FORM 5
CHAPTER 2

Self Test 1

1 (a) $\begin{bmatrix} 16 & 8 & 13 \\ 12 & 10 & 5 \\ 9 & 4 & 2 \end{bmatrix}$ or $\begin{bmatrix} 16 & 12 & 9 \\ 8 & 10 & 4 \\ 13 & 5 & 2 \end{bmatrix}$

(b) $\begin{bmatrix} 58 & 250 & 354 \\ 120 & 498 & 905 \end{bmatrix}$ or $\begin{bmatrix} 58 & 120 \\ 250 & 498 \\ 354 & 905 \end{bmatrix}$

(c) $\begin{bmatrix} 7\,126 & 17\,404 & 14\,510 & 5\,917 \\ 10\,411 & 13\,208 & 12\,414 & 4\,583 \end{bmatrix}$ or $\begin{bmatrix} 7\,126 & 10\,411 \\ 17\,404 & 13\,208 \\ 14\,510 & 12\,414 \\ 5\,917 & 4\,583 \end{bmatrix}$

2 (a) 2×3 (b) 2×1 (c) 3×3

3 (a) $p_{21} = -3$ (b) $p_{13} = 11$ (c) $p_{22} = 5$ (d) $p_{31} = 19$

4 (a) $A \neq B$ because both matrices have different order.
The order of A is 1×2 whereas the order of B is 1×3.

(b) $C = D$ because both matrices have the same order and each of the corresponding elements are equal.

(c) $E \neq F$ because both matrices have different order.
The order of E is 2×3 whereas the order of F is 3×2.

5 $G = H$

$$\begin{bmatrix} x & 9 \\ 3+y & 5 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 9 \\ 4y & 2x+z \\ 2 & 1 \end{bmatrix}$$

$x = 7$

$3 + y = 4y$
$3y = 3$
$y = 1$

$2x + z = 5$
$z = 5 - 2(7)$
$= -9$

6 $M = N$

$$\begin{bmatrix} -5 & 5x+3 \\ 3x & 1 \end{bmatrix} = \begin{bmatrix} -5 & 2y+11 \\ 6y & 1 \end{bmatrix}$$

$3x = 6y$
$x = 2y \ldots ①$

Substitute ① into $5x + 3 = 2y + 11$
$5(2y) - 2y = 11 - 3$
$8y = 8$
$y = 1$

From ①, $x = 2(1)$
$= 2$

Self Test 2

1 (a) No, because both matrices have different orders.

(b) Yes, because both matrices have the same order.

(c) No, because both matrices have different orders.

2 (a) $\begin{bmatrix} 5 & -1 \\ 7 & 3 \end{bmatrix} + \begin{bmatrix} -2 & 1 \\ 6 & -9 \end{bmatrix} = \begin{bmatrix} 5+(-2) & -1+1 \\ 7+6 & 3+(-9) \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 13 & -6 \end{bmatrix}$

(b) $\begin{bmatrix} 8 & 7 \\ 0 & -5 \\ -4 & -2 \end{bmatrix} - \begin{bmatrix} 2 & 2 \\ -7 & 4 \\ 6 & 1 \end{bmatrix} = \begin{bmatrix} 8-2 & 7-2 \\ 0-(-7) & -5-4 \\ -4-6 & 2-1 \end{bmatrix}$

$$= \begin{bmatrix} 6 & 5 \\ 7 & -9 \\ -10 & 1 \end{bmatrix}$$

3 (a) $\begin{bmatrix} 2-(5-7) & -3-(7+2) & 1-(10-6) \\ 5-(-3+4) & -8-(2+12) & 4-(-3-9) \end{bmatrix} = \begin{bmatrix} 4 & -12 & -3 \\ 4 & -22 & 16 \end{bmatrix}$

(b) $\begin{bmatrix} 2+5-(-7) & -3+7-2 & 1+10-(-6) \\ 5+(-3)-4 & -8+2-12 & 4+(-3)-(-9) \end{bmatrix} = \begin{bmatrix} 14 & 2 & 17 \\ -2 & -18 & 10 \end{bmatrix}$

(c) $\begin{bmatrix} -7-5-2 & 2-7-(-3) & -6-10-1 \\ 4-(-3)-5 & 12-2-(-8) & -9-(-3)-4 \end{bmatrix} = \begin{bmatrix} -14 & -2 & -17 \\ 2 & 18 & -10 \end{bmatrix}$

4 $D = \begin{bmatrix} 2 & 3 \\ 1 & -9 \\ -4 & 6 \end{bmatrix} - \begin{bmatrix} 12 & 5 \\ -4 & 3 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -2 & 8 \\ 7 & 4 \\ 5 & -1 \end{bmatrix}$

$$= \begin{bmatrix} 2-12-(-2) & 3-5-8 \\ 1-(-4)-7 & -9-3-4 \\ -4-0-5 & 6-1-(-1) \end{bmatrix}$$

$$= \begin{bmatrix} -8 & -10 \\ -2 & -16 \\ -9 & 6 \end{bmatrix}$$

5 (a) $\begin{bmatrix} 5 & 2x-1 \\ 7y & 5 \end{bmatrix} + \begin{bmatrix} 2 & 8 \\ 6-y & 3 \end{bmatrix} = \begin{bmatrix} 7 & 5x-11 \\ 18 & 8 \end{bmatrix}$

$\begin{bmatrix} 5+2 & 2x-1+8 \\ 7y+6-y & 5+3 \end{bmatrix} = \begin{bmatrix} 7 & 5x-11 \\ 18 & 8 \end{bmatrix}$

$2x + 7 = 5x - 11$
$3x = 18$
$x = 6$

$6y + 6 = 18$
$6y = 12$
$y = 2$

(b) $[5x-2 \quad 12] - [8 \quad 1+y] = [20 \quad -4]$
$[5x-2-8 \quad 12-1-y] = [20 \quad -4]$

$5x - 10 = 20$
$5x = 30$
$x = 6$

$12 - 1 - y = -4$
$11 + 4 = y$
$y = 15$

(c) $\begin{bmatrix} x \\ 4y \end{bmatrix} + \begin{bmatrix} 2x-1 \\ 7x \end{bmatrix} = \begin{bmatrix} 5 \\ 18 \end{bmatrix}$

$\begin{bmatrix} x+2x-1 \\ 4y+7x \end{bmatrix} = \begin{bmatrix} 5 \\ 18 \end{bmatrix}$

$3x - 1 = 5$
$3x = 6$
$x = 2$

$4y + 7(2) = 18$
$4y = 18 - 14$
$y = 1$

Self Test 3

1 (a) $\begin{bmatrix} 6 & -4 \\ -12 & -2 \end{bmatrix}$ (b) $\begin{bmatrix} -6 & 9 \\ 3 & -15 \end{bmatrix}$

(c) $\begin{bmatrix} 20 \\ -10 \end{bmatrix}$ (d) $[-1.2 \quad 0.6 \quad -1.8]$

2 (a) $2[5 \quad -6 \quad 2] - 3[1 \quad 3 \quad 5] - 2[-1 \quad 4 \quad -7]$
$= [10 \quad -12 \quad 4] - [3 \quad 9 \quad 15] - [-2 \quad 8 \quad -14]$
$= [9 \quad -29 \quad 3]$

(b) $\begin{bmatrix} 2 & 5 \\ 12 & 3 \\ 4 & -1 \end{bmatrix} + \frac{2}{5}\begin{bmatrix} 10 & -0.5 \\ -5 & 30 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 2+4 & 5-0.2 \\ 12-2 & 3+12 \\ 4-0.4 & -1+0.8 \end{bmatrix}$

$$= \begin{bmatrix} 6 & 4.8 \\ 10 & 15 \\ 3.6 & -0.2 \end{bmatrix}$$

(c) $0.3\begin{bmatrix} 9 & 0 \\ -5 & 4 \end{bmatrix} - \begin{bmatrix} 8 & 6 \\ -10 & 2 \end{bmatrix} + 2\begin{bmatrix} -0.5 & 2.5 \\ 3.75 & 4.1 \end{bmatrix}$

$= \begin{bmatrix} 2.7-8-1 & 0-6+5 \\ -1.5+10+7.5 & 1.2-2+8.2 \end{bmatrix} = \begin{bmatrix} -6.3 & -1 \\ 16 & 7.4 \end{bmatrix}$

3 (a) $A+\begin{bmatrix}-3\\4\end{bmatrix}+\frac{1}{3}\begin{bmatrix}9\\12\end{bmatrix}=\begin{bmatrix}2\\-7\end{bmatrix}$

$$A=\begin{bmatrix}2\\-7\end{bmatrix}-\begin{bmatrix}-3\\4\end{bmatrix}-\begin{bmatrix}3\\4\end{bmatrix}$$

$$=\begin{bmatrix}2\\-15\end{bmatrix}$$

(b) $\begin{bmatrix}1&2\\10&3\end{bmatrix}-5A-2\begin{bmatrix}-6&5\\3&1\end{bmatrix}=3\begin{bmatrix}1&-1\\3&2\end{bmatrix}$

$$\begin{bmatrix}1&2\\10&3\end{bmatrix}-2\begin{bmatrix}-6&5\\3&1\end{bmatrix}-3\begin{bmatrix}1&-1\\3&2\end{bmatrix}=5A$$

$$5A=\begin{bmatrix}1&2\\10&3\end{bmatrix}-\begin{bmatrix}-12&10\\6&2\end{bmatrix}-\begin{bmatrix}3&-3\\9&6\end{bmatrix}$$

$$=\begin{bmatrix}1+12-3&2-10+3\\10-6-9&3-2-6\end{bmatrix}$$

$$=\begin{bmatrix}10&-5\\-5&-5\end{bmatrix}$$

$$A=\frac{1}{5}\begin{bmatrix}10&-5\\-5&-5\end{bmatrix}$$

$$=\begin{bmatrix}2&-1\\-1&-1\end{bmatrix}$$

4 (a) $2\begin{bmatrix}1&1-2x\\2+3y&3\end{bmatrix}+\begin{bmatrix}2&9\\-y&4\end{bmatrix}=\begin{bmatrix}4&2x-7\\-1&10\end{bmatrix}$

$$\begin{bmatrix}2+2&2-4x+9\\4+6y-y&6+4\end{bmatrix}=\begin{bmatrix}4&2x-7\\-1&10\end{bmatrix}$$

$11-4x=2x-7$ $\qquad$ $4+5y=-1$

$18=6x$ $\qquad$ $5y=-1-4$

$x=3$ $\qquad$ $y=-1$

(b) $[3x\ \ -6\ \ 7y]-3[1-x\ \ 2\ \ 5y+2]=2[-1\ \ -6\ \ y-7]$

$[3x-3+3x\ \ -6-6\ \ 7y-15y-6]=[-2\ \ -12\ \ 2y-14]$

$6x-3=-2$ $\qquad$ $-8y-6=2y-14$

$6x=1$ $\qquad$ $10y=8$

$x=\frac{1}{6}$ $\qquad$ $y=\frac{4}{5}$

(c) $3\begin{bmatrix}-x\\2y\end{bmatrix}+2\begin{bmatrix}3x-4\\-x\end{bmatrix}=\begin{bmatrix}4\\10\end{bmatrix}$

$$\begin{bmatrix}3(-x)+2(3x-4)\\3(2y)+2(-x)\end{bmatrix}=\begin{bmatrix}4\\10\end{bmatrix}$$

$-3x+6x-8=4$

$3x=12$

$x=4$

$6y-2x=10$

$6y-2(4)=10$

$6y=18$

$y=3$

5 (a) $[2\ \ 1]\begin{bmatrix}3&2&-4\\-9&4&0\end{bmatrix}$

Order: $1\times\underline{2}\quad\underline{2}\times 3$ (same)

Multiplication can be performed. The order of the product of matrices is 1×3.

(b) $\begin{bmatrix}6&2\\8&9\\3&9\end{bmatrix}\begin{bmatrix}4\\-5\end{bmatrix}$

Order: $3\times\underline{2}\quad\underline{2}\times 1$ (same)

Multiplication can be performed. The order of the product of matrices is 3×1.

(c) $[-3\ \ 0\ \ 4]\begin{bmatrix}1\\5\end{bmatrix}$

Order: $1\times\underline{3}\quad\underline{2}\times 1$ (not the same)

Multiplication cannot be performed.

(d) $\begin{bmatrix}-1\\6\end{bmatrix}[3\ \ -4]$

Order: $2\times\underline{1}\quad\underline{1}\times 2$ (same)

Multiplication can be performed. The order of the product of matrices is 2×2.

6 (a) $\begin{bmatrix}1\\2\\3\end{bmatrix}[-4\ \ 2]=\begin{bmatrix}1(-4)&1(2)\\2(-4)&2(2)\\3(-4)&3(2)\end{bmatrix}$

$$=\begin{bmatrix}-4&2\\-8&4\\-12&6\end{bmatrix}$$

(b) $[10\ \ -2]\begin{bmatrix}5\\7\end{bmatrix}=[10(5)+(-2)(7)]$

$=[36]$

(c) $\begin{bmatrix}10&-1\\2&6\\5&-3\end{bmatrix}\begin{bmatrix}-1&2\\0&3\end{bmatrix}=\begin{bmatrix}10(-1)+(-1)(0)&10(2)+(-1)(3)\\2(-1)+(6)(0)&2(2)+(6)(3)\\5(-1)+(-3)(0)&5(2)+(-3)(3)\end{bmatrix}$

$$=\begin{bmatrix}-10&17\\-2&22\\-5&1\end{bmatrix}$$

(d) $\begin{bmatrix}-6&3\\8&1\end{bmatrix}\begin{bmatrix}-1&5\\-2&-4\end{bmatrix}=\begin{bmatrix}-6(-1)+3(-2)&-6(5)+3(-4)\\8(-1)+1(-2)&8(5)+1(-4)\end{bmatrix}$

$$=\begin{bmatrix}0&-42\\-10&36\end{bmatrix}$$

7 (a) $[2\ \ -1]\begin{bmatrix}3\\2\end{bmatrix}=[2(3)+(-1)(2)]$

$=[4]$

(b) $\begin{bmatrix}3\\2\end{bmatrix}[2\ \ -1]=\begin{bmatrix}3(2)&3(-1)\\2(2)&2(-1)\end{bmatrix}$

$$=\begin{bmatrix}6&-3\\4&-2\end{bmatrix}$$

(c) Multiplication cannot be performed

(d) $\begin{bmatrix}-7&-1\\1&7\end{bmatrix}\begin{bmatrix}3\\2\end{bmatrix}=\begin{bmatrix}-7(3)+(-1)(2)\\1(3)+7(2)\end{bmatrix}$

$$=\begin{bmatrix}-23\\17\end{bmatrix}$$

(e) $\begin{bmatrix}-2&6\\-3&0\end{bmatrix}\begin{bmatrix}-7&-1\\1&7\end{bmatrix}=\begin{bmatrix}-2(-7)+6(1)&-2(-1)+6(7)\\-3(-7)+0(1)&-3(-1)+0(7)\end{bmatrix}$

$$=\begin{bmatrix}20&44\\21&3\end{bmatrix}$$

(f) $\begin{bmatrix}-7&-1\\1&7\end{bmatrix}\begin{bmatrix}-2&6\\-3&0\end{bmatrix}$

$$=\begin{bmatrix}(-7)(-2)+(-1)(-3)&-7(6)+(-1)(0)\\1(-2)+7(-3)&1(6)+7(0)\end{bmatrix}$$

$$=\begin{bmatrix}17&-42\\-23&6\end{bmatrix}$$

8 (a) $\begin{bmatrix}2&3x\\5y&11\end{bmatrix}\begin{bmatrix}-1\\2\end{bmatrix}=\begin{bmatrix}4\\2\end{bmatrix}$

$$\begin{bmatrix}2(-1)+3x(2)\\5y(-1)+11(2)\end{bmatrix}=\begin{bmatrix}4\\2\end{bmatrix}$$

$$\begin{bmatrix}6x-2\\22-5y\end{bmatrix}=\begin{bmatrix}4\\2\end{bmatrix}$$

$6x=4+2$ $\qquad$ $22-2=5y$

$x=1$ $\qquad$ $y=4$

(b) $\begin{bmatrix} 7 \\ 12 \end{bmatrix}[-3 \quad 2] = \begin{bmatrix} 3x & 14 \\ 9y & 8-4y \end{bmatrix}$

$\begin{bmatrix} 7(-3) & 7(2) \\ 12(-3) & 12(2) \end{bmatrix} = \begin{bmatrix} 3x & 14 \\ 9y & 8-4y \end{bmatrix}$

$3x = -21$ $\qquad$ $24 = 8-4y$

$x = -7$ $\qquad$ $4y = 8-24$

$y = -4$

(c) $\begin{bmatrix} 1 & 2 & 3x-1 \\ 1+4y & 4 & 5 \end{bmatrix}\begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 4x+5 \\ 3-y \end{bmatrix}$

$\begin{bmatrix} 1(-1)+2(0)+(3x-1)(2) \\ (1+4y)(-1)+4(0)+5(2) \end{bmatrix} = \begin{bmatrix} 4x+5 \\ 3-y \end{bmatrix}$

$-1+6x-2 = 4x+5$ $\qquad$ $-1-4y+10 = 3-y$

$2x = 8$ $\qquad$ $3y = 6$

$x = 4$ $\qquad$ $y = 2$

Self Test 4

1 (a) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $\qquad$ (b) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

2 Let $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$.

If A is an identity matrix, then $AB = BA = B$.

$AB = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$= \begin{bmatrix} 0(a)+1(c) & 0(b)+1(d) \\ 1(a)+0(c) & 1(b)+0(d) \end{bmatrix}$

$= \begin{bmatrix} c & d \\ a & b \end{bmatrix} \neq B$ Therefore, A is not an identity matrix.

3 (a) $BI + ID = B + D$

$= \begin{bmatrix} -2 & 1 \\ 6 & -3 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

$= \begin{bmatrix} 0 & 1 \\ 6 & -1 \end{bmatrix}$

(b) $(A-C)I = A - C$

$= \begin{bmatrix} 4 & 2 \\ 3 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 3 & 1 \\ 3 & 0 \end{bmatrix}$

(c) $CID = (CI)(D)$

$= CD$

$= \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

$= \begin{bmatrix} 1(2)+1(0) & 1(0)+1(2) \\ 0(2)+1(0) & 0(0)+1(2) \end{bmatrix}$

$= \begin{bmatrix} 2 & 2 \\ 0 & 2 \end{bmatrix}$

4 If $AB = BA = I$, then matrices A and B are the inverse matrices of one another.

(a) Let $A = \begin{bmatrix} 2 & -5 \\ 1 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 5 \\ -1 & 2 \end{bmatrix}$

$AB = \begin{bmatrix} 2 & -5 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} 3 & 5 \\ -1 & 2 \end{bmatrix}$

$= \begin{bmatrix} 2(3)+(-5)(-1) & 2(5)+(-5)(2) \\ 1(3)+3(-1) & 1(5)+3(2) \end{bmatrix}$

$= \begin{bmatrix} 11 & 0 \\ 0 & 11 \end{bmatrix}$

$AB \neq I$, therefore matrices A and B are not the inverse matrices of one another.

(b) Let $A = \begin{bmatrix} 1 & 2 \\ 0.5 & 1.5 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & -4 \\ -1 & 2 \end{bmatrix}$

$AB = \begin{bmatrix} 1 & 2 \\ 0.5 & 1.5 \end{bmatrix}\begin{bmatrix} 3 & -4 \\ -1 & 2 \end{bmatrix}$

$= \begin{bmatrix} 1(3)+2(-1) & 1(-4)+2(2) \\ 0.5(3)+1.5(-1) & 0.5(-4)+1.5(2) \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

$BA = \begin{bmatrix} 3 & -4 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 0.5 & 1.5 \end{bmatrix}$

$= \begin{bmatrix} 3(1)+(-4)(0.5) & 3(2)+(-4)(1.5) \\ (-1)(1)+2(0.5) & (-1)(2)+2(1.5) \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

$AB = BA = I$, therefore matrices A and B are the inverse matrices of one another.

(c) Let $A = \begin{bmatrix} 2 & 3 \\ 2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -3 \\ -2 & 2 \end{bmatrix}$

$AB = \begin{bmatrix} 2 & 3 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & -3 \\ -2 & 2 \end{bmatrix}$

$= \begin{bmatrix} 2(1)+3(-2) & 2(-3)+3(2) \\ 2(1)+1(-2) & 2(-3)+1(2) \end{bmatrix}$

$= \begin{bmatrix} -4 & 0 \\ 0 & -4 \end{bmatrix}$

$AB \neq I$, therefore matrices A and B are not the inverse matrices of one another.

5 (a) $A = \begin{bmatrix} 3 & 2 \\ 6 & 4 \end{bmatrix}$ $\qquad$ $|A| = 3(4) - 2(6)$

$= 0 \rightarrow$ the inverse matrix does not exist

(b) $B = \begin{bmatrix} 2 & -3 \\ 7 & 6 \end{bmatrix}$ $\qquad$ $|B| = 2(6) - (-3)(7)$

$= 33 \rightarrow$ the inverse matrix exists

$B^{-1} = \frac{1}{33}\begin{bmatrix} 6 & 3 \\ -7 & 2 \end{bmatrix}$

$= \begin{bmatrix} \frac{2}{11} & \frac{1}{11} \\ -\frac{7}{33} & \frac{2}{33} \end{bmatrix}$

(c) $C = \begin{bmatrix} -4 & -3 \\ 8 & 6 \end{bmatrix}$ $\qquad$ $|C| = (-4)(6) - (-3)(8)$

$= 0 \rightarrow$ the inverse matrix does not exist

6 (a) $P^{-1} = \frac{1}{4-5}\begin{bmatrix} 1 & 5 \\ 1 & 4 \end{bmatrix}$

$= \begin{bmatrix} -1 & -5 \\ -1 & -4 \end{bmatrix}$

(b) $Q^{-1} = \frac{1}{6-4}\begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix}$

$= \frac{1}{2}\begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix}$

$= \begin{bmatrix} 1 & 2 \\ \frac{1}{2} & \frac{3}{2} \end{bmatrix}$

(c) $R^{-1} = \frac{1}{24-20}\begin{bmatrix} 6 & -10 \\ -2 & 4 \end{bmatrix}$

$= \frac{1}{4}\begin{bmatrix} 6 & -10 \\ -2 & 4 \end{bmatrix}$

$= \begin{bmatrix} \frac{3}{2} & -\frac{5}{2} \\ -\frac{1}{2} & 1 \end{bmatrix}$

7 If a matrix does not have inverse, then determinant = 0

(a) $4m(-1)-6(2)=0$

$-4m=12$

$m=-3$

(b) $5(4)-(6m+4)(2)=0$

$20-12m-8=0$

$12m=12$

$m=1$

(c) $3(2-3m)-6(4)=0$

$6-9m-24=0$

$9m=-18$

$m=-2$

8 $A^{-1}=\frac{1}{6-(-4)}\begin{bmatrix}3&-4\\1&2\end{bmatrix}$

$\frac{1}{5m}\begin{bmatrix}2n-9&-4\\1&2\end{bmatrix}=\frac{1}{10}\begin{bmatrix}3&-4\\1&2\end{bmatrix}$

Compare the corresponding elements:

$5m=10$ $\quad 2n-9=3$

$m=2$ $\quad 2n=12$

$n=6$

9 (a) $A=\frac{1}{2-(-12)}\begin{bmatrix}1&3\\-4&2\end{bmatrix}$

$=\frac{1}{14}\begin{bmatrix}1&3\\-4&2\end{bmatrix}$

(b) $A=\frac{1}{-3-8}\begin{bmatrix}-3&-2\\-4&1\end{bmatrix}$

$=\frac{1}{-11}\begin{bmatrix}-3&-2\\-4&1\end{bmatrix}$

10 (a) $4x+y=1$

$2x-3y=-10$

$\begin{bmatrix}4&1\\2&-3\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}1\\-10\end{bmatrix}$

$\begin{bmatrix}x\\y\end{bmatrix}=\frac{1}{-12-2}\begin{bmatrix}-3&-1\\-2&4\end{bmatrix}\begin{bmatrix}1\\-10\end{bmatrix}$

$=\frac{1}{-14}\begin{bmatrix}-3(1)+(-1)(-10)\\-2(1)+(4)(-10)\end{bmatrix}$

$=\frac{1}{-14}\begin{bmatrix}7\\-42\end{bmatrix}$

$\therefore x=-\frac{1}{2}, y=3$

(b) $g+2h=1$

$3g+2h=-5$

$\begin{bmatrix}1&2\\3&2\end{bmatrix}\begin{bmatrix}g\\h\end{bmatrix}=\begin{bmatrix}1\\-5\end{bmatrix}$

$\begin{bmatrix}g\\h\end{bmatrix}=\frac{1}{2-6}\begin{bmatrix}2&-2\\-3&1\end{bmatrix}\begin{bmatrix}1\\-5\end{bmatrix}$

$=\frac{1}{-4}\begin{bmatrix}2(1)+(-2)(-5)\\-3(1)+1(-5)\end{bmatrix}$

$=\frac{1}{-4}\begin{bmatrix}12\\-8\end{bmatrix}$

$=\begin{bmatrix}-3\\2\end{bmatrix}$

$\therefore g=-3, h=2$

(c) $p=7-3q\rightarrow p+3q=7$

$2q=4-p\rightarrow p+2q=4$

$\begin{bmatrix}1&3\\1&2\end{bmatrix}\begin{bmatrix}p\\q\end{bmatrix}=\begin{bmatrix}7\\4\end{bmatrix}$

$\begin{bmatrix}p\\q\end{bmatrix}=\frac{1}{2-3}\begin{bmatrix}2&-3\\-1&1\end{bmatrix}\begin{bmatrix}7\\4\end{bmatrix}$

$=\frac{1}{-1}\begin{bmatrix}2(7)+(-3)(4)\\-1(7)+1(4)\end{bmatrix}$

$=-\begin{bmatrix}2\\-3\end{bmatrix}$

$=\begin{bmatrix}-2\\3\end{bmatrix}$

$\therefore p=-2, q=3$

(d) $s+5t=-2$

$\frac{2}{3}s-t=3\rightarrow 2s-3t=9$

$\begin{bmatrix}1&5\\2&-3\end{bmatrix}\begin{bmatrix}s\\t\end{bmatrix}=\begin{bmatrix}-2\\9\end{bmatrix}$

$\begin{bmatrix}s\\t\end{bmatrix}=\frac{1}{-3-10}\begin{bmatrix}-3&-5\\-2&1\end{bmatrix}\begin{bmatrix}-2\\9\end{bmatrix}$

$=\frac{1}{-13}\begin{bmatrix}(-3)(-2)+(-5)(9)\\(-2)(-2)+(1)(9)\end{bmatrix}$

$=\frac{1}{-13}\begin{bmatrix}-39\\13\end{bmatrix}$

$=\begin{bmatrix}3\\-1\end{bmatrix}$

$\therefore s=3, t=-1$

(e) $j=5k-4\rightarrow j-5k=-4$

$j-k=4$

$\begin{bmatrix}1&-5\\1&-1\end{bmatrix}\begin{bmatrix}j\\k\end{bmatrix}=\begin{bmatrix}-4\\4\end{bmatrix}$

$\begin{bmatrix}j\\k\end{bmatrix}=\frac{1}{-1-(-5)}\begin{bmatrix}-1&5\\-1&1\end{bmatrix}\begin{bmatrix}-4\\4\end{bmatrix}$

$=\frac{1}{4}\begin{bmatrix}(-1)(-4)+(5)(4)\\(-1)(-4)+1(4)\end{bmatrix}$

$=\frac{1}{4}\begin{bmatrix}24\\8\end{bmatrix}$

$=\begin{bmatrix}6\\2\end{bmatrix}$

$\therefore j=6, k=2$

(f) $3u-2v=-2$

$4u+3v=-\frac{5}{4}\rightarrow 16u+12v=-5$

$\begin{bmatrix}3&-2\\16&12\end{bmatrix}\begin{bmatrix}u\\v\end{bmatrix}=\begin{bmatrix}-2\\-5\end{bmatrix}$

$\begin{bmatrix}u\\v\end{bmatrix}=\frac{1}{36-(-32)}\begin{bmatrix}12&2\\-16&3\end{bmatrix}\begin{bmatrix}-2\\-5\end{bmatrix}$

$=\frac{1}{68}\begin{bmatrix}(12)(-2)+2(-5)\\(-16)(-2)+3(-5)\end{bmatrix}$

$=\frac{1}{68}\begin{bmatrix}-34\\17\end{bmatrix}$

$=\begin{bmatrix}-\frac{1}{2}\\\frac{1}{4}\end{bmatrix}$

$\therefore u=-\frac{1}{2}, v=\frac{1}{4}$

(g) $2d-e=6$

$d+\frac{1}{2}e=-1\rightarrow 2d+e=-2$

$\begin{bmatrix}2&-1\\2&1\end{bmatrix}\begin{bmatrix}d\\e\end{bmatrix}=\begin{bmatrix}6\\-2\end{bmatrix}$

$\begin{bmatrix}d\\e\end{bmatrix}=\frac{1}{2-(-2)}\begin{bmatrix}1&1\\-2&2\end{bmatrix}\begin{bmatrix}6\\-2\end{bmatrix}$

$=\frac{1}{4}\begin{bmatrix}1(6)+1(-2)\\(-2)(6)+2(-2)\end{bmatrix}$

$$= \frac{1}{4}\begin{bmatrix} 4 \\ -16 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ -4 \end{bmatrix}$$

$$\therefore d = 1, e = -4$$

11 $x = 3y \rightarrow x - 3y = 0$

$x - 10 = 5(y - 10) \rightarrow x - 5y = -40$

$$\begin{bmatrix} 1 & -3 \\ 1 & -5 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ -40 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-5-(-3)}\begin{bmatrix} -5 & 3 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ -40 \end{bmatrix}$$

$$= \frac{1}{-2}\begin{bmatrix} -5(0) + 3(-40) \\ -1(0) + 1(-40) \end{bmatrix}$$

$$= \frac{1}{-2}\begin{bmatrix} -120 \\ -40 \end{bmatrix}$$

$$= \begin{bmatrix} 60 \\ 20 \end{bmatrix}$$

$$\therefore x = 60, y = 20$$

12 (a) Total perimeter of both shapes = 35

I $2(2x + 3y + 5) + 3(3x) = 35$

$13x + 6y = 25$

II $3x = 2y - 1$

$3x - 2y = -1$

(b) $$\begin{bmatrix} 13 & 6 \\ 3 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 25 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-26-18}\begin{bmatrix} -2 & -6 \\ -3 & 13 \end{bmatrix}\begin{bmatrix} 25 \\ -1 \end{bmatrix}$$

$$= \frac{1}{-44}\begin{bmatrix} -2(25) + (-6)(-1) \\ -3(25) + 13(-1) \end{bmatrix}$$

$$= \frac{1}{-44}\begin{bmatrix} -44 \\ -88 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$\therefore x = 1, y = 2$$

13 Let the price of an apple = x

the price of an orange = y

$25x + 25y = 45$

$5x + 5y = 9$

$35x + 15y = 43$

$$\begin{bmatrix} 5 & 5 \\ 35 & 15 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 9 \\ 43 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{75-175}\begin{bmatrix} 15 & -5 \\ -35 & 5 \end{bmatrix}\begin{bmatrix} 9 \\ 43 \end{bmatrix}$$

$$= \frac{1}{-100}\begin{bmatrix} 15(9) + (-5)(43) \\ (-35)(9) + 5(43) \end{bmatrix}$$

$$= \frac{1}{-100}\begin{bmatrix} -80 \\ -100 \end{bmatrix}$$

$$= \begin{bmatrix} 0.8 \\ 1 \end{bmatrix}$$

$\therefore x$ = RM0.80, y = RM1

Stall P: $12x + 10y$ = 12(RM0.80) + 10(RM1)
= RM19.60

Stall Q: $12x + 10y$ = RM20

$\therefore$ Stall P is cheaper.

SPM PRACTICE

Paper 1

1 B $$M = \begin{bmatrix} 3 & 5 \\ 10 & 0 \end{bmatrix} - \begin{bmatrix} 1 & -3 \\ 8 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 8 \\ 2 & -5 \end{bmatrix}$$

2 D $$BA = \begin{bmatrix} 6 & -6 \\ 12 & -13 \end{bmatrix}$$

Order : $m \times (n)$ $(3) \times 2$ 2×2

3 B $p_{22} + q_{12} = -2 + (-3)$

$= -5$

4 B $$3\begin{bmatrix} -2 & 2 \\ 3 & -1 \end{bmatrix} + 2\begin{bmatrix} 2 & 6 \\ 4 & 3 \end{bmatrix} - \begin{bmatrix} 0 & -6 \\ -5 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} -6 & 6 \\ 9 & -3 \end{bmatrix} + \begin{bmatrix} 4 & 12 \\ 8 & 6 \end{bmatrix} - \begin{bmatrix} 0 & -6 \\ -5 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} -6+4-0 & 6+12+6 \\ 9+8+5 & -3+6-2 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 24 \\ 22 & 1 \end{bmatrix}$$

5 B $4X = 2[2 \quad -5] - [4 \quad 10]$

$$X = \frac{1}{4}[4-4 \quad -10-10]$$

$$= \frac{1}{4}[0 \quad -20]$$

$$= [0 \quad -5]$$

6 B $$\frac{1}{m}\begin{bmatrix} -2 & 24 & 6 \\ 0 & 2 & 4 \end{bmatrix} = \begin{bmatrix} 3 & 2 & 5 \\ 6 & 9 & 4 \end{bmatrix} - \begin{bmatrix} 4 & -10 & 2 \\ 6 & 8 & 2n \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 12 & 3 \\ 0 & 1 & 4-2n \end{bmatrix}$$

Compare u_{11}: $\frac{-2}{m} = -1$

$m = 2$

Compare u_{23}: $\frac{4}{2} = 4 - 2n$

$2n = 4 - 2$

$n = 1$

$m + n = 2 + 1$

$= 3$

7 A $$\begin{bmatrix} 3 & 5x+1 \\ y-5 & 6 \end{bmatrix} = \begin{bmatrix} 3 & -9 \\ 6+3x & 6 \end{bmatrix}$$

$5x + 1 = -9$ $\quad$ $y - 5 = 6 + 3x$

$5x = -10$ $\quad$ $y = 6 + 3(-2) + 5$

$x = -2$ $\quad$ $= 5$

8 B $$\frac{1}{2}\begin{bmatrix} -2 & 12 \\ 6 & -n \end{bmatrix} = \begin{bmatrix} -1 & 9 \\ -1 & 5 \end{bmatrix} - \begin{bmatrix} 0 & 3 \\ -4 & 7 \end{bmatrix}$$

$$\begin{bmatrix} -2 & 12 \\ 6 & -n \end{bmatrix} = 2\begin{bmatrix} -1-0 & 9-3 \\ -1-(-4) & 5-7 \end{bmatrix}$$

$$= 2\begin{bmatrix} -1 & 6 \\ 3 & -2 \end{bmatrix}$$

$-n = 2(-2)$

$n = 4$

9 D $4|C| + |D| = 4[(5)(3) - (6)(4)] + [(0)(2) - (7)(-5)]$

$= 4(15 - 24) + 35$

$= -36 + 35$

$= -1$

10 C $$\begin{bmatrix} 2 & 1 \\ 8 & 6 \end{bmatrix}\begin{bmatrix} 2 & 3 & 7 \\ 1 & -5 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 2(2)+1(1) & 2(3)+1(-5) & 2(7)+1(4) \\ 8(2)+6(1) & 8(3)+6(-5) & 8(7)+6(4) \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 1 & 18 \\ 22 & -6 & 80 \end{bmatrix}$$

11 C $A^2 = 9I$

$$\begin{bmatrix} n & 0 \\ -n & -n \end{bmatrix}\begin{bmatrix} n & 0 \\ -n & -n \end{bmatrix} = 9\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} n(n)+0(-n) & n(0)+0(-n) \\ -n(n)+(-n)(-n) & -n(0)+(-n)(-n) \end{bmatrix} = \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix}$$

$$\begin{bmatrix} n^2 & 0 \\ 0 & n^2 \end{bmatrix} = \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix}$$
$$n^2 = 9$$
$$n = \pm 3$$

12 D $\dfrac{1}{-21-(-20)}\begin{bmatrix} -7 & -4 \\ 5 & 3 \end{bmatrix} = \dfrac{1}{-1}\begin{bmatrix} -7 & -4 \\ 5 & 3 \end{bmatrix}$
$$= \begin{bmatrix} 7 & 4 \\ -5 & -3 \end{bmatrix}$$

13 A $m = 4(120) + 2(70)$
$n = 2(120) + 3(70)$
$$\begin{bmatrix} m \\ n \end{bmatrix} = \begin{bmatrix} 4 & 2 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 120 \\ 70 \end{bmatrix}$$

14 D $3\begin{bmatrix} -2 & 3 \\ -4 & 5 \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ 4 & -6 \end{bmatrix} - 5\begin{bmatrix} 2 & 3 \\ 6 & 4 \end{bmatrix}$
$$= \begin{bmatrix} -6 & 9 \\ -12 & 15 \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ 4 & -6 \end{bmatrix} - \begin{bmatrix} 10 & 15 \\ 30 & 20 \end{bmatrix}$$
$$= \begin{bmatrix} -6+2-10 & 9+3-15 \\ -12+4-30 & 15-6-20 \end{bmatrix}$$
$$= \begin{bmatrix} -14 & -3 \\ -38 & -11 \end{bmatrix}$$

15 B $|M| = ad - bc$ and compare with $(3)(2) - (-1)(-2)$
$$\therefore M = \begin{bmatrix} 3 & -1 \\ -2 & 2 \end{bmatrix}$$
$$M^{-1} = \frac{1}{|M|}\begin{bmatrix} 2 & 1 \\ 2 & 3 \end{bmatrix}.$$
$\therefore m = 1, n = 3$

16 C $m = (-2)(5) - (3)(-4) \quad n = -3$
$m = 2$
$m + n = 2 + (-3)$
$= -1$

17 C
$$[p \quad -1]\begin{bmatrix} -2 & 0 \\ -p & 5 \end{bmatrix} = [12 \quad -5]$$
$$[p(-2) + (-1)(-p) \quad p(0) + (-1)(5)] = [12 \quad -5]$$
$$[-p \quad -5] = [12 \quad -5]$$
$$\therefore p = -12$$

18 C Total sales in June = Sum of the product of the number of units and unit price
Multiplication of **A** cannot be performed.
The multiplication of matrices of **B** and **D** produces a matrix in the order of 3×3. Hence, they do not show the total sales.

Paper 2

Section A

1 $3|P| - q_{21} + 5r_{32} = 3[13(-2) - 1(10)] - 7 + 5(4)$
$= 3(-36) - 7 + 20$
$= -95$

2 Let Haikal's age at present = x
Amin's age at present = y

(a) $x - 7 = 2(y - 7)$
$x - 2y = -7$
$x + y = 50$

(b) $\begin{bmatrix} 1 & -2 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -7 \\ 50 \end{bmatrix}$
$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{1-(-2)}\begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} -7 \\ 50 \end{bmatrix}$$
$$= \frac{1}{3}\begin{bmatrix} 1(-7) + 2(50) \\ -1(-7) + 1(50) \end{bmatrix}$$
$$= \frac{1}{3}\begin{bmatrix} 93 \\ 57 \end{bmatrix}$$
$$= \begin{bmatrix} 31 \\ 19 \end{bmatrix}$$
$\therefore x = 31, y = 19$

3 (a) $A^{-1} = \dfrac{1}{9(-2) - xy}\begin{bmatrix} -2 & -x \\ -y & 9 \end{bmatrix}$

Compare A^{-1} above with the A^{-1} given
$-y = -2$ and $-x = 3.5y$
$y = 2$ $x = -3.5(2)$
$x = -7$

(b) The inverse of matrix A does not exist when $|A| = 0$
$-18 - xy = 0$
$xy = -18$

x	y
−18	1
−9	2
−6	3
−3	6
−2	9
−1	18
1	−18
2	−9
3	−6
6	−3
9	−2
18	−1

Maximum value of $x - y$
= Maximum value of x – Minimum value of y
$= 18 - (-1)$
$= 19$

Section B

4 (a) $2x + 4y = 13$
$4x + 4y = 16$
$x + y = 4$

(b) $\begin{bmatrix} 2 & 4 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 13 \\ 4 \end{bmatrix}$
$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{2-4}\begin{bmatrix} 1 & -4 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 13 \\ 4 \end{bmatrix}$$
$$= \frac{1}{-2}\begin{bmatrix} 1(13) + (-4)(4) \\ -1(13) + 2(4) \end{bmatrix}$$
$$= \frac{1}{-2}\begin{bmatrix} -3 \\ -5 \end{bmatrix}$$
$$= \begin{bmatrix} 1.5 \\ 2.5 \end{bmatrix}$$
$\therefore x = \text{RM}1.50, y = \text{RM}2.50$

(c) Total payment $= \text{RM}13 + (0.95)(\text{RM}16)$
$= \text{RM}13 + \text{RM}15.20$
$= \text{RM}28.20$

5 (a) $8(2x) - (2)(6) = 0$
$16x = 12$
$x = \dfrac{3}{4}$

(b) (i) $P + L = 284$
$P = 8 + 3L$

$P - 3L = 8$

(ii) $\begin{bmatrix} 1 & 1 \\ 1 & -3 \end{bmatrix}\begin{bmatrix} P \\ L \end{bmatrix} = \begin{bmatrix} 284 \\ 8 \end{bmatrix}$

$$\begin{bmatrix} P \\ L \end{bmatrix} = \frac{1}{-3-1}\begin{bmatrix} -3 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 284 \\ 8 \end{bmatrix}$$

$$= \frac{1}{-4}\begin{bmatrix} (-3)(284) + (-1)(8) \\ -1(284) + 1(8) \end{bmatrix}$$

$$= \frac{1}{-4}\begin{bmatrix} -860 \\ -276 \end{bmatrix}$$

$$= \begin{bmatrix} 215 \\ 69 \end{bmatrix}$$

$\therefore P = 215, L = 69$

6 (a) $2L + 3B = 2\,895$

$4L - 3B = 1\,299$

$\begin{bmatrix} 2 & 3 \\ 4 & -3 \end{bmatrix}\begin{bmatrix} L \\ B \end{bmatrix} = \begin{bmatrix} 2\,895 \\ 1\,299 \end{bmatrix}$

$$\begin{bmatrix} L \\ B \end{bmatrix} = \frac{1}{-6-12}\begin{bmatrix} -3 & -3 \\ -4 & 2 \end{bmatrix}\begin{bmatrix} 2\,895 \\ 1\,299 \end{bmatrix}$$

$$= \frac{1}{-18}\begin{bmatrix} (-3)(2\,895) + (-3)(1\,299) \\ -4(2\,895) + 2(1\,299) \end{bmatrix}$$

$$= \frac{1}{-18}\begin{bmatrix} -12\,582 \\ -8\,982 \end{bmatrix}$$

$$= \begin{bmatrix} 699 \\ 499 \end{bmatrix}$$

$\therefore L = \text{RM}699, B = \text{RM}499$

(b) $P = \begin{bmatrix} 5L & 10B - 590 \\ 1\,000 - L & 2B \end{bmatrix}$

$$= \begin{bmatrix} 5(699) & 10(499) - 590 \\ 1\,000 - 699 & 2(499) \end{bmatrix}$$

$$= \begin{bmatrix} 3\,495 & 4\,400 \\ 301 & 998 \end{bmatrix}$$