 0$$

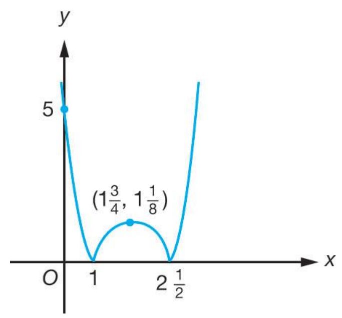
$$(x-1)(2x-5) = 0$$

$$x = 1 \text{ or } \frac{5}{2}$$

Hence, the curve intersects the x-axis at $(1, 0)$ and $\left(2\frac{1}{2}, 0\right)$



(b)



7 (a) $f(x) = -10 + 7x - x^2$

$$= -x^2 + 7x - 10$$

$$= -(x^2 - 7x + 10)$$

$$= -\left[x^2 - 7x + \left(-\frac{7}{2}\right)^2 - \left(-\frac{7}{2}\right)^2 + 10\right]$$

$$= -\left(x^2 - 7x + \frac{49}{4} - \frac{49}{4} + 10\right)$$

$$= -\left[\left(x - \frac{7}{4}\right)^2 - \frac{49}{4} + 10\right]$$

$$= -\left[\left(x - \frac{7}{4}\right)^2 - \frac{9}{4}\right]$$

$$= -\left(x - \frac{7}{4}\right)^2 + \frac{9}{4}$$

Maximum point is $\left(\frac{7}{4}, \frac{9}{4}\right)$, i.e. $\left(3\frac{1}{2}, 2\frac{1}{4}\right)$.

y-intercept = –10

At the x-axis, $y = 0$

$$-x^2 + 7x - 10 = 0$$

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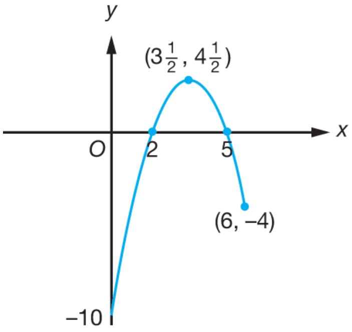
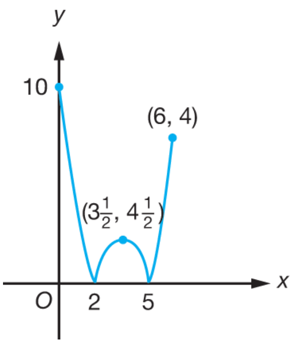
$x^2 - 7x + 10 = 0$
$(x-2)(x-5) = 0$
$x = 2$ or 5
Hence, the curve will intersect the x-axis at the points (2, 0) and (5, 0).

x	0	6
$f(x)$	–10	–4



(b)

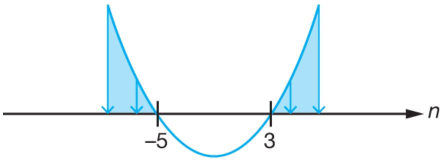


UPSKILL 2.3f

1 $y = m(x-3) - 1$ … (1)
$y = x^2 - 3x$ … (2)

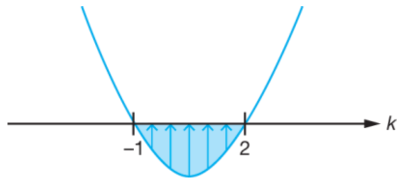
Substitute (2) into (1) :
$x^2 - 3x = m(x-3) - 1$
$x^2 - 3x = mx - 3m - 1$
$x^2 - 3x - mx + 3m + 1 = 0$
$a = 1, b = -3 - m, c = 3m + 1$
$b^2 - 4ac = 0$
$(-3-m)^2 - 4(1)(3m+1) = 0$
$9 + 6m + m^2 - 12m - 4 = 0$
$m^2 - 6m + 5 = 0$
$(m-5)(m-1) = 0$
$m = 5$ or 1

2 $y = nx - 2$ … (1)
$y = 2x^2 - x$ … (2)
Substitute (2) into (1) :
$2x^2 - x = nx - 2$
$2x^2 - x - nx + 2 = 0$
$a = 2, b = -1 - n, c = 2$
$(-1-n)^2 - 4(2)(2) > 0$
$b^2 - 16 > 0$
$1 + 2n + n^2 - 16 > 0$
$n^2 + 2n - 15 > 0$
$(n-3)(n+5) > 0$



The range of values of n is
$n < -5$ or $n > 3$.

3 $y = k(x-1) - 1$ … (1)
$y = x^2 - kx + 1$ … (2)
Substitute (2) into (1) :
$x^2 - kx + 1 = k(x-1) - 1$
$x^2 - kx + 1 = kx - k - 1$
$x^2 - 2kx + k + 2 = 0$
$a = 1, b = -2k, k + 2$
$b^2 - 4ac < 0$
$(-2k)^2 - 4(1)(k+2) < 0$
$4k^2 - 4k - 8 < 0$
$k^2 - k - 2 < 0$
$(k-2)(k+1) < 0$



The range of values of k is
$-1 < k < 2$.

4 $f(x) = 2x^2 - 2tx - 3t + 20$
$a = 2, b = -2t, c = -3t + 20$
$b^2 - 4ac < 0$
$(-2t)^2 - 4(2)(20 - 3t) < 0$
$4t^2 - 8(20 - 3t) < 0$
$t^2 - 2(20 - 3t) < 0$

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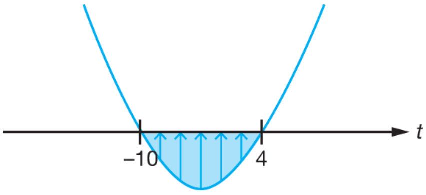
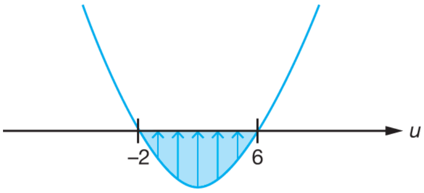
$$t^2 - 40 + 6t < 0$$
$$t^2 + 6t - 40 < 0$$
$$(t-4)(t+10) < 0$$



The range of values of t is $-10 < t < 4$.

5 $g(x) = -2x^2 + (u+6)x - 2u - 6$
$a = -2,\ b = u+6,\ c = -2u - 6$

$$b^2 - 4ac < 0$$
$$(u+6)^2 - 4(-2)(-2u) < 0$$
$$u^2 + 12u + 36 + 8(-2u - 6) < 0$$
$$u^2 + 12u + 36 - 16u - 48 < 0$$
$$u^2 - 4u - 12 < 0$$
$$(u+2)(u-6) < 0$$



The range of values of u is $-2 < u < 6$.

6 (a) When $x - 0$, $h(x) = 0$, thus $c = 0$.
When $x = 120$, $h(120) = 0$.
Thus, $h(120) = a(120)^2 + b(120) = 0$
$$120a + b = 0 \quad \dots (1)$$

When $x = 60$, $h(60) = 70$.
Thus, $h(60) = a(60)^2 + b(60) = 70$
$$3600a + 60b = 70$$
$$360a + 6b = 7 \quad \dots (2)$$

$$360a + 3b = 0 \quad \dots (1) \times 3$$
$$(-)\quad 360a + 6b = 7$$
$$-3b = -7$$
$$b = \frac{-7}{-3}$$
$$b = \frac{7}{3}$$

Substitute $b = \frac{7}{3}$ into (1) :
$$120a + \frac{7}{3} = 0$$
$$120a = -\frac{7}{3}$$
$$a = -\frac{7}{360}$$
Hence, $h(x) = -\frac{7}{360}x^2 + \frac{7}{3}x$

(b) When, $h(x) = 52\frac{1}{2}$,
$$52\frac{1}{2} = -\frac{7}{360}x^2 + \frac{7}{3}x$$
$$18\ 900 = -7x^2 + 840x^2$$
$$7x^2 - 840x + 18\ 900 = 0$$
$$x^2 - 120x + 2700 = 0$$
$$(x-3)(x-90) = 0$$
$$x = 30 \text{ or } x = 90$$

Hence, when the height of the parabolic curve is $52\frac{1}{2}$ m, the distance from P is 30 m or 90 m.

Summative Practice 2

1 $$x(3x-2) = 7 - 5x$$
$$3x^2 - 2x + 5x - 7 = 0$$
$$3x^2 + 3x - 7 = 0$$
$$x = \frac{-3 \pm \sqrt{3^2 - 4(3)(-7)}}{2(3)}$$
$$x = \frac{-3 \pm \sqrt{93}}{6}$$
$$x = 1.107 \text{ or } -2.107$$

2 $$(k-30)(2k+50) - 1\ 400 = 1.61 \times 10\ 000$$
$$2k^2 - 10k - 1\ 500 - 1\ 400 - 16\ 100 = 0$$
$$2k^2 - 10k - 19\ 000 = 0$$
$$k^2 - 5x - 9\ 500 = 0$$
$$k = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(-9\ 500)}}{2(1)}$$
$$k = \frac{5 \pm \sqrt{38\ 025}}{2}$$
$$k = \frac{5 \pm 195}{2}$$

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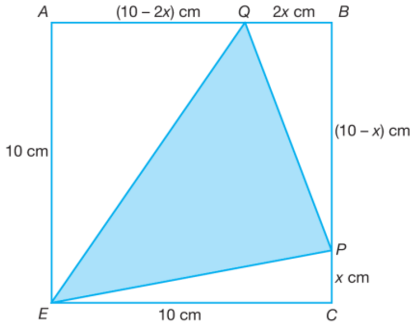
$k = -95$ or 100
$k = -95$ is not accepted.
$k = 100$

3



(a) $A(x) = 10(10) - \frac{1}{2}(10)(x) - \frac{1}{2}(10)(10-2x) - \frac{1}{2}(2x)(10-x)$

$= 100 - 5x - 50 + 10x - 10x + x^2$

$= x^2 - 5x + 50$ [Shown]

(b) $x^2 - 5x + 50 = 44.75$

$4x^2 - 20x + 200 = 179$

$4x^2 - 20x + 21 = 0$

$(2x-7)(2x-3) = 0$

$x = 3.5$ or $x = 1.5$

$x = 3.5$ is not accepted because it does not satisfy $AQ > QB$.
Hence, $x = 1.5$

4 $2x^2 - 8x - 3 = 0$
The roots are α and β.

S.O.R. = $\alpha + \beta = \frac{8}{2} = 4$

P.O.R. = $\alpha\beta = -\frac{3}{2}$

The new roots are $\alpha(1-\beta)$ and $p(1-\alpha)$.

S.O.R = $\alpha(1-\beta) + \beta(1-\alpha)$

$= \alpha - \alpha\beta + \beta - \beta\alpha$

$= \alpha + \beta - 2\alpha\beta$

$= 4 - 2\left(-\frac{3}{2}\right)$

$= 4 + 3$

$= 7$

P.O.R. = $\alpha(1-\beta) \times \beta(1-\alpha)$

$= \alpha\beta[(1-\beta)(1-\alpha)]$

$= \alpha\beta[1 - (\alpha+\beta) + \alpha\beta]$

$= -\frac{3}{2}\left[1 - 4 + \left(-\frac{3}{2}\right)\right]$

$= \frac{27}{4}$

The new quadratic equation is

$x^2 - 7x + \frac{27}{4} = 0$

$4x^2 - 28x + 27 = 0$

5 $x^2 + k = 15x$

$x^2 - 15x + k = 0$

The roots are 2α and 3α.

S.O.R. = 15

$2\alpha + 3\alpha = 15$

$5\alpha = 15$

$\alpha = 3$

P.O.R. = k

$(2\alpha)(3\alpha) = k$

$k = 6\alpha^2$

$k = 6(3)^2$

$k = 54$

6 (a) $hx^2 + kx + 2k = 8x + 4$

$hx^2 + kx - 8x + 2k - 4 = 0$

$hx^2 + (k-8)x + 2k - 4 = 0$

$a = h, b = k - 8, c = 2k - 4$

The roots are k and $\frac{1}{h}$.

S.O.R. = $-\frac{b}{a}$

$k + \frac{1}{h} = -\frac{(k-8)}{h}$

$\frac{hk+1}{h} = -\frac{(k-8)}{h}$

$hk + 1 = -k + 8$

$hk + k = 7$ … (1)

P.O.R. = $\frac{c}{a}$

$\frac{k}{h} = \frac{2k-4}{h}$

$k = 2k - 4$

$k = 4$

From (1) :

$4h + 4 = 7$

$4h = 3$

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$h=\frac{3}{4}$

(b) New roots are $2h = 2\left(\frac{3}{4}\right)=\frac{3}{2}$ and

$-k=-4$.

S.O.R. = $\frac{3}{2}+(-4)=-\frac{5}{2}$

P.O.R. = $\frac{3}{2}(-4)=-6$

The new quadratic equation is

$x^2+\frac{5}{2}x-6=0$

$2x^2+5x-12=0$

7 $x^2+2x-5=0$

The roots are α and β.

$\alpha+\beta=-2$

$\alpha\beta=-5$

$x^2+4x+q=0$

The roots are $\frac{p}{\alpha}$ and $\frac{p}{\beta}$.

S.O.R. = $\frac{p}{\alpha}+\frac{p}{\beta}$

$=\frac{p(\alpha+\beta)}{\alpha\beta}$

$=\frac{p(-2)}{-5}$

$=\frac{2}{5}p$

S.O.R. = $-\frac{b}{a}=-4$

By comparison, $\frac{2}{5}p=-4$

$p=\frac{5}{2}(-4)$

$p=-10$

P.O.R. = $\left(\frac{p}{\alpha}\right)\left(\frac{p}{\beta}\right)$

$=\frac{p^2}{\alpha\beta}$

$=\frac{p^2}{-5}$

P.O.R. = $\frac{c}{a}=q$

By comparison,

$q=\frac{p^2}{-5}$

$q=\frac{(-10)^2}{-5}$

$q=\frac{100}{-5}$

$q=-20$

8 $x^2+2mx+1=0$

The roots are α and β.

$\alpha+\beta=-2m$

$\alpha\beta=1$

$x^2+4x-n=0$

The roots are 2α and 2β.

$2\alpha+2\beta=-\frac{b}{a}$

$2(\alpha+\beta)=-4$

$2(-2m)=-4$

$-4m=-4$

$m=\frac{-4}{-4}=1$

$(2\alpha)(2\beta)=\frac{c}{a}$

$4\alpha\beta=-n$

$4(1)=-n$

$n=-4$

9 $(x+m)^2=kx$

$x^2+2mx+m^2-kx=0$

$x^2+(2m-k)x+m^2=0$

The roots are 1 and 16.

S.O.R. = $-\frac{b}{a}$

$1+16=-(2m-k)$

$17=-2m+k$ … (1)

P.O.R. = $\frac{c}{a}$

$1\times16=m^2$

$m=\pm4$

From (1) :

When m = 4,

$17=-2(4)+k$

$k=17+8$

$k=25$

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From (1) :
When $m = -4$,
$17 = -2(-4) + k$
$17 = 8 + k$
$k = 9$

10 $x^2 + 15 = 8x$
$x^2 - 8x + 15 = 0$
The roots are $(h+1)$ and $(k-2)$.

S.O.R. = $-\dfrac{b}{a}$
$(h+1) + (k-2) = 8$
$h + k - 1 = 8$
$h + k = 9$
$h = 9 - k$ … (1)

P.O.R. = $\dfrac{c}{a}$
$(h+1)(k-2) = 15$
$hk - 2h + k - 2 = 15$
$hk - 2h + k = 17$ … (2)

Substitute (1) into (2) :
$k(9-k) - 2(9-k) + k = 17$
$9k - k^2 - 18 + 2k + k = 17$
$-k^2 + 12k - 35 = 0$
$k^2 - 12k + 35 = 0$
$(k-7)(k-5) = 0$
$k = 7$ or 5

From (1) :
When $k = 7$,
$h = 9 - k = 9 - 7 = 2$
When $k = 5$,
$h = 9 - 5 = 4$

Hence, $k = 7$, $h = 2$ or $k = 5$, $h = 4$

11 $y = 2(x-2)^2 + 3q$
$y = 2(x^2 - 4x + 4) + 3q$
$y = 2x^2 - 8x + 8 + 3q$
S.O.R. = $-\dfrac{-8}{2} = 4$ … (1)
P.O.R. = $\dfrac{8+3q}{2}$ … (2)

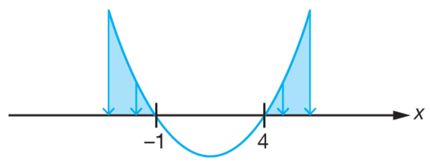
$y = x^2 + x - px - 5$
$y = x^2 + (1-p)x - 5$
S.O.R. = $-\dfrac{(1-p)}{1} = p - 1$ … (3)

P.O.R. = -5 … (4)

Equating (1) and (3) :
$p - 1 = 4$
$p = 5$

Equating (2) and (4) :
$\dfrac{8+3q}{2} = -5$
$3q + 8 = -10$
$3q = -10 - 8$
$3q = -18$
$q = -6$

12 $-x(x-4) < x - 4$
$-x^2 + 4x - x + 4 < 0$
$-x^2 + 3x + 4 < 0$
$x^2 - 3x - 4 > 0$
$(x-4)(x+1) > 0$



The range of values of x is
$x < -1$ or $x > 4$.

13 (a) $x(x-4) = 2$
$x^2 - 4x - 2 = 0$

(b) S.O.R. = $-\dfrac{b}{a} = -\left(\dfrac{-4}{1}\right) = 4$

(c) $b^2 - 4ac$
$= (-4)^2 - 4(-2)$
$= 16 + 8$
$= 24$ (> 0)
Hence, the roots are real and distinct.

14 $3x^2 - 2mx = 5 - 4p$
$3x^2 - 2mx + 4p - 5 = 0$
$a = 3$, $b = -2m$, $c = 4p - 5$
$b^2 - 4ac = 0$
$(-2m)^2 - 4(3)(4p-5) = 0$
$4m^2 - 48p + 60 = 0$
$m^2 - 12p + 15 = 0$
$12p = m^2 + 15$
$p = \dfrac{m^2 + 15}{12}$

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15 $9x^2 + qx + 1 = 4x$

$9x^2 + qx - 4x + 1 = 0$

$a = 9, b = q - 4, c = 1$

$b^2 - 4ac = 0$

$(q-4)^2 - 4(9)(1) = 0$

$q^2 - 8q + 16 - 36 = 0$

$q^2 - 8q - 20 = 0$

$(q+2)(q-10) = 0$

$q = -2$ or 10

16 $f(x) = 2x^2 - px + p + 6$

$b^2 - 4ac = 0$

$(-p)^2 - 4(2)(p+6) = 0$

$p^2 - 8p - 48 = 0$

$(p+4)(p-12) = 0$

$p = -4$ or 12

17 $g(x) = x^2 + 2kx + 2 - k$

$a = 1, b = 2k, c = 2 - k$

$b^2 - 4ac = 0$

$(2k)^2 - 4(1)(2-k) = 0$

$4k^2 - 8 + 4k = 0$

$k^2 + k - 2 = 0$

$(k+2)(k-1) = 0$

$k = -2$ or 1

18 (a) $x^2 - 2px + 2p + 3 = 0$

$a = 1, b = -2p, c = 2p + 3$

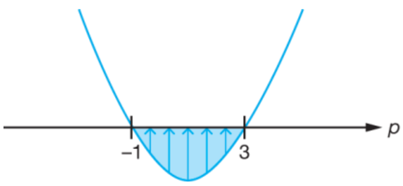
$b^2 - 4ac < 0$

$(-2p)^2 - 4(1)(2p+3) < 0$

$4p^2 - 8p - 12 < 0$

$p^2 - 2p - 3 < 0$

$(p-3)(p+1) < 0$



The range of values of p is $-1 < p < 3$.

(b) $x^2 + 2p^2 + 3p + 2 = 2px + 4x$

$x^2 - 2px - 4x + 2p^2 + 3p + 2 = 0$

$a = 1, b = -2p - 4, c = 2p^2 + 3p + 2$

$b^2 - 4ac > 0$

$(-2p-4)^2 - 4(1)(2p^2 + 3p + 2) > 0$

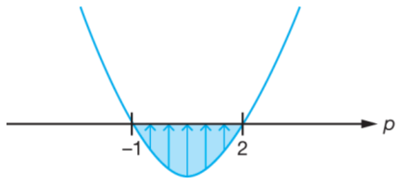
$4p^2 + 16p + 16 - 8p^2 - 12p - 8 > 0$

$-4p^2 + 4p + 8 > 0$

$-p^2 + p + 2 > 0$

$p^2 - p - 2 < 0$

$(p-2)(p+1) < 0$



The range of values of p is $-1 < p < 2$.

19 (a) $f(x) = (1-m)x^2 - 4mx + 9$

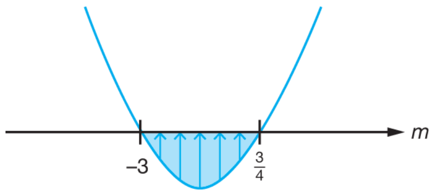
$b^2 - 4ac < 0$

$(-4m)^2 - 4(1-m)(9) < 0$

$16m^2 + 36m - 36 < 0$

$4m^2 + 9m - 9 < 0$

$(4m-3)(m+3) < 0$



The range of values of m is

$-3 < m < \frac{3}{4}$

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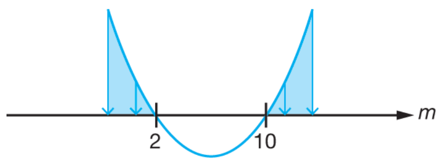
(b) $f(x) = 4x^2 - (m+2)x + m - 1$

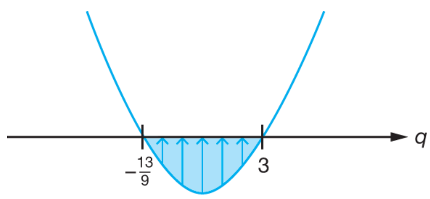
$$b^2 - 4ac > 0$$
$$[-(m+2)]^2 - 4(4)(m-1) > 0$$
$$m^2 + 4m + 4 - 16m + 16 > 0$$
$$m^2 - 12m + 20 > 0$$
$$(m-2)(m-10) > 0$$



The range of values of m is
$m < 2$ or $m > 10$

20 $3x^2 - 3x + 4 + q(2x^2 - x - 1) = 0$

$$3x^2 - 3x + 4 + 2qx^2 - qx - q = 0$$
$$(2q+3)x^2 + (-3-q)x + 4 - q = 0$$
$$b^2 - 4ac < 0$$
$$(-3-q)^2 - 4(2q+3)(4-q) < 0$$
$$9 + 6q + q^2 - 4(-2q^2 + 5q + 12) < 0$$
$$9 + 6q + q^2 + 8q^2 - 20q - 48 < 0$$
$$9q^2 - 14q - 39 < 0$$
$$(q-3)(9q+13) < 0$$



The range of values of q is
$-\frac{13}{9} < q < 3$

21 (a) $f(x) = -x^2 + 4kx - 5k^2 - 1$

$$f(x) = -(x^2 - 4kx + 5k^2 + 1)$$
$$f(x) = -[x^2 - 4kx + (-2k)^2 - (-2k)^2 + 5k^2 + 1]$$
$$f(x) = -[x^2 - 4kx + (4k^2) - (4k^2) + 5k^2 + 1]$$
$$f(x) = -[(x-2k)^2 + k^2 + 1]$$
$$f(x) = -(x-2k)^2 - k^2 - 1$$

The maximum value of $f(x)$ is $-k^2 - 1$
when $x - 2k = 0 \Rightarrow x = 2k$.

But it is given that the maximum value of $f(x)$ is $-r^2 - 2k$.

By comparison,
$$-k^2 - 1 = -r^2 - 2k$$
$$r^2 = k^2 - 2k + 1$$
$$r^2 = (k-1)^2$$
$$r = k - 1 \text{ [Shown]}$$

(b) The axis of symmetry is $x = 2k$.
But it is given that the axis of symmetry is $x = r^2 - 1$.
By comparison,
$r^2 - 1 = 2k$ ... (1)
Substitute $r = k - 1$ into (1) :
$$(k-1)^2 - 1 = 2k$$
$$k^2 - 2k + 1 - 1 - 2k = 0$$
$$k^2 - 4k = 0$$
$$k(k-4) = 0$$
Given that $k \neq 0$, thus $k = 4$
Therefore $r = k - 1 = 4 - 1 = 3$

22 (a) $f(x) = a(x-p)^2 + q$
Since $f(x)$ has a maximum value, therefore $a < 0$.

(b) $f(x) = a(x-2)^2 + 3$
But it is given that $f(x) = a(x-p)^2 + q$.
By comparison, $p = 2$ and $q = 3$.

(c) $f(x) = -2(x-2)^2 + 3$
$$= -2(x^2 - 4x + 4) + 3$$
$$= -2x^2 + 8x - 5$$

(d) (i) $f(x) = 2x^2 - 8x + 5$ ← The sign of each term is changed.

(ii) $f(x) = -2x^2 - 8x - 5$ ← The sign of the coefficient of x is changed

23 (a) $h(x) = -x^2 + 6x - 8$
$$= -(x^2 - 6x + 8)$$
$$= -\left[x^2 - 6x + \left(\frac{-6}{2}\right)^2 - \left(\frac{-6}{2}\right)^2 + 8\right]$$
$$= -(x^2 - 6x + 9 - 9 + 8)$$
$$= -[(x-3)^2 - 1]$$
$$= -(x-3)^2 + 1$$
Thus, $p = 1$

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(b) The maximum point is (3, 1).
y-intercept = –8
At the x-axis, y = 0

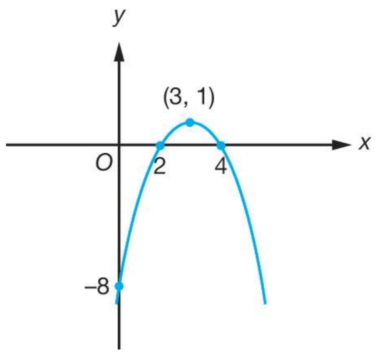
$-x^2+8x-8=0$

$x^2-8x+8=0$

$(x-2)(x-4)=0$

$x=2$ or 4

Thus, the curve will intersect the x-axis at (2, 0) and (4, 0).



24 (a) $f(x)=2x-3-4x^2$

$=-4x^2+2x-3$

$=-4\left(x^2-\frac{1}{2}x+\frac{3}{4}\right)$

$=-4\left[x^2-\frac{1}{2}x+\left(-\frac{1}{(2)(2)}\right)^2-\left(-\frac{1}{(2)(2)}\right)^2+\frac{3}{4}\right]$

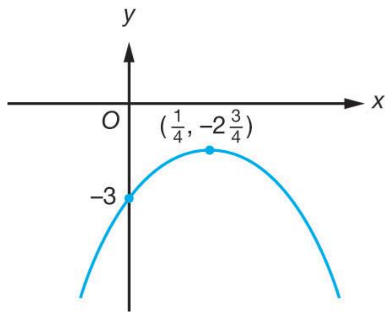
$=-4\left(x^2-\frac{1}{2}x+\frac{1}{16}-\frac{1}{16}+\frac{3}{4}\right)$

$=-4\left[\left(x-\frac{1}{4}\right)^2+\frac{11}{16}\right]$

$=-4\left(x-\frac{1}{4}\right)^2-\frac{11}{4}$

Maximum value = $-\frac{11}{4}$ when $x=\frac{1}{4}$

(b) y-intercept = –3



25 (a) $f(x)=4-3x-x^2$

$=-x^2-3x+4$

$=-(x^2+3x-4)$

$=-\left[x^2+3x+\left(\frac{3}{2}\right)^2-\left(\frac{3}{2}\right)^2-4\right]$

$=-\left(x^2+3x+\frac{9}{4}-\frac{9}{4}-4\right)$

$=-\left[\left(x+\frac{3}{2}\right)^2-\frac{25}{4}\right]$

$=-\left(x+\frac{3}{2}\right)^2+\frac{25}{4}$

Hence, the maximum point is $\left(-\frac{3}{2},\frac{25}{4}\right)$, i.e. $\left(-1\frac{1}{2},6\frac{1}{4}\right)$.

(b) y-intercept = 4
At the x-axis, y = 0

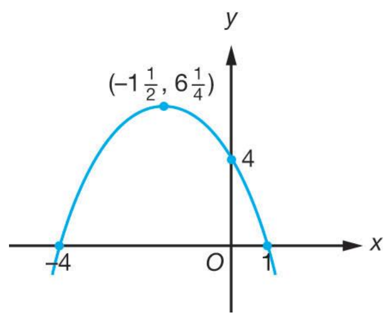
$-x^2-3x+4=0$

$x^2+3x-4=0$

$(x+4)(x-1)=0$

$x=-4$ or x = 1

Thus, the curve will intersect the x-axis at the points (–4, 0) and (1, 0).



(c) $x=-1\frac{1}{2}$

26 (a) $f(x)=x^2-x-6$

$=(x+2)(x-3)$

y-intercept = –6
x-intercept = –2 and 3

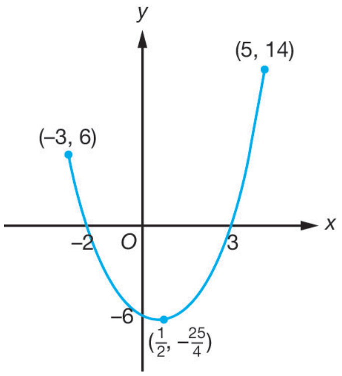
$f(x)=x^2-x-6$

$=x^2-x+\left(-\frac{1}{2}\right)^2-\left(-\frac{1}{2}\right)^2-6$

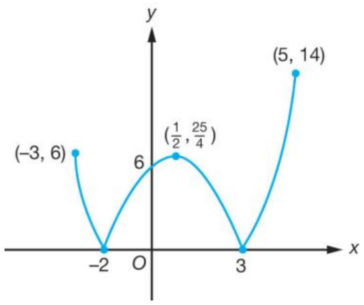
$=x^2-x+\frac{1}{4}-\frac{1}{4}-6$

$$=\left(x-\frac{1}{2}\right)^2-\frac{25}{4}$$

x	–3	5
$f(x)$	6	14



(b)



27 (a) $h(x)=-x^2-4kx+5k$

$$=-\left(x^2+4kx-5k\right)$$

$$=-\left[x^2+4kx+\left(\frac{4k}{2}\right)^2-\left(\frac{4k}{2}\right)^2-5k\right]$$

$$=-\left(x^2+4kx+4k^2-4k^2-5k\right)$$

$$=-\left[(x-2k)^2-4k^2-5k\right]$$

$$=-(x-2k)^2+4k^2+5k$$

Maximum value = 6

$$4k^2+5k=6$$

$$4k^2+5k-6=0$$

$$(4k-3)(k+2)=0$$

$$k=\frac{3}{4}\ \text{ or }\ -2$$

(b) $h(x)=-x^2-4(-2)x-10$

$$=-x^2+8x-10$$

$$=-\left(x^2-8x+10\right)$$

$$=-\left[x^2-8x+\left(\frac{-8}{2}\right)-\left(\frac{-8}{2}\right)+10\right]$$

$$=\left(x^2-8x+16-16+10\right)$$

$$=(x-4)^2-6$$

Maximum point is $(4,-6)$.

y-intercept = –10

At the x-axis, y = 0.

$$-x^2+8x-10=0$$

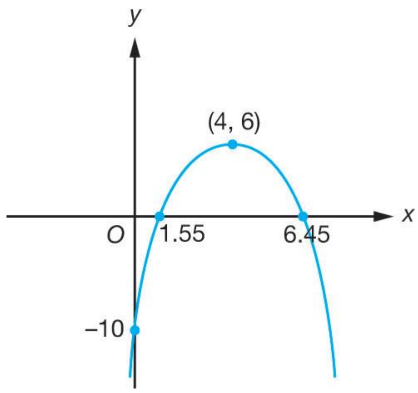
$$x^2-8x+10=0$$

$$x=\frac{8\pm\sqrt{(-8)^2-4(1)(10)}}{2(1)}$$

$$x=\frac{8\pm\sqrt{24}}{2}$$

$$x=6.45\ \text{ or }\ 1.55$$

Hence, the curve will intersect the x-axis at (1.55, 0) and (6.45, 0).



28 (a) $f(x)=x^2+hx+5$

$$=\left(x+hx+\left(\frac{h}{2}\right)^2-\left(\frac{h}{2}\right)^2+5\right)$$

$$=\left(x+\frac{h}{2}\right)^2-\frac{h^2}{4}+5$$

Given $h(x)=(x+k)^2+\dfrac{11}{4}$.

By comparison,

$$-\frac{h^2}{4}+5=\frac{11}{4}$$

$$-\frac{h^2}{4}=-\frac{9}{3}$$

$h=-3$ [Given $h<0$]

$$k=\frac{h}{2}$$

$$k=-\frac{3}{2}$$

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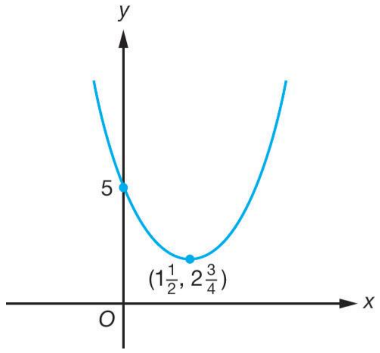
(b) When $h = -3$, $k = -\frac{3}{2}$,

$f(x) = \left(x - \frac{3}{2}\right)^2 + \frac{11}{4}$

Minimum point $\left(\frac{3}{2}, \frac{11}{4}\right)$ i.e. $\left(1\frac{1}{2}, 2\frac{3}{4}\right)$

y-intercept = 5



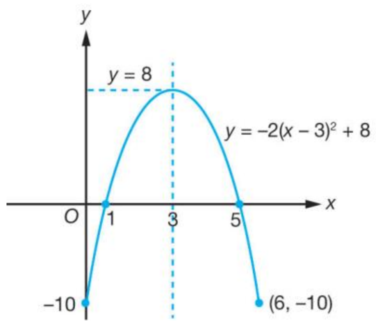
29 (a) The midpoint between (1, 0) and (5, 0) is $\left(\frac{1+5}{2}, 0\right)$, i.e. $(3, 0)$.

The maximum value is 8.
Therefore, (3, 8) is the maximum point.
Hence, $f(x) = -2(x-3)+8$

But it is given that $f(x) = -2(x-h)-2k$.

By comparison, $h = 3$ and $-2k = 8 \Rightarrow k = -4$.

(b) y-intercept is 8 and the x-intercepts are 1 and 5.



(c) $f(x) = -2(x-3)^2 + 8$

If the graph is reflected in the x-axis, the sign of each term is changed.

$f(x) = 2(x-3)^2 - 8$

(d) $f(x) = a(x-h)+k$

If the graph is reflected in the y-axis, the sign of h is changed.

$f(x) = -2(x+3)^2 + 8$

30 $y = px + 4$ … (1)

$y = x^2 - 4x + 5$ … (2)

Substitute (2) into (1):

$x^2 - 4x + 5 = px + 4$

$x^2 - 4x - px + 1 = 0$

$a = 1, b = -4 - p, c = 1$

$b^2 - 4ac = 0$

$(-p-4)^2 - 4 = 0$

$p^2 + 8x + 16 - 4 = 0$

$p^2 + 8x + 12 = 0$

$(p+2)(p+6) = 0$

$p = -2$ or -6

31 $y = h - 2x$ … (1)

$y^2 + xy + 8 = 0$ … (2)

Substitute (1) into (2):

$(h-2x)^2 + x(h-2x) + 8 = 0$

$h^2 - 4hx + 4x^2 + hx - 2x^2 + 8 = 0$

$2x^2 - 3hx + h^2 + 8 = 0$

$a = 2, b = -3h, ch^2 + 8$

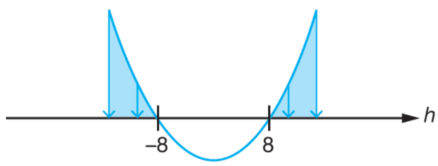
$b^2 - 4ac > 0$

$(-3h)^2 - 4(2)(h^2 + 8) > 0$

$9h^2 - 8h^2 - 64 > 0$

$h^2 - 64 > 0$

$(h+8)(h-8) > 0$



The range of values of h is $h < -8$ or $h > 8$.

32 $y = x + k$ … (1)

$y^2 + x^2 = 2$ … (2)

Substitute (1) into (2):

$(x+k)^2 + x^2 = 2$

$x^2 + 2kx + k^2 + x^2 - 2 = 0$

$2x^2 + 2kx + k^2 - 2 = 0$

$a = 2, b = 2k, c = k^2 - 2$

$b^2 - 4ac < 0$

$(2k)^2 - 4(2)(k^2 - 2) < 0$

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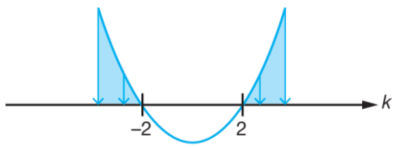
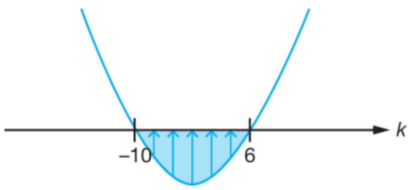
$$4k^2 - 8k^2 + 16 < 0$$
$$-4k^2 + 16 < 0$$
$$4k^2 - 16 > 0$$
$$k^2 - 4 > 0$$
$$(k+2)(k-2) > 0$$



The range of values of k is
$k < -2$ or $k > 2$.

33 $f(x) = x^2 + (k-2)x + 16 - 2k$

$$b^2 - 4ac < 0$$
$$(k-2)^2 - 4(1)(16 - 2k < 0$$
$$k^2 - 4k + 4 - 64 + 8k < 0$$
$$k^2 + 4k - 60 < 0$$
$$(k-6)(k+10) < 0$$



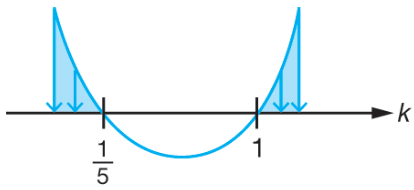
The range of values of k is
$-10 < k < 6$.
But it is given that $m < k < n$.
By comparison, $m = -10$ and $n = 6$.

SPM Spot

1 $$x^2 + x = 3kx - k^2$$
$$x^2 + x - 3kx + k^2 = 0$$
$$x^2 + (1-3k)x + k^2 = 0$$
$$a = 1,\ b = 1 - 3k,\ c = k^2$$

If the quadratic equation has two real and distinct roots,

$$b^2 - 4ac > 0$$
$$(1-3k)^2 - 4(1)(k^2) > 0$$
$$1 - 6k + 9k^2 - 4k^2 > 0$$
$$5k^2 - 6k + 1 > 0$$
$$(5k-1)(k-1) > 0$$



The range of values of k is
$k < \frac{1}{5}$ or $k > 1$.

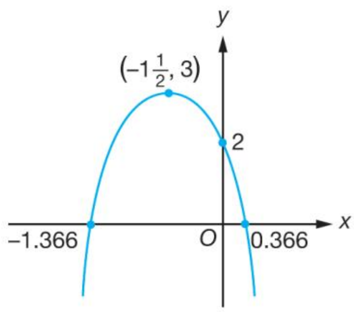
2 (a) $f(x) = -4x^2 - 4x + 2$

$$= -4\left(x^2 + x - \frac{1}{2}\right)$$
$$= -4\left[x^2 + x + \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2 - \frac{1}{2}\right]$$
$$= -4\left(x^2 + x + \frac{1}{4} - \frac{1}{4} - \frac{1}{2}\right)$$
$$= -4\left[\left(x + \frac{1}{2}\right)^2 - \frac{3}{4}\right]$$
$$= -4\left(x + \frac{1}{2}\right)^2 + 3$$

The maximum point is $\left(-\frac{1}{2}, 3\right)$.

The y-intercept is 2.
At the x-axis, $y = 0$.

$$-4x^2 - 4x + 2 = 0$$
$$2x^2 + 2x - 1 = 0$$
$$x = \frac{-2 \pm \sqrt{2^2 - 4(2)(-1)}}{2(2)}$$
$$x = \frac{-2 \pm \sqrt{12}}{4}$$
$$x = -1.366 \text{ or } 0.3660$$



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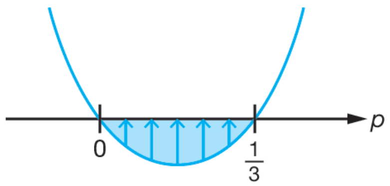
(b) $g(x) = 3x^2 - 6px + p$

If $g(x)$ is always positive, then its graph is always above the x-axis (no x-intercept).

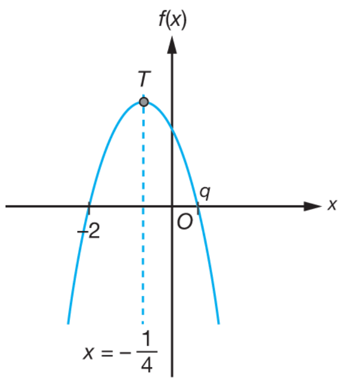
$$b^2 - 4ac < 0$$
$$(-6p)^2 - 4(3)p < 0$$
$$36p^2 - 12p < 0$$
$$12p(3p - 1) < 0$$



Hence, the range of values of p is

$0 < p < \frac{1}{3}$.

3 (a)



$$\frac{-2+q}{2} = -\frac{1}{4}$$
$$-8 + 4q = -2$$
$$4q = 6$$
$$q = \frac{3}{2}$$

For $f(x) < 0$, the part of the graph below the x-axis is taken into consideration,

which is $x < -2$ or $x > \frac{3}{2}$.

(b) $f(x) = -2x^2 - hx + 2k - 5$

$a = -2$, $b = -h$, $c = 2k - 5$

When the quadratic curve intersects the x-axis at two different points

$$b^2 - 4ac > 0$$
$$(-h)^2 - 4(-2)(2k-5) > 0$$
$$h^2 + 16k - 40 > 0$$
$$16k > 40 - h^2$$
$$k > \frac{40 - h^2}{16}$$

[Shown]

(c) $f(x) = -2x^2 - hx + 2k - 5$

$$= -2\left(x^2 + \frac{h}{2}x - k + \frac{5}{2}\right)$$
$$= -2\left[x^2 + \frac{h}{2}x + \left(\frac{h}{4}\right)^2 - \left(\frac{h}{4}\right)^2 - k + \frac{5}{2}\right]$$
$$= -2\left[\left(x + \frac{h}{4}\right)^2 - \frac{h^2}{16} - k + \frac{5}{2}\right]$$
$$= -2\left(x + \frac{h}{4}\right)^2 + \frac{h^2}{8} + 2k - 5$$

$$x = -\frac{h}{4}$$
$$-\frac{1}{4} = -\frac{h}{4}$$
$$h = 1$$

Hence, the maximum value of $f(x)$

$$= \frac{1^2}{8} + 2k - 5$$
$$= -\frac{39}{8} + 2k$$

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