Fully-Worked Solutions

FORM 4

CHAPTER 9 Solution of Triangles

Self Test 1

1 (a) Let $\angle NKL = \theta$

$$\frac{\sin\ \theta}{15.8} = \frac{\sin\ 34°}{9}$$

$\sin\theta = 0.9817$

$\theta = \sin^{-1}(0.9817)$

$= 79.02°$

(b) Let $\angle MLN = \alpha$

$$\frac{\sin\ \alpha}{15.8} = \frac{\sin\ 34°}{11.4}$$

$\sin\alpha = 0.775$

$\alpha = \sin^{-1}(0.775)$

$= 50.81°$

2 $\angle ABC = 180° - 123°$

$= 57°$

$\angle ABD = 57° - 25°$

$= 32°$

$\therefore\ \angle ADB = 180° - 123° - 32°$

$= 25°$

(a) $\dfrac{\sin\ 25°}{5.2} = \dfrac{\sin\ 32°}{AD}$

$AD = \sin 32° \times \dfrac{5.2}{\sin\ 25°}$

$= 6.52$ cm

$BC = AD = 6.52$ cm

(b) $\dfrac{BD}{\sin\ 123°} = \dfrac{5.2}{\sin\ 25°}$

$BD = 10.32$ cm

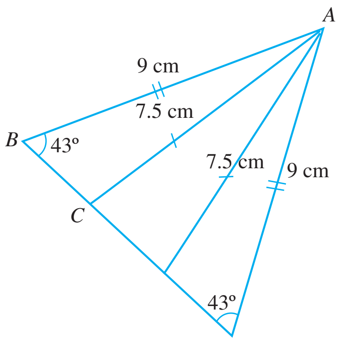
3 (a) $\dfrac{\sin\ 43°}{7.5} = \dfrac{\sin\ C}{9}$

$\sin C = 0.8184$

$C = 180° - 54.92°$

$= 125.08°$

(b)



(c) If only one triangle is formed, the angle $ACB = 90°$.

$\dfrac{\sin\ 43°}{AC} = \dfrac{\sin\ 90°}{9}$

$AC = \sin 43° \times \dfrac{9}{\sin\ 90°}$

$= 6.138$ cm

Self Test 2

1 $\sin\angle ABC = \dfrac{12}{13}$ (Obtuse angle)

(a) $\angle ABC = \sin^{-1}\left(\dfrac{12}{13}\right)$

$= 180° - 67.38°$

$= 112.62°$

$AC^2 = 5^2 + 6^2 - 2(5)(6)\cos(112.62°)$

$= 84.077$

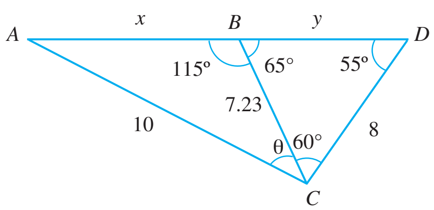
$AC = 9.17$ cm

(b) $\dfrac{\sin\ \angle ADC}{9.17} = \dfrac{\sin\ 70°}{14}$

$\sin\angle ADC = 0.6155$

$\angle ADC = 37.99°$

2 (a)



$\dfrac{y}{\sin\ 60°} = \dfrac{8}{\sin\ 65°}$

$y = 7.64$

$\angle ABC = 180° - 65° = 115°$

$\angle BDC = 180° - 65° - 60° = 55°$

$\dfrac{\sin\ 55°}{BC} = \dfrac{\sin\ 65°}{8}$

$BC = 7.23$ cm

$10^2 = x^2 + 7.23^2 - 2(x)(7.23)\cos(115°)$

$100 = x^2 + 7.23^2 + 6.111x$

$x^2 + 6.111x - 47.73 = 0$

$x = 4.5$

(b) $\dfrac{\sin\ 115°}{10} = \dfrac{\sin\ \theta}{4.5}$

$\sin\theta = 0.4078$

$\theta = 24.07°$

$\therefore\ \angle ACD = 24.07° + 60° = 84.07°$

3 $\angle ABD = 180° - 120° = 60°$

(a) $\dfrac{AD}{\sin\ 60°} = \dfrac{8}{\sin\ 50°}$

$AD = 9.044$ cm

(b) $CD^2 = 7^2 + 8^2 - 2(7)(8)\cos(120°)$

$= 169$

$CD = 13$ cm

Self Test 3

1 (a) $\dfrac{\sin\ 37°}{7.5} = \dfrac{\sin\ C}{5}$

$\sin C = 0.4012$

$C = 23.65°$

$\angle ABC = 180° - 37° - 23.65° = 119.35°$

Area of $\triangle = \dfrac{1}{2}(5)(7.5)\sin(119.35°)$

$= 16.34\text{ cm}^2$

(b) $\cos S = \dfrac{7^2 + 8^2 - 9^2}{2(7)(8)}$
$= 0.2857$
$S = 73.4°$
Area of $\triangle = \dfrac{1}{2}(7)(8)\sin(73.4°)$
$= 26.83 \text{ cm}^2$

2 (a) $\tan 47° = \dfrac{UR}{250}$
$UR = 268.09$ cm
$\tan \angle TSR = \dfrac{TR}{SR}$
$\tan 72° = \dfrac{TR}{250}$
$TR = 769.42$ cm
$\therefore TU = 769.42 - 268.09$
$= 501.33$ cm

(b) Area of $\triangle URS = \dfrac{1}{2}(UR)(RS)$
$= \dfrac{1}{2}(268.09)(250)$
$= 33\,511.25 \text{ cm}^2$

(c) $\angle SUR = 90° - 47°$
$= 43°$
$\angle TUS = 180° - 43°$
$= 137°$
$SU = \sqrt{UR^2 + SR^2}$
$= \sqrt{268.09^2 + 250^2}$
$= 366.57$ cm
Area of $\triangle UST = \dfrac{1}{2}(501.33)(366.57)\sin(137°)$
$= 62\,666.28 \text{ cm}^2$

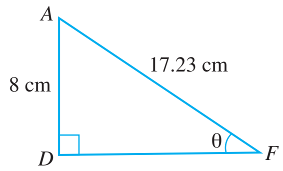
3 (a) $\angle ABP = 90° - 75° = 15°$
$\angle ABR = 180° - 15° - 50° = 115°$
Thus, the area of $\triangle ABR = \dfrac{1}{2}(7)(14)\sin(115°)$
$= 44.41 \text{ cm}^2$

(b) $\cos 50° = \dfrac{BQ}{14}$
$BQ = 9$ cm
$QR = \sqrt{14^2 - 9^2} = 10.72$ cm
$\sin 75° = \dfrac{PB}{7}$
$PB = 6.76$ cm
$\therefore PQ = 6.76 + 9 = 15.76$ cm
Area of rectangle $= 15.76 \times 10.72$
$= 168.95 \text{ cm}^2$
Percentage of area of $\triangle ABR = \dfrac{44.41}{168.95} \times 100\%$
$= 26.29\%$

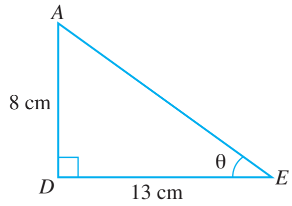
Self Test 4

1 (a) $AE = \sqrt{8^2 + 13^2} = 15.26$ cm
Area of $CDEF = 104$
$13(EF) = 104$
$EF = 8$ cm

(b) The angle between the line AF and base $CDEF = \angle AFD$
$AF^2 = AE^2 + EF^2$
$= (15.26)^2 + 8^2$
$AF = 17.23$ cm
$\sin \angle AFD = \dfrac{8}{17.23}$
$= 0.4643$
$\angle AFD = 27.67°$

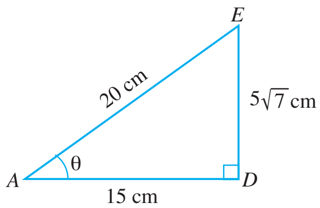


(c) The angle between the plane $CDEF$ and plane $ABFE = \angle AED$
$\tan \angle AED = \dfrac{8}{13}$
$= 0.6154$
$\angle AED = 31.61°$

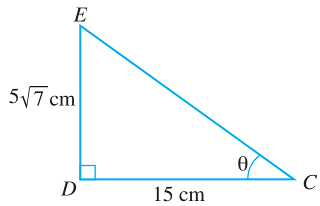


2 (a) $ED = \sqrt{20^2 - 15}$
$= \sqrt{175}$
$= 5\sqrt{7}$ cm

(b) The angle between the line AE and base $ABCD = \angle EAD$
$\cos \theta = \dfrac{15}{20}$
$\theta = 41.41°$
$\therefore \angle EAD = 41.41°$



(c) The angle between the plane EBC and base $ABCD = \angle ECD$
$\tan \angle ECD = \dfrac{5\sqrt{7}}{15}$
$= 0.8819$
$\angle ECD = 41.41°$



3 (a) $\cos \angle ACD = \dfrac{4^2 + 6^2 - 8^2}{2(4)(6)}$
$= -0.25$
$\angle ACD = 104.48°$

(b) $\angle BCA = 180° - 104.48°$
$= 75.52°$
$\dfrac{x}{\sin 75.52°} = \dfrac{4}{\sin 60°}$
$x = 4.47$ cm

(c) Area of $\triangle ABC = \dfrac{1}{2}(4.47)(4)\sin(180° - 60° - 75.52°)$
$= \dfrac{1}{2}(4.47)(4)\sin(44.48°)$
$= 6.26 \text{ cm}^2$
$\dfrac{\sin 104.48°}{8} = \dfrac{\sin \angle CAD}{6}$
$\sin \angle CAD = 0.7262$
$\angle CAD = 46.57°$
$\angle BAD = 44.48° + 46.57°$
$= 91.05°$
Thus, the area of $\triangle ABD = \dfrac{1}{2}(4.47)(8)\sin(91.05°)$
$= 17.88 \text{ cm}^2$
Ratio of the area of $\triangle ABC : \triangle ABD = 6.26 : 17.88$
$= 7 : 20$

SPM Practice

Paper 2

1 (a) (i) $\dfrac{\sin R}{32.3} = \dfrac{\sin 83°}{37}$

$\sin R = 0.8665$

$R = 60.05°$

$\therefore \angle QPR = 180° - 60.05° - 83°$

$= 36.95°$

(ii) $\dfrac{QR}{\sin 36.95°} = \dfrac{37}{\sin 83°}$

$QR = 22.41 \text{ cm}$

(b) (i) $\dfrac{\sin A}{8.8} = \dfrac{\sin 42°}{6.5}$

$\sin A = 0.9059$

$A = 64.9°$

or

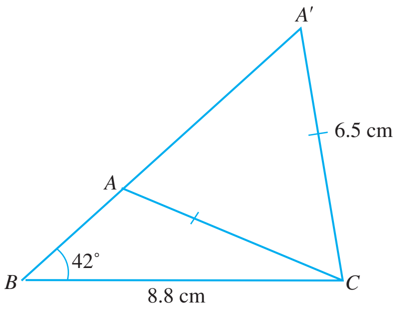
$A = 180° - 64.9° = 115.1°$ (ambiguous case)

$\therefore \angle C = 180° - 64.9° - 42° = 73.1°$

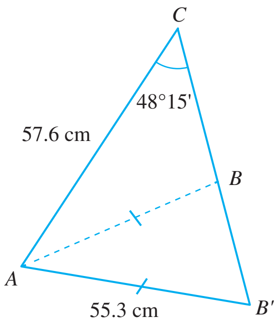
or

$\angle C = 180° - 115.1° - 42° = 22.9°$

(ii)



2 (a) (i)



(ii) $\dfrac{\sin 48°15'}{55.3} = \dfrac{\sin \angle ABC}{57.6}$

$\sin \angle ABC = 0.7771$

$\angle ABC = 51°, 180° - 51°$

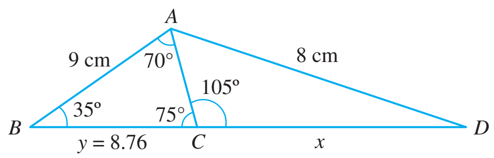
$= 51°, 129°$

$\therefore \angle CAB = 180° - 48°15' - 51° = 80°45'$

or

$\angle CAB = 180° - 48°15' - 129° = 2°45'$

(b)



$\angle ABC = 180° - 70° - 75°$

$= 35°$

$\angle ACD = 180° - 75°$

$= 105°$

$\dfrac{\sin 70°}{y} = \dfrac{\sin 75°}{9}$

$y = 8.76$

$\dfrac{AC}{\sin 35°} = \dfrac{9}{\sin 75°}$

$AC = 5.34 \text{ cm}$

$\dfrac{\sin 105°}{8} = \dfrac{\sin D}{5.34}$

$\sin D = 0.6448$

$D = 40.15°$

$\therefore \angle CAD = 180° - 105° - 40.15°$

$= 34.85°$

$\dfrac{x}{\sin 34.85°} = \dfrac{8}{\sin 105°}$

$x = 4.73$

3 (a) (i) $\tan 44° = \dfrac{PN}{30}$

$PN = 28.97 \text{ cm}$

(ii) $\tan 35° = \dfrac{28.97}{AN}$

$AN = 41.37 \text{ cm}$

$AB = \sqrt{41.37^2 + 30^2}$

$= 51.1 \text{ cm}$

(iii) $PB = \sqrt{28.97^2 + 30^2}$

$= 41.7 \text{ cm}$

$PA = \sqrt{41.37^2 + 28.97^2}$

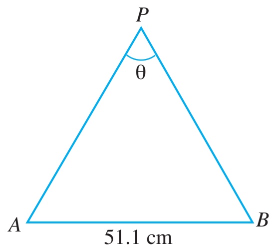
$= 50.5 \text{ cm}$

$\cos\theta$

$= \dfrac{41.7^2 + 50.5^2 - 51.1^2}{2(41.7)(50.5)}$

$= 0.3984$

$\theta = 66.52°$



(b) (i) $\dfrac{AB}{\sin 65°} = \dfrac{5}{\sin 50°}$

$AB = 5.92 \text{ cm}$

$\angle DAB = 180° - 65° - 50° = 65°$

Area of $\triangle ABD = \dfrac{1}{2}(5)(5.92)\sin(65°)$

$= 13.41 \text{ cm}^2$

Area of $\triangle ACD = \dfrac{1}{2}(5)(2 \times 5.92)\sin(65°)$

$= 26.82 \text{ cm}^2$

Thus, the area of $\triangle ABD$: area of $\triangle ACD = 1 : 2$

(ii) $\dfrac{BD}{\sin 65°} = \dfrac{5}{\sin 50°}$

$BD = 5.92 \text{ cm}$

$CD^2 = 5.92^2 + 5.92^2 - 2(5.92)(5.92)\cos(130°)$

$= 115.15$

$CD = 10.73 \text{ cm}$

4 (a) $KA = KB$

$= \sqrt{KC^2 + BC^2}$

$= \sqrt{5^2 + 12^2}$

$= \sqrt{169}$

$= 13 \text{ cm}$

(b) The angle between the line KA and plane $ABCD$ is $\angle KAN$.

$KN = \sqrt{KC^2 + CN^2}$

$= \sqrt{5^2 - 3^2}$

$= 4 \text{ cm}$

$\sin \angle KAN = \dfrac{KN}{KA}$

$= \dfrac{4}{13}$

$= 0.3077$

$\angle KAN = 17.92°$

(c) The angle between the plane KAB and plane $ABCD$ is $\angle KMN$.

$$\tan \angle KMN = \frac{KN}{MN}$$
$$= \frac{4}{12}$$
$$= 0.3333$$
$$\angle KMN = 18.43°$$

(d) $$\cos \angle AKB = \frac{KA^2 + KB^2 - AB^2}{2(KA)(KB)}$$
$$= \frac{13^2 + 13^2 - 6^2}{2(13)(13)}$$
$$= 0.8935$$
$$\angle AKB = 26.68°$$

(e) $$\text{Area of } \triangle AKB = \frac{1}{2} \times 13 \times 13 \times \sin 26.68°$$
$$= 37.9 \text{ cm}^2$$