FORM 5
CHAPTER 6

Self Test 1

1 (a) Quadrant II, Corresponding reference angle = 180° – 162°
= 18°

(b) Quadrant III, Corresponding reference angle = 222.5° – 180°
= 42.5°

(c) Quadrant IV, Corresponding reference angle = 360° – 305° 32'
= 54° 28'

(d) Quadrant II, Corresponding reference angle = 180° – 96°
= 84°

2 (a) $\sin 123° = \sin (180° - 123°)$
$= \sin 57°$

(b) $\cos 154° = -\cos (180° - 154°)$
$= -\cos 26°$

(c) $\tan 105° = -\tan (180° - 105°)$
$= -\tan 75°$

(d) $\sin 205.6° = -\sin (205.6° - 180°)$
$= -\sin 25.6°$

(e) $\cos 238.2° = -\cos (238.2° - 180°)$
$= -\cos 58.2°$

(f) $\tan 194.3° = \tan (194.3° - 180°)$
$= \tan 14.3°$

(g) $\sin 297° 21' = -\sin (360° - 297° 21')$
$= -\sin 62° 39'$

(h) $\cos 342° 53' = \cos (360° - 342° 53')$
$= \cos 17° 7'$

(i) $\tan 314° 6' = -\tan (360° - 314° 6')$
$= -\tan 45° 54'$

(j) $\sin 332° 26' = -\sin (360° - 332° 26')$
$= -\sin 27° 34'$

3 (a) $\sin \theta = -0.6$
$\cos \theta = -0.8$
$\tan \theta = \frac{-0.6}{-0.8}$
$= \frac{3}{4}$

(b) $\sin \theta = -0.28$
$\cos \theta = 0.96$
$\tan \theta = \frac{-0.28}{0.96}$
$= -\frac{7}{24}$

4 (a) $\sin 123° = \sin (180° - 123°)$
$= \sin 57°$
$= 0.8387$

(b) $\cos 237° = -\cos (237° - 180°)$
$= -\cos 57°$
$= -0.5446$

(c) $\tan 303° = -\tan (360° - 303°)$
$= -\tan 57°$
$= -1.5399$

(d) $\cos 303° = \cos (360° - 303°)$
$= \cos 57°$
$= 0.5446$

(e) $\tan 237° = \tan (237° - 180°)$
$= \tan 57°$
$= 1.5399$

(f) $\tan 123° = -\tan (180° - 123°)$
$= -\tan 57°$
$= -1.5399$

(g) $\cos 123° = -\cos (180° - 123°)$
$= -\cos 57°$
$= -0.5446$

(h) $\sin 237° = -\sin (237° - 180°)$
$= -\sin 57°$
$= -0.8387$

5 (a) $\sin 135° = \sin (180° - 135°)$
$= \sin 45°$
$= \frac{1}{\sqrt{2}}$

(b) $\cos 240° = -\cos (240° - 180°)$
$= -\cos 60°$
$= -\frac{1}{2}$

(c) $\tan 315° = -\tan (360° - 315°)$
$= -\tan 45°$
$= -1$

(d) $\sin 330° = -\sin (360° - 330°)$
$= -\sin 30°$
$= -\frac{1}{2}$

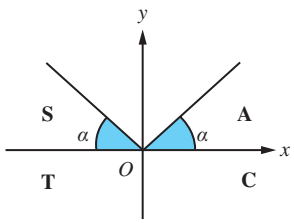
(e) $\sin 300° = -\sin (360° - 300°)$
$= -\sin 60°$
$= -\frac{\sqrt{3}}{2}$

(f) $\tan 150° = -\tan (180° - 150°)$
$= -\tan 30°$
$= -\frac{1}{\sqrt{3}}$

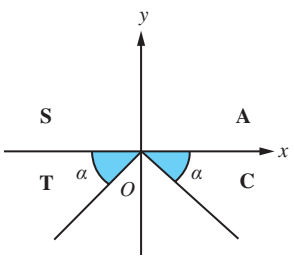
(g) $\cos 150° = -\cos (180° - 150°)$
$= -\cos 30°$
$= -\frac{\sqrt{3}}{2}$

(h) $\tan 240° = \tan (240° - 180°)$
$= \tan 60°$
$= \sqrt{3}$

6 (a) $\sin \theta = 0.4337$
Corresponding reference angle, $\alpha = \sin^{-1} 0.4337$
$= 25.7°$
Actual angle, $\theta = 25.7°$ or $180° - 25.7°$
$= 25.7°$ or $154.3°$



(b) $\sin \theta = -0.632$
Corresponding reference angle, $\alpha = \sin^{-1} 0.632$
$= 39.2°$
Actual angle, $\theta = 180° + 39.2°$ or $360° - 39.2°$
$= 219.2°$ or $320.8°$



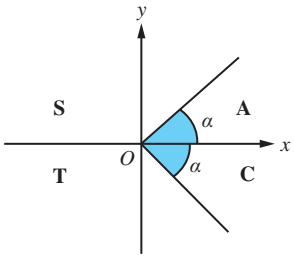
(c) $\cos\theta = 0.8329$

Corresponding reference angle, $\alpha = \cos^{-1} 0.8329$

$= 33.6°$

Actual angle, $\theta = 33.6° \text{ or } 360° - 33.6°$

$= 33.6° \text{ or } 326.4°$



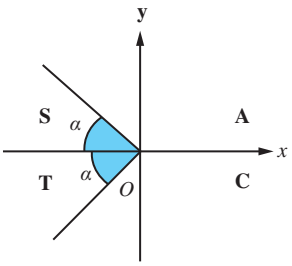
(d) $\cos\theta = -0.284$

Corresponding reference angle, $\alpha = \cos^{-1} 0.284$

$= 73.5°$

Actual angle, $\theta = 180° - 73.5° \text{ or } 180° + 73.5°$

$= 106.5° \text{ or } 253.5°$



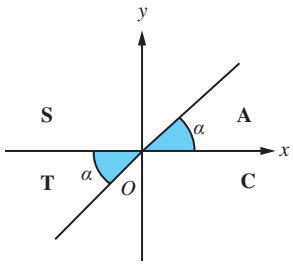
(e) $\tan\theta = 1.732$

Corresponding reference angle, $\alpha = \tan^{-1} 1.732$

$= 60°$

Actual angle, $\theta = 60° \text{ or } 180° + 60°$

$= 60° \text{ or } 240°$



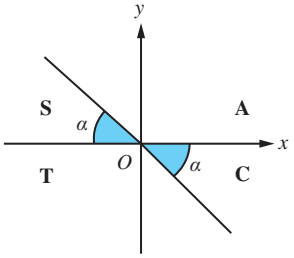
(f) $\tan\theta = -2.605$

Corresponding reference angle, $\alpha = \tan^{-1} 2.605$

$= 69°$

Actual angle, $\theta = 180° - 69° \text{ or } 360° - 69°$

$= 111° \text{ or } 291°$

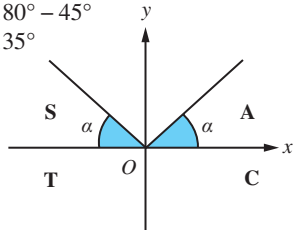


7 (a) $\sin\theta = \frac{1}{\sqrt{2}}$

Corresponding reference angle, $\alpha = 45°$

Actual angle, $\theta = 45° \text{ atau } 180° - 45°$

$= 45° \text{ atau } 135°$

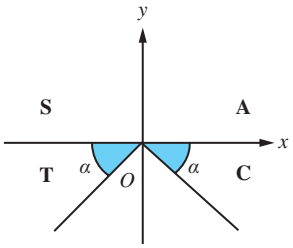


(b) $\sin\theta = -\frac{\sqrt{3}}{2}$

Corresponding reference angle, $\alpha = 60°$

Actual angle, $\theta = 180° + 60° \text{or } 360° - 60°$

$= 240° \text{ or } 300°$

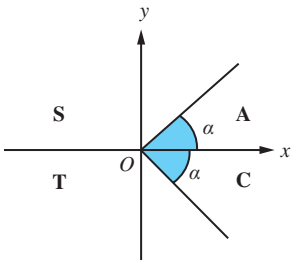


(c) $\cos\theta = \frac{1}{2}$

Corresponding reference angle, $\alpha = 60°$

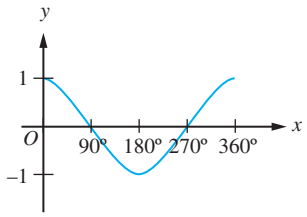
Actual angle, $\theta = 60° \text{or } 360° - 60°$

$= 60° \text{ or } 300°$



(d) $\cos\theta = -1$

$\theta = 180°$

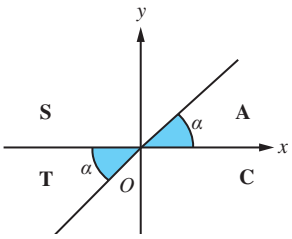


(e) $\tan\theta = \frac{1}{\sqrt{3}}$

Corresponding reference angle, $\alpha = 30°$

Actual angle, $\theta = 30° \text{ or } 180° + 30°$

$= 30° \text{ or } 210°$

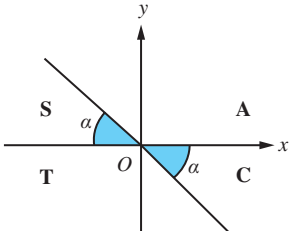


(f) $\tan\theta = -1$

corresponding reference angle, $\alpha = 45°$

Actual angle, $\theta = 180° - 45° \text{ or } 360° - 45°$

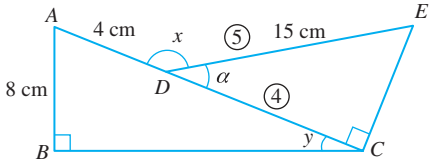
$= 135° \text{ or } 315°$



8 $\cos\alpha = \frac{4}{5}$

$\frac{DC}{15} = \frac{4}{5}$

$DC = 12$



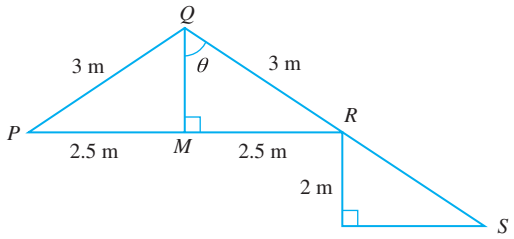
(a) $BC = \sqrt{(4+12)^2 - 8^2}$
$= \sqrt{192}$
$= 13.86$ cm

(b) $\tan x + \sin y = -\tan \alpha + \frac{8}{16}$
$= -\frac{3}{4} + \frac{1}{2}$
$= -\frac{1}{4}$

(c) $\sin y = \frac{1}{2}$ $\quad\quad$ $\cos \alpha = \frac{4}{5}$
$y = \sin^{-1}\frac{1}{2}$ $\quad\quad$ $\alpha = \cos^{-1}\frac{4}{5}$
$= 30°$ $\quad\quad$ $= 36° 52'$
$x = 180° - 36° 52'$
$= 143° 8'$

$x - y = 143° 8' - 30°$
$= 113° 8'$

9



(a) value $\angle PQR = 2\theta$
$= 2\left(\sin^{-1}\frac{2.5}{3}\right)$
$= 2(56.44°)$
$= 112.89°$

(b) $\tan \angle PRS = \tan(56.44° + 90°)$
$= -0.6634$

(c) Height of main pole $= 2 + QM$
$= 2 + \sqrt{3^2 - 2.5^2}$
$= 2 + \sqrt{2.75}$
$= 3.66$ m

Self Test 2

1 (a) Amplitude $= \frac{5}{3}$; Period $= 360°$

(b) Amplitude $= 3$; Period $= \frac{360°}{\frac{2}{3}}$
$= 540°$

(c) Amplitude = None; Period $= \frac{180°}{3}$
$= 60°$

(d) Amplitude $= 2$; Period $= \frac{360°}{4}$
$= 90°$

(e) Amplitude $= 0.5$; Period $= \frac{360°}{3}$
$= 120°$

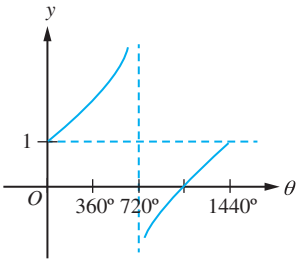
(f) Amplitude = none; Period $= \frac{180°}{2}$
$= 90°$

2 (a) Maximum value $= \frac{1}{4} - 3$; Minimum value $= -\frac{1}{4} - 3$
$= -2.75$ $\quad\quad$ $= -3.25$

(b) Maximum value $= 3 + 1$; Minimum value $= -3 + 1$
$= 4$ $\quad\quad$ $= -2$

(c) Period $= \frac{180°}{\frac{1}{8}}$
$= 1\,440°$

From the sketched graph for $0° \leqslant x \leqslant 360°$,

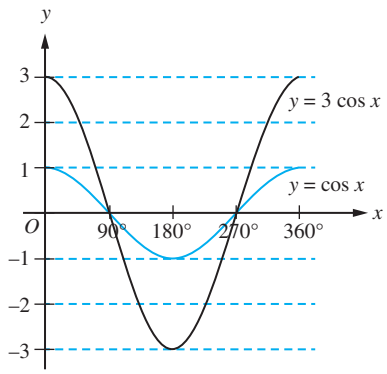


Maximum value $= 2\tan\frac{360}{8} + 1$
$= 3$

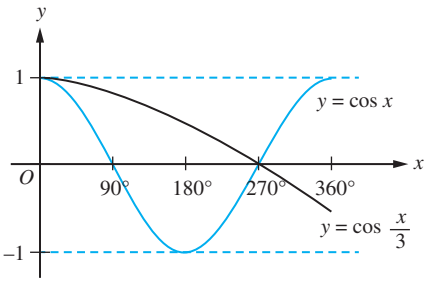
Minimum value $= 1$

(d) Maximum value $= 2 - 5$; Minimum value $= -2 - 5$
$= -3$ $\quad\quad$ $= -7$

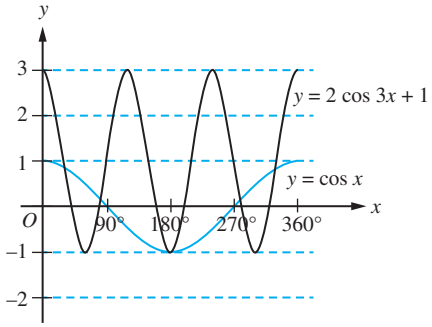
3 (a) $y = 3\cos x$



(b) $y = \cos\frac{x}{3}$



(c) $y = 2\cos 3x + 1$



4 (a) $a = 2 - (-1)$; $b = \frac{1}{2}$; $c = -1$
$= 3$

(b) $a = \frac{5.5 - (-0.5)}{2}$; $b = 3$; $c = \frac{5.5 + (-0.5)}{2}$
$= 3$ $\quad\quad$ $= 2.5$

(c) Period $= 360°$ $\quad$ $c = 3$
$\frac{180°}{b} = 360°$

$b = \frac{180°}{360°}$

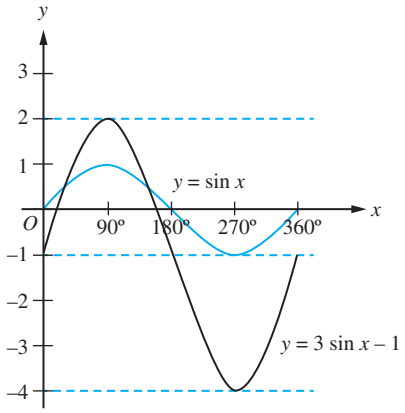
$= \frac{1}{2}$

(d) $a = 1$; $\frac{360°}{b} = 240°$; $c = 0$

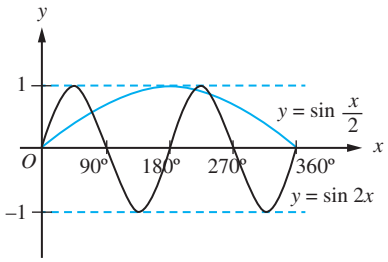
$b = \frac{360°}{240°}$

$= 1.5$

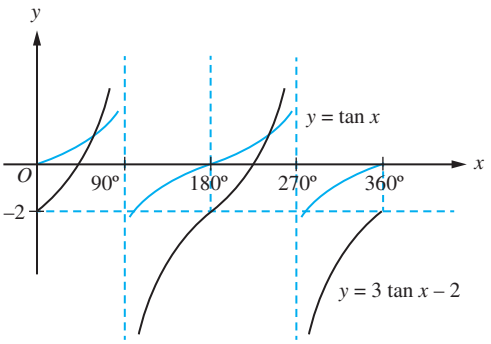
5 (a) $y = \sin x$ and $y = 3 \sin x - 1$



(b) $y = \sin 2x$ and $y = \sin \frac{x}{2}$



(c) $y = \tan x$ and $y = 3 \tan x - 2$



(d) $y = \tan \frac{x}{4}$ and $y = \tan 2x$

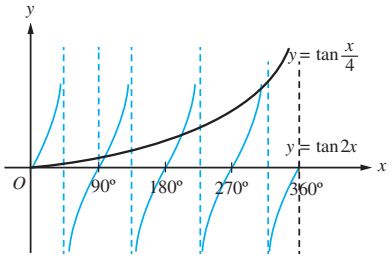
For $y = \tan \frac{x}{4}$,

Period $= \frac{180°}{\frac{1}{4}}$

$= 720°$

Asymptote $= \frac{720°}{2}$

$= 360°$



6 (a) $a = 19 - 12$; Period = 12 hours; $c = 12$

$= 7$

$\frac{360°}{b} = 12$

$b = 30$

(b) Range of temperature in a day $= 19 - 5$

$= 14$ °C

$t = 13$

$T = 7 \sin (30 \times 13) + 12$

$= 15.5$ °C

SPM PRACTICE

Paper 1

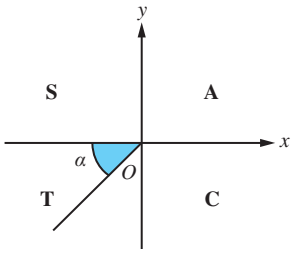
1 A $\cos \theta = -0.7721$ and $180° \leq \theta \leq 360°$

Corresponding reference angle, $\alpha = \cos^{-1} 0.7721$

$= 39° 27'$

Actual angle, $\theta = 180° + 39° 27'$

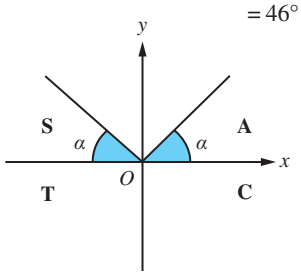
$= 219° 27'$



2 D $\sin A = 0.7193$

Corresponding reference angle, $\alpha = \sin^{-1} 0.7193$

$= 46°$



Actual angle, $A = 46°$ or $180° - 46°$

$= 46°$ or $134°$

$\tan 46° = 1.0355$ or $\tan 134° = -1.0355$

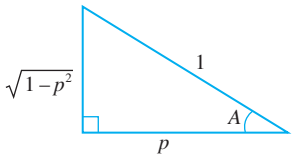
3 B $\cos A$ is positive, therefore A is in Quadrant I or IV

$\sin A$ may be positive or negative

$\sin A = \frac{\text{Opposite side}}{\text{Hypotenuse}}$

$= \frac{\sqrt{1 - p^2}}{1}$

$= \sqrt{1 - p^2}$



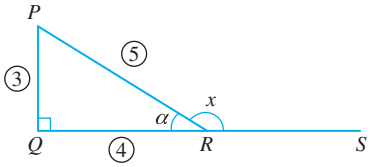
4 B Corresponding reference angle, $\alpha = 45°$

The value of $\sin x$ and $\cos x$ are equivalent in quadrants I and III

Actual angle, $x = 45°$ or $180° + 45°$

$= 45°$ or $225°$

5 D



$\cos x - \tan x = -\cos \alpha - (-\tan \alpha)$

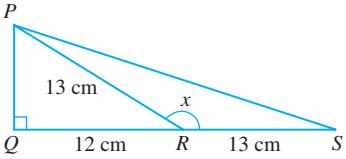
$= -\frac{4}{5} + \frac{3}{4}$

$= -\frac{1}{20}$

6 B $PQ = \sqrt{13^2 - 12^2}$

$= 5$ cm

$\tan x = -\frac{5}{12}$



7 A $AD = 3$ cm

$AC = 3 + 31$

$= 34$ cm

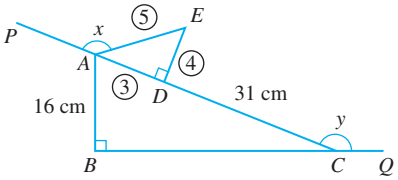
$BC = \sqrt{34^2 - 16^2}$

$= \sqrt{900}$

$= 30$ cm

$\tan y = -\frac{16}{30}$

$= -\frac{8}{15}$



8 B θ in quadrant III

Corresponding reference angle, α

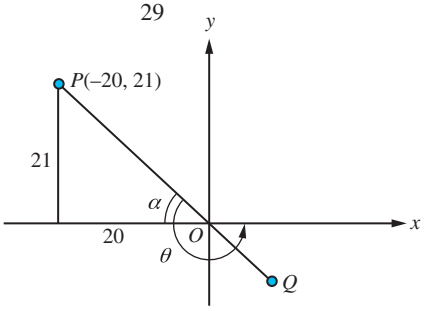
Hypotenuse, $OP = \sqrt{20^2 + 21^2}$

$= \sqrt{841}$

$= 29$

$\cos\theta = -\cos\alpha$

$= -\frac{20}{29}$



9 C

10 C When $\sin x = \cos x$, the corresponding reference angle $= 45°$

Both $\sin x$ and $\cos x$ are negative in quadrant III.

Actual angle, $x = 180° + 45°$

$= 225°$

Paper 2

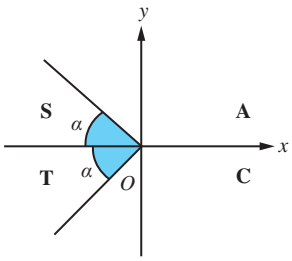
Section A

1 $\cos x = -\frac{\sqrt{3}}{2}$

Corresponding reference angle, $\alpha = 30°$ (special angle)

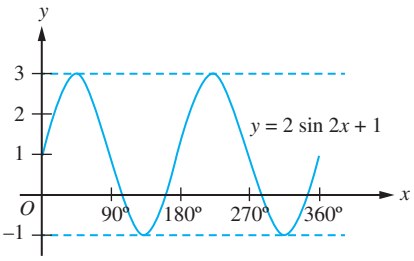
Actual angle, $x = 180° - 30°$ or $180° + 30°$

$= 150°$ or $210°$



2 $b = 2$, period $= \frac{360°}{2}$

$= 180°$



3 (a) $\sin\theta$ = coordinate-y

$= 0.5736$

(b) $\cos\theta$ = coordinate-x

$= -0.8192$

(c) $\tan\theta = \frac{\sin\theta}{\cos\theta}$

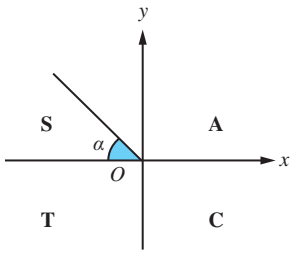
$= \frac{0.5736}{-0.8192}$

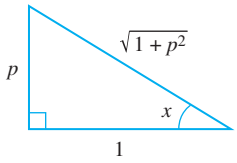
$= -0.7002$

4 $\tan x$ is negative in Quadrant II, therefore $90° \leq x \leq 180°$.

(a) $\sin x = \frac{p}{\sqrt{1+p^2}}$

(b) $\cos x = -\frac{1}{\sqrt{1+p^2}}$



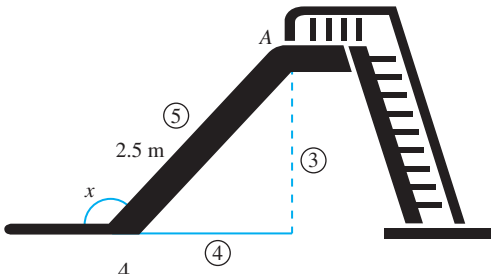


5 $a = \frac{800 - 200}{2}$; $\frac{360°}{b} = 4$; $c = \frac{800 + 200}{2}$

$= 300$ $b = 90°$ $= 500$

$y = 300\cos 90x + 500$

6



(a) $\cos x = -\frac{4}{5}$

(b) $\frac{\text{Vertical distance of } A \text{ from ground}}{3} = \frac{2.5}{5}$

Vertical distance of A from ground $= 0.5 \times 3$

$= 1.5$ m

Section B

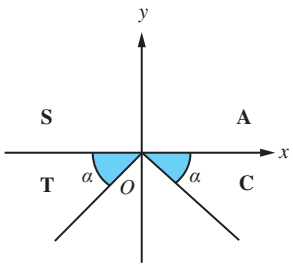
7 (a) $\sin x = -0.3568$

Corresponding reference angle, $\alpha = \sin^{-1} 0.3568$

$= 20°\ 54'$

Actual angle, $x = 180° + 20°\ 54'$ or $360° - 20°\ 54'$

$= 200°\ 54'$ or $339°\ 6'$



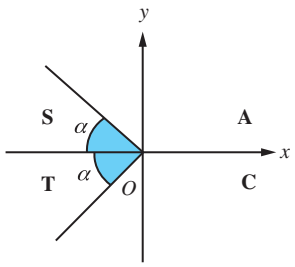
(b) $\cos x = -0.6562$

Corresponding reference angle, $\alpha = \cos^{-1} 0.6562$

$= 48°\ 59'$

Actual angle, $x = 180° - 48°\ 59'$ or $180° + 48°\ 59'$

$= 131°\ 1'$ or $228°\ 59'$



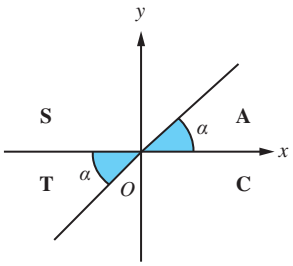
(c) $\tan x = 1.4234$

Corresponding reference angle, $\alpha = \tan^{-1} 1.4234$

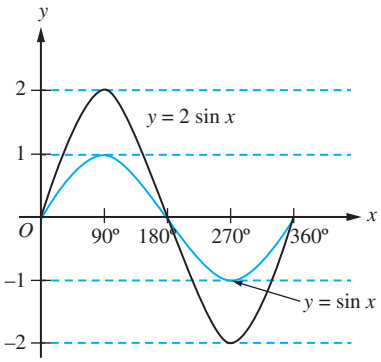
$= 54° 55'$

Actual angle, $x = 54° 55'$ or $180° + 54° 55'$

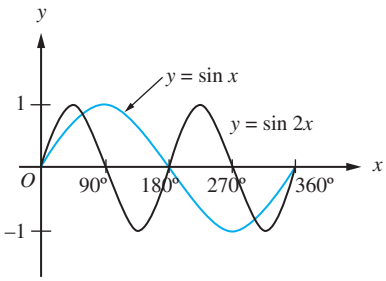
$= 54° 55'$ or $234° 55'$



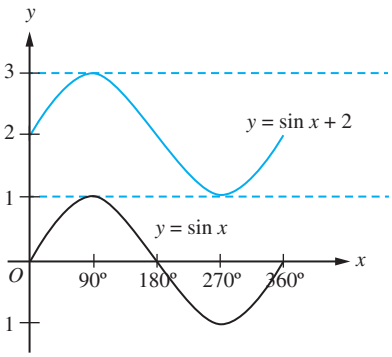
8 (a) $y = 2 \sin x$



(b) $y = \sin 2x$



(c) $y = \sin x + 2$



a increases, the graph is compressed horizontally.

b increases, the period decreases and the number of complete cycles in 360° increases.

c increases, the vertical movement of graph increases.

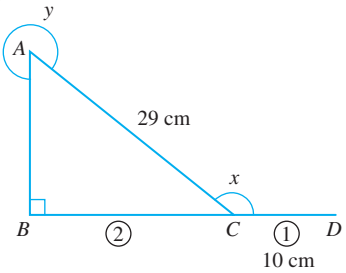
9 (a) $BC = 2 \times 10$ cm

$= 20$ cm

(i) $AB = \sqrt{29^2 - 20^2}$

$= \sqrt{441}$

$= 21$ cm



(ii) x in quadrant II ($\tan x$ is negative)

y in quadrant IV ($\sin y$ is negative)

$$\tan x - \sin y = -\frac{21}{20} - \left(-\frac{20}{29}\right)$$

$$= -\frac{209}{580}$$

(b) Amplitude $= 5$

$$\text{Period} = \frac{360°}{3}$$

$= 120°$

Maximum value $= 5 + 9$

$= 14$

Minimum value $= -5 + 9$

$= 4$

10 (a) Amplitude $= 25$

(b) $$\text{Period} = \frac{360}{40}$$

$= 9$ s

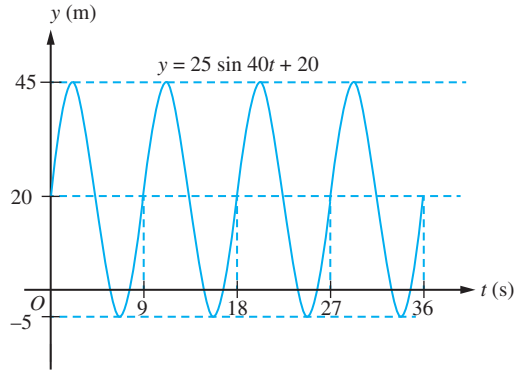
(c) Vertical movement of graph $= 20$

(d) Maximum value $= 25 + 20$

$= 45$

(e) Minimum value $= -25 + 20$

$= -5$



Section C

11 (a) (i) $PC = \sqrt{5^2 - 4^2}$

$= 3$ cm

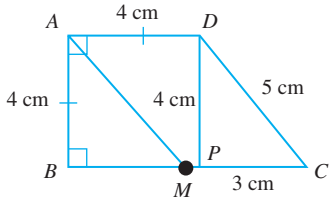
$$BM = \frac{4 + 3}{2}$$

$= 3.5$ cm

$AM = \sqrt{4^2 + 3.5^2}$

$= \sqrt{28.25}$

$= 5.315$ cm



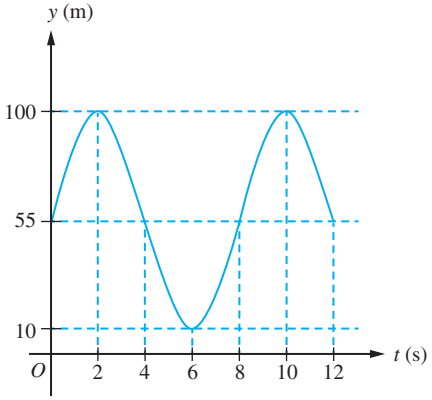
(ii) $\angle AMC$ is in quadrant II

$$\cos \angle AMC + \sin \angle AMC = -\frac{3.5}{5.315} + \frac{4}{5.315}$$

$$= \frac{100}{1\,063} \text{ or } 0.0941$$

(b) (i) Sine function

(ii)



Amplitude, $a = 100 - 55$

$= 45$

$\frac{360°}{b} = 8$

$b = \frac{360°}{8}$

$= 45$

$c = 55$

$\therefore y = 45 \sin 45t + 55$

(iii) Maximum value = 80 cm

Minimum value = 30 cm

$a = \frac{80 - 30}{2}$ $\quad$ $c = \frac{80 + 30}{2}$

$= 25$ $\quad$ $= 55$

The information about period is not given, therefore assume that period remains unchanged.

Trigonometric function, $y = 25 \sin 45t + 55$

$t = 25$, $y = 25 \sin 45(25) + 55$

$= 72.68$ cm