FORM 4

CHAPTER 1

Self Test 1

1 (a) $2x - 9$ is not a quadratic expression because the highest power of the variable is one.
(b) $x^2 + 3x - 4$ is a quadratic expression in one variable.
(c) $3 + 6x - 5x^{-2}$ is not a quadratic expression because there is a variable with a power which is not a whole number.
(d) $x^2 + 2x^3$ is not a quadratic expression because the highest power of the variable is three.
(e) $3x^2 - 2y + 5$ is not a quadratic expression in one variable because there are two variables, x and y.
(f) $(x - \sqrt{3})(2 + x) = 2x + x^2 - 2\sqrt{3} - \sqrt{3}x$
is a quadratic expression in one variable.

2 (a) $a = 3, b = 0, c = -12$
(b) $a = 5, b = 1, c = -2$
(c) $a = -1, b = 7, c = \frac{2}{3}$
(d) $a = 2, b = 3, c = 0$
(e) $a = \frac{4}{5}, b = -\frac{1}{2}, c = 5$
(f) $5t(t - 6) = 5t^2 - 30t$
$a = 5, b = -30, c = 0$

3 (a) $f(x) = x^2 - 2x + 8$
$a = 1,\ b = -2,\ c = 8$
a is positive, the shape of the graph is $\cup$, this function has a minimum point.
$$x = -\frac{b}{2a} = -\frac{-2}{2(1)} = 1$$
$$f(1) = (1)^2 - 2(1) + 8 = 7$$
$\therefore$ Minimum point $(1, 7)$
(b) $f(x) = x^2 + 6x$
$a = 1, \quad b = 6, \quad c = 0$
a is positive, the shape of the graph is $\cup$, this function has a minimum point.
$$x = -\frac{b}{2a} = -\frac{6}{2(1)} = -3$$
$$f(-3) = (-3)^2 + 6(-3) = -9$$
$\therefore$ Minimum point $(-3, -9)$
(c) $f(x) = x^2 - 9$
$a = 1, \quad b = 0, \quad c = -9$
a is positive, the shape of the graph is $\cup$, this function has a minimum point.
$$x = -\frac{b}{2a} = 0$$
$$f(0) = (0)^2 - 9 = -9$$
$\therefore$ Minimum point $(0, -9)$
(d) $f(x) = 7 + 4x - x^2$
$a = -1, \quad b = 4, \quad c = 7$
a is negative, the shape of the graph is $\cap$, this function has a maximum point.
$$x = -\frac{b}{2a} = -\frac{4}{2(-1)} = 2$$
$$f(2) = 7 + 4(2) - (2)^2 = 11$$
$\therefore$ Maximum point $(2, 11)$
(e) $f(x) = (3 - x)(11 - x)$
$= 33 - 3x - 11x + x^2$
$= x^2 - 14x + 33$
$a = 1, \quad b = -14, \quad c = 33$
a is positive, the shape of the graph is $\cup$, this function has a minimum point.
$$x = -\frac{b}{2a} = -\frac{-14}{2(1)} = 7$$
$$f(7) = (3 - 7)(11 - 7) = -16$$
$\therefore$ Minimum point $(7, -16)$
(f) $f(x) = 4x(1 - 3x)$
$= 4x - 12x^2$
$a = -12, \quad b = 4, \quad c = 0$
a is negative, the shape of the graph is $\cap$, this function has a maximum point.
$$x = -\frac{b}{2a} = -\frac{4}{2(-12)} = \frac{1}{6}$$
$$f\left(\frac{1}{6}\right) = 4\left(\frac{1}{6}\right) - 12\left(\frac{1}{6}\right)^2 = \frac{1}{3}$$
$\therefore$ Maximum point $\left(\frac{1}{6}, \frac{1}{3}\right)$

4 (a) $x = \frac{2 + 6}{2} = 4$
(b) $x = 0$
(c) $x = \frac{-1 + 4}{2} = 1.5$
(d) $x = \frac{-10 + 2}{2} = -4$

5 (a) $10 = (3)^2 - 5(3) + c$
$c = 16$
(b) $-5 = -3(1)^2 - 7(1) + c$
$c = 5$
(c) x- intercept $= 2, f(x) = 0$
$0 = (2)^2 + 12(2) + c$
$c = -28$

6 (a) The shape of the graph is $\cup$, thus $k > 0$
As the curve of the graph $g(x)$ is wider, therefore $k < 5$
$\therefore 0 < k < 5$

(b) The shape of the graph is ∩, thus $k < 0$
As the curve of the graph $g(x)$ is narrower, therefore $k < -1$
$\therefore k < -1$

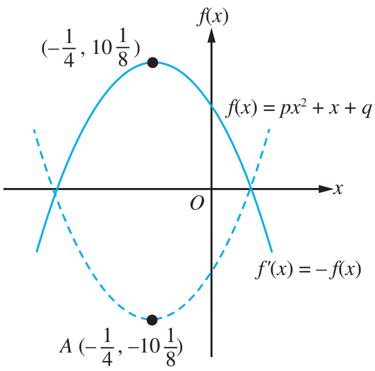
7 (a) The shape of the graph is ∪ $\rightarrow p > 0, p = 1$ or 2

$$x = -\frac{b}{2a}$$
$$-\frac{1}{4} = -\frac{1}{2p}$$
$$p = 2$$

(b) Substitute $\left(-\frac{1}{4}, -10\frac{1}{8}\right)$ into

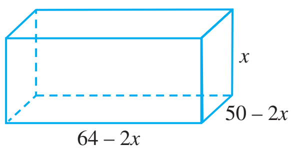
$$f(x) = 2x^2 + x + q$$
$$\frac{-81}{8} = 2\left(-\frac{1}{4}\right)^2 + \left(-\frac{1}{4}\right) + q$$
$$q = \frac{-81}{8} - \frac{1}{8} + \frac{1}{4}$$
$$= -10$$

(c)



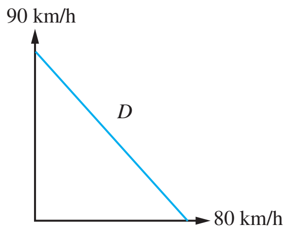
$f(x) = -2x^2 - x + 10$

8



Base area $= (64 - 2x)(50 - 2x)$
$= 2(32 - x)(2)(25 - x)$
$= 4(800 - 57x + x^2)\text{cm}^2$

9 (a)



$$D^2 = (80t)^2 + (90t)^2$$
$$D = \sqrt{6\,400t^2 + 8\,100t^2}$$
$$D = \sqrt{14\,500}\,t$$

(b) $\sqrt{14\,500}t = 200$
$14\,500t^2 = 40\,000$
$29t^2 = 80$

10 (a) $x^2 - 4x + 2 = 0$
When $x = -2$
LEFT $= (-2)^2 - 4(-2) + 2$
$= 4 + 8 + 2$
$= 14$
$\neq$ RIGHT
Thus, $x = -2$ is not the root of the quadratic equation $x^2 - 4x + 2 = 0$

When $x = 1$
LEFT $= (1)^2 - 4(1) + 2$
$= 1 - 4 + 2$
$= -1$
$\neq$ RIGHT
Thus, $x = 1$ is not the root of the quadratic equation $x^2 - 4x + 2 = 0$

(b) $-3x^2 - x + 2 = 2, x = 0, x = 3$
When $x = 0$
LEFT $= -3(0)^2 - (0) + 2$
$= 2$
$=$ RIGHT
Thus, $x = 0$ is the root of the quadratic equation $-3x^2 - x + 2 = 2$
When $x = 3$
LEFT $= -3(3)^2 - (3) + 2$
$= -27 - 3 + 2$
$= -28$
$\neq$ RIGHT
Thus, $x = 3$ is not the root of the quadratic equation $-3x^2 - x + 2 = 2$

(c) $2(1 + x)(2x - 3) = 0, x = 1, x = \frac{3}{2}$

When $x = 1$
LEFT $= 2(1 + 1)(2 - 3)$
$= -4$
$\neq$ RIGHT
Thus, $x = 1$ is not the root of the quadratic equation $2(1 + x)(2x - 3) = 0$
When $x = \frac{3}{2}$

LEFT $= 2\left(1 + \frac{3}{2}\right)\left[2\left(\frac{3}{2}\right) - 3\right]$
$= 0$
$=$ RIGHT

Thus, $x = \frac{3}{2}$ is the root of the quadratic equation $2(1 + x)(2x - 3) = 0$

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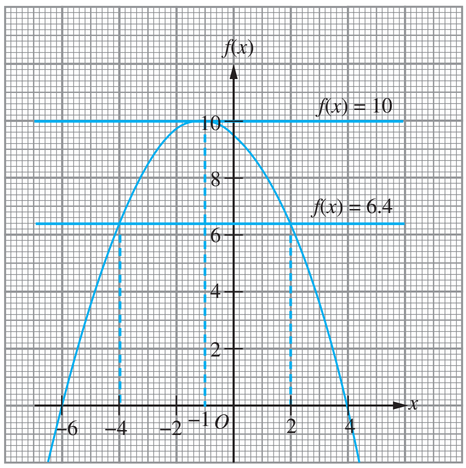


(a) $f(x) = 0$
$x = -6, 4$
(b) $f(x) = 10$
$x = -1$
(c) $f(x) = 6.4$
$x = -4, 2$

12 (a) $x^2 - 9x + 20 = 0$
$(x - 4)(x - 5) = 0$
$x = 4, 5$

(b) $x^2 - x - 12 = 0$
$(x - 4)(x + 3) = 0$
$x = 4, -3$

(c) $x^2 - 10x + 9 = 0$
$(x - 1)(x - 9) = 0$
$x = 1, 9$

(d) $x^2 + 5x - 14 = 0$
$(x + 7)(x - 2) = 0$
$x = -7, 2$

(e) $x^2 + 2x - 24 = 0$
$(x + 6)(x - 4) = 0$
$x = -6, 4$

(f) $x^2 - 9x + 18 = 0$
$(x - 3)(x - 6) = 0$
$x = 3, 6$

13 (a) $4x^2 + 8x + 3 = 0$
$(2x + 1)(2x + 3) = 0$
$x = -\frac{1}{2}, -\frac{3}{2}$

(b) $2x^2 + 5x - 12 = 0$
$(2x - 3)(x + 4) = 0$
$x = \frac{3}{2}, -4$

(c) $6x^2 - 7x + 2 = 0$
$(3x - 2)(2x - 1) = 0$
$x = \frac{2}{3}, \frac{1}{2}$

(d) $10 + 13x - 3x^2 = 0$
$3x^2 - 13x - 10 = 0$
$(3x + 2)(x - 5) = 0$
$x = -\frac{2}{3}, 5$

(e) $5x^2 - 29x + 20 = 0$
$(5x - 4)(x - 5) = 0$
$x = \frac{4}{5}, 5$

(f) $4x^2 - 9x - 9 = 0$
$(4x + 3)(x - 3) = 0$
$x = -\frac{3}{4}, 3$

14 (a) $14 + 5x = \frac{3}{x}$
$14x + 5x^2 = 3$
$5x^2 + 14x - 3 = 0$
$(5x - 1)(x + 3) = 0$
$x = -3, \frac{1}{5}$

(b) $(m - 2)^2 = 28 - 9m$
$m^2 - 4m + 4 - 28 + 9m = 0$
$m^2 + 5m - 24 = 0$
$(m - 3)(m + 8) = 0$
$m = 3, -8$

(c) $\frac{w + 1}{5} = \frac{7 - w}{6w}$
$6w(w + 1) = 5(7 - w)$
$6w^2 + 6w - 35 + 5w = 0$
$6w^2 + 11w - 35 = 0$
$(3w - 5)(2w + 7) = 0$
$w = \frac{5}{3}, -\frac{7}{2}$

(d) $3p - 4 = -\frac{7}{p + 2}$
$(3p - 4)(p + 2) = -7$
$3p^2 + 6p - 4p - 8 + 7 = 0$
$3p^2 + 2p - 1 = 0$
$(3p - 1)(p + 1) = 0$
$p = \frac{1}{3}, -1$

(e) $t^2 - 3t(t + 1) - 10 = t(4 - t)$
$t^2 - 3t^2 - 3t - 10 = 4t - t^2$
$t^2 + 7t + 10 = 0$
$(t + 2)(t + 5) = 0$
$t = -2, -5$

(f) $\frac{16 + 3q}{q} = q - 3$
$16 + 3q = q^2 - 3q$
$q^2 - 3q - 3q - 16 = 0$
$q^2 - 6q - 16 = 0$
$(q - 8)(q + 2) = 0$
$q = 8, -2$

15 (a) $(7x - 1)^2 + (5x + 8)^2 = (8x + 9)^2$
$49x^2 - 14x + 1 + 25x^2 + 80x + 64 = 64x^2 + 144x + 81$
$10x^2 - 78x - 16 = 0$
$5x^2 - 39x - 8 = 0$

(b) $5x^2 - 39x - 8 = 0$
$(5x + 1)(x - 8) = 0$
$x = 8$ ($x = -\frac{1}{5}$ is rejected))
$\therefore$ Length, $AB = 7(8) - 1$
$= 55$ cm
Width, $BC = 5(8) + 8$
$= 48$ cm

16 $x(x + 2) = 675$
$x^2 + 2x - 675 = 0$
$(x + 27)(x - 25) = 0$
$x = -27, 25$
The odd numbers are $= -27, -25$ atau $25, 27$

17 Let $m =$ the age of Mazni this year
$a =$ the age of the child this year
$m - 2 = 4(a - 2)$
$m = 4a - 8 + 2$
$= 4a - 6$
$(m + 5)(a + 5) = 585$
$(4a - 6 + 5)(a + 5) = 585$
$4a^2 + 20a - a - 5 - 585 = 0$
$4a^2 + 19a - 590 = 0$
$(4a + 59)(a - 10) = 0$
$a = 10$ (the negative value is rejected)
$m = 4(10) - 6$
$= 34$

18 (a) $J = x(3x + 5) + (x + 3)(7x - 3)$
$= 3x^2 + 5x + 7x^2 - 3x + 21x - 9$
$= 10x^2 + 23x - 9$

(b) $10x^2 + 23x - 9 = 4(100) - 44$
$10x^2 + 23x - 365 = 0$
$(10x + 73)(x - 5) = 0$
$x = 5$ ($x = -7.3$ is rejected)
Total amount paid by Zamidi
$= 2 \times$ RM$[3(5) + 5] + 3 \times$ RM$[7(5) - 3]$
= RM40 + RM96
= RM136

SPM PRACTICE

Paper 1

1 **A**

2 **B**

3 **C**
$8 = 3(3)^2 - 2(3) + c$
$c = 8 - 27 + 6$
$= -13$

4 **B**
The graph of a quadratic function is a parabola and it is symmetrical.

5 **C**
$a = 1 \quad b = -6 \quad c = 5$

a is positive, the shape of graph is $\cup$

Equation of axis of symmetry, $x = -\dfrac{b}{2a}$

$= -\dfrac{(-6)}{2(1)}$

$= 3$

y-intercept $= 5$

After the reflection on the y-axis, the equation of axis of symmetry is $x = -3$, the shape of the graph and the y-intercept remain unchanged.

6 D

$x^2 - 6x + 5 = -3$

$x^2 - 6x + 8 = 0$

$(x - 4)(x - 2) = 0$

$x = 4$ or 2

7 C

Volume $= 1\,050$

$(x + 15)(x)(1.5) = 1\,050$

$x^2 + 15x = 700$

$x^2 + 15x - 700 = 0$

$(x + 35)(x - 20) = 0$

$x = 20$ ($x = -35$ is rejected)

Length $= 20 + 15$

$= 35$ m

8 A

Let u = initial speed

$d = 200$ km

$ut = 200$

$u = \dfrac{200}{t}$ ... ①

$\left(\dfrac{200}{t} + 10\right)(t - 1) = 200$

$200 - \dfrac{200}{t} + 10t - 10 = 200$

$10t^2 - 10t - 200 = 0$

$t^2 - t - 20 = 0$

$(t - 5)(t + 4) = 0$

$t = 5$ ($t = -4$ is rejected)

From ① $u = \dfrac{200}{5}$

$= 40$ km/h

9 C

The maximum height is on the axis of symmetry,

$t = -\dfrac{10}{2(-5)}$

$= 1$

$h(1) = -5(1)^2 + 10(1) + 20$

$= 25$

10 C

$x^2 - 4(x + 1) = 17$

$x^2 - 4x - 4 - 17 = 0$

$x^2 - 4x - 21 = 0$

$(x - 7)(x + 3) = 0$

$x = 7$ ($x = -3$ is rejected)

Paper 2

Section A

1 $\dfrac{4}{x + 6} + \dfrac{5}{x + 8} = 1$

$4(x + 8) + 5(x + 6) = (x + 6)(x + 8)$

$4x + 32 + 5x + 30 = x^2 + 14x + 48$

$x^2 + 14x + 48 - 9x - 62 = 0$

$x^2 + 5x - 14 = 0$

$(x + 7)(x - 2) = 0$

$x = -7, 2$

2 (a) Equation of axis of symmetry, $x = \dfrac{1 + 4.5}{2}$

$= 2.75$

OR $x = -\dfrac{(-11)}{2(2)}$

$= 2.75$

(b) $f(0) = 2(0)^2 - 11(0) - c$

$5 = -c$

$c = -5$

(c) The minimum point is on the axis of symmetry

$f(2.75) = 2(2.75)^2 - 11(2.75) + 5$

$= -10.125$

Coordinates of minimum point $(2.75, -10.125)$

3 $(20)(40) - (20 - 2x)(40 - 3x) = 256$

$800 - 256 = 800 - 60x - 80x + 6x^2$

$6x^2 - 140x + 256 = 0$

$3x^2 - 70x + 128 = 0$

$(3x - 64)(x - 2) = 0$

$x = 2$ ($x = 21\frac{1}{3}$ is rejected)

4 Let s = the age of Samuel this year

a = the age of his son this year

$s - 6 = 2(a - 6)^2$

$s = 2(a^2 - 12a + 36) + 6$

$= 2a^2 - 24a + 72 + 6$

$= 2a^2 - 24a + 78$ ...①

$s + 4 = 4(a + 4)$

$s = 4a + 16 - 4$

$= 4a + 12$ ...②

Substitute ② into ①

$4a + 12 = 2a^2 - 24a + 78$

$2a^2 - 24a + 78 - 4a - 12 = 0$

$2a^2 - 28a + 66 = 0$

$a^2 - 14a + 33 = 0$

$(a - 3)(a - 11) = 0$

The age of his son this year, $a = 11$ ($a = 3$ is rejected)

The age of Samuel this year, $s = 4(11) + 12$

$= 56$

5 (a) $A = (7x + 1)(6x - 7)$

$= 42x^2 - 49x + 6x - 7$

$= 42x^2 - 43x - 7$

(b) $42x^2 - 43x - 7 = 75$

$42x^2 - 43x - 7 - 75 = 0$

$42x^2 - 43x - 82 = 0$

$(42x + 41)(x - 2) = 0$

$x = 2$ (the negative value of x is rejected)

Scale 1 cm : 5 m

Length $= [7(2) + 1] \times 5$ m

$= 75$ m

Width $= [6(2) - 7] \times 5$ m

$= 25$ m

Section B

6 (a) $\pi(3x + 5)^2 + \pi(5x - 8)^2 = 770$

$\dfrac{22}{7}(9x^2 + 30x + 25 + 25x^2 - 80x + 64) = 770$

$34x^2 - 50x + 89 = 245$

$34x^2 - 50x - 156 = 0$

$17x^2 - 25x - 78 = 0$ (shown)

(b) (i) $17x^2 - 25x - 78 = 0$

$(17x + 26)(x - 3) = 0$

$x = 3$ (the negative value of x is rejected because radius can never be negative)

(ii) Radius of circle with centre $P = 3(3) + 5$

$= 14$ cm

Radius of circle with centre $Q = 5(3) - 8$

$= 7$ cm

7 (a) $h(0) = -16(0)^2 + 96(0) + c$

$c = 200$

(b) Equation of axis of symmetry, $t = -\dfrac{96}{2(-16)}$

$= 3$

(c) Maximum height,

$h(3) = -16(3)^2 + 96(3) + 200$

$= 344$ m

(d) $-16t^2 + 96t + 200 = 200$

$t^2 - 6t = 0$

$t(t - 6) = 0$

$t = 0, 6$

The rocket returns to the launching platform at $t = 6$ s

(e) Distance between the launching platform and the maximum point

$= 344 - 200$

$= 144$ m

8 (a) The maximum value on the symmetrical axis.

$x = -\dfrac{1\,200}{2(-2)}$

$= 300$

$P(300) = -2(300)^2 + 1\,200(300) - 135\,000$

$= 45\,000$

Selling price = RM300 per pair gives a maximum profit of RM45 000.

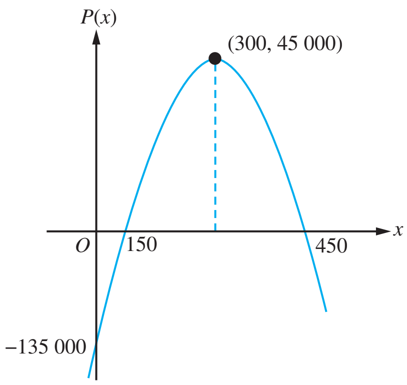
(b) $P(x) = 0$

$2x^2 - 1\,200x + 135\,000 = 0$

$x^2 - 600 + 67\,500 = 0$

$(x - 150)(x - 450) = 0$

$x = 150, 450$



(c) If each pair of sport shoes is sold at RM150 or RM450, the company will not make any profit.

Section C

9 (a) (i) Length of arc AB

$= \dfrac{40°}{360°} \times 2 \times 3.142 \times \dfrac{2.12}{2}$

$= 0.74$ m

Length of arc PQ

$= 0.74 + 2(0.24)$

$= 1.22$ m

(ii) $\dfrac{40°}{360°} \times 2 \times 3.142 \times \left(\dfrac{2.12}{2} + x\right) = 9.82$

$1.06 + x = 14.0643$

$x = 13$ m

(b) $f(x) = ax^2 + bx$

$f(x)$ does not have a constant value ($c = 0$), thus the graph of function f intersects the origin. The distance from the origin is 15 m.

Thus, $x = 0, x = 15$

$x(x - 15) = 0$

$f(x) = a(x^2 - 15x)$

$= ax^2 - 15ax$

Equation of axis of symmetry, $x = \dfrac{0 + 15}{2}$

$= 7.5$

$f(7.5) = 3.375$

$a(x^2 - 15x) = 3.375$

$a[7.5^2 - 15(7.5)] = 3.375$

$a = \dfrac{3.375}{7.5^2 - 15(7.5)}$

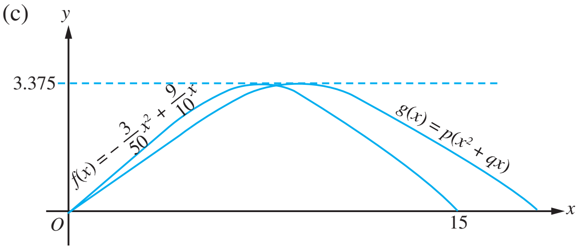
$= -\dfrac{3}{50}$

$\therefore b = -15a$

$= -15\left(-\dfrac{3}{50}\right)$

$= \dfrac{9}{10}$

(c)



The curve of the quadratic graph $g(x)$ must be wider than graph $f(x)$, thus

$|p| < \dfrac{3}{50}$ and $p < 0$

$p < \dfrac{3}{50}$ or $-p < \dfrac{3}{50}$

$p > -\dfrac{3}{50}$

Range of $p : -\dfrac{3}{50} < p < 0$

$q < -15$