FORM 4

CHAPTER 5

Self Test 1

1 (a) (i) $V = \{P, Q, R, S, T, U\}$
$n(V) = 6$
(ii) $E = \{(P, Q), (P, T), (P, U), (Q, T), (Q, S), (R, T), (R, S), (S, T), (T, U)\}$
$n(E) = 9$
(iii) Sum of degrees, $\Sigma d(v) = 2E$
$= 18$

(b) (i) $V = \{P, Q, R, S, T, U, V, W, X, Y\}$
$n(V) = 10$
(ii) $E = \{(P, Q), (Q, R), (Q, X), (R, Y), (R, U), (S, T), (T, U) (U, Y), (U, V), (V, W), (W, Y), (W, X)\}$
$n(E) = 12$
(iii) Sum of degrees, $\Sigma d(v) = 2E$
$= 24$

2 (a) (i) $V = \{1, 2, 3, 4, 5\}$
$n(V) = 5$
(ii) $E = \{(1, 2), (1, 2), (1, 3), (1, 5), (2, 3), (3, 4), (3, 4), (3, 5), (4, 4), (4, 5), (5, 5)\}$
$n(E) = 11$
(iii) Sum of degrees, $\Sigma d(v) = 2E$
$= 22$

(b) (i) $V = \{A, B, C, D\}$
$n(V) = 4$
(ii) $E = \{(A, A), (A, B), (A, C), (A, D), (B, C), (B, D), (C, D) (C, D), (C, D)\}$
$n(E) = 9$
(iii) Sum of degrees, $\Sigma d(v) = 2E$
$= 18$

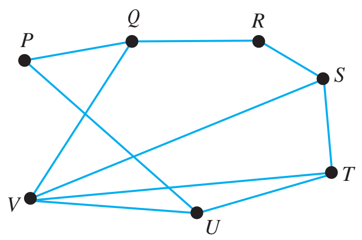
3 (a) $d(P) = 3$
$d(Q) = 2$
$d(R) = 2$
$d(S) = 1$
$\Sigma d(v) = 3 + 2 + 2 + 1$
$= 8$

(b) $d(P) = 4$
$d(Q) = 3$
$d(R) = 2$
$d(S) = 2$
$d(T) = 3$
$d(U) = 4$
$d(V) = 2$
$\Sigma d(v) = 4 + 3 + 2 + 2 + 3 + 4 + 2$
$= 20$

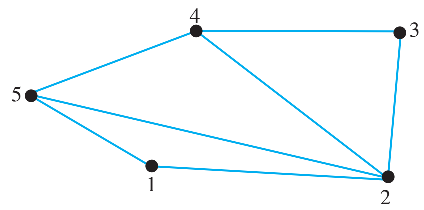
(c) $d(1) = 5$
$d(2) = 1$
$d(3) = 4$
$d(4) = 3$
$d(5) = 5$
$\Sigma d(v) = 5 + 1 + 4 + 3 + 5$
$= 18$

(d) $d(1) = 5$
$d(2) = 7$
$d(3) = 3$
$d(4) = 5$
$d(5) = 4$
$\Sigma d(v) = 5 + 7 + 3 + 5 + 4$
$= 24$

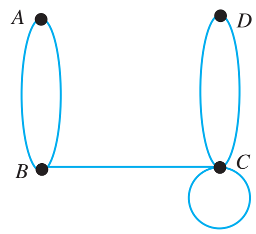
4 (a)



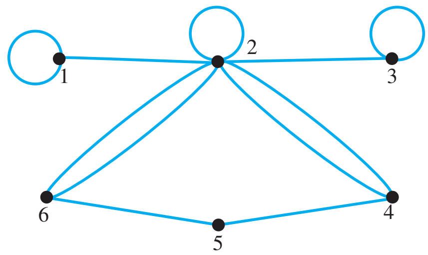
(b)



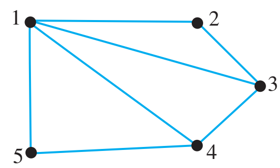
5 (a)



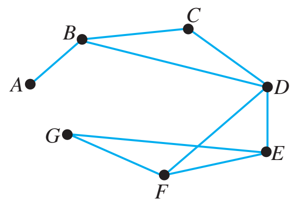
(b)



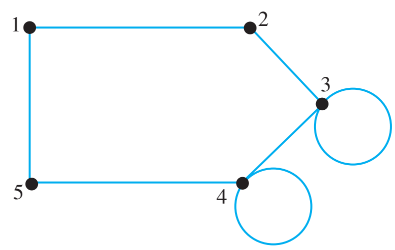
6 (a)



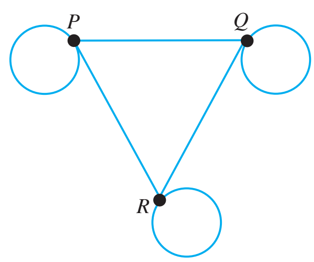
(b)



7 (a)



(b)



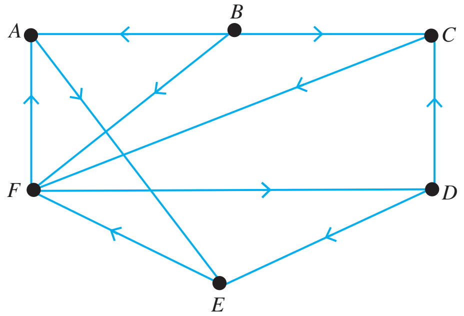
8 (a) $\Sigma d(v) = 1 + 3 + 2 + 4 + 1 + 2$
$= 13$
The sum of degrees is odd,
therefore the graph cannot be drawn.

(b) $\Sigma d(v) = 1 + 3 + 2 + 3 + 3 + 2$
$= 14$
The sum of degrees is even,
therefore the graph can be drawn.

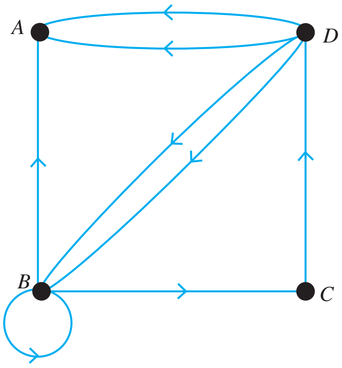
(c) $\sum d(v) = 3 + 2 + 3 + 1 + 4$
$= 13$
The sum of degrees is odd,
therefore, the graph cannot be drawn.

(d) $\sum d(v) = 2 + 2 + 2 + 4$
$= 10$
The sum of degrees is even,
therefore, the graph can be drawn.

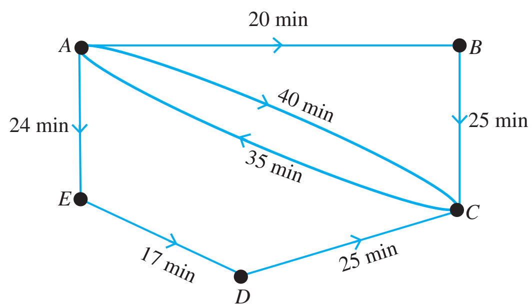
9 (a)



(b)



10 (a)

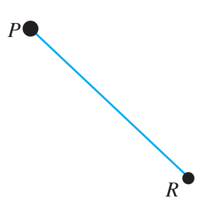


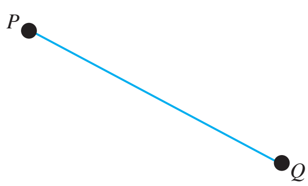
(b) $A \to C$
$A \to B \to C$
$A \to E \to D \to C$

(c) Duration $A \to C = 40$ minutes
Duration $A \to B \to C = (20 + 25)$ minutes
$= 45$ minutes
Duration $A \to E \to D \to C = (24 + 17 + 25)$ minutes
$= 66$ minutes
Difference between the shortest and the longest travelling duration
$= (66 - 40)$ minutes
$= 26$ minutes

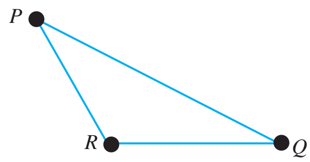
11 (Accept any correct subgraphs)

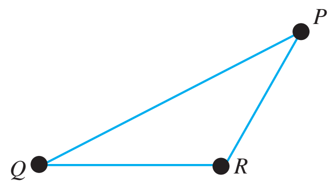
(a) (i) (ii)



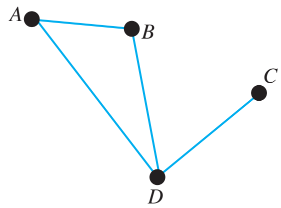


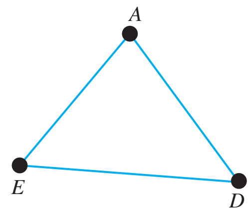
(iii) (iv)



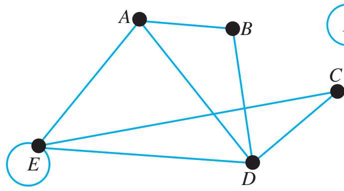


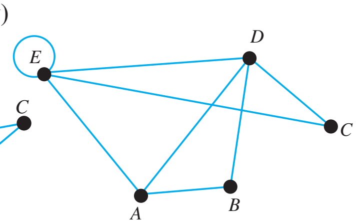
(b) (i) (ii)





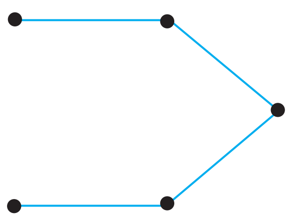
(iii) (iv)

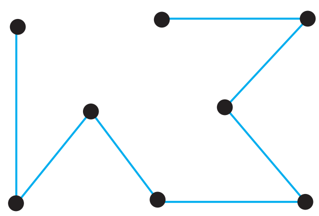




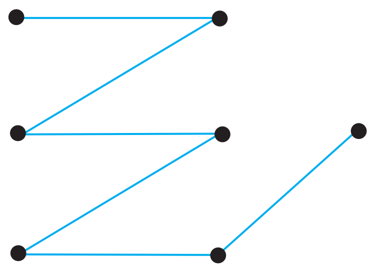
12 (a) Not a tree because each pair of vertices can be connected in various ways.
(b) Not a tree because vertices 3 and 5 are not connected with other vertices in the graph.
(c) Not a tree because it is a multiple edges graph.
(d) A tree because there are 5 vertices, 4 edges and each pair of vertices is connected by one edge only.

13 (a) Vertices = 5 (b) Vertices = 8
Therefore, edges = 4 Therefore, edges = 7

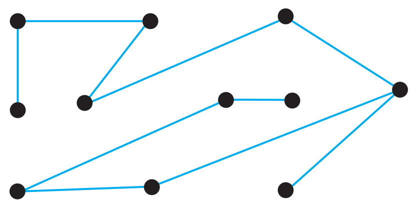




(c) Edges = 6
Therefore, vertices = 7

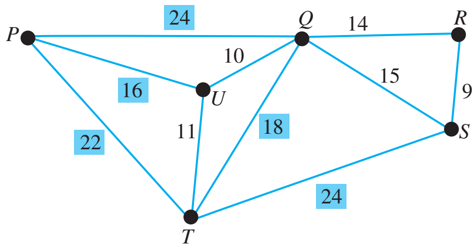


(d) Edges = 10
Therefore, vertices = 11

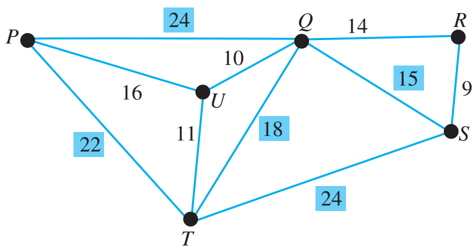


14 (a) Number of vertices = 6, therefore the number of edges = 5
Number of edges to be removed = 10 – 5
= 5

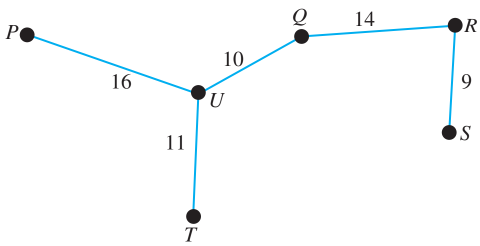
① Remove edges with the highest weight.



② Vertex P need to be connected to other vertices. Edge PU cannot be removed. Choose the edge with the highest weight from the rest of edges.

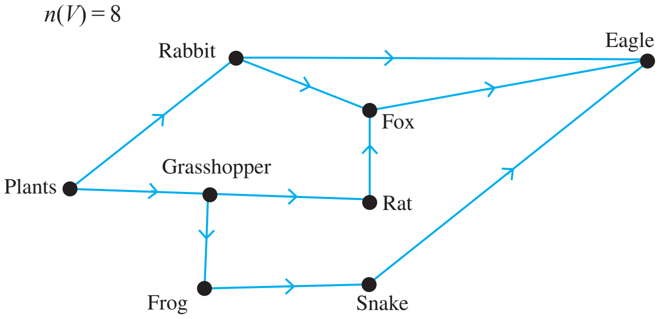


③ Draw the tree.



(b) Total weight $= 16 + 10 + 11 + 14 + 9$
$= 60$

15 V = {Plants, rabbit, eagle, fox, grasshopper, frog, snake, rat}
$n(V) = 8$



16 (a) $12 + 36 = 48$

(b) $x_1 = 18$
$x_1 + x_3 + 48 = 94$
$x_3 = 94 - 18 - 48$
$= 28$
$x_2 = 12 + x_3$
$= 12 + 28$
$= 40$

SPM PRACTICE

Paper 1

1 **D**

2 **B** Network is a graph with **at least** a pair of dots/vertices that are connected.

3 **B** The graph that can be drawn has the sum of degrees that is even only.

A $\sum d(v) = 4 + 3 + 2 + 1 + 1 + 2$
$= 13$

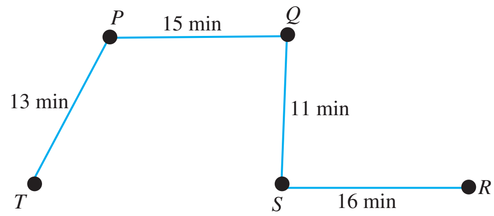
B $\sum d(v) = 1 + 1 + 1 + 3 + 2$
$= 8$

C $\sum d(v) = 3 + 5 + 4 + 1$
$= 13$

D $\sum d(v) = 2 + 2 + 4 + 1$
$= 9$

4 **B** $d(A) = 2, d(B) = 5, d(C) = 3, d(D) = 4$

5 **B** Find a tree graph. The number of vertices = 5, therefore the number of edges = 4. Remove 4 edges with the highest weight.



Shortest duration = (13 + 15 + 11 + 16) min
= 55 min

Paper 2

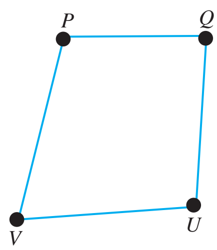
Section A

1 (a) $E = \{(P, Q), (P, U), (Q, Q), (Q, R), (R, S), (S, T), (T, R), (T, S), (T, P), (U, S), (U, T), (U, P)\}$
$n(E) = 12$

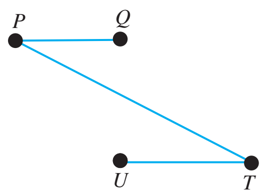
(b) Routes from P to S
$P \rightarrow Q \rightarrow R \rightarrow S$: Distance = (700 + 450 + 300) m
= 1 450 m
$P \rightarrow U \rightarrow S$: Distance = (400 + 1 350) m
= 1 750 m
$P \rightarrow U \rightarrow T \rightarrow S$: Distance = (400 + 850 + 650) m
= 1 900 m
$P \rightarrow U \rightarrow T \rightarrow R \rightarrow S$: Distance = (400 + 850 + 400 + 300) m
= 1 950 m
Shortest distance: $P \rightarrow Q \rightarrow R \rightarrow S$ with a distance of 1 450 m.

2 Subgraphs

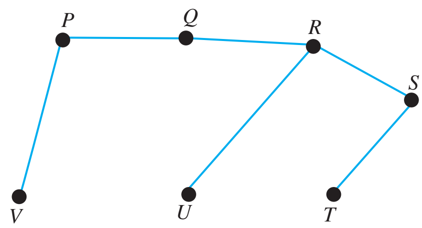
(i)



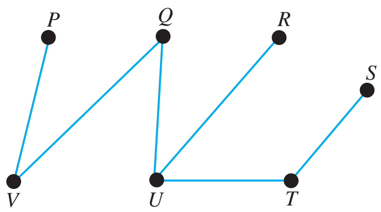
(ii)



(iii)

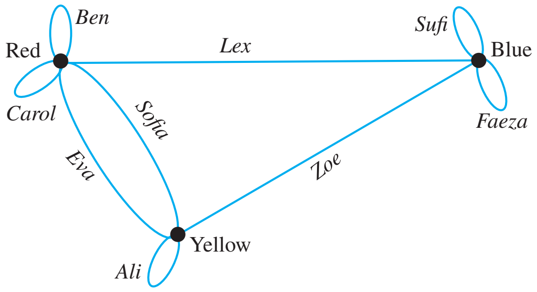


(iv)

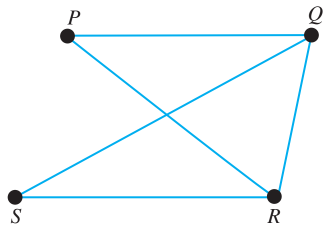


3 (a) Colour is chosen as vertices. Each colour is the favourite of more than a student.

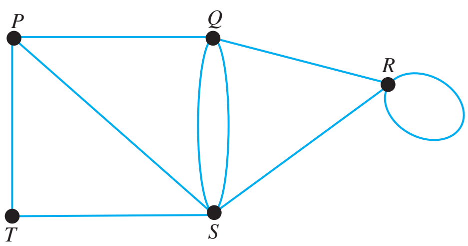
(b)



4 (a) A simple graph with the degrees of vertices 2, 3, 3, 2



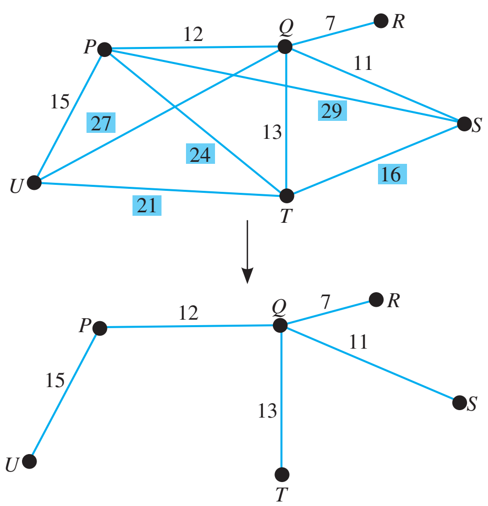
(b) Graph with loops and multiple edges with the degrees of vertices 3, 4, 4, 5, 2



5 Vertices = 6, therefore edges = 5

Number of edges to be removed = 10 – 5

= 5



Minimum total weight = 15 + 12 + 13 + 7 + 11

= 58

Section B

6 (a) All the possible routes

$P \to Q \to S$ ✓

$P \to Q \to R \to S$

$P \to Q \to R \to T \to S$

$P \to Q \to U \to T \to S$

$P \to Q \to U \to T \to R \to S$

$P \to R \to S$ ✓

$P \to R \to T \to S$

$P \to U \to T \to S$ ✓

$P \to U \to T \to R \to S$

$P \to U \to Q \to S$

$P \to U \to Q \to R \to S$

$P \to U \to Q \to R \to T \to S$

✓ Routes that have the least number of toll booths.

Total toll charges $P \to Q \to S$ = RM(1.20 + 2.40)

= RM3.60

Total toll charges $P \to R \to S$ = RM(3 + 1)

= RM4

Total toll charges $P \to U \to T \to S$ = RM(1.50 + 1.70 + 1.50)

= RM4.70

Answer: Route with the lowest toll charges = Route $P \to Q \to S$

(b) Total distance = $PQ + QS$

= 12 + 27

= 39 km

Duration = $\frac{39}{95} \times 60$ minutes

= 24.6 minutes

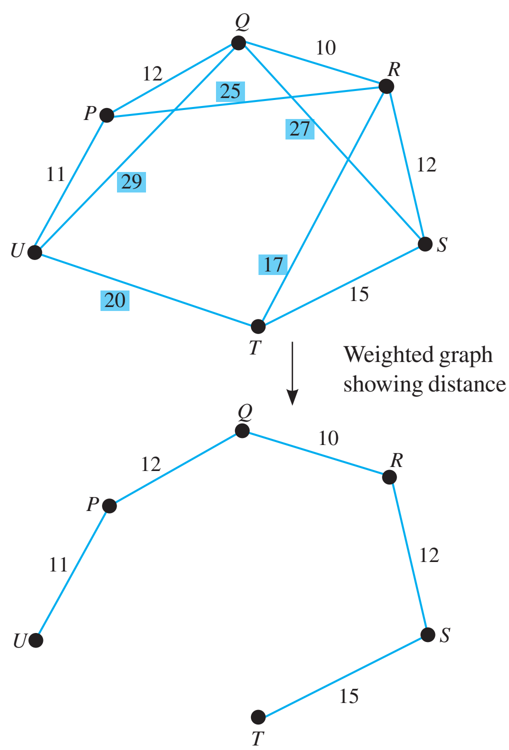
Arrival time in town S = 0930 + 0025

= 0955 hour

(c) Draw a tree: Number of vertices = 6, therefore the number of edges = 5

Number of edges to be removed = 10 – 5

= 5



Minimum distance between P and S = 12 + 10 + 12

= 34 km