Fully-Worked Solutions

SPM MODEL PAPER

Paper 1

1 $f(x) = 3|1 - x|, -1 \leqslant x \leqslant 2$

(a)
$$f(a) = 0$$
$$3|1 - a| = 0$$
$$1 - a = 0$$
$$a = 1$$

(b) $f(x) = 6$
$$3|1 - x| = 6$$
$$|1 - x| = 2$$
$1 - x = -2$ or $1 - x = 2$
$x = 3$ (Not accepted) $\quad x = -1$
$\therefore x = -1$

(c) Range: $0 \leqslant f(x) \leqslant 6$

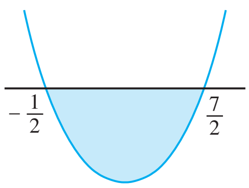
(d) Relation: Many-to-one

2 $f(x) = \dfrac{x^2 - 1}{2}$, $g(x) = 5 - 2x$

(a) Let $g^{-1}(1) = y$
$$g(y) = 1$$
$$5 - 2y = 1$$
$$y = 2$$
$$\therefore fg^{-1}(1) = f(2) = \frac{2^2 - 1}{2} = \frac{3}{2}$$

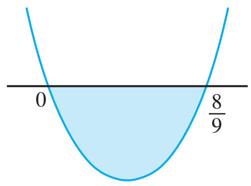
(b) $gf(x) = -3$
$$g\left(\frac{x^2 - 1}{2}\right) = -3$$
$$5 - x^2 + 1 = -3$$
$$x^2 = 9$$
$$x = -3, x = 3$$

3 (a)
$$-4(y + 1)(y - 4) - 9 > 0$$
$$-4(y^2 - 3y - 4) - 9 > 0$$
$$-4y^2 + 12y + 16 - 9 > 0$$
$$-4y^2 + 12y + 7 > 0$$
$$4y^2 - 12y - 7 < 0$$
$$(2y + 1)(2y - 7) < 0$$



$$\therefore -\frac{1}{2} < y < \frac{7}{2}$$

(b) $2x^2 + 3kx + k = 0$
$$b^2 - 4ac < 0$$
$$(3k)^2 - 4(2)(k) < 0$$
$$9k^2 - 8k < 0$$
$$k(9k - 8) < 0$$



$$\therefore 0 < k < \frac{8}{9}$$

4 $f(x) = p(x + q)^2 + r$

(a) Maximum value $= -3$
Thus, $p < 0$

(b) Axis of symmetry, $x = -2$
$$x + q = 0$$
$$x = -q = -2$$
$$\therefore q = 2$$

(c) Maximum value $= -3$
$\therefore r = -3$

5 (a)
$$\frac{16^{x-1}}{8} = 4^{3-2x}$$
$$\frac{(2^4)^{x-1}}{2^3} = (2^2)^{3-2x}$$
$$2^{4x-4-3} = 2^{6-4x}$$
$$4x - 7 = 6 - 4x$$
$$8x = 13$$
$$x = \frac{13}{8}$$

(b)
$$\frac{1 + \sqrt{2}}{2\sqrt{2} - 3} \times \frac{2\sqrt{2} + 3}{2\sqrt{2} + 3}$$
$$= \frac{(1 + \sqrt{2})(2\sqrt{2} + 3)}{(2\sqrt{2})^2 - 3^2}$$
$$= \frac{2\sqrt{2} + 3 + 2(2) + 3\sqrt{2}}{4(2) - 9}$$
$$= \frac{5\sqrt{2} + 7}{-1}$$
$$= -7 - 5\sqrt{2}$$

(c)
$$2\log_8 p - \log_2 q = 0$$
$$\log_8 p^2 = \log_2 q$$
$$\frac{\log_2 p^2}{\log_2 2^3} = \log_2 q$$
$$\frac{\log_2 p^2}{3} = \log_2 q$$
$$\log_2 p^2 = 3\log_2 q$$
$$p^2 = q^3$$
$$p = q^{\frac{3}{2}}$$

6 (a)
$$\frac{x}{-16} = \frac{2}{-4}$$
$$x = 8$$

(b) $a=-16, r=-\frac{1}{2}$

$S_{15}-S_4$

$=\dfrac{-16\left[1-\left(-\frac{1}{2}\right)^{15}\right]}{1-\left(-\frac{1}{2}\right)}-\dfrac{-16\left[1-\left(-\frac{1}{2}\right)^{4}\right]}{1-\left(-\frac{1}{2}\right)}$

$=-10.667+10$

$=-0.667$

7 (a) $a=27$

$S_\infty=81$

$\dfrac{27}{1-r}=81$

$1-r=\dfrac{1}{3}$

$r=\dfrac{2}{3}$

(b) 32, 27, 22

$T_n<0$

$32+(n-1)(-5)<0$

$32-5n+5<0$

$-5n<-37$

$5n>37$

$n>7.4$

$\therefore n=8$

8 (a) $2AT=3TB$

Let point $T(x, y)$

$(x, y)=\left(\dfrac{0+6}{5}, \dfrac{-10+0}{5}\right)$

$=\left(\dfrac{6}{5}, -2\right)$

$\therefore T\left(\dfrac{6}{5}, -2\right)$

(b) Area of $\triangle=\dfrac{21}{2}$

$\dfrac{1}{2}\begin{vmatrix}-1 & 2 & h & -1\\ 2 & 5 & 2 & 2\end{vmatrix}=\dfrac{21}{2}$

$|(-5+4+2h)-(4+5h-2)|=21$

$|-1+2h-2-5h|=21$

$|-3-3h|=21$

$-3-3h=-21$ or $-3-3h=21$

$-3h=-18$ $\quad -3h=24$

$h=6$ $\quad h=-8$

(Not accpeted)

$\therefore h=6$

9 (a) $y=\dfrac{2w}{100x}$

$100xy=2w$

$xy=\dfrac{2w}{100}$

$\lg xy=\lg\dfrac{2w}{100}$

$\lg x+\lg y=\lg 2w-\lg 10^2$

$\lg y=-\lg x+\lg 2w-2$

Gradient, $m=-1$

y-intercept, $c=\lg 2w-2$

(b) $c=\lg 2w-2$

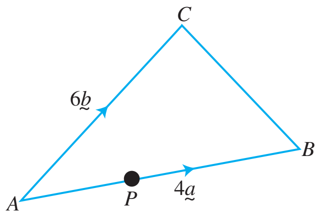
$3=\lg 2w-2$

$\lg 2w=5$

$2w=10^5$

$w=\dfrac{10^5}{2}=50\ 000$

10 (a)



(i) $\overrightarrow{CB}=\overrightarrow{CA}+\overrightarrow{AB}$

$=-6\underset{\sim}{b}+4\underset{\sim}{a}$

(ii) $\overrightarrow{PC}=\overrightarrow{PA}+\overrightarrow{AC}$

$=\dfrac{2}{5}\overrightarrow{BA}+\overrightarrow{AC}$

$=\dfrac{2}{5}(-4\underset{\sim}{a})+6\underset{\sim}{b}$

$=-\dfrac{8}{5}\underset{\sim}{a}+6\underset{\sim}{b}$

(b) $\underset{\sim}{m}=\begin{pmatrix}2\\3\end{pmatrix}=2\underset{\sim}{i}+3\underset{\sim}{j}$

$\underset{\sim}{n}=\begin{pmatrix}5\\t\end{pmatrix}=5\underset{\sim}{i}+t\underset{\sim}{j}$

(i) $2\underset{\sim}{m}+\underset{\sim}{n}=2\begin{pmatrix}2\\3\end{pmatrix}+\begin{pmatrix}5\\t\end{pmatrix}$

$=\begin{pmatrix}4\\6\end{pmatrix}+\begin{pmatrix}5\\t\end{pmatrix}$

$=9\underset{\sim}{i}+(6+t)\underset{\sim}{j}$

(ii) $|2\underset{\sim}{m}+\underset{\sim}{n}|=15$

$\sqrt{9^2+(6+t)^2}=15$

$81+(6+t)^2=225$

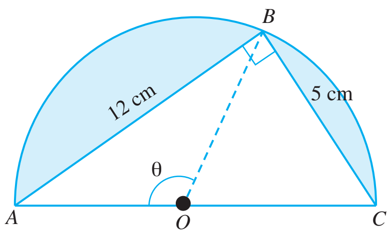
$(6+t)^2=144$

$6+t=\pm 12$

$6+t=-12$ or $6+t=12$

$t=-18$ $\quad t=6$

11



$AC=\sqrt{12^2+5^2}$

$=13$ cm

Thus, radius of circle $=\dfrac{13}{2}=6.5$ cm

(a) $\angle AOB=\theta$

$\cos\theta=\dfrac{6.5^2+6.5^2-12^2}{2(6.5)(6.5)}$

$=-0.7041$

$\theta=\cos^{-1}(-0.7041)$

$=2.352$ rad

(b) Area of shaded region

= Area of semicircle – Area of $\triangle ABC$

$=\dfrac{1}{2}\pi(6.5)^2-\dfrac{1}{2}\times 12\times 5$

$=66.366-30$

$=36.37$ cm^2

12 (a) (i) ${}^6P_4=360$

(ii) A / I __ __

${}^2P_2\times{}^4P_2\times 3=72$

(b) (i) $Z = 0.35$

$\frac{x - 25}{4} = 0.35$

$x = 26.4$ cm

(ii) $P(20 < X < 28)$

$= P\left(\frac{20 - 25}{4} < Z < \frac{28 - 25}{4}\right)$

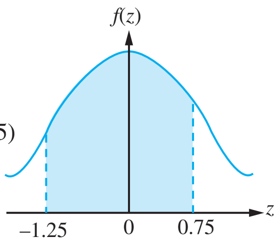
$= P(-1.25 < Z < 0.75)$

$= 1 - P(Z < -1.25) - P(Z > 0.75)$

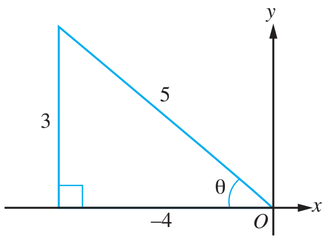
$= 1 - P(Z > 1.25) - P(Z > 0.75)$

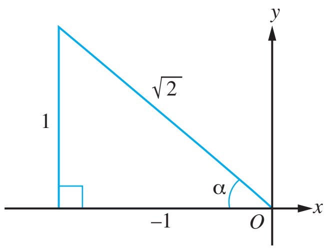
$= 1 - 0.1056 - 0.2266$

$= 0.6678$



13 (a) Sin is positive and cos is negative in quadrant II





(i) $\cot \alpha = \frac{1}{\tan \alpha}$

$= \frac{1}{\frac{1}{-1}}$

$= -1$

(ii) $\sin(\theta + \alpha) = \sin\theta\cos\alpha + \cos\theta\sin\alpha$

$= \frac{3}{5}\left(-\frac{1}{\sqrt{2}}\right) + \left(-\frac{4}{5}\right)\left(\frac{1}{\sqrt{2}}\right)$

$= -\frac{3}{5\sqrt{2}} - \frac{4}{5\sqrt{2}}$

$= -\frac{7}{5\sqrt{2}}$

$= -\frac{7\sqrt{2}}{10}$

(b) $6 \sin x = \frac{1}{2} \sin 2x$

$6 \sin x = \frac{1}{2}(2 \sin x \cos x)$

$6 \sin x - \sin x \cos x = 0$

$\sin x(6 - \cos x) = 0$

$\sin x = 0 \quad \cos x = 6$ (Undefined)

$x = 0°, 180°, 360°$

14 (a) Stationary point, $\frac{dy}{dx} = 0$ at $x = -1$

(i) $2 + 4ax = 0$

$2 + 4a(-1) = 0$

$4a = 2$

$a = \frac{1}{2}$

(ii) $\int 2 + 2x\, dx = 2x + x^2 + c$

Substitute $x = -1$, $y = 8$,

$8 = 2(-1) + (-1)^2 + c$

$c = 9$

$\therefore y = x^2 + 2x + 9$

(b) $12y + x = 5$

$y = -\frac{x}{12} + \frac{5}{12} \Rightarrow m = -\frac{1}{12}$

$y = -3(x + 2)(x - 2)$

$= -3(x^2 - 4)$

$= -3x^2 + 12$

$\frac{dy}{dx} = -6x$

$m_{normal} = -\frac{1}{12} \Rightarrow m_{tangent} = 12$

$\frac{dy}{dx} = -6x$

$12 = -6x$

$x = -2$

$y = -3(-2)^2 + 12$

$= 0$

Equation of normal: $\frac{y - 0}{x + 2} = -\frac{1}{12}$

$12y = -x - 2$

15 (a) $\frac{\sin 29°}{8.3} = \frac{\sin \angle ACB}{14}$

$\sin \angle ACB = 0.8178$

$\angle ACB = 54.87°$

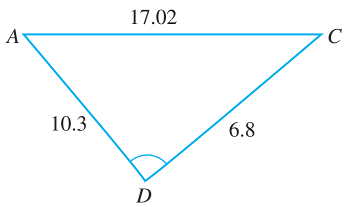
(b) $\angle ABC = 180° - 54.87° - 29°$

$= 96.13°$

$\frac{\sin 29°}{8.3} = \frac{\sin 96.13°}{AC}$

$AC = 17.02$ cm

(c)



$\cos \angle ADC = \frac{10.3^2 + 6.8^2 - 17.02^2}{2(10.3)(6.8)}$

$= -0.9805$

$\angle ADC = 168.67°$

Area of quadrilateral $ABCD$

$=$ Area of $\triangle ABC$ + Area of $\triangle ADC$

$= \frac{1}{2}(17.02)(14) \sin 29° + \frac{1}{2}(10.3)(6.8) \sin 168.67°$

$= 57.76 + 6.88$

$= 64.64$ cm^2

Paper 2

1 $3x - 5y + 4z = -12$ .... ①

$2x - 6y + 8z = -20$ .... ②

$4x - y - 8z = 6$ .... ③

② + ③: $6x - 7y = -14$ .... ④

① × 2: $6x - 10y + 8z = -24$ .... ⑤

⑤ + ③: $10x - 11y = -18$ .... ⑥

5 × ④: $30x - 35y = -70$ .... ⑦

3 × ⑥: $30x - 33y = -54$ .... ⑧

⑦ − ⑧: $-2y = -16$

$y = 8$

Substitute $y = 8$ into ④,

$6x - 7(8) = -14$

$6x = 42$

$x = 7$

Substitute $x = 7$, $y = 8$ into ③,

$4(7) - 8 - 8z = 6$

$8z = 14$

$z = \dfrac{7}{4}$

2 $f(x) = t - 7x - 2x^2$

(a) $f(0) = 4$

$t - 7(0) - 2(0)^2 = 4$

$t = 4$

$f(x) = -2x^2 - 7x + 4$

$= -(2x^2 + 7x - 4)$

$= -(2x - 1)(x + 4)$

Axis of symmetry, $x = -\dfrac{b}{2a}$

$= -\dfrac{(-7)}{2(-2)}$

$= -\dfrac{7}{4}$

$f\left(-\dfrac{7}{4}\right) = 4 - 7\left(-\dfrac{7}{4}\right) - 2\left(-\dfrac{7}{4}\right)^2$

$= \dfrac{81}{8}$

$\therefore$ Coordinates of point $P = \left(-\dfrac{7}{4}, \dfrac{81}{8}\right)$

(b) (i) $f(x) = -(2x - 1)(x + 4)$

$m > n$, $\therefore m = \dfrac{1}{2}, n = -4$

(ii) The range if $f(x) > 0$ is $-4 < x < \dfrac{1}{2}$

3 (a) $\cos(60° + A) + \sin(30° + A)$

$= \cos 60° \cos A - \sin 60° \sin A + \sin 30° \cos A + \cos 30° \sin A$

$= \dfrac{1}{2}\cos A - \dfrac{\sqrt{3}}{2}\sin A + \dfrac{1}{2}\cos A + \dfrac{\sqrt{3}}{2}\sin A$

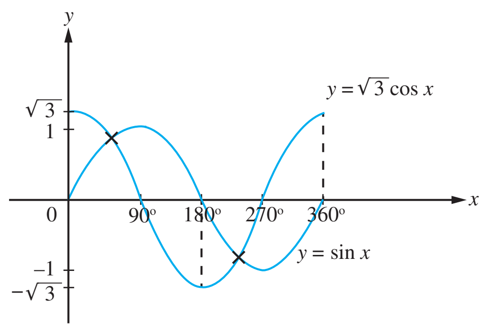
$= \cos A$

(b) $\tan x = \sqrt{3}$

$\dfrac{\sin x}{\cos x} = \sqrt{3}$

$\sin x = \sqrt{3} \cos x$

$\therefore y = \sin x$



Number of solutions = 2

4 (a) Area of $\triangle ABC = \dfrac{1}{2}(AB)(BC)$

$23.5 = \dfrac{1}{2}\left(4\sqrt{3} + 1\right)(BC)$

$\dfrac{47}{2} = \dfrac{1}{2}\left(4\sqrt{3} + 1\right)(BC)$

$BC = \dfrac{47}{4\sqrt{3} + 1}$

$= \dfrac{47\left(4\sqrt{3} - 1\right)}{\left(4\sqrt{3} + 1\right)\left(4\sqrt{3} - 1\right)}$

$= \dfrac{47\left(4\sqrt{3} - 1\right)}{16(3) - 1}$

$= \left(4\sqrt{3} - 1\right)$ cm

(b) $AC = \sqrt{AB^2 + BC^2}$

$= \sqrt{\left(4\sqrt{3} + 1\right) + \left(4\sqrt{3} - 1\right)^2}$

$= \sqrt{48 + 8\sqrt{3} + 1 + 48 - 8\sqrt{3} + 1}$

$= \sqrt{98}$

$= \sqrt{49(2)}$

$= 7\sqrt{2}$ cm

5 (a) $p + 5, p, p - 4$

$\dfrac{p}{p + 5} = \dfrac{p - 4}{p}$

$p^2 = (p - 4)(p + 5)$

$p^2 = p^2 + p - 20$

$p = 20$

(b) $9, 9x^3, 9x^6$

$a = 9, r = \dfrac{9x^6}{9x^3} = x^3$

$S_\infty = 1$

$\dfrac{9}{1 - x^3} = 1$

$1 - x^3 = 9$

$x^3 = -8$

$x^3 = (-2)^3$

$x = -2$

6 D = midpoint of AB

$= \left(\dfrac{-8 + 2}{2}, \dfrac{-3 + 5}{2}\right)$

$= (-3, 1)$

$E = \left(\dfrac{7 - 16}{3}, \dfrac{-1 - 6}{3}\right)$

$= \left(-3, -\dfrac{7}{3}\right)$

(a) $\overrightarrow{CD} = \overrightarrow{CO} + \overrightarrow{OD}$

$= -\begin{pmatrix} 7 \\ -1 \end{pmatrix} + \begin{pmatrix} -3 \\ 1 \end{pmatrix}$

$= \begin{pmatrix} -7 \\ 1 \end{pmatrix} + \begin{pmatrix} -3 \\ 1 \end{pmatrix}$

$= \begin{pmatrix} -10 \\ 2 \end{pmatrix}$

$= -10\underset{\sim}{i} + 2\underset{\sim}{j}$

(b) $\overrightarrow{DE} = \overrightarrow{DO} + \overrightarrow{OE}$

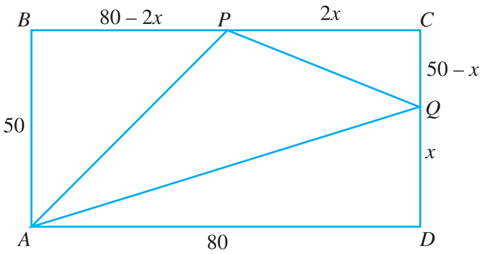
$= -\begin{pmatrix} -3 \\ 1 \end{pmatrix} + \begin{pmatrix} -3 \\ -\frac{7}{3} \end{pmatrix}$

$= \begin{pmatrix} 0 \\ -\frac{10}{3} \end{pmatrix}$

$|\overrightarrow{DE}| = \sqrt{\left(-\frac{10}{3}\right)^2}$

$= \frac{10}{3}$ units

7 (a)



Area of $\triangle APQ$

$= (50)(80) - \frac{1}{2}(50)(80 - 2x) - \frac{1}{2}(80)(x) - \frac{1}{2}(2x)(50 - x)$

$= 4\,000 - 2\,000 + 50x - 40x - 50x + x^2$

$= 2\,000 - 40x + x^2$ (Shown)

Minimum area, $\frac{dA}{dx} = 0$

$\frac{dA}{dx} = -40 + 2x$

$-40 + 2x = 0$

$2x = 40$

$x = 20$

Minimum area $= 20^2 - 40(20) + 2\,000$

$= 1\,600 \text{ cm}^2$

(b) $\int_{-2}^{1} 3f(x) - 2g(x) - 7\, dx$

$= \int_{-2}^{1} 3f(x)\, dx - \int_{-2}^{1} 2g(x)\, dx - \int_{-2}^{1} 7\, dx$

$= 3\int_{-2}^{1} f(x)\, dx - 2\int_{-2}^{1} g(x)\, dx - \int_{-2}^{1} 7\, dx$

$= 3(6) - 2(-3) - [7x]_{-2}^{1}$

$= 18 + 6 - [7 - 7(-2)]$

$= 24 - 21$

$= 3$

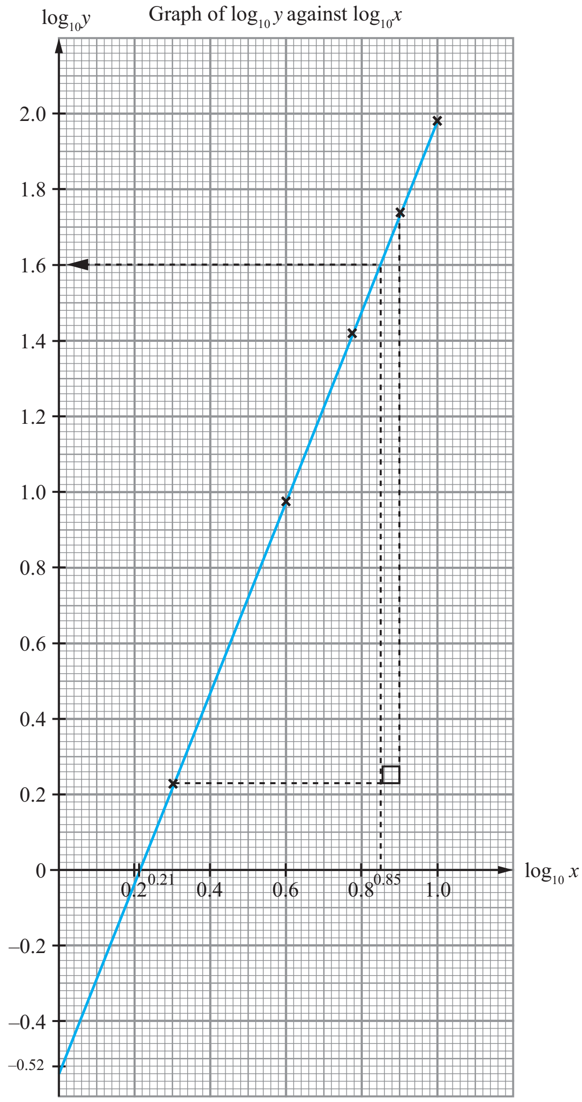
8 (a) $y = x^k p$

$\log_{10} y = \log_{10}(x^k p)$

$= \log_{10} x^k + \log_{10} p$

$= k(\log_{10} x) + \log_{10} p$

$\log_{10} x$	0.3	0.6	0.78	0.9	1
$\log_{10} y$	0.23	0.98	1.42	1.74	1.98



(b) $k = m =$ gradient

$k = \frac{1.74 - 0.23}{0.9 - 0.3}$

$= 2.52$

$\log_{10} p = y$-intercept

$\log_{10} p = -0.52$

$p = 10^{-0.52}$

$= 0.302$

(c) (i) $x = 7$,

$\log_{10} x = \log_{10} 7$

$= 0.85$

$\log_{10} y = 1.6$

$y = 10^{1.6}$

$= 39.8$

(ii) $y = 1, \log_{10} y = \log_{10} 1 = 0$

$\log_{10} x = 0.21$

$x = 10^{0.21}$

$= 1.62$

9 (a) $AB = 2BC$

$BC = \sqrt{(12 - 2)^2 + (-2 + 7)^2}$

$= \sqrt{100 + 25}$

$= \sqrt{125}$

$AB = \sqrt{(k-12)^2+(18+2)^2}$

$$\sqrt{(k-12)^2+(18+2)^2} = 2\sqrt{125}$$
$$(k-12)^2+20^2 = 4(125)$$
$$(k-12)^2+400 = 500$$
$$(k-12)^2 = 100$$
$$k-12 = -10 \quad \text{or} \quad k-12 = 10$$
$$k = 2 \qquad\qquad k = 22$$

Based on the diagram, $k = 22$

(b) Let the coordinates of $D = (x, y)$

$$m_{CD} \times m_{BC} = -1$$
$$\frac{y+7}{x-2} \times \frac{-2-(-7)}{12-2} = -1$$
$$\frac{y+7}{x-2} \times \frac{5}{10} = -1$$
$$\frac{y+7}{x-2} = -2$$
$$y+7 = -2x+4$$
$$y = -2x-3 \text{ .... ①}$$

$$m_{BC} = m_{AD}$$
$$\frac{y-18}{x-22} = \frac{1}{2}$$
$$y-18 = \frac{1}{2}(x-22)$$
$$y = \frac{1}{2}x+7 \text{ .... ②}$$

① = ②:

$$-2x-3 = \frac{1}{2}x+7$$
$$-4x-6 = x+14$$
$$-5x = 20$$
$$x = -4$$

$\therefore D(-4, 5)$

(c) Area of $ABCD$

$$= \frac{1}{2}\begin{vmatrix} 22 & 12 & 2 & -4 & 22 \\ 18 & -2 & -7 & 5 & 18 \end{vmatrix}$$
$$= \frac{1}{2}\left|(-44-84+10-72)-(216-4+28+110)\right|$$
$$= \frac{1}{2}\left|-190-350\right|$$
$$= 270 \text{ units}^2$$

(d) Given $P(x, y)$

$$AP = 5$$
$$\sqrt{(x-22)^2+(y-18)^2} = 5$$
$$(x-22)^2+(y-18)^2 = 5^2$$
$$x^2-44x+484+y^2-36y+324 = 25$$
$$x^2+y^2-44x-36y+783 = 0$$

10 (a) X = number of defective switches, $p = 0.04$, $n = 20$

(i) $P(X \geqslant 1) = 1 - P(X = 0)$
$$= 1 - {}^{20}C_0\,(0.04)^0\,(0.96)^{20}$$
$$= 1 - 0.442$$
$$= 0.558$$

(ii) Probability $= (0.558)^5 = 0.0541$

(b) (i) $P(23 < X < 30) = P\left(\frac{23-26}{2} < Z < \frac{30-26}{2}\right)$
$$= P(-1.5 < Z < 2)$$
$$= 1 - P(Z < -1.5) - P(Z > 2)$$
$$= 1 - P(Z > 1.5) - P(Z > 2)$$
$$= 1 - 0.0668 - 0.0228$$
$$= 0.9104$$
$$= 91.04\%$$

(ii) $P(X > 25) = 0.25$
$$P\left(Z > \frac{25-\mu}{2}\right) = 0.25$$
$$\frac{25-\mu}{2} = 0.674$$
$$25-\mu = 1.348$$
$$\mu = 23.65$$

11 $y = (x-2)^2+5$

(a) $x = 0,\ y = (0-2)^2+5$
$$= 9$$

$\therefore r = 9$

Minimum point is $P(2, 5)$

(b) $\frac{dy}{dx} = 2(x-2)$

$x = 1,\ \frac{dy}{dx} = 2(1-2) = -2$

Equation of tangent: $\frac{y+4}{x-1} = -2$
$$y+4 = -2x+2$$
$$y = -2x-2$$

(c) Area $= \int_{-1}^{2} (x-2)^2+5\,dx$
$$= \left[\frac{(x-2)^3}{3}+5x\right]_{-1}^{2}$$
$$= 10 - \left[\frac{(-3)^3}{3}+5(-1)\right]$$
$$= 24 \text{ units}^2$$

(d) When $y = 9$,
$$9 = (x-2)^2+5$$
$$(x-2)^2 = 4$$
$$x-2 = -2 \quad \text{or} \quad x-2 = 2$$
$$x = 0 \qquad\qquad x = 4$$

Volume $= \pi\int_0^4 9^2\,dx - \pi\int_0^4 \left[(x-2)^2+5\right]^2 dx$
$$= \pi\left[81x\right]_0^4 - \pi\int_0^4 (x-2)^4+10(x-2)^2+25\,dx$$
$$= 324\pi - \pi\left[\frac{(x-2)^5}{5}+\frac{10\,(x-2)^3}{3}+25x\right]_0^4$$
$$= 324 - \pi\left[\left(\frac{32}{5}+\frac{80}{3}+100\right)-\left(-\frac{32}{5}-\frac{80}{5}\right)\right]$$
$$= 324\pi - \frac{2\,492}{15}\pi$$
$$= \frac{2\,368}{15}\pi \text{ units}^3$$

12 $a = 2t-6$

(a) $v = \int 2t-6\,dt$

$v = t^2-6t+c$

$t = 0,\ v = 0 \Rightarrow c = 0$

$\therefore v = t^2-6t$

The object stops instantaneously, $v = 0$

$t^2-6t = 0$

$t(t-6) = 0$

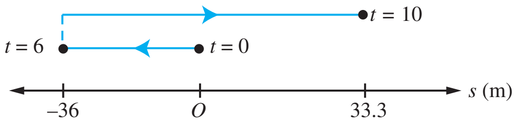
$t = 6$ seconds

(b) Minimum velocity, $a = \frac{dv}{dt} = 0$
$$\frac{dv}{dt} = 2t-6$$
$$2t-6 = 0$$
$$t = 3$$
$$t = 3,\ v = 3^2-6(3)$$
$$= -9 \text{ m s}^{-1}$$

(c) $s = \int t^2 - 6t\, dt$
$= \dfrac{t^3}{3} - 3t^2 + c$
$t = 0, s = 0 \Rightarrow c = 0$
$\therefore s = \dfrac{t^3}{3} - 3t^2$

Object returns to the origin. $s = 0$
$\dfrac{t^3}{3} - 3t^2 = 0$
$t^3 - 9t^2 = 0$
$t^2(t - 9) = 0$
$t = 9$ seconds

(d) $t = 0, s = 0$
$t = 6, s = \dfrac{6^3}{3} - 3(6) = -36$ m
$t = 10, s = \dfrac{10^3}{3} - 3(10) = 33.3$ m



Total distance $= 2(36) + 33.3 = 105.3$ m

(e) Velocity decreases means the object decelerates.
$a < 0$
$2t - 6 < 0$
$t < 3$

13 (a) (i) $x = \dfrac{2\ 150}{2\ 000} \times 100 = 107.5$

(ii) $98 = \dfrac{1\ 999}{y} \times 100$
$y = 2\ 039.80$

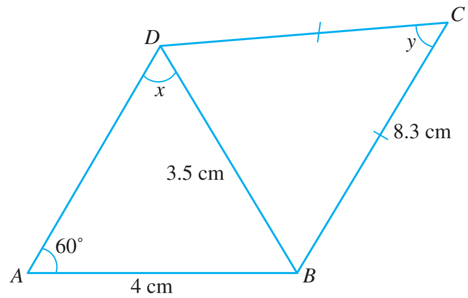
(iii) $120 = \dfrac{z}{3\ 200} \times 100$
$z = 3\ 840$

(b) Composite index = 110
$110 = \dfrac{107.5(8) + 98(5) + 120(6) + 120t}{8 + 5 + 6 + t}$
$110 = \dfrac{2\ 070 + 120t}{19 + t}$
$2\ 090 + 110t = 2\ 070 + 120t$
$20t = 10$
$t = 2$

(c) $\dfrac{\text{RM}2\ 800}{P} \times 100 = 110$
$P = \text{RM}2\ 545.45$

(d) $I_{2019/2017} = \dfrac{I_{2019}}{I_{2018}} \times \dfrac{I_{2018}}{I_{2017}} \times 100$
$= \dfrac{120}{100} \times \dfrac{116}{100} \times 100$
$= 139.2$

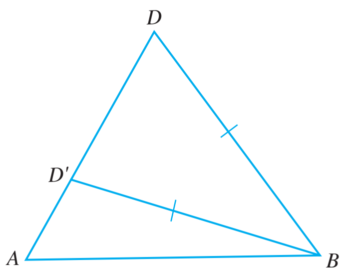
14



(a) (i) $\dfrac{\sin 60°}{3.5} = \dfrac{\sin x}{4}$
$\sin x = 0.9897$
$x = 81.77°$

(ii) $\cos y = \dfrac{8.3^2 + 8.3^2 - 3.5^2}{2(8.3)(8.3)}$
$= 0.9111$
$y = 24.34°$

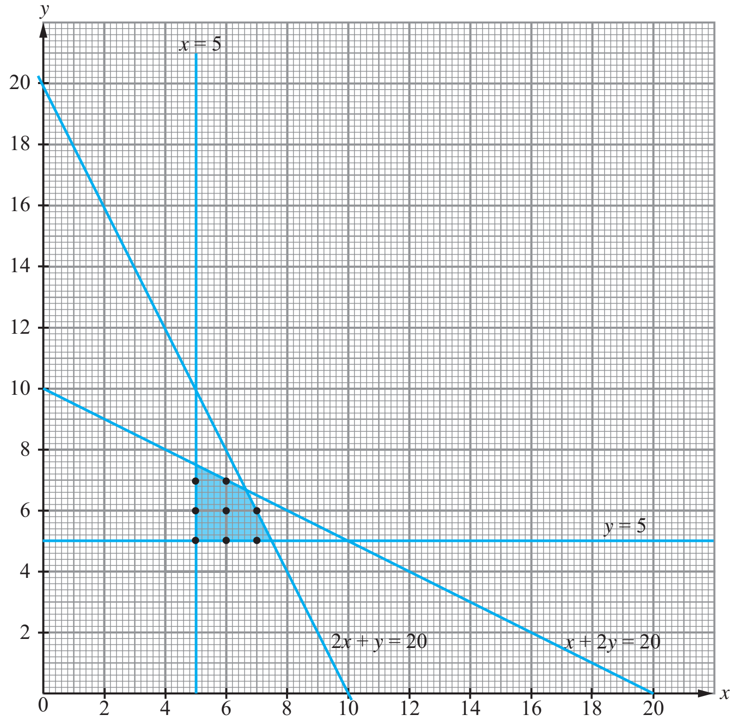
(b)



(c) Area of quadrilateral
= Area of $\triangle ABD$ + Area of $\triangle BCD$
$= \dfrac{1}{2}(3.5)(4) \sin 38.23° + \dfrac{1}{2}(8.3)(8.3) \sin 24.34°$
$= 4.33 + 14.2$
$= 18.53 \text{ cm}^2$

15 (a) $x \geqslant 5$
$y \geqslant 5$
$3x + 6y \leqslant 60 \Rightarrow x + 2y \leqslant 20$
$6x + 3y \leqslant 60 \Rightarrow 2x + y \leqslant 20$

(b), (c) (i)



(c) (ii) $y = 6, 5 \leqslant x \leqslant 7$

(iii) Sales revenue, $P = 4x + 3y$

P is maximum at point (7, 6)

$x = 7, y = 6, P = 4(7) + 3(6)$

$= \text{RM}46$