## FORM 4

### CHAPTER 3

## Self Test 1

1. (a) A statement because it is true.
   (b) Not a statement because the truth value cannot be determined.
   (c) A statement because it is true.
   (d) A statement because it is false.
2. (a) False
   (b) True
   (c) True
   (d) True
3. (a) Not all the quadratic functions have two roots.
   (b) The probability of an event that may occur is not less than 1.
   (c) 3 is not a factor of 418.
4. (a) $2x + 3$ is a linear expression in one variable and all triangles have three sides.
   (b) 3 or 4 is a factor of 16.

5. (a) False (False and True)
   (b) False (False or False)
   (c) True (True and True)
   (d) True (True or False)
6. (a) If $x + y = 90°$, then $\sin x = \cos y$.
   (b) If the radius of a sphere is 7 cm, then the total surface area of the sphere is $196\pi$ cm$^2$.
7. (a) Antecedent: $x + 5 = 8$
       Consequent: $x = 3$
   (b) Antecedent: The weather is bad.
       Consequent: The football match is postponed to next week.
8. (a) $m = 11$ if and only if $m^2 = 121$.
   (b) $ABCD$ is a square if and only if $\angle A = \angle B = \angle C = \angle D = 90°$.
9. (a) Implication 1: If $\sin \theta = \cos (90 - \theta)$, then $0° \leqslant \theta \leqslant 90°$.
       Implication 2: If $0° \leqslant \theta \leqslant 90°$, then $\sin \theta = \cos (90 - \theta)$.
   (b) Implication 1: If $x^n$ is a quadratic expression, then $n = 2$.
       Implication 2: If $n = 2$, then $x^n$ is a quadratic expression.

10.

| Implication | Converse | Inverse | Contrapositive |
|---|---|---|---|
| (a) If $\angle A$ is an acute angle, then $\angle A < 90°$. | If $\angle A < 90°$, then $\angle A$ is an acute angle. | If $\angle A$ is not an acute angle, then $\angle A \geqslant 90°$. | If $\angle A \geqslant 90°$, then $\angle A$ is not an acute angle. |
| (b) If $x$ is a negative number, then the cube of $x$ is a negative number. | If the cube of $x$ is a negative number, then $x$ is a negative number. | If $x$ is not a negative number, then the cube of $x$ is not a negative number. | If the cube of $x$ is not a negative number, then $x$ is not a negative number. |

11. (a)

| | Statement | Antecedent | Consequent | Truth value |
|---|---|---|---|---|
| Implication | If 4 is a factor of 28, then 28 can be divided exactly by 4. | True | True | True |
| Converse | If 28 can be divided exactly by 4, then 4 is a factor of 28. | True | True | True |
| Inverse | If 4 is not a factor of 28, then 28 cannot be divided exactly by 4. | False | False | True |
| Contrapositive | If 28 cannot be divided exactly by 4, then 4 is not a factor of 28. | False | False | True |

(b) If $1_2 + 1_2 = 10_2$, then $5_6 + 5_6 = 10_6$

| | Statement | Antecedent | Consequent | Truth value |
|---|---|---|---|---|
| Implication | If $1_2 + 1_2 = 10_2$, then $5_6 + 5_6 = 10_6$ | True | False | False |
| Converse | If $5_6 + 5_6 = 10_6$, then $1_2 + 1_2 = 10_2$ | False | True | True |
| Inverse | If $1_2 + 1_2 \neq 10_2$, then $5_6 + 5_6 \neq 10_6$ | False | True | True |
| Contrapositive | If $5_6 + 5_6 \neq 10_6$, then $1_2 + 1_2 \neq 10_2$ | True | False | False |

12. (a) True.
    (b) False, plants such as Rafflesia, Venus flytrap are the plants that do not carry out photosynthesis.
    (c) True.
    (d) False. The probability is 1.
13. (a) The sum of the interior angles of a pentagon is not 540°.
        False. The sum of the interior angles of a pentagon
        $= (5 - 2)(180°)$
        $= 3(180°)$
        $= 540°$
    (b) If 4 is a factor of 40, then 8 is a factor of 40.
        True.
    (c) If $2m$ is not an even number, then $m$ is not an even number.
        True.
    (d) If a quadratic function does not have two equal roots, then the graph of the function does not intersect the $x$-axis at one point. True.

## Self Test 2

1. (a) Deductive argument
   (b) Inductive argument
   (c) Deductive argument
   (d) Inductive argument
2. (a) Not valid because it does not comply with the valid form of deductive argument.
       Not sound because the argument is invalid and the conclusion is false (Snakes are reptiles).
   (b) Valid because it complies with the valid form of deductive argument.
       Not sound because premise 1 is false (not all cats are black)
   (c) Valid because it complies with the valid form of deductive argument.
       Sound because the premises and conclusion are true.
   (d) Not valid because it does not comply with the valid form of deductive argument.
       Not sound because the argument is invalid and the conclusion is false.

(e) Valid because it complies with the valid form of deductive argument.
Sound because the premises and conclusion are true.

**3** (a) Puan Shamini wears batik clothing every Monday.

(b) $\dfrac{x+3}{2} \neq 3$

(c) $x+6 > 5$

**4** (a) All Malaysians have blue identity card.
(b) If the sum of angles $x$ and $y$ is $360°$, then $x$ and $y$ are a pair of conjugate angles.
(c) If straight line $y = mx + c$ is parallel to $y$-axis, then $m = \infty$.

**5** (a) The conclusion is false. The argument is weak and not cogent.
(b) The conclusion is true. All premises are true. The argument is strong and cogent.
(c) The conclusion is true. The premise is false. The argument is strong but not cogent.
(d) The conclusion is true. All premises are true. The argument is strong and cogent.

**6** (a) $n^2 + 5$, $n = 1, 2, 3, ...$     (b) $\dfrac{n}{1+2n}$, $n = 1, 2, 3, ...$

**7** (a) $T_n = n$-th day
$T_1 = x$
$T_2 = x + 2$
$T_3 = x + 2 + 2$
    $= x + 2(2)$
$T_4 = x + 3(2)$
$\vdots$
$T_n = x + 2(n-1)$, $n = 1, 2, 3, ...$
(b) Total number of festive cookies sold
    $= 7x + 2(1 + 2 + 3 + 4 + 5 + 6)$
$77 = 7x + 2(21)$
$7x = 77 - 42$
   $= 35$
 $x = 5$
On the last day, $T_7 = 5 + 2(7-1)$
              $= 5 + 12$
              $= 17$ boxes

**8** Premise 1: If the sum of the stamps can be divided exactly by 4 and 5, then the sum of the stamps can be divided exactly by 20.
Premise 2: If $x$ can be divided exactly by 20, then $x$ is the sum of stamps collected by Eva.
Premise 3: 120 and 140 can be divided exactly by 20.
Conclusion: 120 is the minimum stamps collected by Eva.

**9** (a) $45, 50, 55, 60, ……$
$45 = 40 + 1(5)$
$50 = 40 + 2(5)$
$55 = 40 + 3(5)$
$60 = 40 + 4(5)$
$\vdots$
$40 + 5n$, $n = 1, 2, 3, 4, ...$
(b) $40 + 5n \geqslant 120$
    $5n \geqslant 80$
     $n \geqslant 16$
Minimum value of $n = 16$
Susan will achieve the typing speed of 120 words per minute on the 16th day. She has succeeded in achieving her targeted typing speed before the typing competition.

## SPM PRACTICE

### Paper 1

**1** B
**2** C
**3** B

**4** B
  A     False and true $\rightarrow$ false
  B     True or false $\rightarrow$ true
  C     False or false $\rightarrow$ false
  D     True and false $\rightarrow$ false
**5** C
**6** C
**7** C
**8** A

### Paper 2
#### Section A

**1** (a) Some             (b) All
(c) Some             (d) Some

**2** (a) Implication 1 : If the volume of a sphere is $972\pi\,\text{mm}^3$, then the radius of the sphere is 9 mm.
   Implication 2 : If the radius of a sphere is 9 mm, then the volume of the sphere is $972\pi\,\text{mm}^3$.
(b) Implication 1 : If Krishnan further studies to London, then Krishnan is awarded with scholarship from *Jabatan Perkhidmatan Awam* (JPA).
   Implication 2 : If Krishnan is awarded with scholarship from *Jabatan Perkhidmatan Awam* (JPA), then Krishnan further studies to London.

**3** (a) If $x$ is a multiple of 3, then $x$ is a multiple of 12.
False because 6 is a multiple of 3 but is not a multiple of 12.
(b) If Kamal does not sleep late, then he will not be late for school.
False. Kamal may be late due to other factors such as the car breaks down, heavy traffic and etc.

**4** (a) Kites do not have a pair of opposite sides that are parallel.
(b) Valid because it complies with the valid form of deductive argument.
Not sound because the conclusion is false.

**5** (a) Implication 1: If two straight lines are perpendicular to each other, then the product of the gradients of the two straight lines is $-1$.
   Implication 2 : If the product of the gradients of two straight lines is $-1$, then the two straight lines are perpendicular to each other.
(b) $n^{n+1} - 3n$, $n = 1, 2, 3, ...$

#### Section B

**6** (a) Premise 2: Set $M$ has 5 elements.
(b) The conclusion is true. All premises are true. The argument is strong and cogent.
(c) (i) Let $T_n$ = Area of the $n^{\text{th}}$ semicircle
$$T_1 = \frac{1}{2}\pi(1 \times 7)^2 = \frac{1}{2}\pi(2^0 \times 7)^2$$
$$T_2 = \frac{1}{2}\pi(2 \times 7)^2 = \frac{1}{2}\pi(2^1 \times 7)^2$$
$$T_3 = \frac{1}{2}\pi(4 \times 7)^2 = \frac{1}{2}\pi(2^2 \times 7)^2$$
$$T_4 = \frac{1}{2}\pi(8 \times 7)^2 = \frac{1}{2}\pi(2^3 \times 7)^2$$
$$\vdots$$
$$T_n = \frac{1}{2}\pi(2^{n-1} \times 7)^2$$
(ii) $T_7 = \dfrac{1}{2}\left(\dfrac{22}{7}\right)(2^{7-1} \times 7)^2$

      $= 315\,392\ \text{cm}^2$

**7** (a) (i) Not a statement
(ii) Statement
(iii) Statement
(b) $5^3 = 5 \times 3$ (False)
A regular octagon has 8 axes of symmetry. (True)

A false compound statement: $5^3 = 5 \times 3$ and a regular octagon has 8 axes of symmetry.

(c) Deductive argument

(d) (i) If $b^2 - 4ac < 0$, then the quadratic equation $ax^2 + bx + c = 0$ has no root.

(ii) 7 is an odd number.
The argument is not sound because Premise 1 is false. 3 is an odd number but 2 is a prime number.

**8** (a) Let $T_n$ = the total mass of processed rice at the $n^{th}$ hour
$T_1 = 10$
$T_2 = 10 + 9$
$T_3 = 10 + 2(9)$
$T_4 = 10 + 3(9)$
$\vdots$
$T_n = 10 + 9(n - 1)$, $n = 1, 2, 3, ...$

(b) 8 a.m. – 10 p.m. = 14 hours
$T_{14} = 10 + 9(14 - 1)$
$= 127$ tonnes

(c) Price per kg $= \dfrac{RM20}{5 \text{ kg}}$

$= RM4/kg$
Total mass = 127 000 kg
Total revenue = 127 000 kg × RM4/kg
$= RM508\ 000$

(d) Number of months to break even on the capital
$= \dfrac{RM1\ 000\ 000}{RM508\ 000}$
$= 1.96$
$\approx 2$ months