Fully-Worked Solutions

FORM 5
CHAPTER 7

Self Test 1

1 Size of class interval $= \frac{40 - 17}{5}$

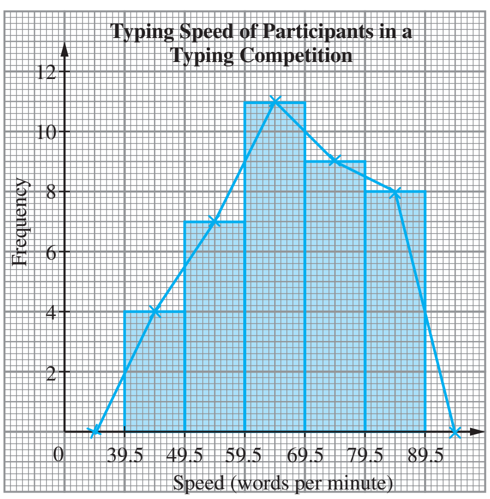
$= 4.6 \approx 5$

Age (years)	Tally	Frequency	Lower limit	Upper limit	Midpoint	Lower boundary	Upper boundary
16 – 20	|||	3	16	20	18	15.5	20.5
21 – 25	𝍸 |||	8	21	25	23	20.5	25.5
26 – 30	𝍸 ||||	9	26	30	28	25.5	30.5
31 – 35	𝍸 |	6	31	35	33	30.5	35.5
36 – 40	||||	4	36	40	38	35.5	40.5

2

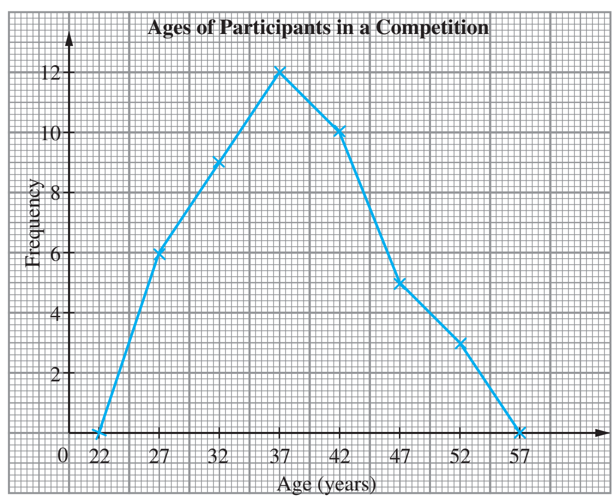
Number of books	Midpoint	Lower limit	Upper limit	Lower boundary	Upper boundary	Frequency	Cumulative frequency
22 – 26	24	22	26	21.5	26.5	15	15
27 – 31	29	27	31	26.5	31.5	19	34
32 – 36	34	32	36	31.5	36.5	26	60
37 – 41	39	37	41	36.5	41.5	24	84
42 – 46	44	42	46	41.5	46.5	10	94
47 – 51	49	47	51	46.5	51.5	6	100

3



4

Age (years)	Frequency	Midpoint
20 – 24	0	22
25 – 29	6	27
30 – 34	9	32
35 – 39	12	37
40 – 44	10	42
45 – 49	5	47
50 – 54	3	52
55 – 59	0	57



5 (a) Area A = Uniform
Area B = Left-skewed

(b) The dispersion of the prices of houses in both areas A and B are the same.

(c) Area B represents the urban area while area A represents the suburban area. Most of the double storey terrace houses in area B are sold at higher prices. There are still many low priced double storey terrace houses in the suburban area.

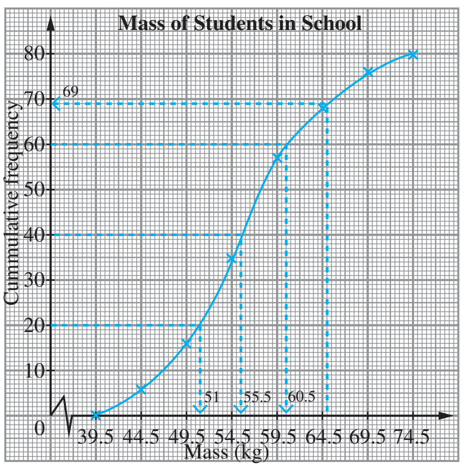
6 (a) Sports club A: Bell-shaped
Sports club B: Left-skewed

(b) The dispersion of ages of members for the two sports clubs are the same.

(c) Sports club A represents extreme sports club
Sports club B represents field bowling club.
Most of the members in Sports Club A are less than 54.5 years old while most of the members in Sports Club B are more than 54.5 years old.

7 (a)

Mass (kg)	Frequency	Upper boundary	Cumulative frequency
35 – 39	0	39.5	0
40 – 44	6	44.5	6
45 – 49	10	49.5	16
50 – 54	19	54.5	35
55 – 59	22	59.5	57
60 – 64	11	64.5	68
65 – 69	8	69.5	76
70 – 74	4	74.5	80



(b) (i) First quartile = 51 kg

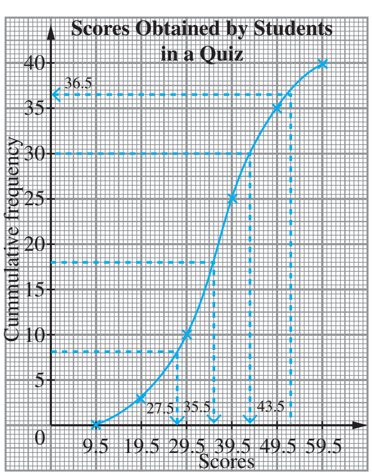
(ii) median = 55.5 kg

(iii) Third quartile = 60.5 kg

(iv) $\dfrac{80-69}{80} \times 100\% = 13.75\%$

8 (a)

Scores	Frequency	Upper boundary	Cumulative frequency
0 – 9	0	9.5	0
10 – 19	3	19.5	3
20 – 29	7	29.5	10
30 – 39	15	39.5	25
40 – 49	10	49.5	35
50 – 59	5	59.5	40



(b) (i) $P_{20} = \left(\dfrac{20}{100} \times 40\right)$ the value
= the 8th value
= 27.5

(ii) $P_{45} = \left(\dfrac{45}{100} \times 40\right)$ the value
= the 18th value
= 35.5

(iii) $P_{75} = \left(\dfrac{75}{100} \times 40\right)$ the value
= the 30th value
= 43.5

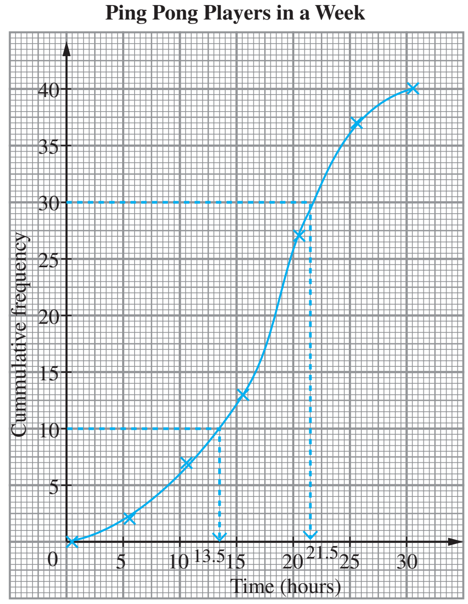
(c) Percentage of students who obtained scores of 52 and below
$= \dfrac{36}{40} \times 100\%$
$= 90\%$

Self Test 2

1

Time (hour)	Frequency	Upper boundary	Cumulative frequency
	0	0.5	0
1 – 5	2	5.5	2
6 – 10	5	10.5	7
11 – 15	6	15.5	13
16 – 20	14	20.5	27
21 – 25	10	25.5	37
26 – 30	3	30.5	40

Total Practice Time for School Ping Pong Players in a Week



From the ogive,
Range = 30.5 – 0.5
= 30 hours

Interquartile range = 21.5 – 13.5
= 8 hours

The dispersion of the total practice time in a week is 30 hours. 50% of the players have the difference in total practice time of 8 hours.

2 (a)

Score	Frequency, f	Midpoint, x	fx	fx^2
21 – 40	2	30.5	61.0	1 860.50
41 – 60	5	50.5	252.5	12 751.25
61 – 80	13	70.5	916.5	64 613.25
81 – 100	21	90.5	1 900.5	171 995.25
101 – 120	8	110.5	884.0	97 682.00
121 – 140	1	130.5	130.5	17 030.25
	$\Sigma f = 50$		**$\Sigma fx = 4\,145$**	**$\Sigma fx^2 =$ 365 932.50**

mean, $\bar{x} = \dfrac{\Sigma fx}{\Sigma f}$

$= \dfrac{4\,145}{50}$

$= 82.9$

variance $\sigma^2 = \dfrac{\Sigma fx^2}{\Sigma f} - (\bar{x})^2$

$= \dfrac{365\,932.50}{50} - (82.9)^2$

$= 446.24$

standard deviation, $\sigma = \sqrt{446.24}$

$= 21.12$

(b)

Time (minutes)	Frequency, f	Midpoint, x	fx	fx^2
61 – 70	5	65.5	327.5	21 451.25
71 – 80	8	75.5	604.0	45 602.00
81 – 90	15	85.5	1 282.5	109 653.75
91 – 100	12	95.5	1 146.0	109 443.00
101 – 110	6	105.5	633.0	66 781.50
111 – 120	4	115.5	462.0	53 361.00
	$\Sigma f = 50$		**$\Sigma fx = 4\,455$**	**$\Sigma fx^2 =$ 406 292.5**

mean, $\bar{x} = \dfrac{\Sigma fx}{\Sigma f}$

$= \dfrac{4\,455}{50}$

$= 89.1$

variance $\sigma^2 = \dfrac{\Sigma fx^2}{\Sigma f} - (\bar{x})^2$

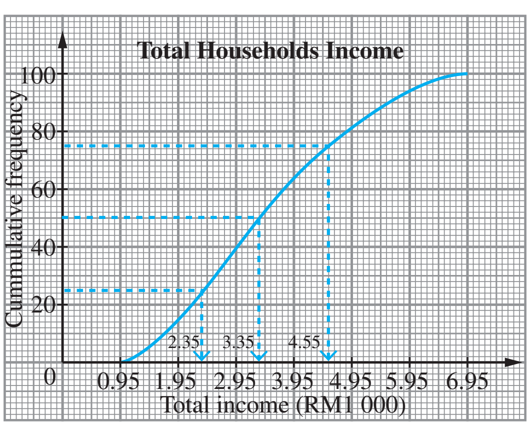
$= \dfrac{406\,292.5}{50} - (89.1)^2$

$= 187.04$

standard deviation, $\sigma = \sqrt{187.04}$

$= 13.68$

3



(a) From the ogive, the minimum value = 0.95, the maximum value = 6.95

Position of $Q_1 = \dfrac{1}{4} \times 100$

$= 25$

$\therefore Q_1 = 2.35$

Position of $Q_2 = \dfrac{1}{2} \times 100$

$= 50$

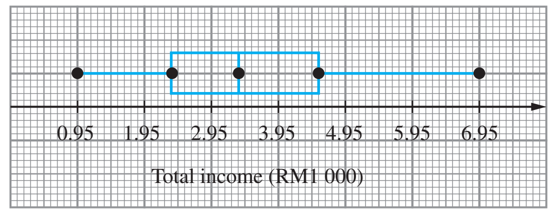
$\therefore Q_2 = 3.35$

Position of $Q_3 = \dfrac{3}{4} \times 100$

$= 75$

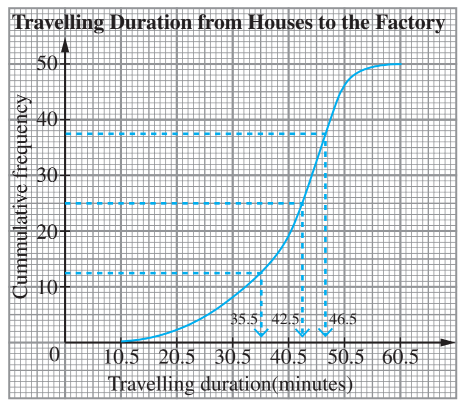
$\therefore Q_3 = 4.55$

Box plot



(b) Data distribution shape: Right-skewed

4



(a) From the ogive, the minimum value = 10.5, the maximum value = 60.5

Position of $Q_1 = \dfrac{1}{4} \times 50$

$= 12.5$

$\therefore Q_1 = 35.5$

Position of $Q_2 = \dfrac{1}{2} \times 50$

$= 25$

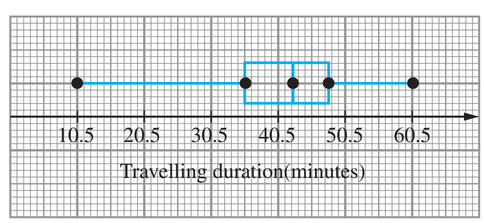
$\therefore Q_2 = 42.5$

Position of $Q_3 = \dfrac{3}{4} \times 50$

$= 37.5$

$\therefore Q_3 = 46.5$

Box plot



(b) The shape of data distribution: Left-skewed

5 Hei Yin

Time (minutes)	Frequency, f	Midpoint, x	fx	fx^2
48.1 – 49.0	1	48.55	48.55	2 357.1025
49.1 – 50.0	5	49.55	247.75	12 276.0125
50.1 – 51.0	13	50.55	657.15	33 218.9325
51.1 – 52.0	7	51.55	360.85	18 601.8175
52.1 – 53.0	3	52.55	157.65	8 284.5075
53.1 – 54.0	1	53.55	53.55	2 867.6025
	$\Sigma f = 30$		**$\Sigma fx =$ 1 525.50**	**$\Sigma fx^2 =$ 77 605.975**

Mean, $\bar{x}$

$= \frac{\Sigma fx}{\Sigma f}$

$= \frac{1\,525.50}{30}$

$= 50.85$ seconds

Standard deviation, σ

$= \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$

$= \sqrt{\frac{77\,605.975}{30} - (50.85)^2}$

$= 1.069$ seconds

William

Time (minutes)	Frequency, f	Midpoint, x	fx	fx^2
48.1 – 49.0	2	48.55	97.10	4 714.2050
49.1 – 50.0	7	49.55	346.85	17 186.4175
50.1 – 51.0	9	50.55	454.95	22 997.7225
51.1 – 52.0	6	51.55	309.30	15 944.4150
52.1 – 53.0	4	52.55	210.20	11 046.0100
53.1 – 54.0	2	53.55	107.1	5 735.205
	$\Sigma f = 30$		**$\Sigma fx =$ 1 525.5**	**$\Sigma fx^2 =$ 77 623.975**

Mean, $\bar{x}$

$= \frac{\Sigma fx}{\Sigma f}$

$= \frac{1\,525.50}{30}$

$= 50.85$ seconds

Standard deviation, σ

$= \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$

$= \sqrt{\frac{77\,623.975}{30} - (50.85)^2}$

$= 1.320$ seconds

$\therefore$ Hei Yin shows better and consistent performance. The standard deviation of Hei Yin's record is smaller than William.

6 (a) Garden A

Diameter (cm)	Frequency, f	Midpoint, x	fx	fx^2
5.0 – 5.9	1	5.45	5.45	29.7025
6.0 – 6.9	8	6.45	51.60	332.8200
7.0 – 7.9	9	7.45	67.05	499.5225
8.0 – 8.9	12	8.45	101.40	856.83
9.0 – 9.9	10	9.45	94.50	893.0250
10.0 – 10.9	1	10.45	10.45	109.2025
	$\Sigma f = 41$		**$\Sigma fx =$ 330.45**	**$\Sigma fx^2 =$ 2 721.1025**

Mean, $\bar{x}$

$= \frac{\Sigma fx}{\Sigma f}$

$= \frac{330.45}{41}$

$= 8.0598$ cm

Standard deviation, σ

$= \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$

$= \sqrt{\frac{2\,721.1025}{41} - \left(\frac{330.45}{41}\right)^2}$

$= 1.1869$ cm

Garden B

Diameter (cm)	Frequency, f	Midpoint, x	fx	fx^2
5.0 – 5.9	3	5.45	16.35	89.1075
6.0 – 6.9	7	6.45	45.15	291.2175
7.0 – 7.9	10	7.45	74.50	555.0250
8.0 – 8.9	6	8.45	50.70	428.4150
9.0 – 9.9	5	9.45	47.25	446.5125
10.0 – 10.9	2	10.45	20.90	218.4050
	$\Sigma f = 33$		**$\Sigma fx =$ 254.85**	**$\Sigma fx^2 =$ 2 028.6825**

Mean, $\bar{x}$

$= \frac{\Sigma fx}{\Sigma f}$

$= \frac{254.85}{33}$

$= 7.7227$ cm

Standard deviation, σ

$= \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$

$= \sqrt{\frac{2\,028.6825}{33} - \left(\frac{254.85}{33}\right)^2}$

$= 1.3546$ cm

(b) The flowers from garden A have a larger diameter than those from garden B. The diameter of the flowers from garden A has a smaller dispersion compared to the flowers from garden B. This indicates that fertiliser P used in garden A can produce larger and more consistent flowers. Fertiliser Q used in garden B results in smaller flowers compared to those in garden A.
Fertiliser P is more effective than fertiliser Q.

7 Company A:
Minimum value = 24.925 mm, Q_1 = 24.975 mm,
median = 25 mm, Q_3 = 25.025 mm,
Maximum value = 25.075 mm

Company B:
Minimum value = 24.86 mm, Q_1 = 24.94 mm,
median = 24.955 mm, Q_3 = 24.97 mm,
Maximum value = 25.025 mm

Company C:
Minimum value = 24.90 mm, Q_1 = 24.975 mm,
median = 25.025mm, Q_3 = 25.05 mm,
Maximum value = 25.10 mm

Acceptable thickness: 24.9 mm ~ 25.1mm

Company A will be chosen by the furniture factory as the supplier of plywood for the year 2024 because the minimum and the maximum thickness are in the required range. The thickness of the plywood from company A is more consistent than company C.

SPM PRACTICE

Paper 1

1 C

Mean, $\bar{x} = \frac{5(30.5) + 4(50.5) + 3(70.5) + 5(90.5) + 2(110.5) + 1(130.50)}{5 + 4 + 3 + 5 + 2 + 1}$

$= 68.5$ kg

2 B

Score	10 – 29	30 – 49	50 – 69	70 – 89	90 – 109
Midpoint	19.5	39.5	59.5	79.5	99.5
Cumulative frequency	x	10	18	25	30
Frequency	x	$10 - x$	8	7	5

$$63.5 = \frac{x(19.5) + (10 - x)(39.5) + 8(59.5) + 7(79.5) + 5(99.5)}{30}$$

$$63.5(30) = 19.5x + 395 - 39.5x + 476 + 556.5 + 497.5$$
$$1\,905 = 1\,925 - 20x$$
$$20x = 20$$
$$x = 1$$

3 **A**

Midpoint of marks	25.5	35.5	45.5	55.5	65.5	75.5
Frequency	10	14	16	25	10	5

$$\text{Mean}, \bar{x} = \frac{10(25.5) + 14(35.5) + 16(45.5) + 25(55.5) + 10(65.5) + 5(75.50)}{10 + 14 + 16 + 25 + 10 + 5}$$
$$= 48.75$$

Standard deviation, σ

$$= \sqrt{\frac{10(25.5^2) + 14(35.5^2) + 16(45.5^2) + 25(55.5^2) + 10(65.5^2) + 5(75.5^2)}{10 + 14 + 16 + 25 + 10 + 5} - (48.75)^2}$$
$$= 13.94$$

4 **B**
Position of P_{25} = position of the first quartile

5 **B**

6 **B**
Symmetric distribution

7 **B**
There are 160 patients whose LDL reading is less than 4.14 mmol/L.
Therefore, the total patients whose LDL reading more than 4.14 mmol/L is 40.

$$\text{Percentage} = \frac{40}{200} \times 100\%$$
$$= 20\%$$

8 **A**
False statement. Classes *R* and *S* have the same median, not mean.

9 **C**
Graph C shows the histogram. The rest are not histogram.

10 **B**

$$\text{Percentage} = \frac{10}{50} \times 100\%$$
$$= 20\%$$

Paper 2

Section A

1 mean, $\bar{x}$

$$= \frac{5(27) + 13(32) + 23(37) + 34(42) + 62(47) + 43(52) + 20(57)}{5 + 13 + 23 + 34 + 62 + 43 + 20}$$
$$= 45.6$$

Standard deviation, σ

$$= \sqrt{\frac{5(27^2) + 13(32^2) + 23(37^2) + 34(42^2) + 62(47^2) + 43(52^2) + 20(57^2)}{5 + 13 + 23 + 34 + 62 + 43 + 20} - (45.6)^2}$$
$$= 7.334$$

2 (a) Left-skewed

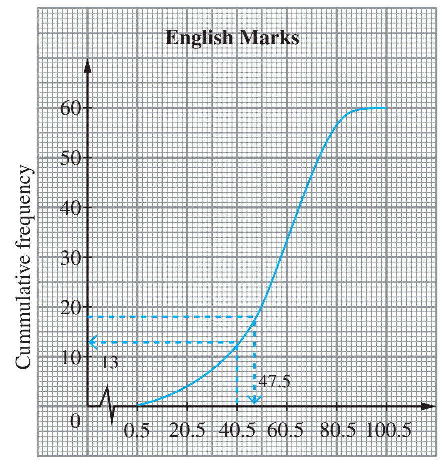
(b) Range = 160 – 30
= 130

Interquartile range = 130 – 84
= 46

3 (a) $P_{30} = \text{the}\left(\frac{30}{100} \times 60\right)\text{th value}$
= the 18th value
= 47.5%

(b) Number of students who passed = 60 – 13
= 47

$$\% \text{ of students who passed} = \frac{47}{60} \times 100\%$$
$$= 78.3\%$$



4 Company *A*: Median length = 92 cm, interquartile range = 13 cm

Company *B*: Median length = 89 cm, interquartile range = 5 cm

With assumption that the data from both companies are having symmetric distribution, thus, Q_1 and Q_3 for company *A* are 85.5 and 98.5 respectively while Q_1 and Q_3 for Company *B* are 86.5 and 91.5 respectively.

Mr Ong should choose the koi fish fingerlings from company *A*. This is because the koi fish fingerlings from company *A* show higher median length than company *B*. The median value from company *A* is also larger than the value of third quartile from company *B*. If Mr Ong buys the koi fish fingerlings from company *A*, he has at least 50% chance to grow the size of the fish more than 92 cm as compared to company *B*. This means that, in average, koi fish from company *A* have the potential to grow larger compared to the koi fish from company *B*, which have a median length of 89 cm and an interquartile range of 5 cm. With this choice, Mr Ong has a higher chance of acquiring koi fish with the potential to reach a larger size in the competition for the longest koi fish.

Section B

5 (a) $\text{Saiz of class interval} = \frac{48 - 18}{7}$
$= 4.3 \approx 5$

Age (years)	Tally	Frequency
16 – 20	|||	3
21 – 25	~~||||~~ |||	8
26 – 30	~~||||~~	5
31 – 35	~~||||~~ ||||	9
36 – 40	~~||||~~ ||	7
41 – 45	~~||||~~	5
46 – 50	|||	3

(b)

Age (years)	Frequency, f	Midpoint, x	fx	fx^2
16 – 20	3	18	54	972
21 – 25	8	23	184	4 232
26 – 30	5	28	140	3 920
31 – 35	9	33	297	9 801
36 – 40	7	38	266	10 108
41 – 45	5	43	215	9 245
46 – 50	3	48	144	6 912
	$\Sigma f = 40$		$\Sigma fx = 1\,300$	$\Sigma fx^2 = 45\,190$

mean, $\bar{x} = \frac{\Sigma fx}{\Sigma f}$

$= \frac{1\,300}{40}$

$= 32.5$

variance $\sigma^2 = \frac{\Sigma fx^2}{\Sigma f} - (\bar{x})^2$

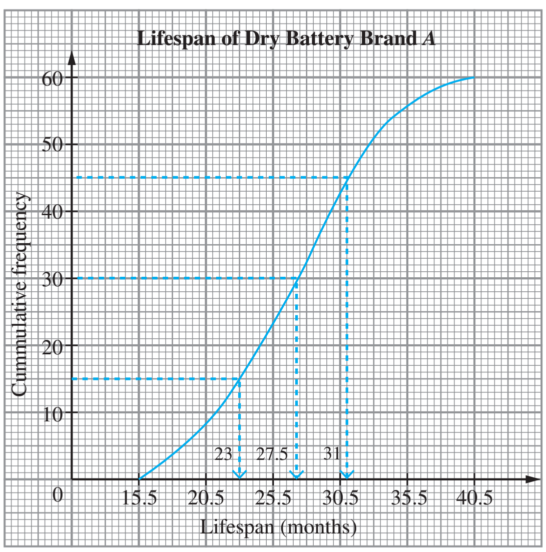
$= \frac{45\,190}{40} - \left(\frac{1\,300}{40}\right)^2$

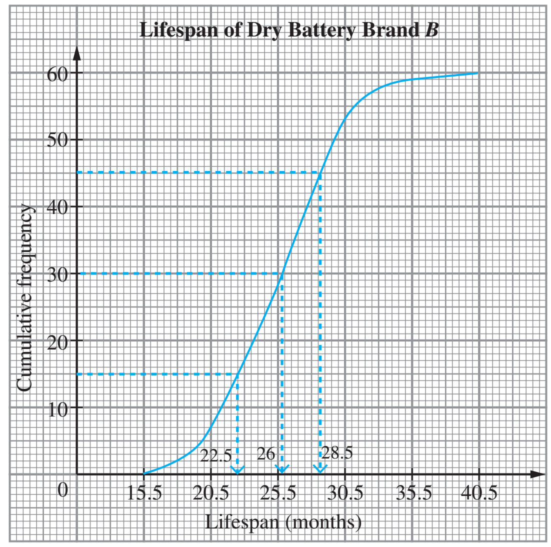
$= 73.5$

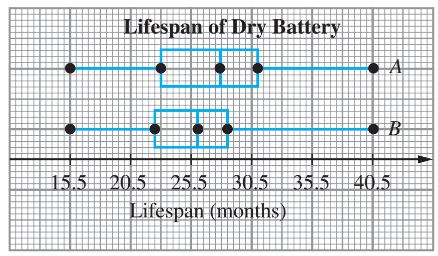
standard deviation, $\sigma = \sqrt{73.5}$

$= 8.57$

6 (a)







(b) Car battery A has a longer mean of lifespan. The probability of the lifespan of car battery A more than 27.5 months is 50%.

Even though car battery B has a shorter lifespan (1.5 months shorter than car battery A), its performance is more consistent than car battery A.